Unified Growth Theory and Comparative Development

Ömer Özak

Department of Economics
Southern Methodist University

Economic Growth and Comparative Development

Unified Growth Theory

Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces the triggered the transition from stagnation to growth
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces the triggered the transition from stagnation to growth
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces the triggered the transition from stagnation to growth
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces the triggered the transition from stagnation to growth
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces the triggered the transition from stagnation to growth
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
  - Historical origins of vast and persistent inequality across countries
  - Forces that triggered the transition of DCs from stagnation to growth
  - Hurdles faced by LDCs in their take-off from stagnation to growth
  - Factors that hindered convergence across countries
  - Importance of deep-rooted factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

Unified Growth Theory

- Captures the:
  - Process of development in its entirety
  - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
  - Hurdles faced by LDCs in their transitions from stagnation to growth
  - Persistent effect of initial biogeographical factors on the growth process
- Encompasses:
  - Existing hypotheses about the role of geographical, cultural, institutional and genetic factors in comparative development

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

The Fundamental Challenge

- A unified theory of economic growth that accounts for the:
  - Epoch of Malthusian stagnation
  - Take-off from the Malthusian Regime
  - Emergence of human capital as a significant factor in the growth process
  - Demographic transition
  - Shift to sustained economic growth
  - Emergence of inequality in income per capita across countries

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:
- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:

- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:

- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:

- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:

- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:

- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:
- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
  - Escape from a stable Malthusian equilibrium:
- Hypothetical mechanisms:
  - Shock in an economy with multiple stable equilibria
    - Inconsistent with a gradual increase in TFP growth
  - A gradual escape from an absorbing (stable) equilibrium
    - Contradiction to the essence of a stable equilibrium
  - Vanishing Malthusian equilibrium

Origins of the Phase Transition

- Evolution of a latent state variable – the demand for human capital
  - Ultimately changes the dynamical system qualitatively:
    - The Malthusian equilibrium vanishes endogenously
    - The economy gravitates towards the emerging Modern Growth Regime

Origins of the Phase Transition

- Evolution of a latent state variable – the demand for human capital
  - Ultimately changes the dynamical system qualitatively:
    - The Malthusian equilibrium vanishes endogenously
    - The economy gravitates towards the emerging Modern Growth Regime

Origins of the Phase Transition

- Evolution of a latent state variable – the demand for human capital
  - Ultimately changes the dynamical system qualitatively:
    - The Malthusian equilibrium vanishes endogenously
    - The economy gravitates towards the emerging Modern Growth Regime

Origins of the Phase Transition

- Evolution of a latent state variable – the demand for human capital
  - Ultimately changes the dynamical system qualitatively:
    - The Malthusian equilibrium vanishes endogenously
    - The economy gravitates towards the emerging Modern Growth Regime

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita
- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita
- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita
- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita
- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
  - Faster rates of technological progress
  - Faster rate of population growth
  - Insignificant investment in human capital
  - Onset of growth in income per capita

- Transition from the Post-Malthusian to Modern Growth Regime:
  - Faster rate of technological progress
  - Faster rate of human capital accumulation
  - Decline in population growth
  - Faster growth of income per capita

Suggestive Evidence

- The underlying forces that govern these transitions:
  - The effect of changes in the technological environment on:
    - population size and quality
  - The effect of changes in the size and the quality of the population on:
    - the rate of technological progress

Suggestive Evidence

- The underlying forces that govern these transitions:
  - The effect of changes in the technological environment on:
    - population size and quality
  - The effect of changes in the size and the quality of the population on:
    - the rate of technological progress

Suggestive Evidence

- The underlying forces that govern these transitions:
  - The effect of changes in the technological environment on:
    - population size and quality
  - The effect of changes in the size and the quality of the population on:
    - the rate of technological progress

Suggestive Evidence

- The underlying forces that govern these transitions:
  - The effect of changes in the technological environment on:
    - population size and quality
  - The effect of changes in the size and the quality of the population on:
    - the rate of technological progress

Suggestive Evidence

- The underlying forces that govern these transitions:
  - The effect of changes in the technological environment on:
    - population size and quality
  - The effect of changes in the size and the quality of the population on:
    - the rate of technological progress

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor (measured in efficiency units)
  - Land

Factor Supply

- Land is fixed over time
  - Surface of planet earth
- Efficiency units of labor evolves endogenously
  - Determined by households' decisions about the number and level of human capital of their children

Factor Supply

- Land is fixed over time
  - Surface of planet earth
- Efficiency units of labor evolves endogenously
  - Determined by households' decisions about the number and level of human capital of their children

Factor Supply

- Land is fixed over time
  - Surface of planet earth
- Efficiency units of labor evolves endogenously
  - Determined by households' decisions about the number and level of human capital of their children

Factor Supply

- Land is fixed over time
  - Surface of planet earth
- Efficiency units of labor evolves endogenously
  - Determined by households' decisions about the number and level of human capital of their children

Main Elements

- The Malthusian Structure
- Sources of Technological Progress
- Origins of Human Capital Formation
- Triggers of the Demographic Transition

Main Elements

- The Malthusian Structure
- Sources of Technological Progress
- Origins of Human Capital Formation
- Triggers of the Demographic Transition

Main Elements

- The Malthusian Structure
- Sources of Technological Progress
- Origins of Human Capital Formation
- Triggers of the Demographic Transition

Main Elements

- The Malthusian Structure
- Sources of Technological Progress
- Origins of Human Capital Formation
- Triggers of the Demographic Transition

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
  - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production – Land
  - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
  - Reflecting diminishing returns to labor & positive effect of income on population

Production

- The output produced in period t

$$Y_t = H_t^{\alpha}(A_t X)^{1-\alpha}$$

  - $H_t \equiv$ efficiency units of labor
  - $A_t \equiv$ technological level
  - $X \equiv$ land

- Output per worker produced at time t

$$y_t = h_t^{\alpha} x_t^{1-\alpha}$$

  - $h_t \equiv H_t/L_t \equiv$ efficiency units per-worker
  - $x_t \equiv (A_t X)/L_t \equiv$ effective resources per worker

Production

- The output produced in period t

$$Y_t = H_t^{\alpha}(A_t X)^{1-\alpha}$$

  - $H_t \equiv$ efficiency units of labor
  - $A_t \equiv$ technological level
  - $X \equiv$ land

- Output per worker produced at time t

$$y_t = h_t^{\alpha} x_t^{1-\alpha}$$

  - $h_t \equiv H_t/L_t \equiv$ efficiency units per-worker
  - $x_t \equiv (A_t X)/L_t \equiv$ effective resources per worker

The Malthusian Structure – Effects of Technological Progress

- Very short-run (for a given population):
  - $A_t \uparrow \implies y_t \uparrow$ (above $\bar{y}$)
- Short-run (initial adjustment of population):
  - $y_t \uparrow \implies L_t \uparrow$
- Long-run (population reaches a new steady-state):
  - $L_t \uparrow \implies y \downarrow$ (back to $\bar{y}$)

The Malthusian Structure – Effects of Technological Progress

- Very short-run (for a given population):
  - $A_t \uparrow \implies y_t \uparrow$ (above $\bar{y}$)
- Short-run (initial adjustment of population):
  - $y_t \uparrow \implies L_t \uparrow$
- Long-run (population reaches a new steady-state):
  - $L_t \uparrow \implies y \downarrow$ (back to $\bar{y}$)

The Malthusian Structure – Effects of Technological Progress

- Very short-run (for a given population):
  - $A_t \uparrow \implies y_t \uparrow$ (above $\bar{y}$)
- Short-run (initial adjustment of population):
  - $y_t \uparrow \implies L_t \uparrow$
- Long-run (population reaches a new steady-state):
  - $L_t \uparrow \implies y \downarrow$ (back to $\bar{y}$)

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Earlier stages of development
  - Population size positively affects technological progress:

$$L_t \uparrow \implies A_t \uparrow$$

  - Channels:
    - Supply of innovations
    - Demand for innovations
    - Diffusion of knowledge
    - Division of labor
    - Extent of trade

Sources of Technological Progress

- Later Stages of Development
  - Human capital positively affects technological progress

$$e_t \uparrow \quad \Longrightarrow \quad A_t \uparrow$$

  - Educated individuals have an advantage in adopting and advancing new technologies

Sources of Technological Progress

- Later Stages of Development
  - Human capital positively affects technological progress

$$e_t \uparrow \quad \Longrightarrow \quad A_t \uparrow$$

    - Educated individuals have an advantage in adopting and advancing new technologies

Sources of Technological Progress

- Later Stages of Development
  - Human capital positively affects technological progress

$$e_t \uparrow \quad \Longrightarrow \quad A_t \uparrow$$

    - Educated individuals have an advantage in adopting and advancing new technologies

Technological Progress

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t)$$

- $g_{t+1} \equiv$ rate of tech progress
- $e_t \equiv$ education
- $L_t \equiv$ population size

Technological Progress

$$g_{t+1} = g(e_t, L_t)$$

- $g_e(e_t, L_t) > 0$ and $g_{ee}(e_t, L_t) < 0$
  - Education has a positive and diminishing effect of technological progress
- $g_L(e_t, L_t) > 0$ and $g_{LL}(e_t, L_t) < 0$
  - The scale of the economy has a positive and diminishing effect on technological progress
- $g(0, L) > 0$ for $L > 0$
  - Technological progress is positive at the outset

Technological Progress

$$g_{t+1} = g(e_t, L_t)$$

- $g_e(e_t, L_t) > 0$ and $g_{ee}(e_t, L_t) < 0$
  - Education has a positive and diminishing effect of technological progress
- $g_L(e_t, L_t) > 0$ and $g_{LL}(e_t, L_t) < 0$
  - The scale of the economy has a positive and diminishing effect on technological progress
- $g(0, L) > 0$ for $L > 0$
  - Technological progress is positive at the outset

Technological Progress

$$g_{t+1} = g(e_t, L_t)$$

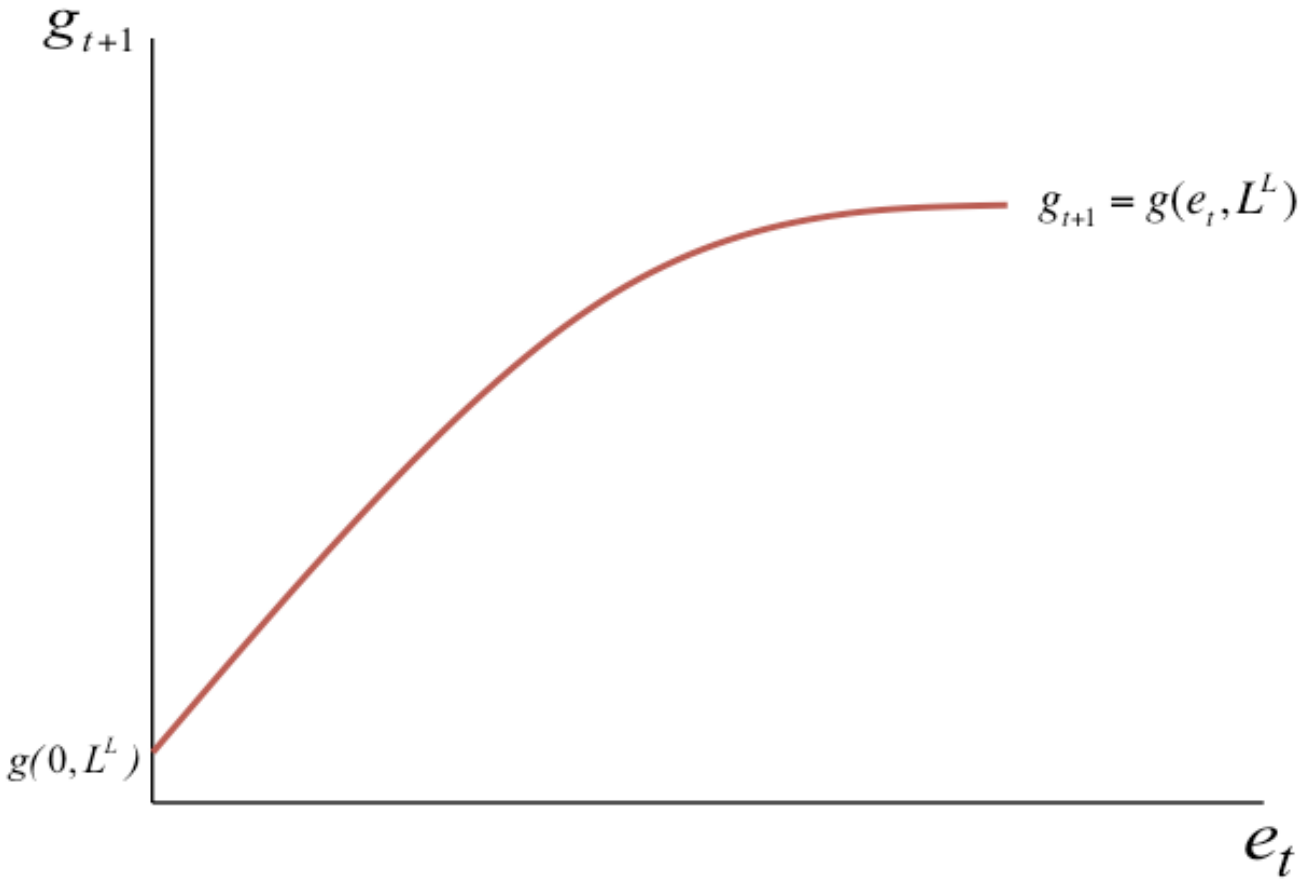
- $g_e(e_t, L_t) > 0$ and $g_{ee}(e_t, L_t) < 0$
  - Education has a positive and diminishing effect of technological progress
- $g_L(e_t, L_t) > 0$ and $g_{LL}(e_t, L_t) < 0$
  - The scale of the economy has a positive and diminishing effect on technological progress
- $g(0, L) > 0$ for $L > 0$
  - Technological progress is positive at the outset

Technological Progress

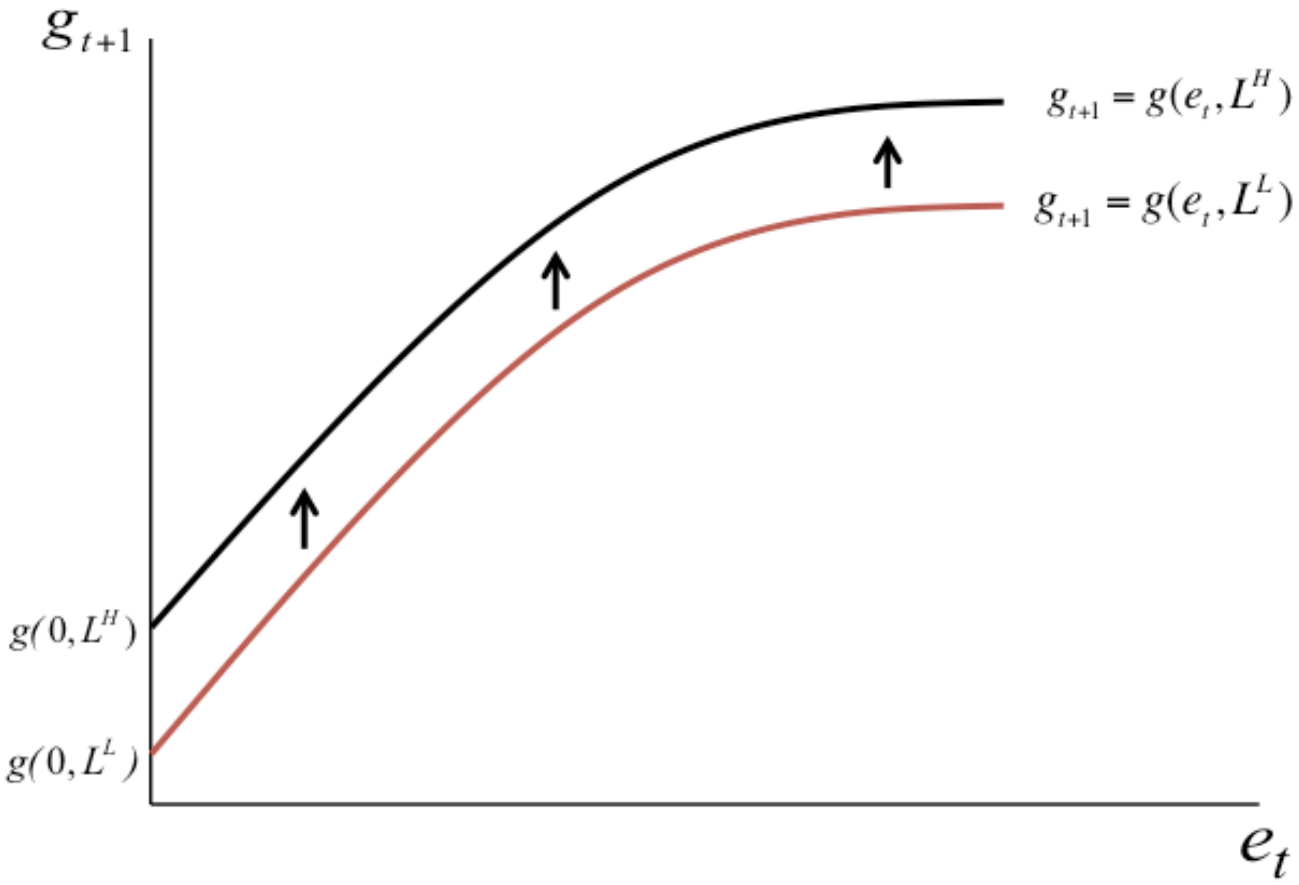
$$g_{t+1} = g(e_t, L_t)$$

- $g_e(e_t, L_t) > 0$ and $g_{ee}(e_t, L_t) < 0$
  - Education has a positive and diminishing effect of technological progress
- $g_L(e_t, L_t) > 0$ and $g_{LL}(e_t, L_t) < 0$
  - The scale of the economy has a positive and diminishing effect on technological progress
- $g(0, L) > 0$ for $L > 0$
  - Technological progress is positive at the outset

Technological Progress



The Effect of Population Size on Technological Progress



Origins of Human Capital Formation

- The increase in the rate of technological progress increases the demand for human capital
  - Human capital permits individuals to better cope with the changes in the technological environment
  - The introduction of new technologies is skill-biased in the short-run, although the nature of the technology can be skill-biased or skill-saving in the long run

Origins of Human Capital Formation

- The increase in the rate of technological progress increases the demand for human capital
  - Human capital permits individuals to better cope with the changes in the technological environment
  - The introduction of new technologies is skill-biased in the short-run, although the nature of the technology can be skill-biased or skill-saving in the long run

Origins of Human Capital Formation

- The increase in the rate of technological progress increases the demand for human capital
  - Human capital permits individuals to better cope with the changes in the technological environment
  - The introduction of new technologies is skill-biased in the short-run, although the nature of the technology can be skill-biased or skill-saving in the long run

Human Capital Formation

Human capital of an individual who joins the labor force in period $t+1$

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $e_{t+1} \equiv$ the individual education level (determined by parental investment, subject to their subsistence constraint, in period t)
- $g_{t+1} \equiv$ rate of tech progress

Human Capital Formation

Human capital of an individual who joins the labor force in period $t+1$

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $e_{t+1} \equiv$ the individual education level (determined by parental investment, subject to their subsistence constraint, in period t)
- $g_{t+1} \equiv$ rate of tech progress

Human Capital Formation

Human capital of an individual who joins the labor force in period $t+1$

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $e_{t+1} \equiv$ the individual education level (determined by parental investment, subject to their subsistence constraint, in period t)
- $g_{t+1} \equiv$ rate of tech progress

Human Capital Formation

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $h_e(e,g) > 0$ and $h_{ee}(e,g) < 0$
  - HC is increasing (in decreasing rates) in the parental time investment in the education of the child
- $h_g(e,g) < 0$ and $h_{gg}(e,g) > 0$
  - Obsolescence of HC in a changing technological environment
- $h_{eg}(e,g) > 0$
  - Education lessens the obsolescence of HC in a changing technological environment
- $h(0,g) > 0$
  - Basic level of human capital

Human Capital Formation

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $h_e(e,g) > 0$ and $h_{ee}(e,g) < 0$
  - HC is increasing (in decreasing rates) in the parental time investment in the education of the child
- $h_g(e,g) < 0$ and $h_{gg}(e,g) > 0$
  - Obsolescence of HC in a changing technological environment
- $h_{eg}(e,g) > 0$
  - Education lessens the obsolescence of HC in a changing technological environment
- $h(0,g) > 0$
  - Basic level of human capital

Human Capital Formation

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

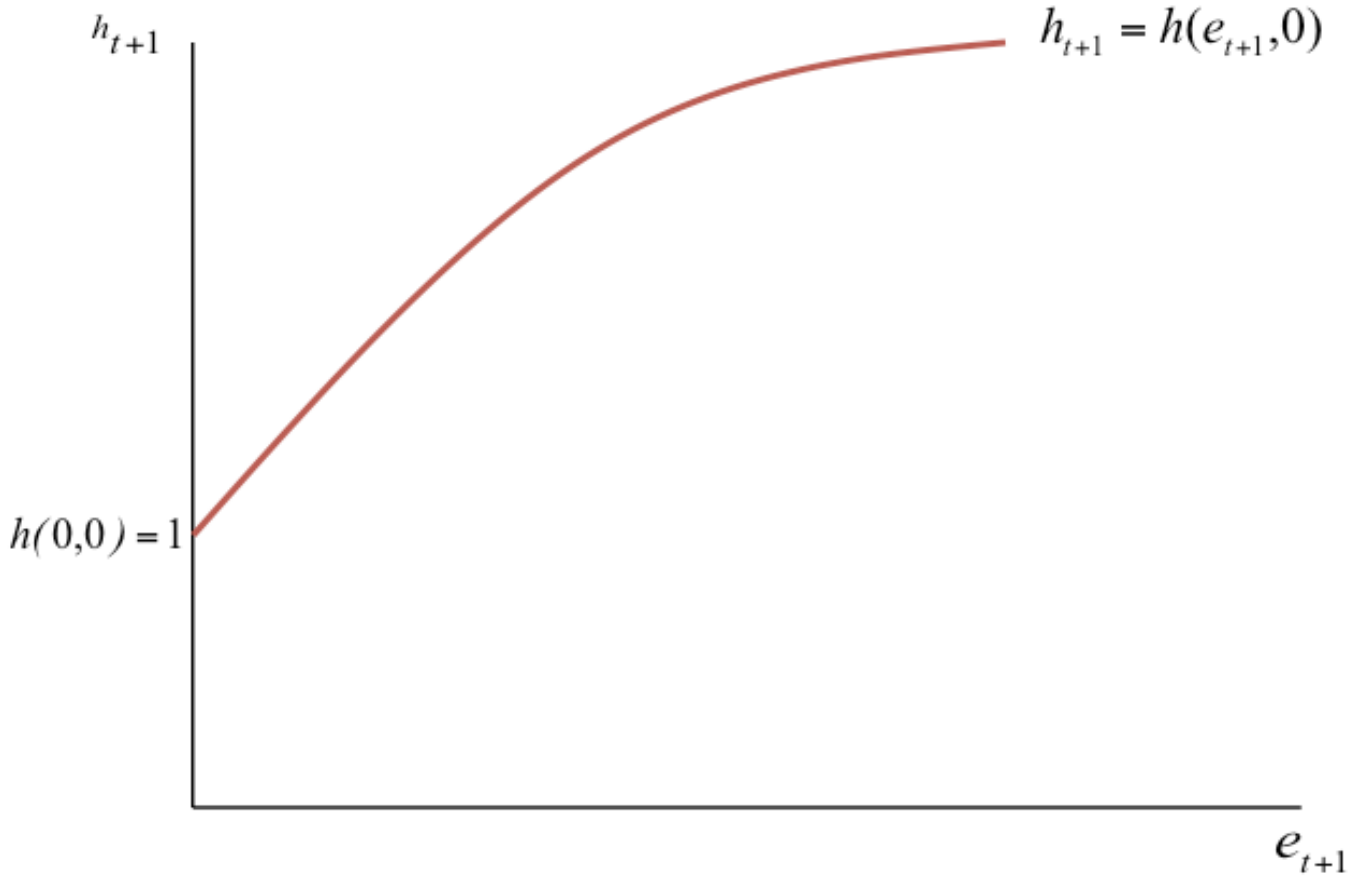
- $h_e(e,g) > 0$ and $h_{ee}(e,g) < 0$
  - HC is increasing (in decreasing rates) in the parental time investment in the education of the child
- $h_g(e,g) < 0$ and $h_{gg}(e,g) > 0$
  - Obsolescence of HC in a changing technological environment
- $h_{eg}(e,g) > 0$
  - Education lessens the obsolescence of HC in a changing technological environment
- $h(0,g) > 0$
  - Basic level of human capital

Human Capital Formation

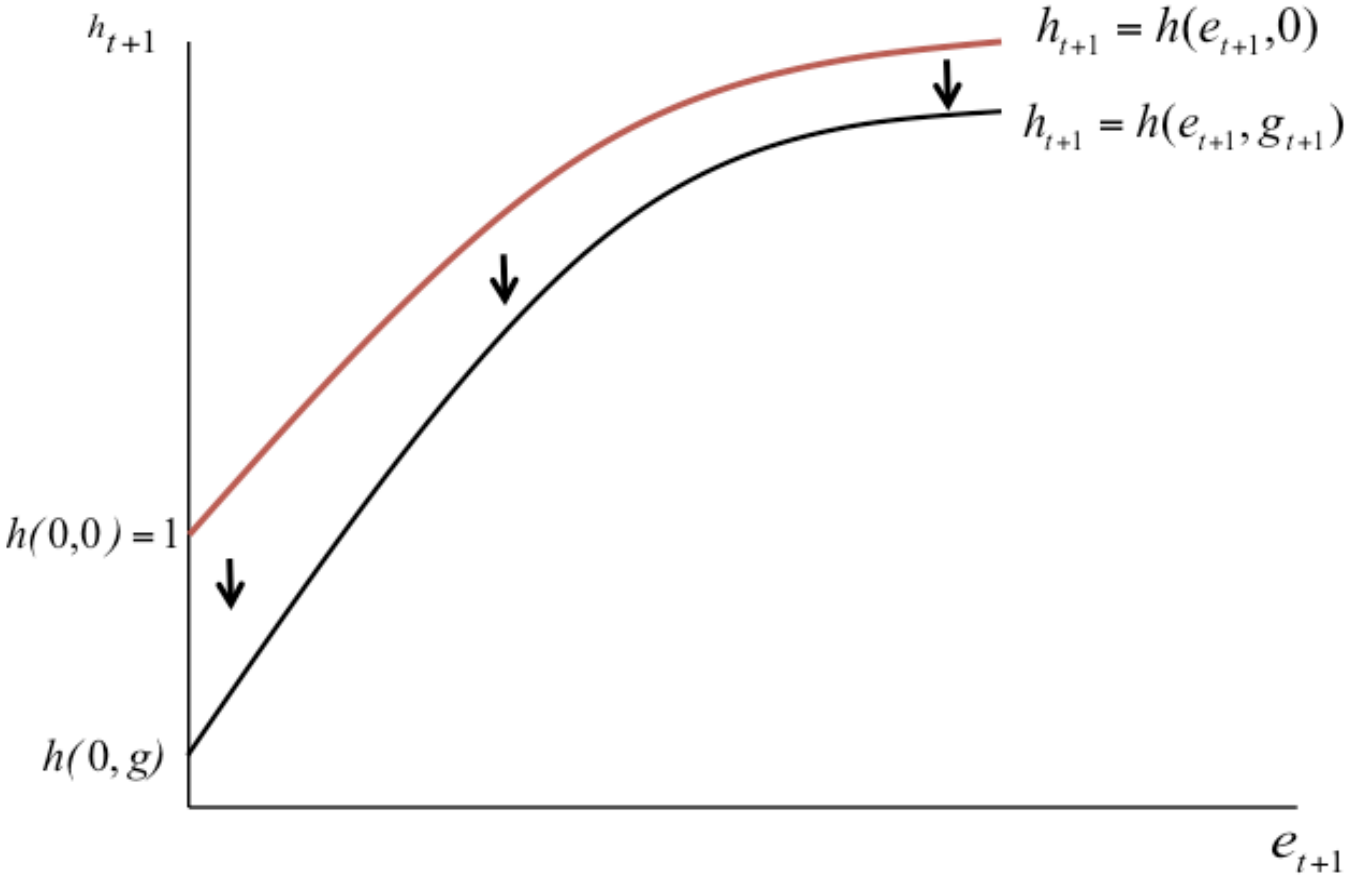
$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $h_e(e,g) > 0$ and $h_{ee}(e,g) < 0$
  - HC is increasing (in decreasing rates) in the parental time investment in the education of the child
- $h_g(e,g) < 0$ and $h_{gg}(e,g) > 0$
  - Obsolescence of HC in a changing technological environment
- $h_{eg}(e,g) > 0$
  - Education lessens the obsolescence of HC in a changing technological environment
- $h(0,g) > 0$
  - Basic level of human capital

Human Capital Formation



Human Capital Formation



Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children

- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
  - An income effect – more income to spend on children
  - Substitution effects
    - The opportunity cost of raising children increases
    - Return to investment in child quality increases

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:
- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:
- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:
- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:

- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:

- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
  - The income effect dominates (moderate demand for human capital):
    - Population growth & human capital formation increase:

- Later part of the second phase of industrialization:
  - The substitution effect dominates (significant demand for human capital):
    - Population growth declines & human capital formation increases further

Individuals

- Live for 2 periods

- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}

- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Individuals

- Live for 2 periods
- Childhood (1st Period):
  - Consume a fraction of parental time endowment
  - The required time increases with child quality
    - $\tau \equiv$ time required to raise a child, regardless of quality
    - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
  - Allocate the time endowment between childrearing and work
  - Choose the optimal mixture of child quantity and quality
  - Consume

Preferences

- The utility function of individual t (adult at time t)

$$u^t = (1-\gamma)\ln(c_t) + \gamma \ln(n_t h_{t+1})$$

  - $c_t \equiv$ consumption of individual t
  - $n_t \equiv$ number of children of individual t
  - $h_{t+1} \equiv$ level of human capital of each child

Budget and Subsistence Consumption Constraints

$$z_t n_t(\tau + e_{t+1}) + c_t \leq z_t$$

- $z_t \equiv$ potential income of individual t
- $\tau \equiv$ time required to raise a child, regardless of quality
- $\tau + e_{t+1} \equiv$ time needed to raise a child with education e_{t+1}

$$z_t \equiv y_t = h_t^{\alpha} x_t^{1-\alpha} = z(e_t, g_t, x_t)$$

- Subsistence consumption constraint:

$$c_t \geq \tilde{c}$$

Budget and Subsistence Consumption Constraints

$$z_t n_t(\tau + e_{t+1}) + c_t \leq z_t$$

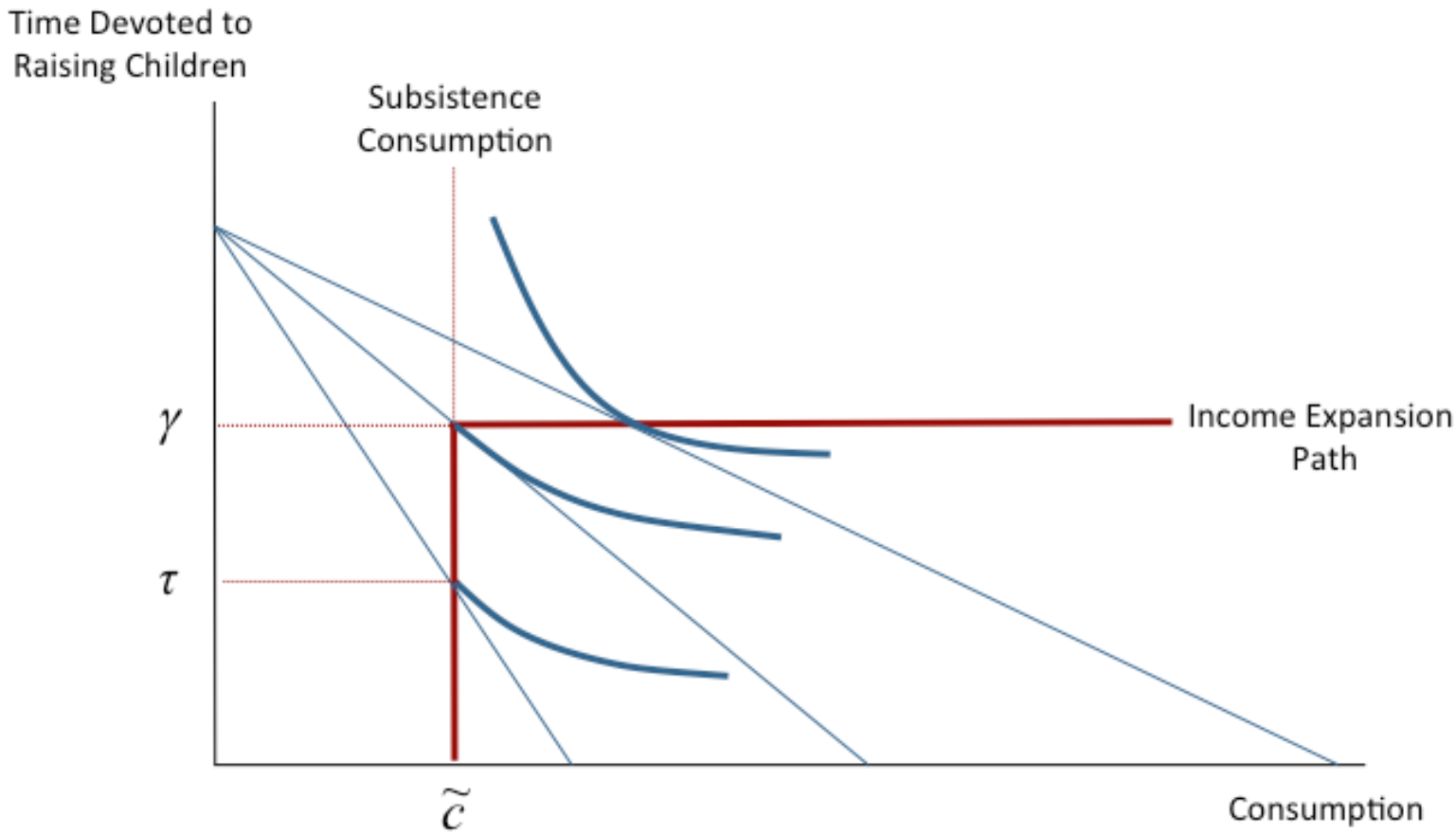
- $z_t \equiv$ potential income of individual t
- $\tau \equiv$ time required to raise a child, regardless of quality
- $\tau + e_{t+1} \equiv$ time needed to raise a child with education e_{t+1}

$$z_t \equiv y_t = h_t^{\alpha} x_t^{1-\alpha} = z(e_t, g_t, x_t)$$

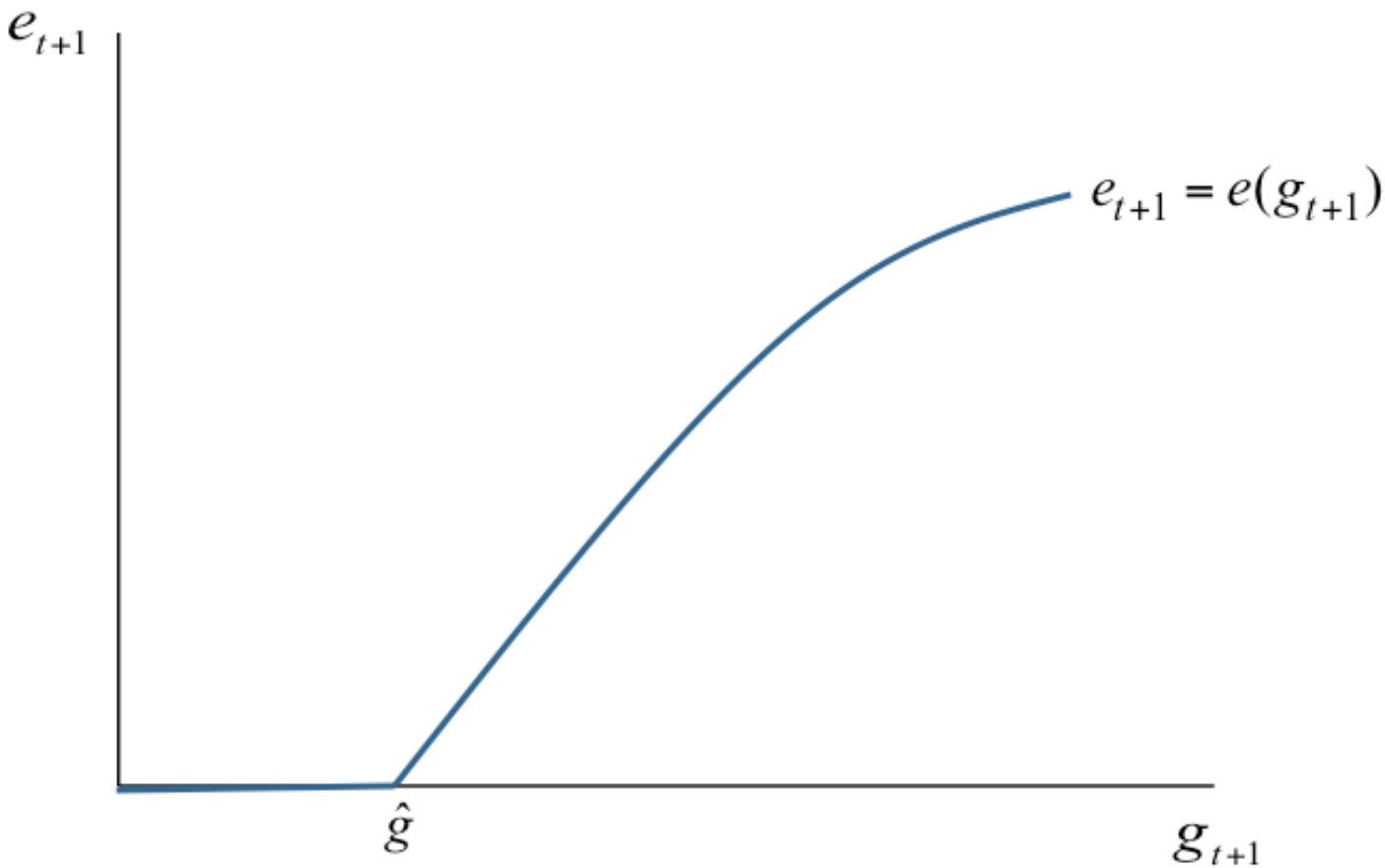
- Subsistence consumption constraint:

$$c_t \geq \tilde{c}$$

Constraint and Optimization



Optimal Investment in Child Quality



Optimization: Quantity and Quality of Children

- Time devoted to children:

$$n_t(\tau + e_{t+1}) = \begin{cases} \gamma & \text{if} \quad z_t \geq \tilde{z} \\ 1 - \frac{\tilde{c}}{z_t} & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

- $z_t = \tilde{z}$ is the highest level of potential income such that the subsistence constraint is still binding

$$e_{t+1} = e(g_{t+1}) \implies$$

$$n_t = \begin{cases} \frac{\gamma}{\tau + e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if} \quad z_t \geq \tilde{z} \\ \frac{1-[\tilde{c}/z_t]}{\tau + e(g_{t+1})} \equiv n^a(g_{t+1}, z(e_t, g_t, x_t)) & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

Optimization: Quantity and Quality of Children

- Time devoted to children:

$$n_t(\tau + e_{t+1}) = \begin{cases} \gamma & \text{if} \quad z_t \geq \tilde{z} \\ 1 - \frac{\tilde{c}}{z_t} & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

- $z_t = \tilde{z}$ is the highest level of potential income such that the subsistence constraint is still binding

$$e_{t+1} = e(g_{t+1}) \implies$$

$$n_t = \begin{cases} \frac{\gamma}{\tau + e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if} \quad z_t \geq \tilde{z} \\ \frac{1-[\tilde{c}/z_t]}{\tau + e(g_{t+1})} \equiv n^a(g_{t+1}, z(e_t, g_t, x_t)) & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

Optimization: Quantity and Quality of Children

- Time devoted to children:

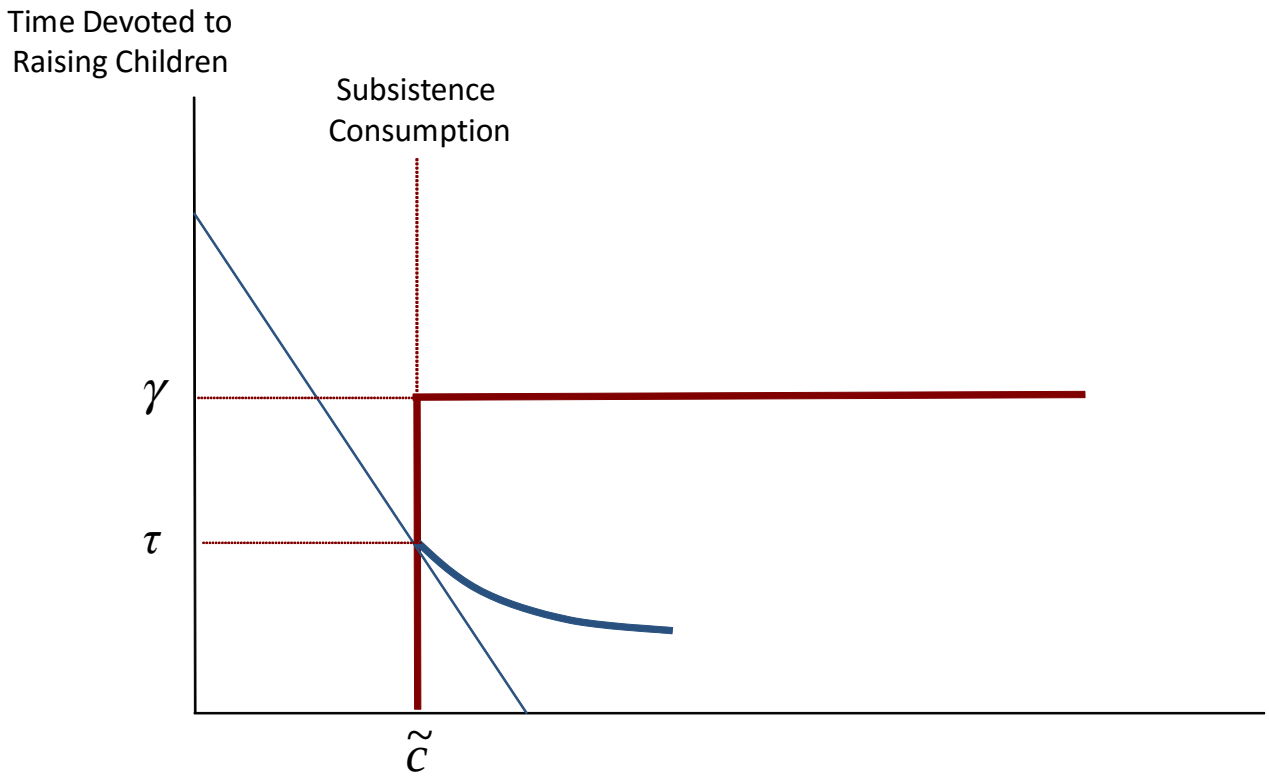
$$n_t(\tau + e_{t+1}) = \begin{cases} \gamma & \text{if} \quad z_t \geq \tilde{z} \\ 1 - \frac{\tilde{c}}{z_t} & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

- $z_t = \tilde{z}$ is the highest level of potential income such that the subsistence constraint is still binding

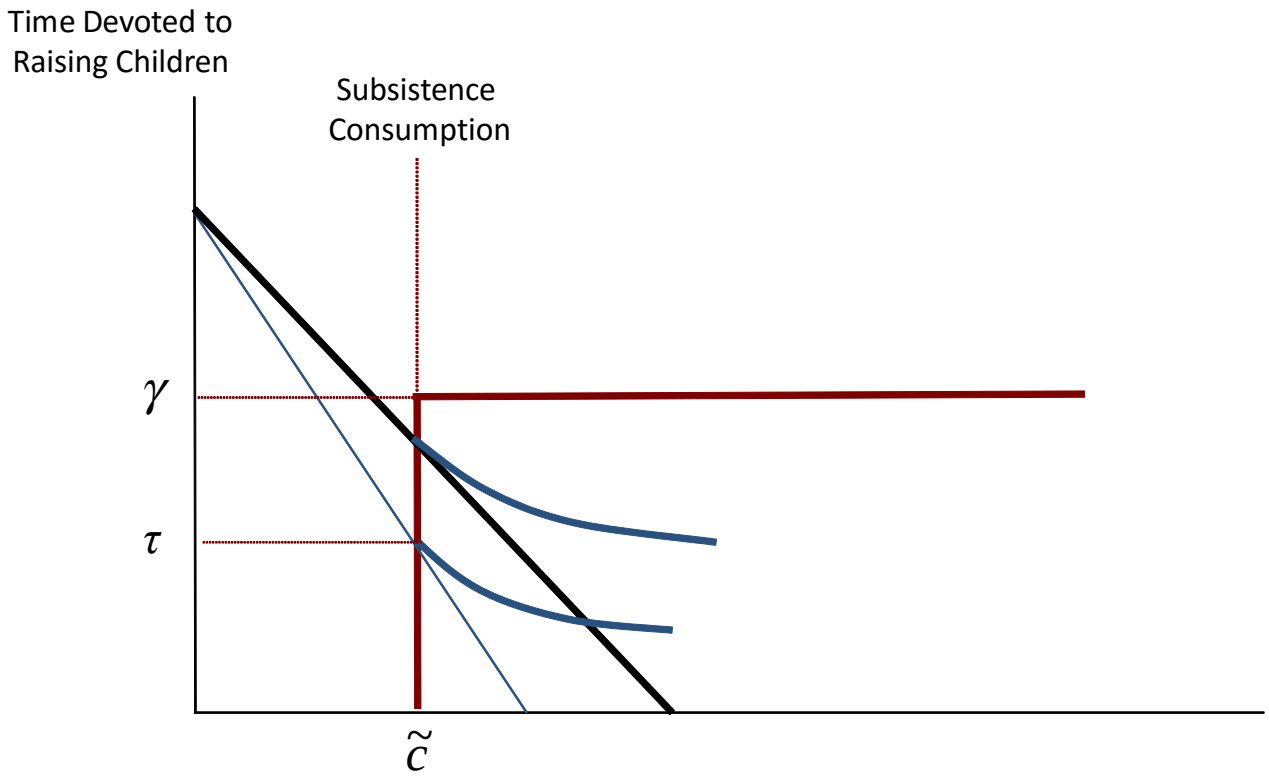
$$e_{t+1} = e(g_{t+1}) \implies$$

$$n_t = \begin{cases} \frac{\gamma}{\tau + e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if} \quad z_t \geq \tilde{z} \\ \frac{1-[\tilde{c}/z_t]}{\tau + e(g_{t+1})} \equiv n^a(g_{t+1}, z(e_t, g_t, x_t)) & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

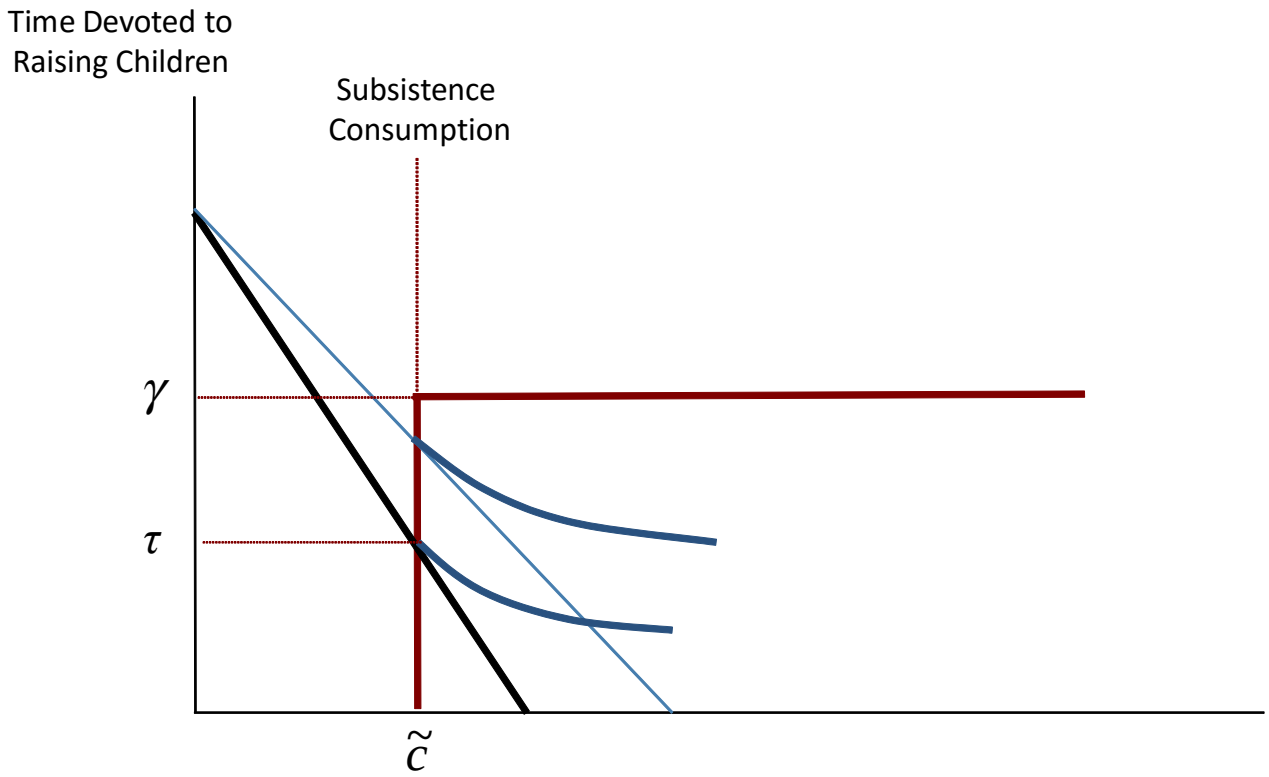
Optimization – Malthusian Epoch



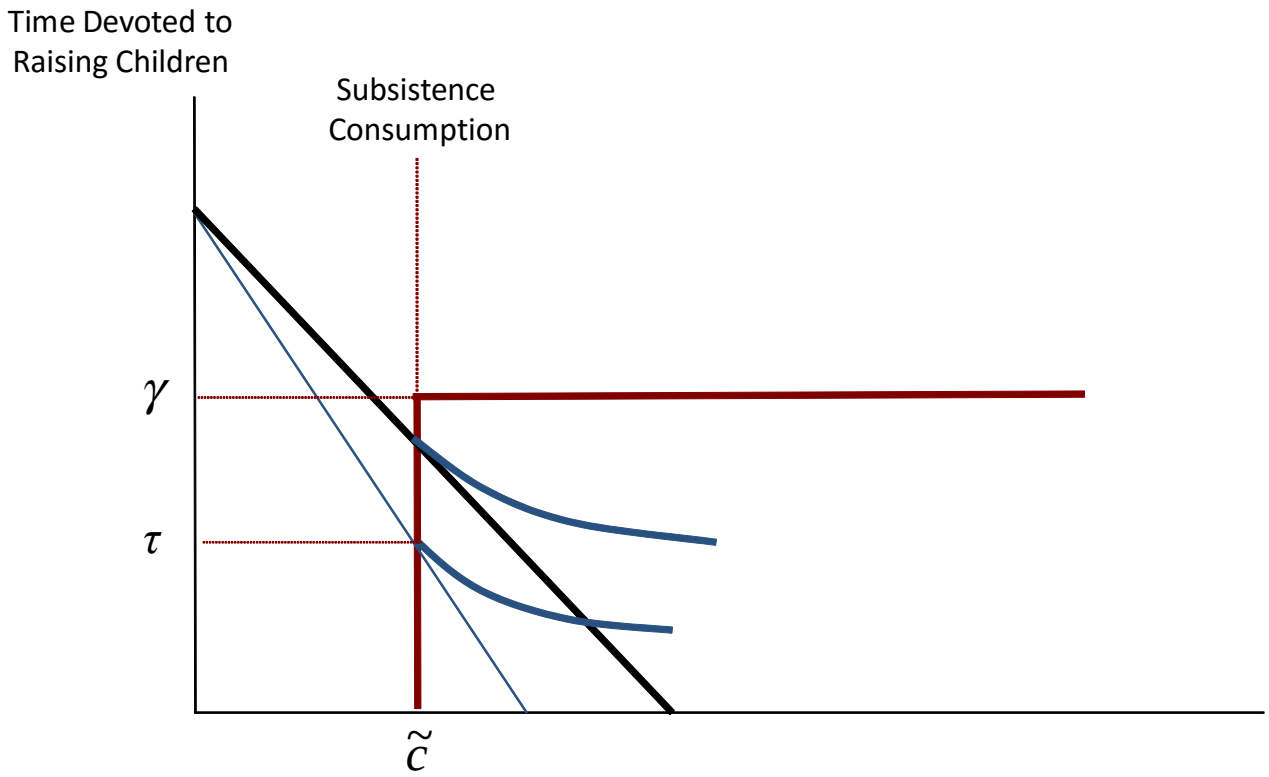
Optimization – Malthusian Epoch



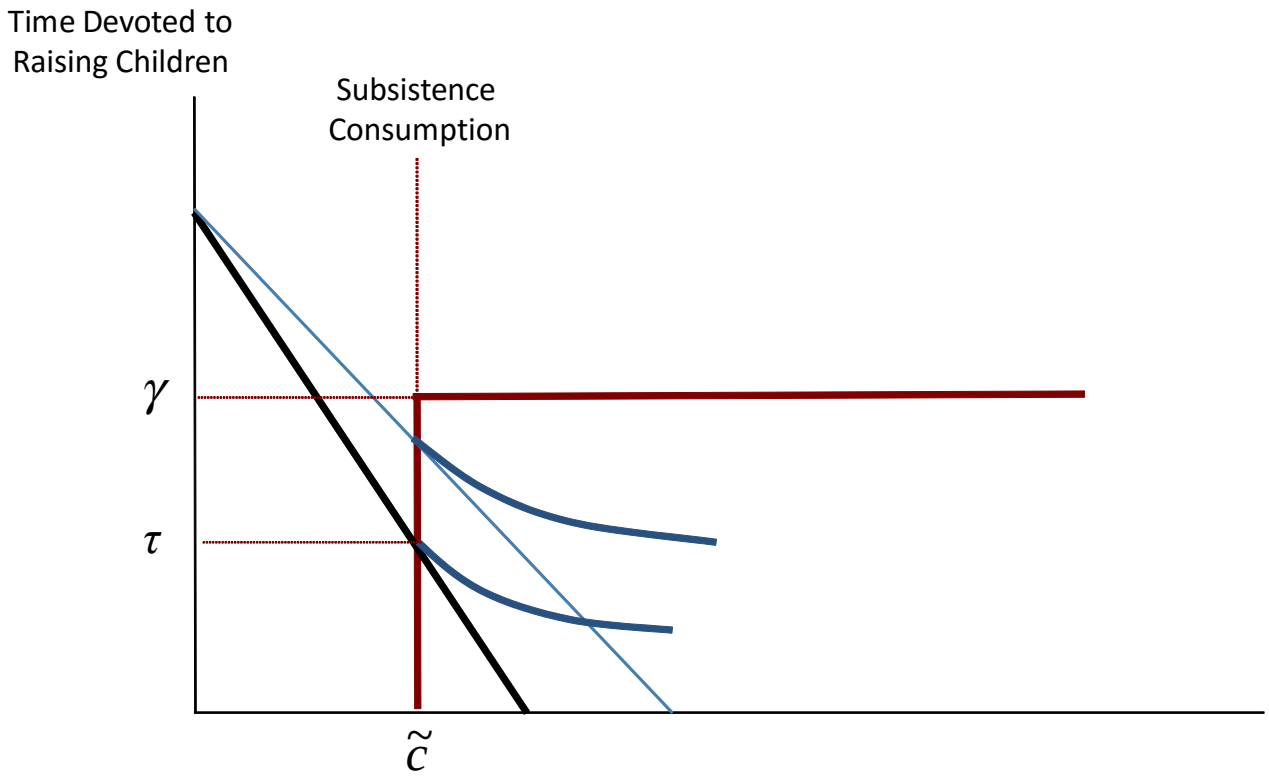
Optimization – Malthusian Epoch



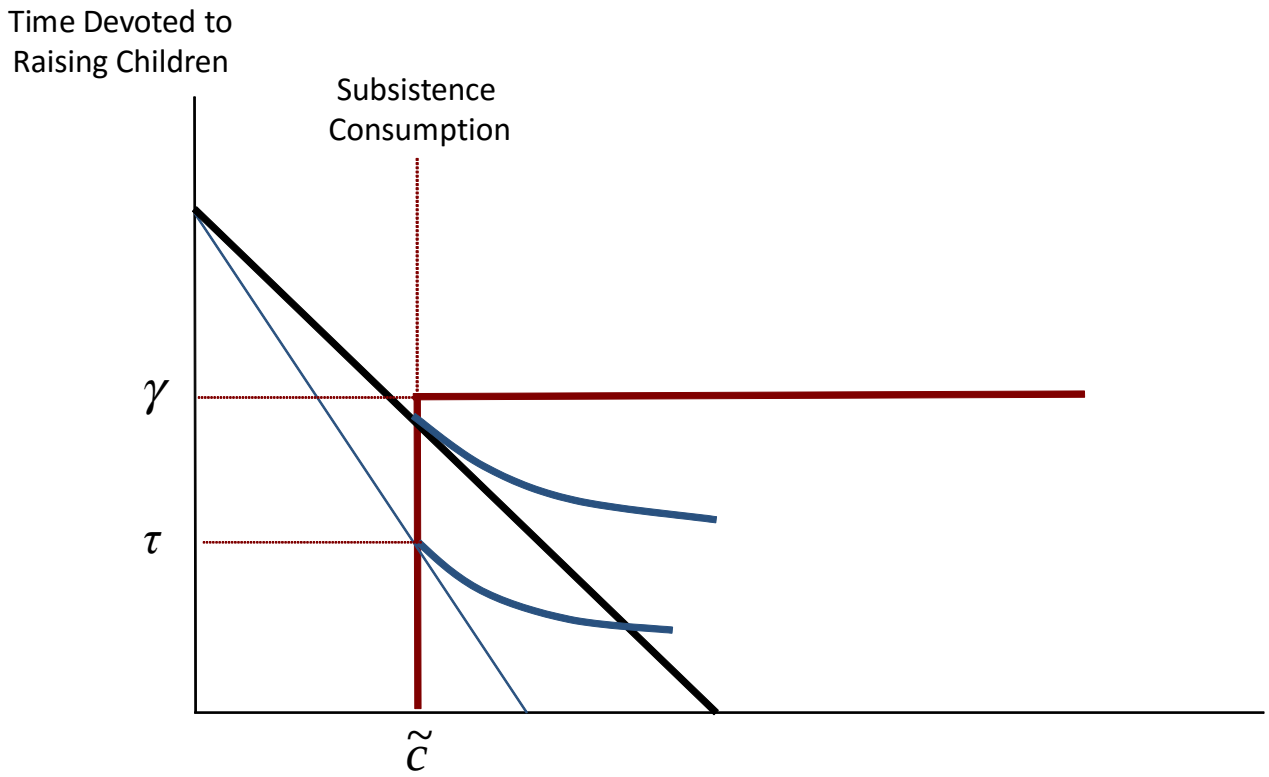
Income Expansion Path – Malthusian Epoch



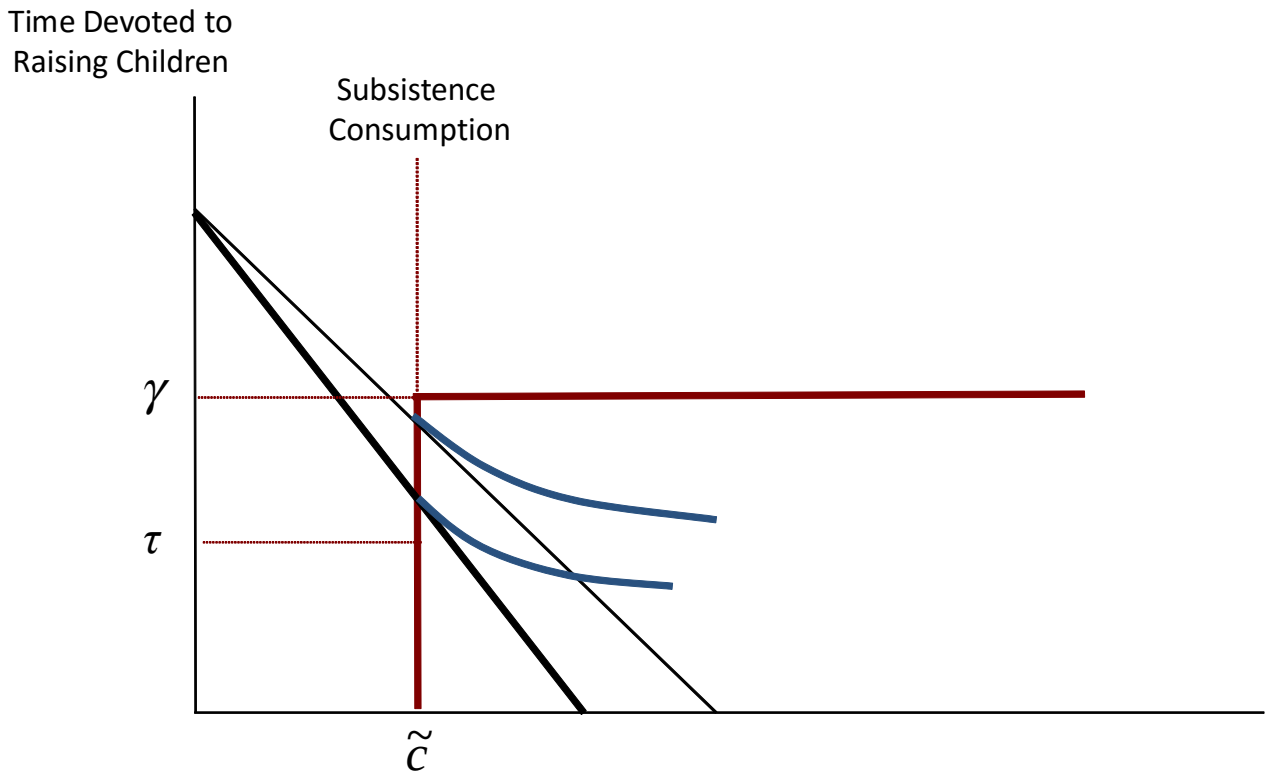
Income Expansion Path – Malthusian Epoch



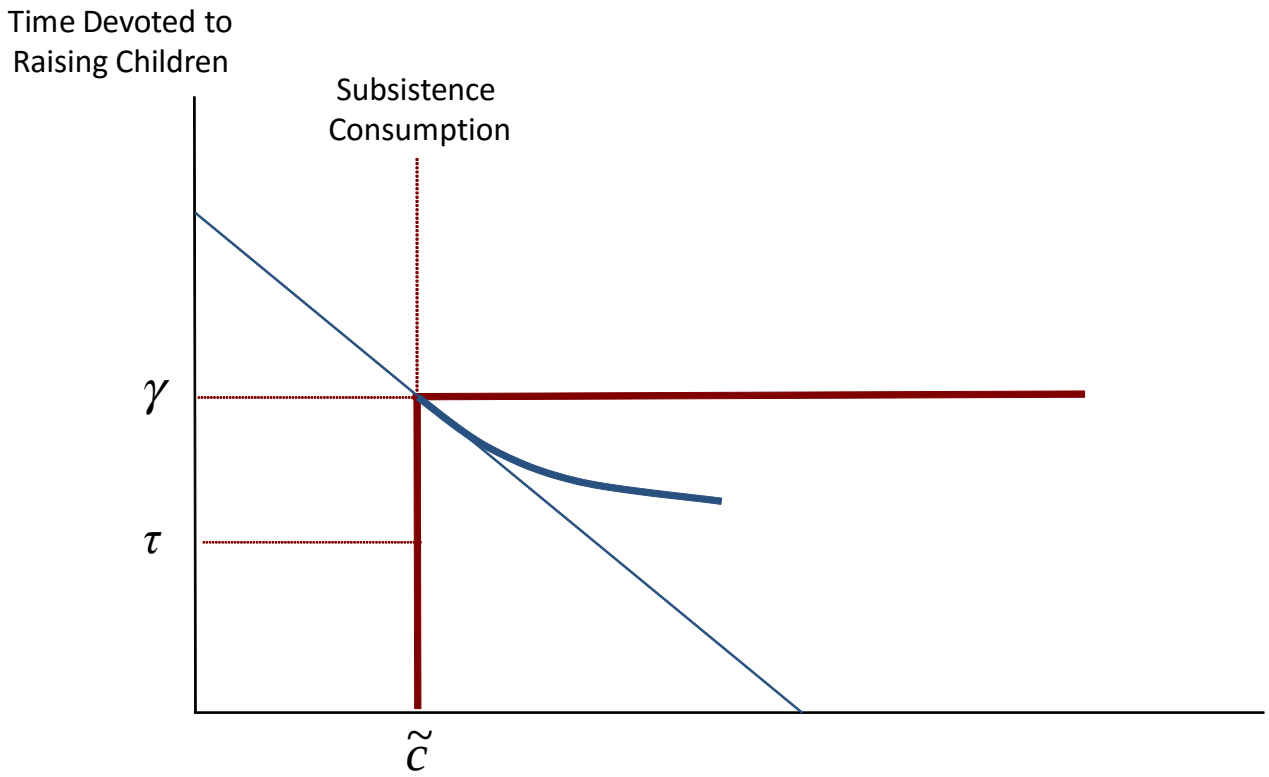
Income Expansion Path – Malthusian Epoch



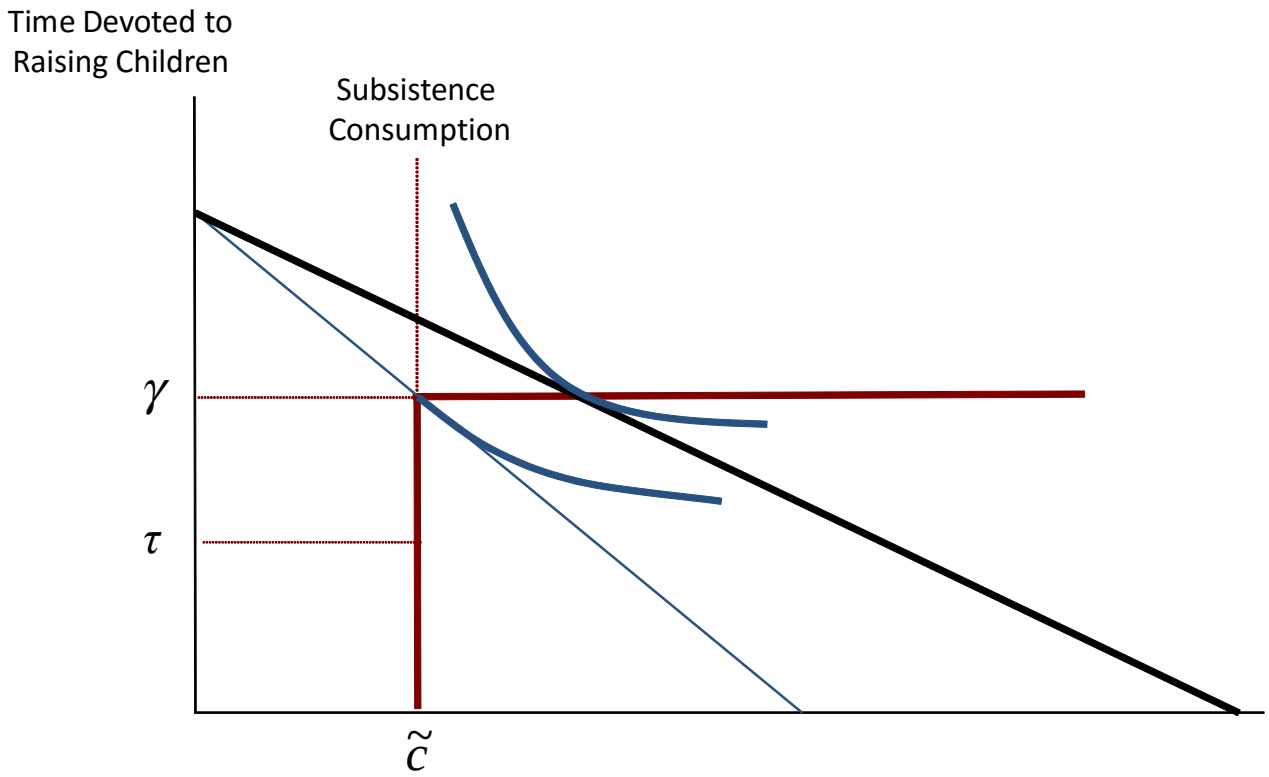
Income Expansion Path – Malthusian Epoch



Income Expansion Path – Post-Demographic Transition



Income Expansion Path – Post-Demographic Transition



Population Dynamics

$$L_{t+1} = n_t L_t$$

$$L_{t+1} = \begin{cases} n^b(g_{t+1})L_t & \text{if} \quad z_t \geq \tilde{z} \\ n^a(g_{t+1}, z(e_t, g_t, x_t))L_t & \text{if} \quad z_t \leq \tilde{z} \end{cases}$$

Dynamics of the Level of Resources per Worker

$$x_{t+1} = \frac{A_{t+1}X}{L_{t+1}} = \frac{(1+g_{t+1})A_tX}{n_tL_t} = \frac{1+g_{t+1}}{n_t}x_t$$

$$x_{t+1} = \begin{cases} \frac{[1+g(e_t,L_t)][\tau^q+\tau^e e(g(e_t,L_t))]}{\gamma}x_t \equiv \phi^b(e_t;L)x_t & z_t \geq \tilde{z} \\ \frac{[1+g(e_t,L_t)][\tau+e(g(e_t,L_t))]}{1-[\tilde{c}/z(e_t,g_t,x_t)]}x_t \equiv \phi^a(e_t,g_t,x_t,L_t)x_t & z_t \leq \tilde{z}, \end{cases}$$

The Dynamical System

A sequence $\{x_t, e_t, g_t, L_t\}_{t=0}^{\infty}$ such that:

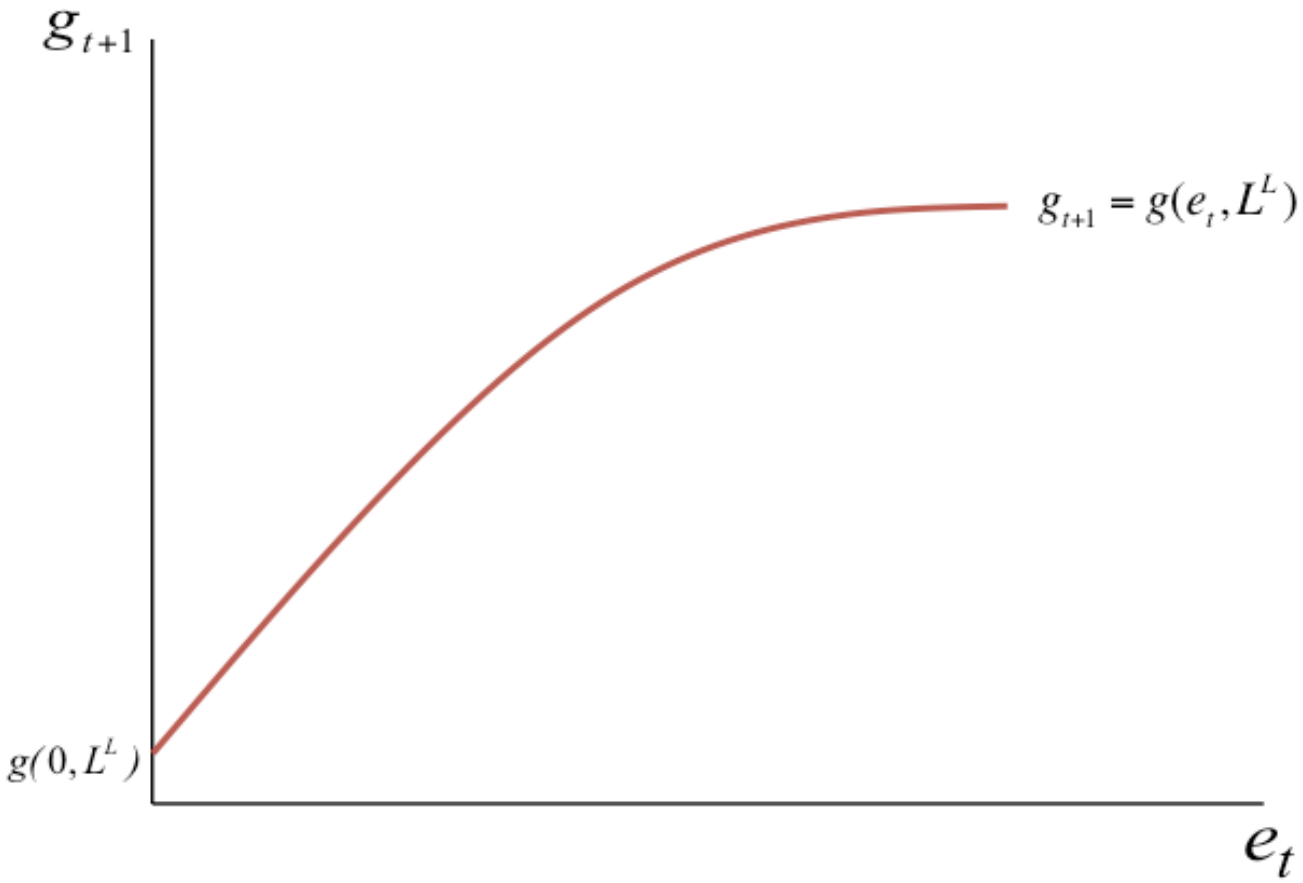
$$\begin{cases} x_{t+1} = \phi(e_t, g_t, x_t, L_t)x_t \\ e_{t+1} = e(g(e_t, L_t)) \\ g_{t+1} = g(e_t, L_t) \\ L_{t+1} = n(e_t, g_t, x_t, L_t)L_t \end{cases}$$

The Conditional Evolution of Technology and Education

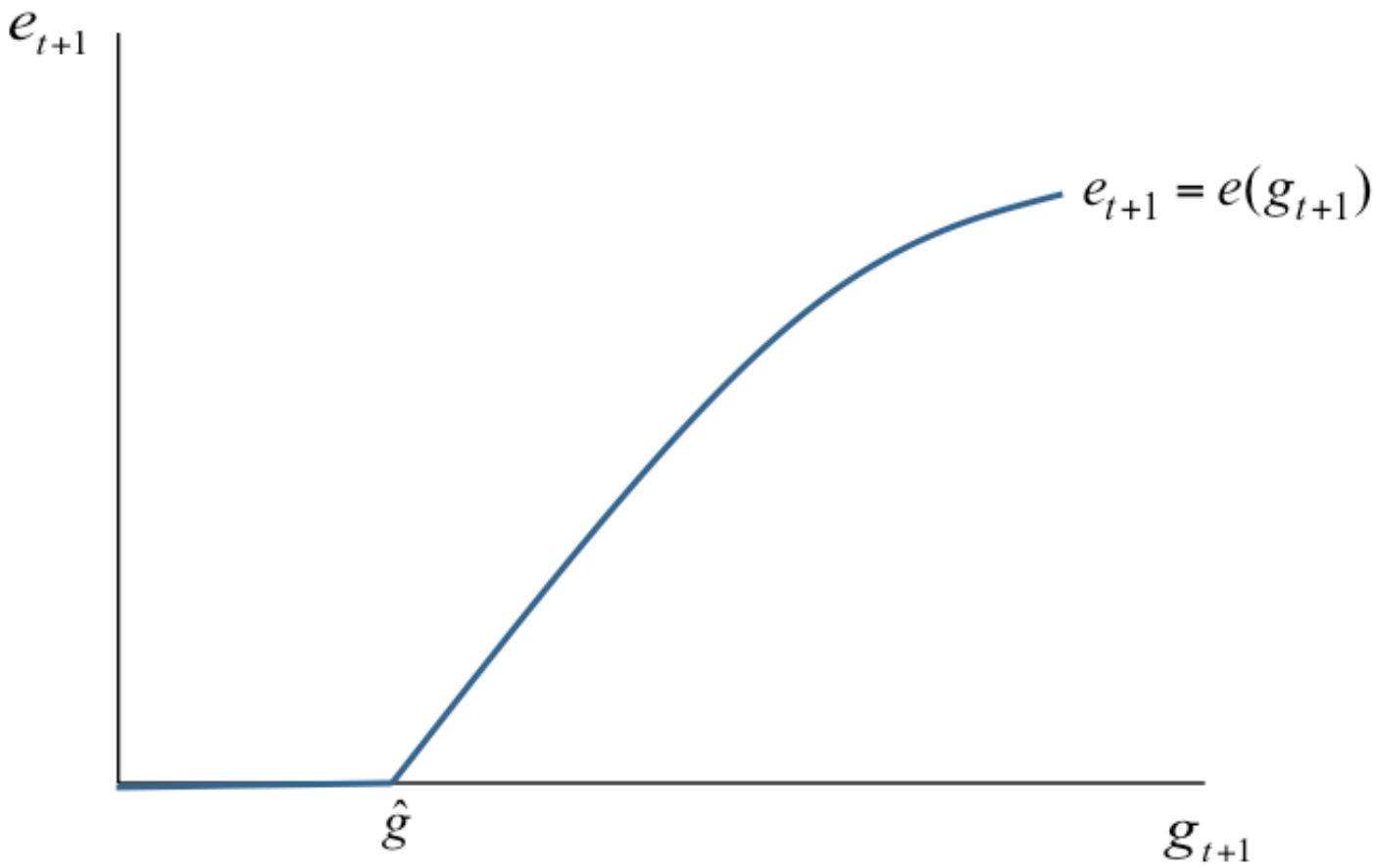
A sequence $\{g_t, e_t; L\}_{t=0}^{\infty}$ such that:

$$\begin{cases} g_{t+1} = g(e_t; L) \\ \\ e_{t+1} = e(g_{t+1}) \end{cases}$$

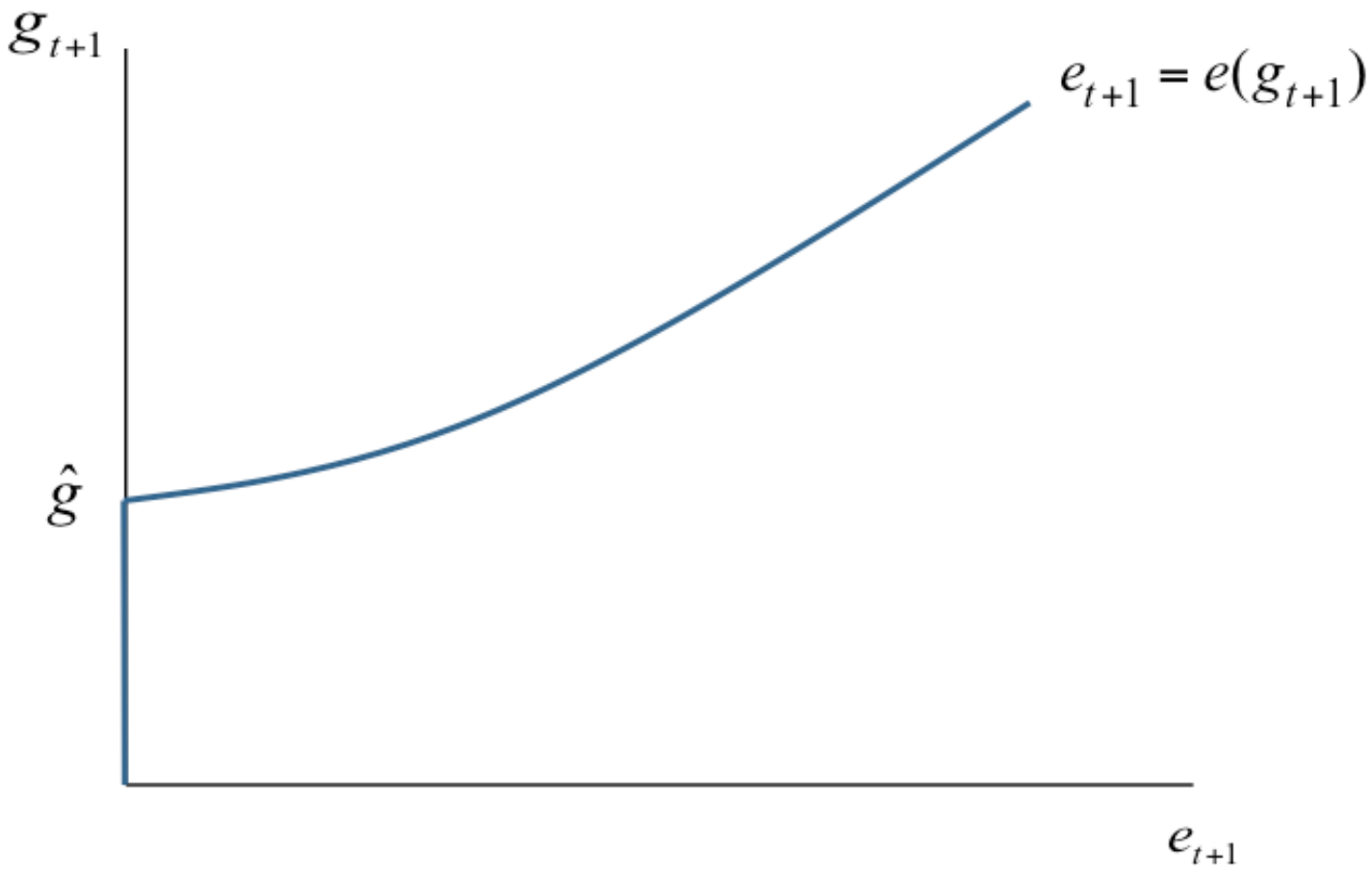
The Effect of Education on Technology



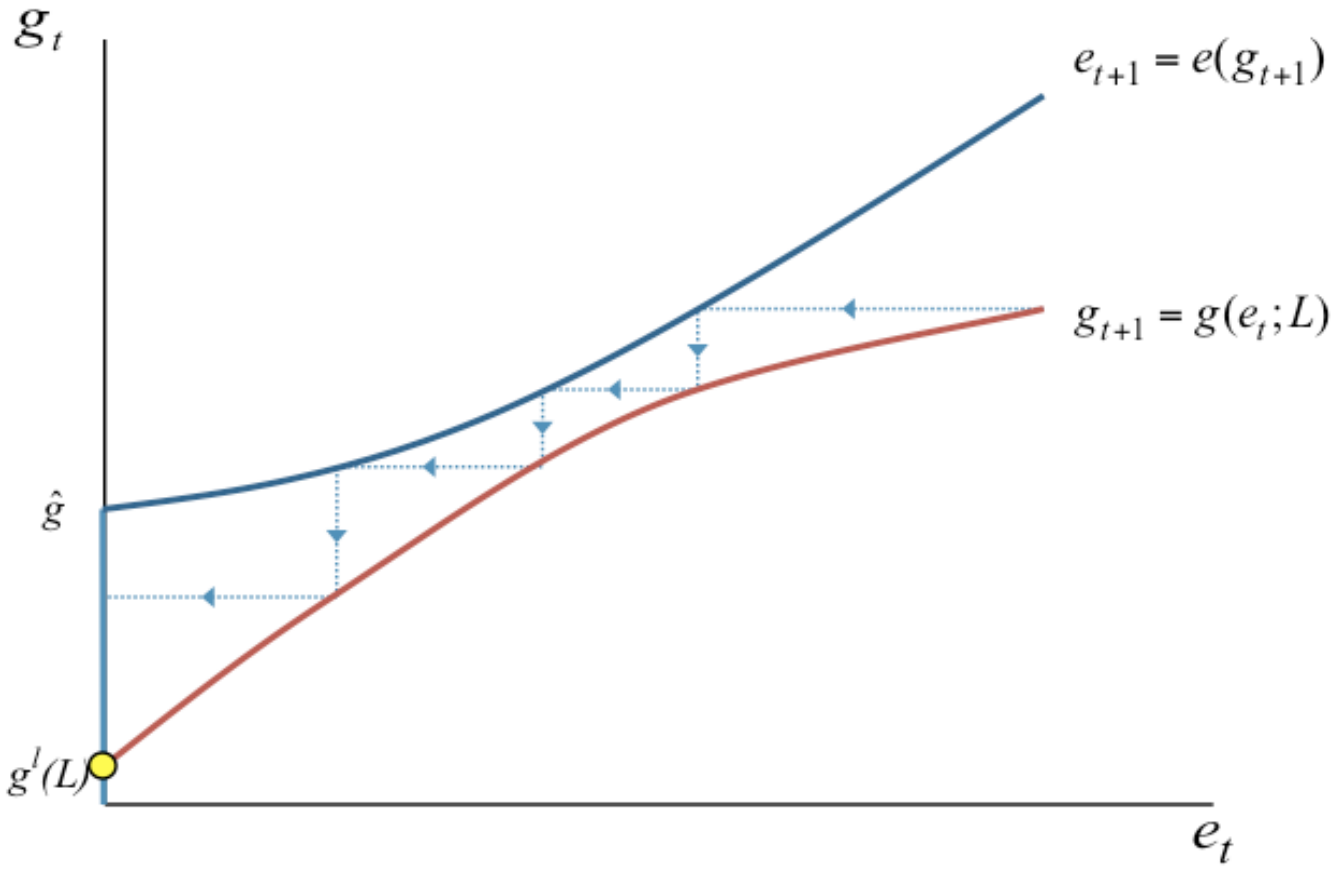
The Effect of Technology on Education



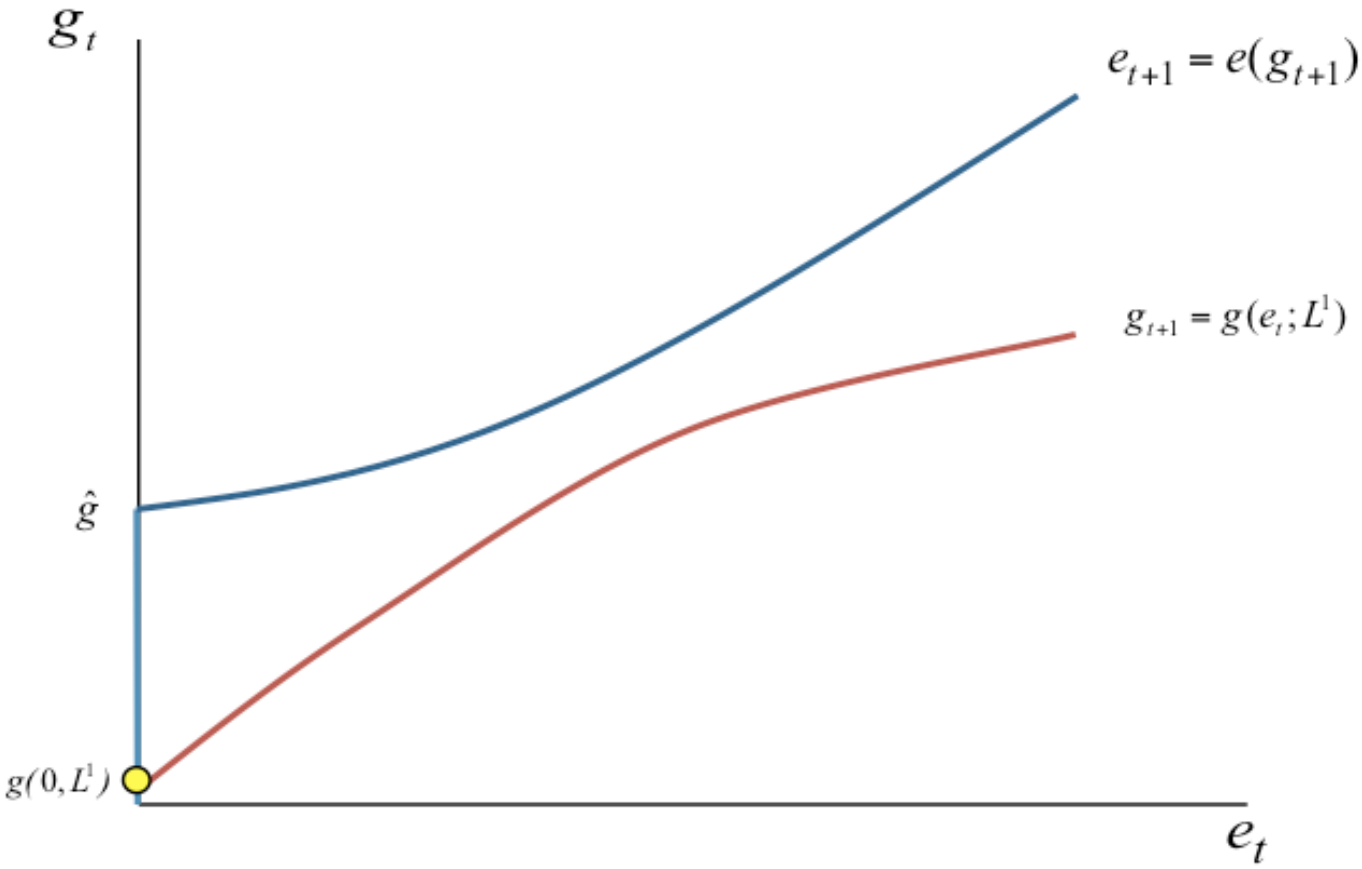
The Effect of Technology on Education: Flipped Axis



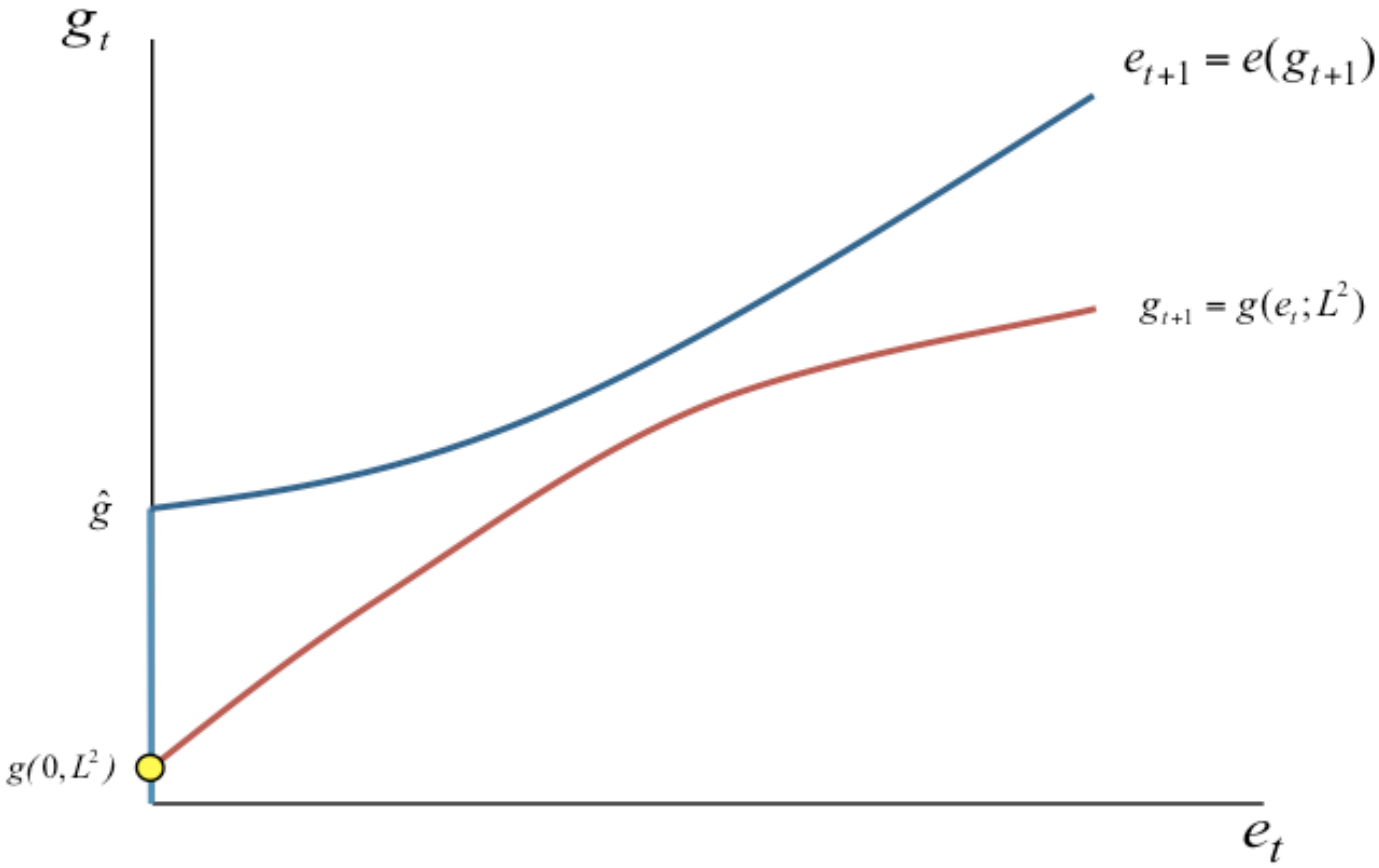
The Evolution of Education and Technology: For a Given Population Size



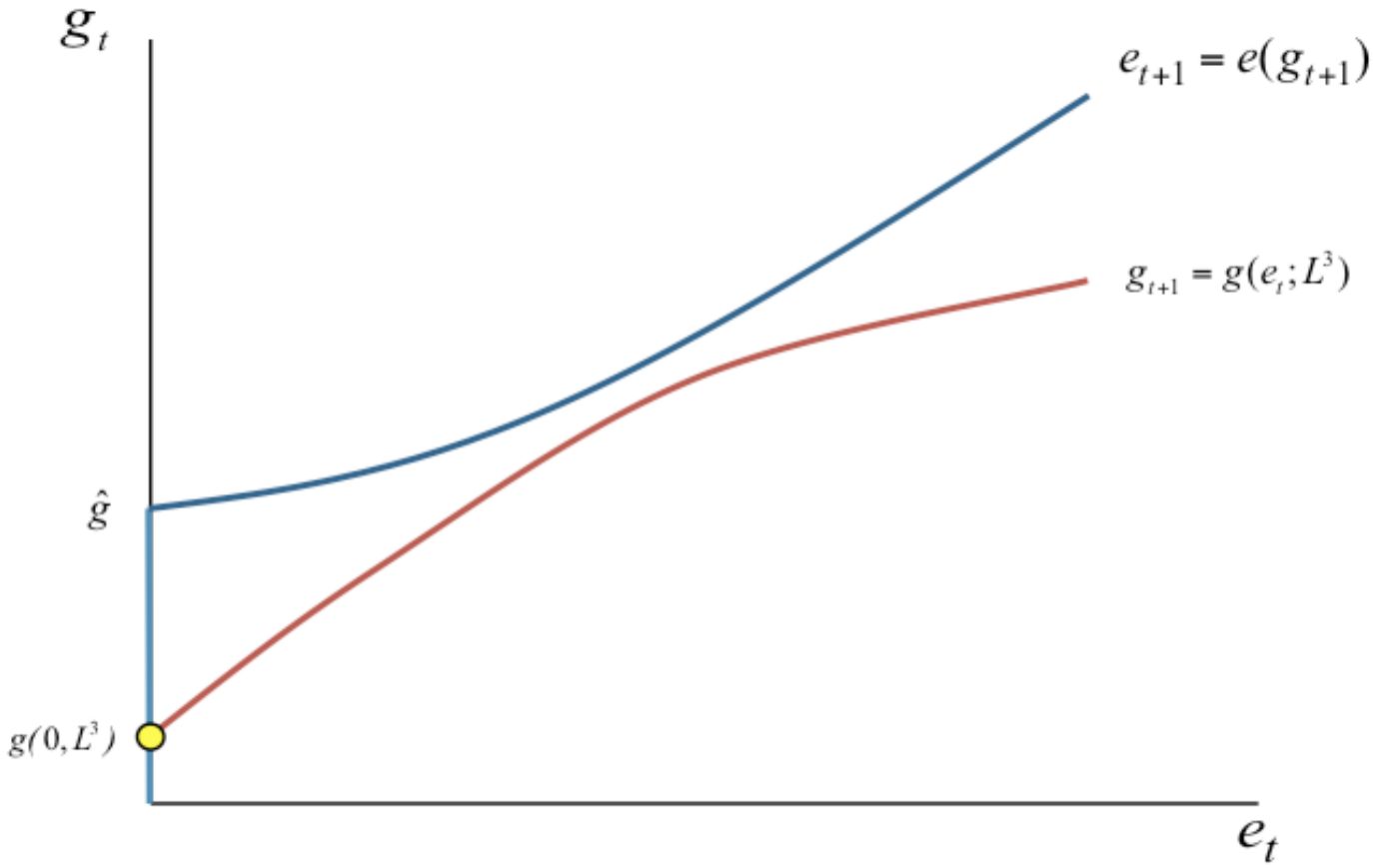
The Evolution of Education and Technology



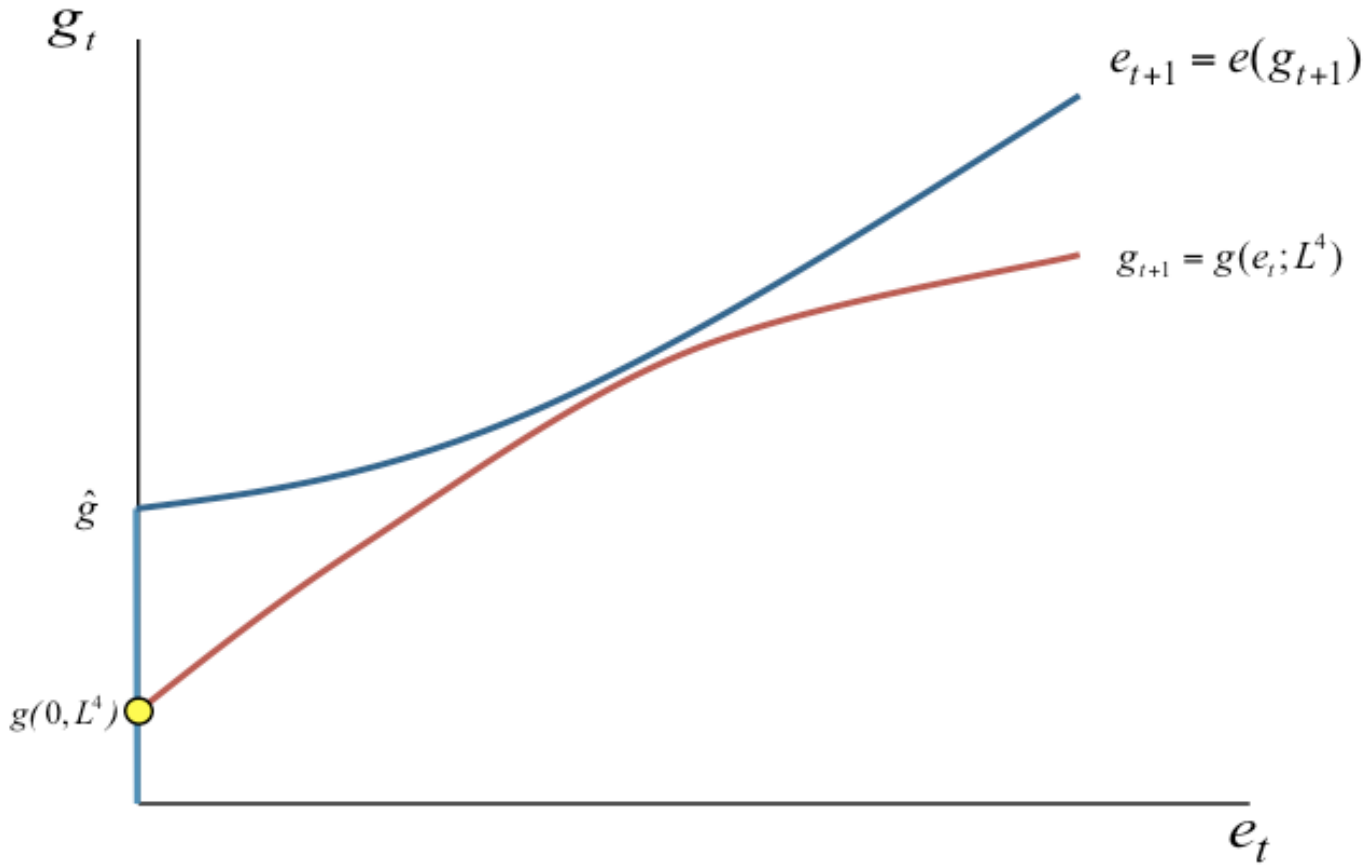
The Evolution of Education and Technology



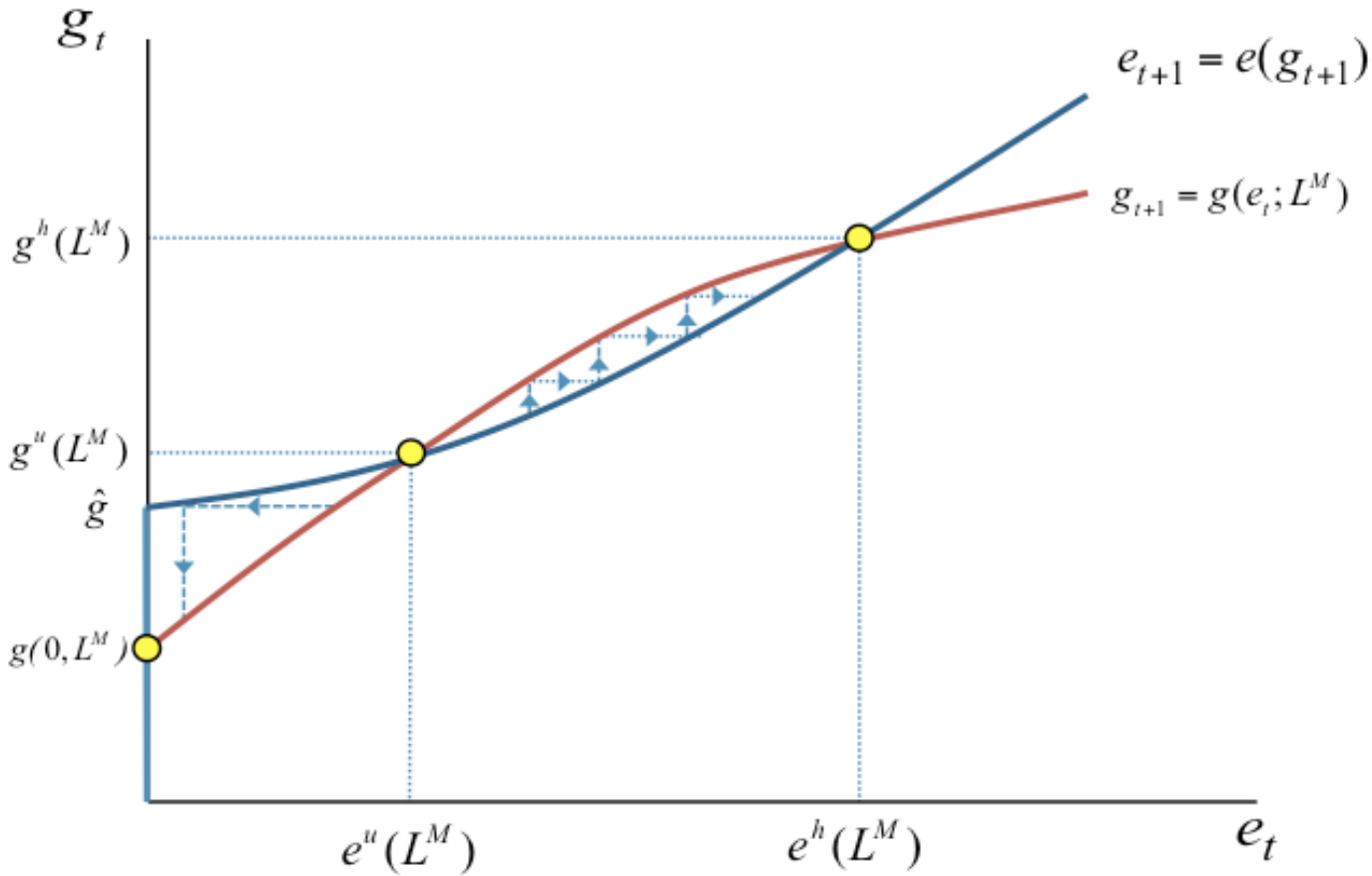
The Evolution of Education and Technology



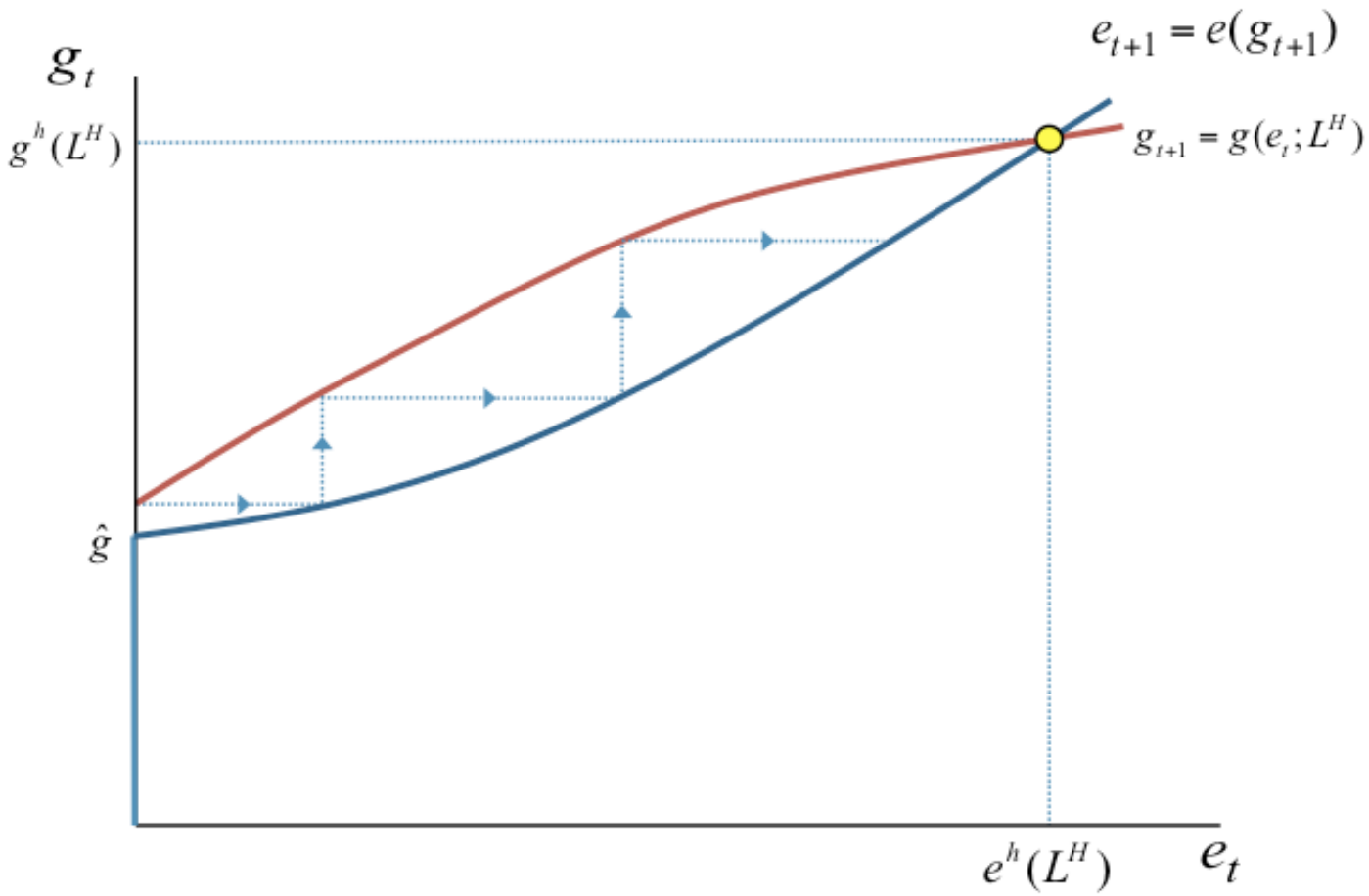
The Evolution of Education and Technology



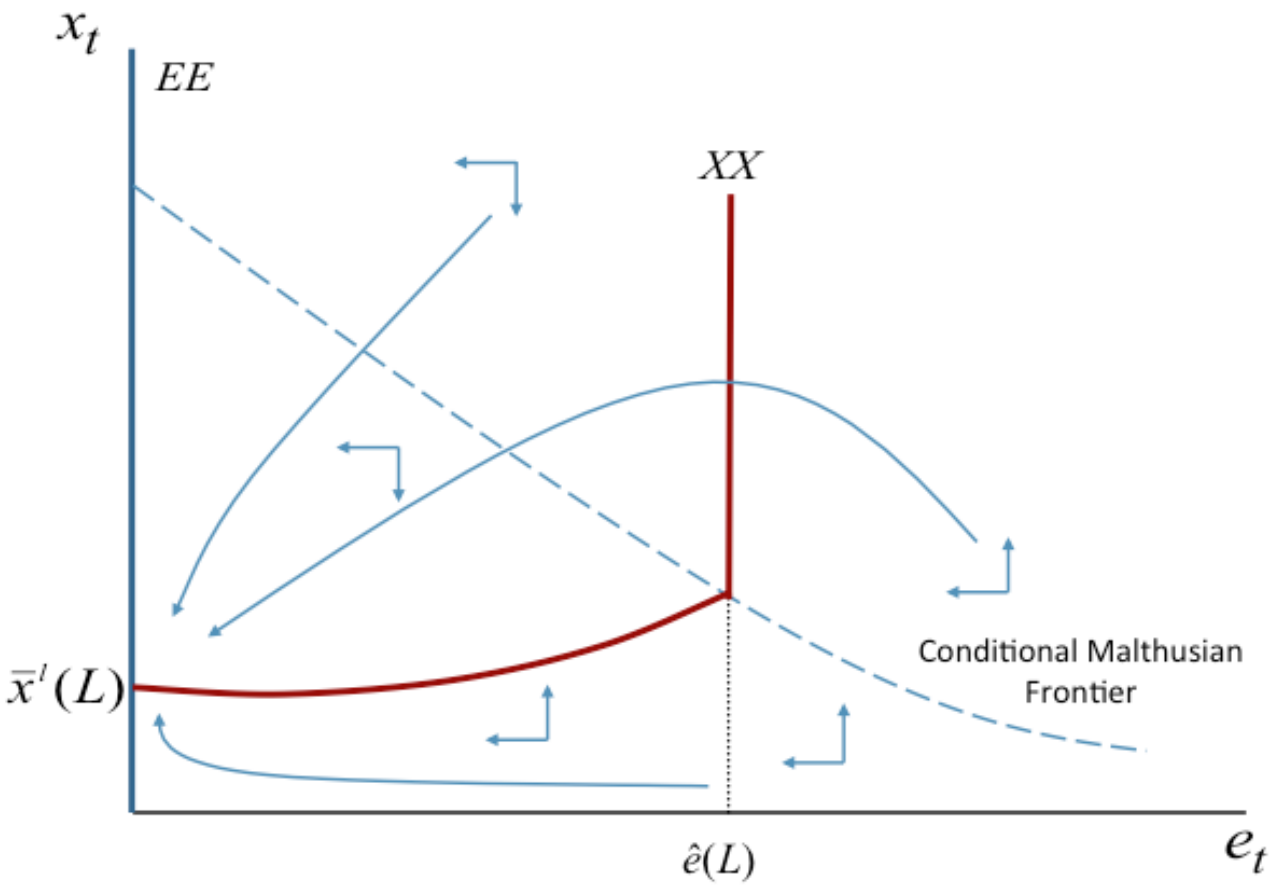
The Evolution of Education and Technology



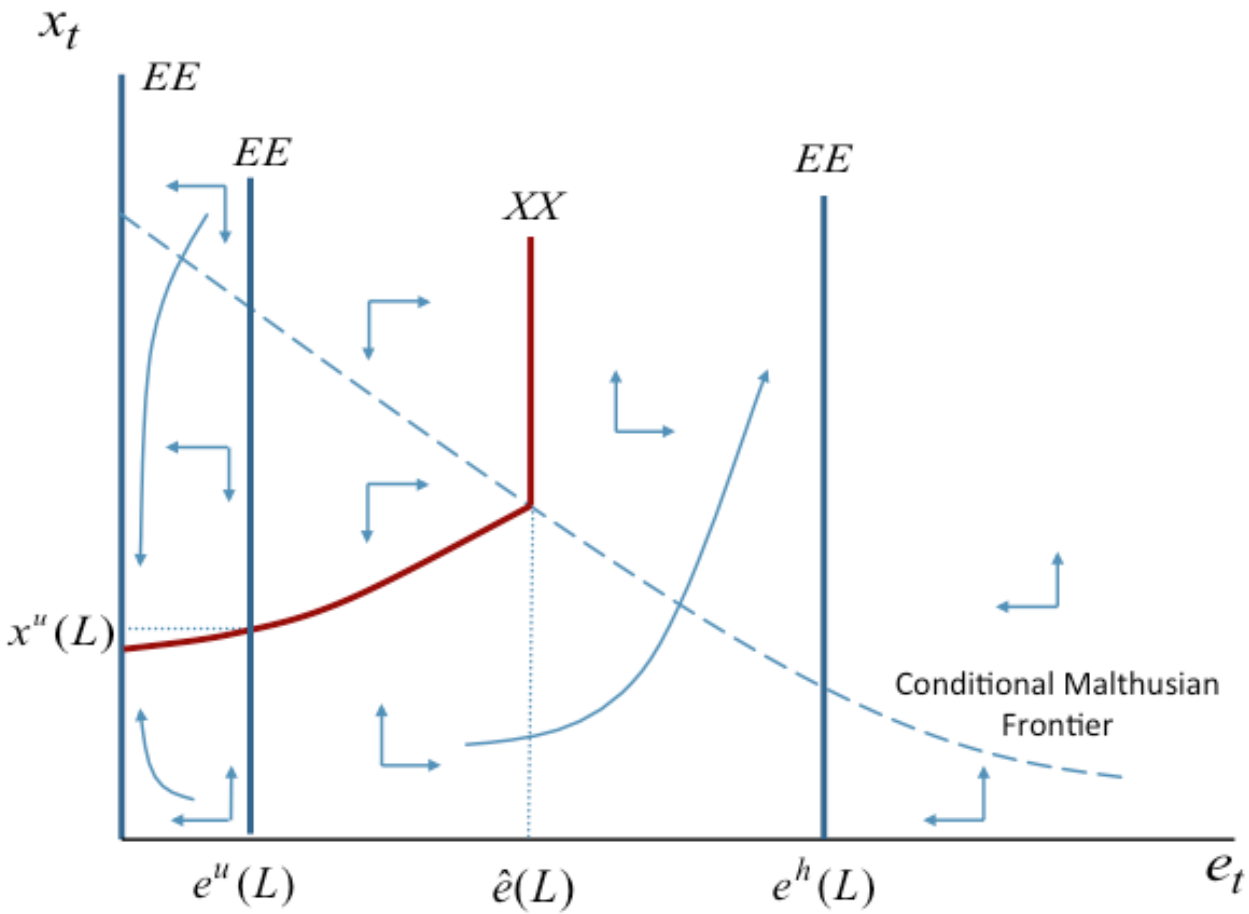
The Evolution of Education and Technology



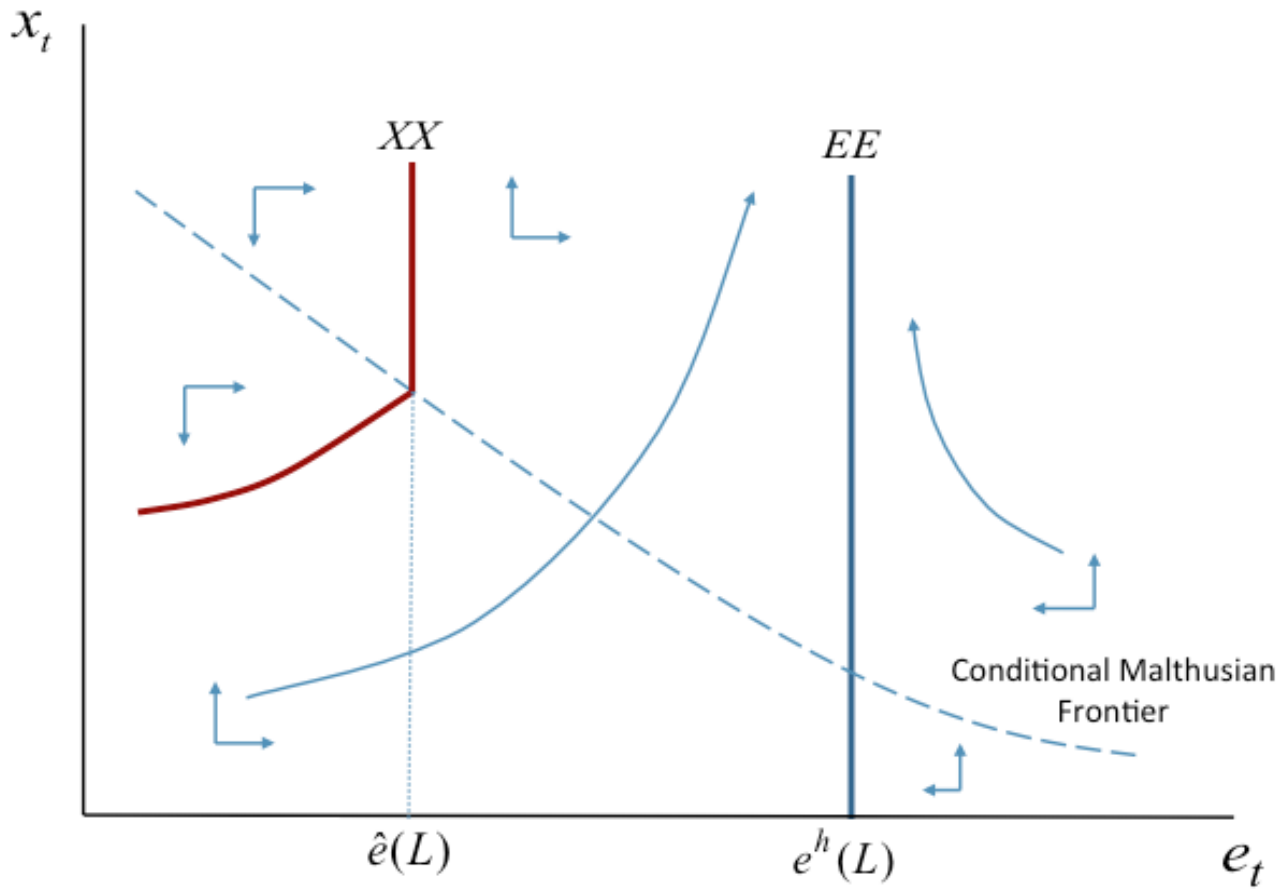
The Evolution of Education and Resources Per Worker: Small Population



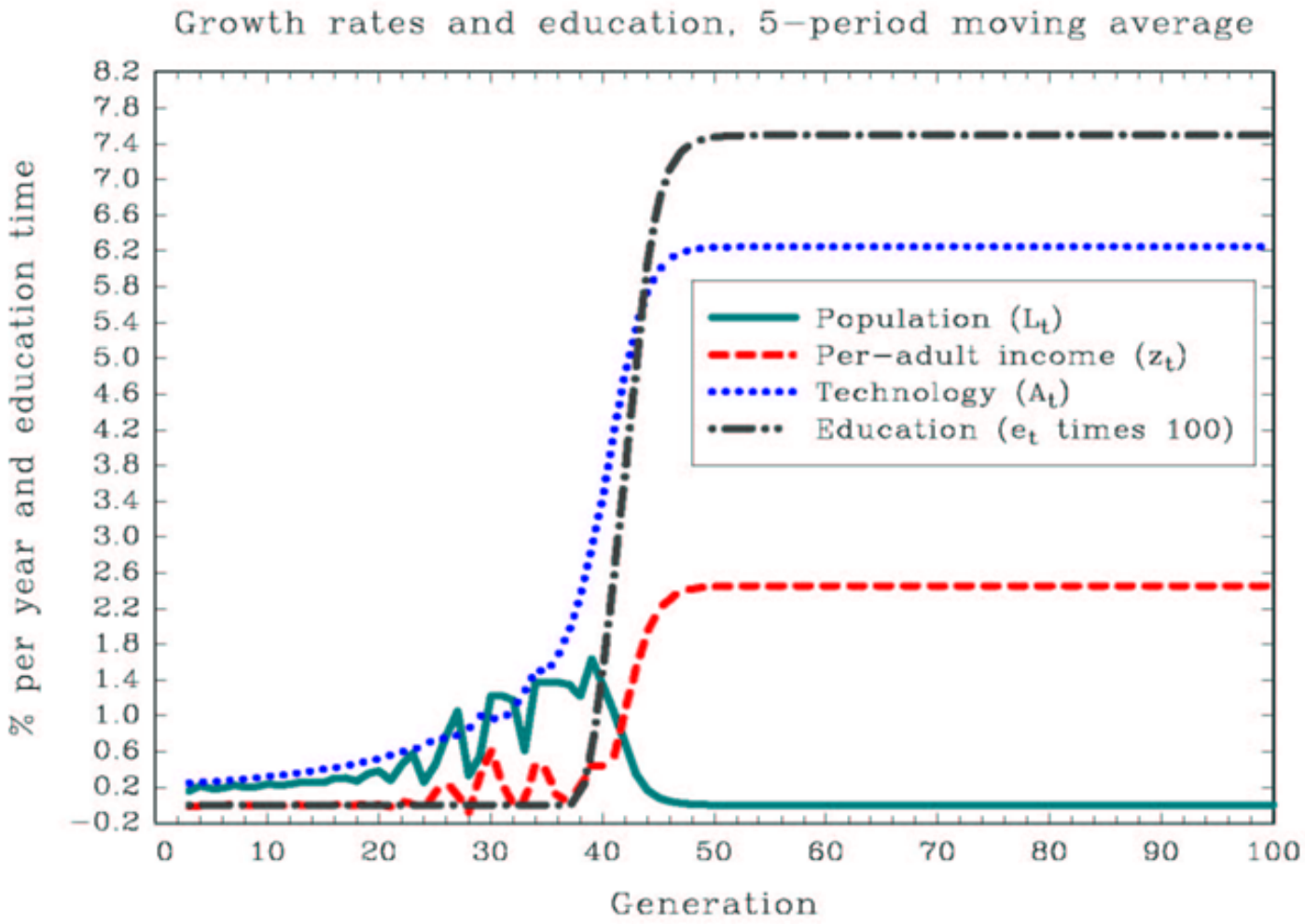
The Evolution of Education and Resources Per Worker: Intermediate Population



The Evolution of Education and Resources Per Worker: Large Population



Simulation



Source: Lagerlof (RED 2006)

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- The Malthusian interaction between technology & population
  - Acceleration in technological progress
    - $\Longrightarrow$ Industrial demand for human capital
  - Human capital formation
    - $\Longrightarrow$ Decline in fertility rates
    - $\Longrightarrow$ Further technological progress
  - Decline in population growth
    - $\Longrightarrow$ Economic growth is freed from counterbalancing effects of population
  - Technological progress, human capital & decline in population growth
    - $\Longrightarrow$ Sustained economic growth

- Variations in the timing of the take-off contributed significantly to the divergence in income per capita in the past two centuries
- Differences in the economic performance across countries reflect:
  - Variations in country-specific characteristics that affect:
    - The pace of technological progress
    - The intensity of human capital formation

- Variations in the timing of the take-off contributed significantly to the divergence in income per capita in the past two centuries
- Differences in the economic performance across countries reflect:
  - Variations in country-specific characteristics that affect:
    - The pace of technological progress
    - The intensity of human capital formation

- Variations in the timing of the take-off contributed significantly to the divergence in income per capita in the past two centuries
- Differences in the economic performance across countries reflect:
  - Variations in country-specific characteristics that affect:
    - The pace of technological progress
    - The intensity of human capital formation

- Variations in the timing of the take-off contributed significantly to the divergence in income per capita in the past two centuries
- Differences in the economic performance across countries reflect:
  - Variations in country-specific characteristics that affect:
    - The pace of technological progress
    - The intensity of human capital formation

Variations in Country-Specific Characteristics Conducive for Technological Progress

$$g^i_{t+1} = g(e^i_t, L^i_t, \Omega^i_t)$$

$\Omega^i_t \equiv$ characteristics affecting tech progress in country i:

- Protection of intellectual property rights (policy)
- The stock of knowledge within a society
- The propensity of a country to trade (geography & policy)
  - Technological diffusion
  - Specialization and technological progress via learning by doing

Variations in Country-Specific Characteristics Conducive for Technological Progress

$$g^i_{t+1} = g(e^i_t, L^i_t, \Omega^i_t)$$

$\Omega^i_t \equiv$ characteristics affecting tech progress in country i:

- Protection of intellectual property rights (policy)
- The stock of knowledge within a society
- The propensity of a country to trade (geography & policy)
  - Technological diffusion
  - Specialization and technological progress via learning by doing

Variations in Country-Specific Characteristics Conducive for Technological Progress

$$g_{t+1}^i = g(e_t^i, L_t^i, \Omega_t^i)$$

$\Omega_t^i \equiv$ characteristics affecting tech progress in country i:

- Protection of intellectual property rights (policy)
- The stock of knowledge within a society
- The propensity of a country to trade (geography & policy)
  - Technological diffusion
  - Specialization and technological progress via learning by doing

Variations in Country-Specific Characteristics Conducive for Technological Progress

$$g_{t+1}^i = g(e_t^i, L_t^i, \Omega_t^i)$$

$\Omega_t^i \equiv$ characteristics affecting tech progress in country i:

- Protection of intellectual property rights (policy)
- The stock of knowledge within a society
- The propensity of a country to trade (geography & policy)
  - Technological diffusion
  - Specialization and technological progress via learning by doing

Variations in Country-Specific Characteristics Conducive for Technological Progress

- Cultural and religious composition of society
  - Attitude toward knowledge creation and diffusion (e.g., The Inquisition)
- The composition of interest groups in society
  - Incentives to block or promote technological innovation (e.g., Luddites; landowners)
- Cultural and genetic diversity
  - Wider spectrum of traits are more likely to contain the ones complementary to the adoption or implementation of new technologies
- Abundance of natural resources
  - complementary for industrialization (e.g., Coal & Steam engine)

Variations in Country-Specific Characteristics Conducive for Technological Progress

- Cultural and religious composition of society
  - Attitude toward knowledge creation and diffusion (e.g., The Inquisition)
- The composition of interest groups in society
  - Incentives to block or promote technological innovation (e.g., Luddites; landowners)
- Cultural and genetic diversity
  - Wider spectrum of traits are more likely to contain the ones complementary to the adoption or implementation of new technologies
- Abundance of natural resources
  - complementary for industrialization (e.g., Coal & Steam engine)

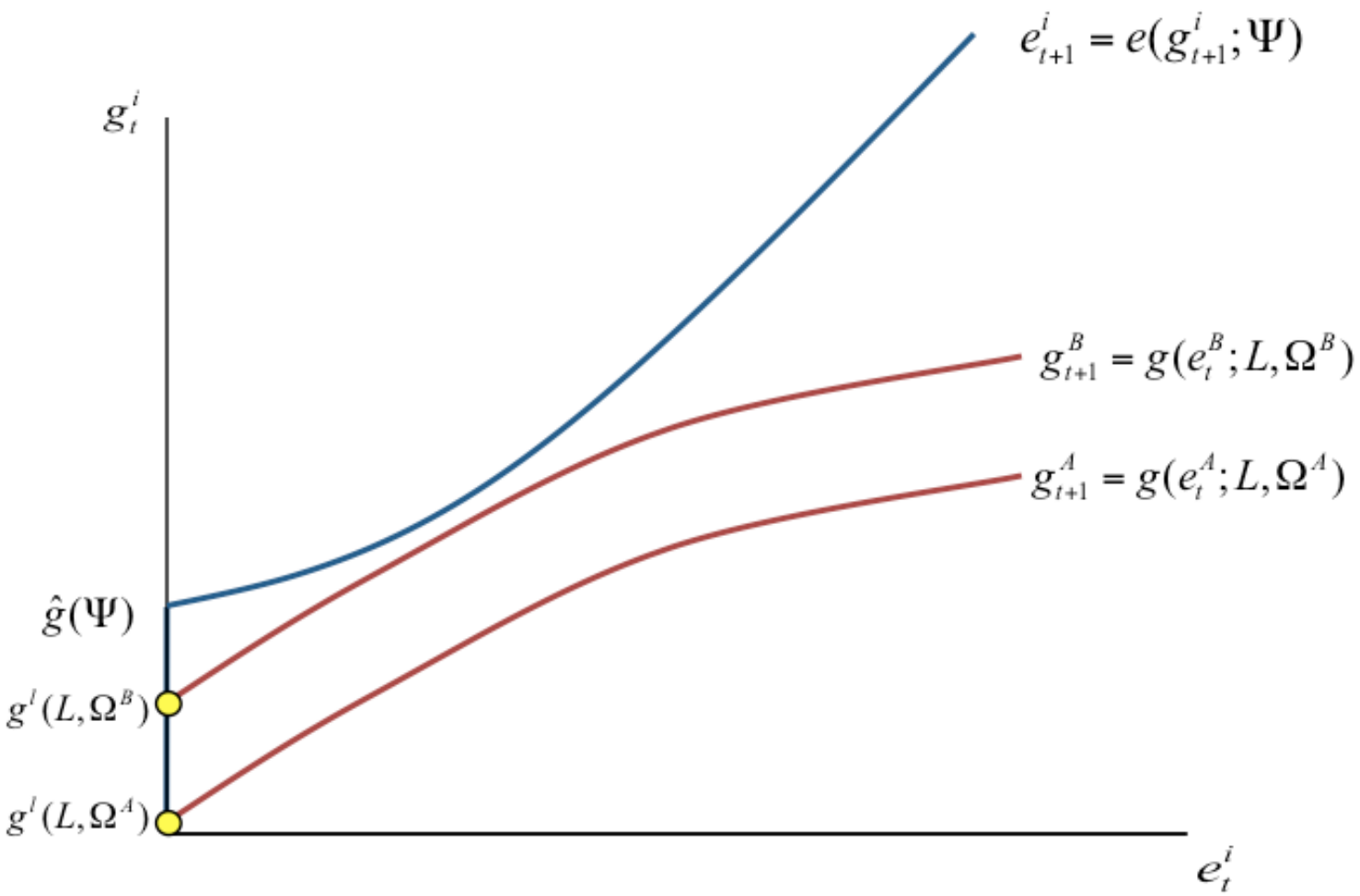
Variations in Country-Specific Characteristics Conducive for Technological Progress

- Cultural and religious composition of society
  - Attitude toward knowledge creation and diffusion (e.g., The Inquisition)
- The composition of interest groups in society
  - Incentives to block or promote technological innovation (e.g., Luddites; landowners)
- Cultural and genetic diversity
  - Wider spectrum of traits are more likely to contain the ones complementary to the adoption or implementation of new technologies
- Abundance of natural resources
  - complementary for industrialization (e.g., Coal & Steam engine)

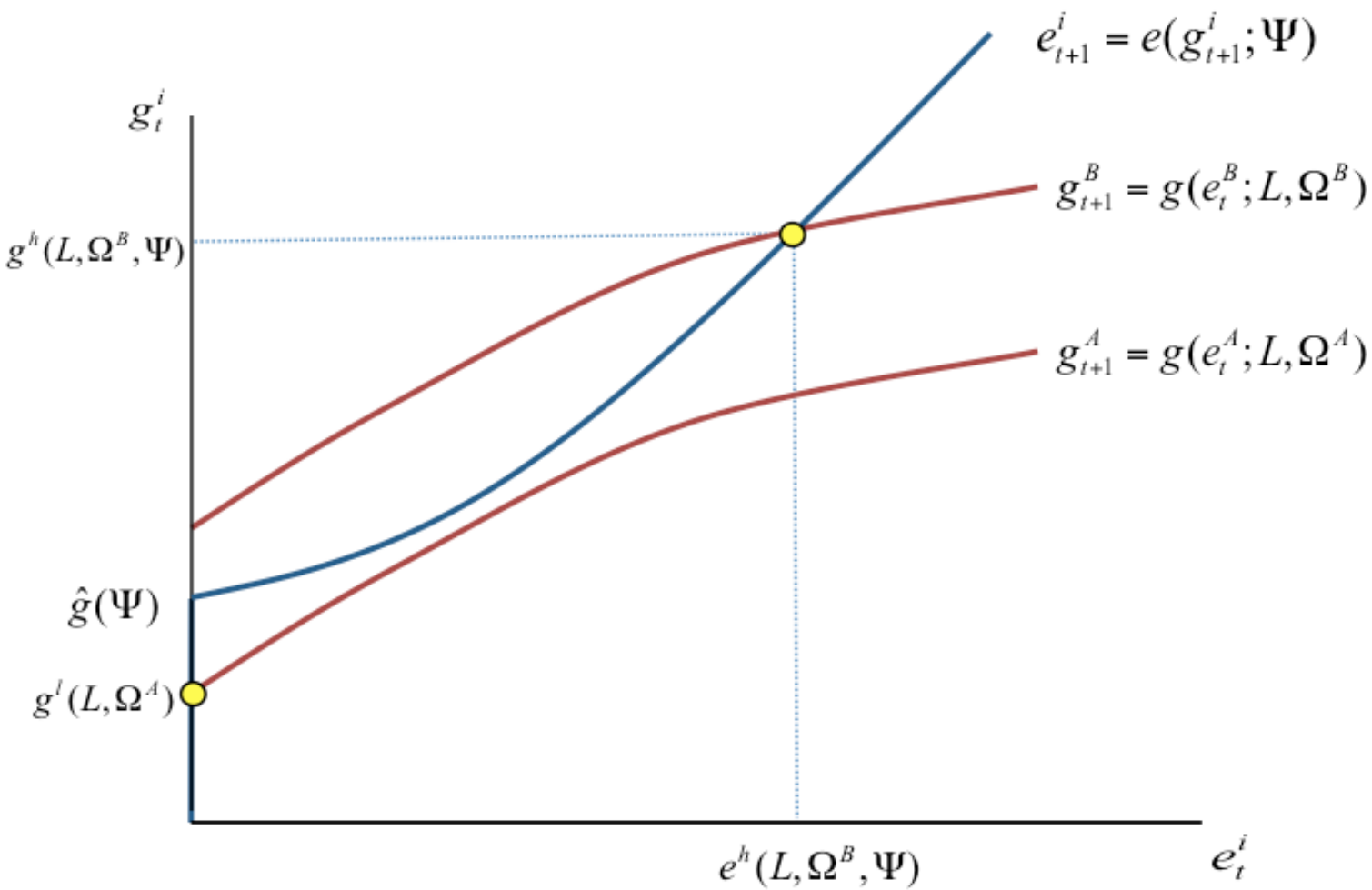
Variations in Country-Specific Characteristics Conducive for Technological Progress

- Cultural and religious composition of society
  - Attitude toward knowledge creation and diffusion (e.g., The Inquisition)
- The composition of interest groups in society
  - Incentives to block or promote technological innovation (e.g., Luddites; landowners)
- Cultural and genetic diversity
  - Wider spectrum of traits are more likely to contain the ones complementary to the adoption or implementation of new technologies
- Abundance of natural resources
  - complementary for industrialization (e.g., Coal & Steam engine)

Variations in Country-Specific Characteristics Conducive for Technological Progress



Earlier Take-off in Country B



Variation in Characteristics Conducive for Human Capital Formation

- For country-specific characteristics Ψ_t^i

$$e_{t+1}^i = e(g_{t+1}^i; \Psi_t^i) \begin{cases} = 0 & \text{if} \quad g_{t+1}^i \leq \hat{g}(\Psi_t^i), \\ > 0 & \text{if} \quad g_{t+1}^i > \hat{g}(\Psi_t^i) \end{cases}$$

Variation in Characteristics Conducive for Human Capital Formation

- For country-specific characteristics Ψ_t^i

$$e_{t+1}^i = e(g_{t+1}^i; \Psi_t^i) \begin{cases} = 0 & \text{if} \quad g_{t+1}^i \leq \hat{g}(\Psi_t^i), \\ > 0 & \text{if} \quad g_{t+1}^i > \hat{g}(\Psi_t^i) \end{cases}$$

Variation in Characteristics Conducive for Human Capital Formation

- Ability of individuals to finance the cost of education and the forgone earnings
  - Extent of human capital formation
- The availability, accessibility, and quality of public education (policy & interest groups)
  - Extent of human capital formation
- Cultural and religious composition of society
  - Attitude towards education affects the availability, quality and desirability of education
- The stock of knowledge in society
  - Productivity of human capital formation

Variation in Characteristics Conducive for Human Capital Formation

- Ability of individuals to finance the cost of education and the forgone earnings
  - Extent of human capital formation
- The availability, accessibility, and quality of public education (policy & interest groups)
  - Extent of human capital formation
- Cultural and religious composition of society
  - Attitude towards education affects the availability, quality and desirability of education
- The stock of knowledge in society
  - Productivity of human capital formation

Variation in Characteristics Conducive for Human Capital Formation

- Ability of individuals to finance the cost of education and the forgone earnings
  - Extent of human capital formation
- The availability, accessibility, and quality of public education (policy & interest groups)
  - Extent of human capital formation
- Cultural and religious composition of society
  - Attitude towards education affects the availability, quality and desirability of education
- The stock of knowledge in society
  - Productivity of human capital formation

Variation in Characteristics Conducive for Human Capital Formation

- Ability of individuals to finance the cost of education and the forgone earnings
  - Extent of human capital formation
- The availability, accessibility, and quality of public education (policy & interest groups)
  - Extent of human capital formation
- Cultural and religious composition of society
  - Attitude towards education affects the availability, quality and desirability of education
- The stock of knowledge in society
  - Productivity of human capital formation

Variation in Characteristics Conducive for Human Capital Formation

- The propensity of a country to trade
  - Skill-intensity in production and its effect on the demand for human capital
- The effect of geographical attributes on health
  - Return to investment in human capital (e.g., Malaria, Hookworm)
- Composition of religious groups within a society and their attitude towards literacy (e.g., Judaism, Protestantism)
- Social status associated with education

Variation in Characteristics Conducive for Human Capital Formation

- The propensity of a country to trade
  - Skill-intensity in production and its effect on the demand for human capital
- The effect of geographical attributes on health
  - Return to investment in human capital (e.g., Malaria, Hookworm)
- Composition of religious groups within a society and their attitude towards literacy (e.g., Judaism, Protestantism)
- Social status associated with education

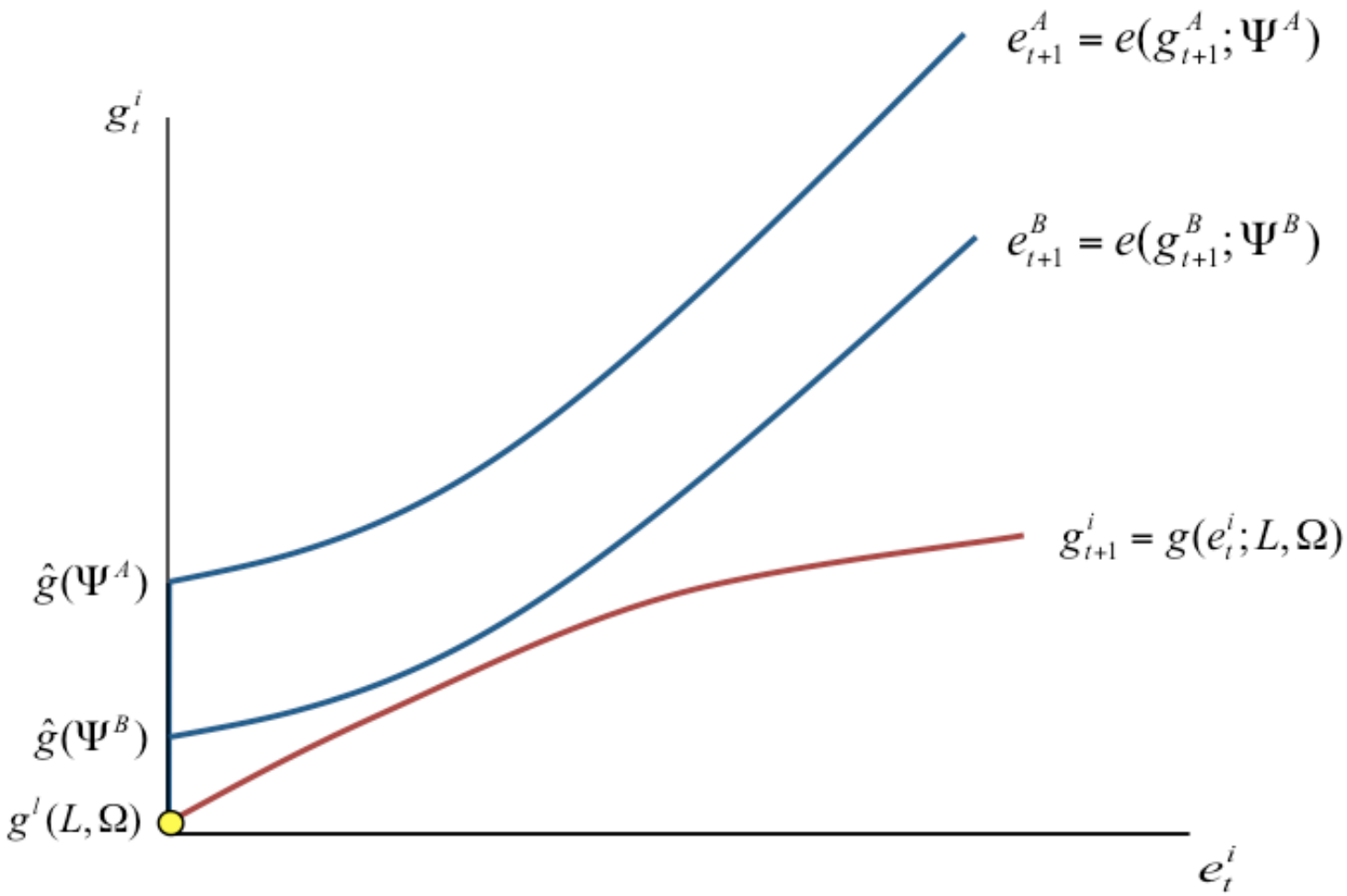
Variation in Characteristics Conducive for Human Capital Formation

- The propensity of a country to trade
  - Skill-intensity in production and its effect on the demand for human capital
- The effect of geographical attributes on health
  - Return to investment in human capital (e.g., Malaria, Hookworm)
- Composition of religious groups within a society and their attitude towards literacy (e.g., Judaism, Protestantism)
- Social status associated with education

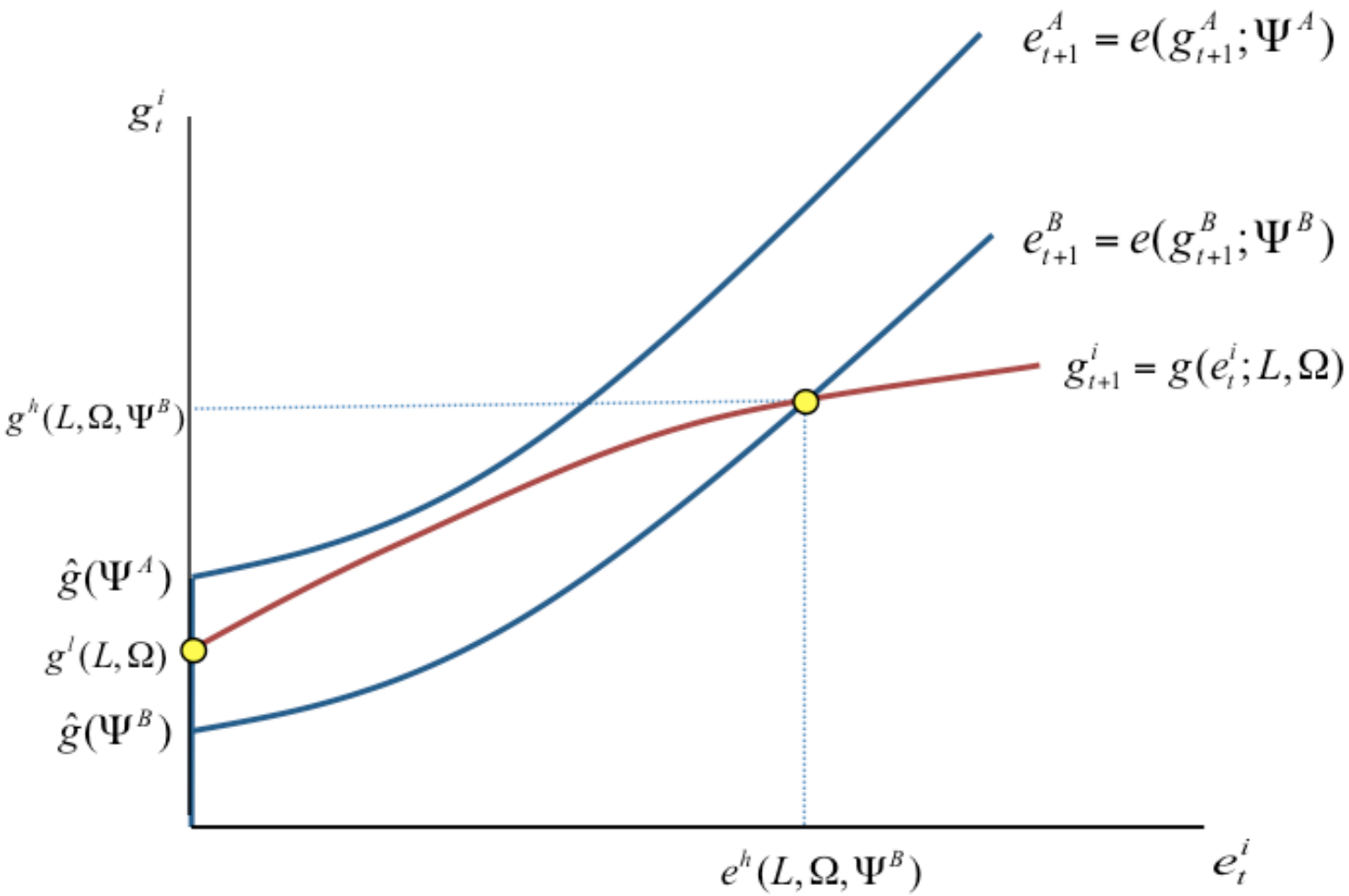
Variation in Characteristics Conducive for Human Capital Formation

- The propensity of a country to trade
  - Skill-intensity in production and its effect on the demand for human capital
- The effect of geographical attributes on health
  - Return to investment in human capital (e.g., Malaria, Hookworm)
- Composition of religious groups within a society and their attitude towards literacy (e.g., Judaism, Protestantism)
- Social status associated with education

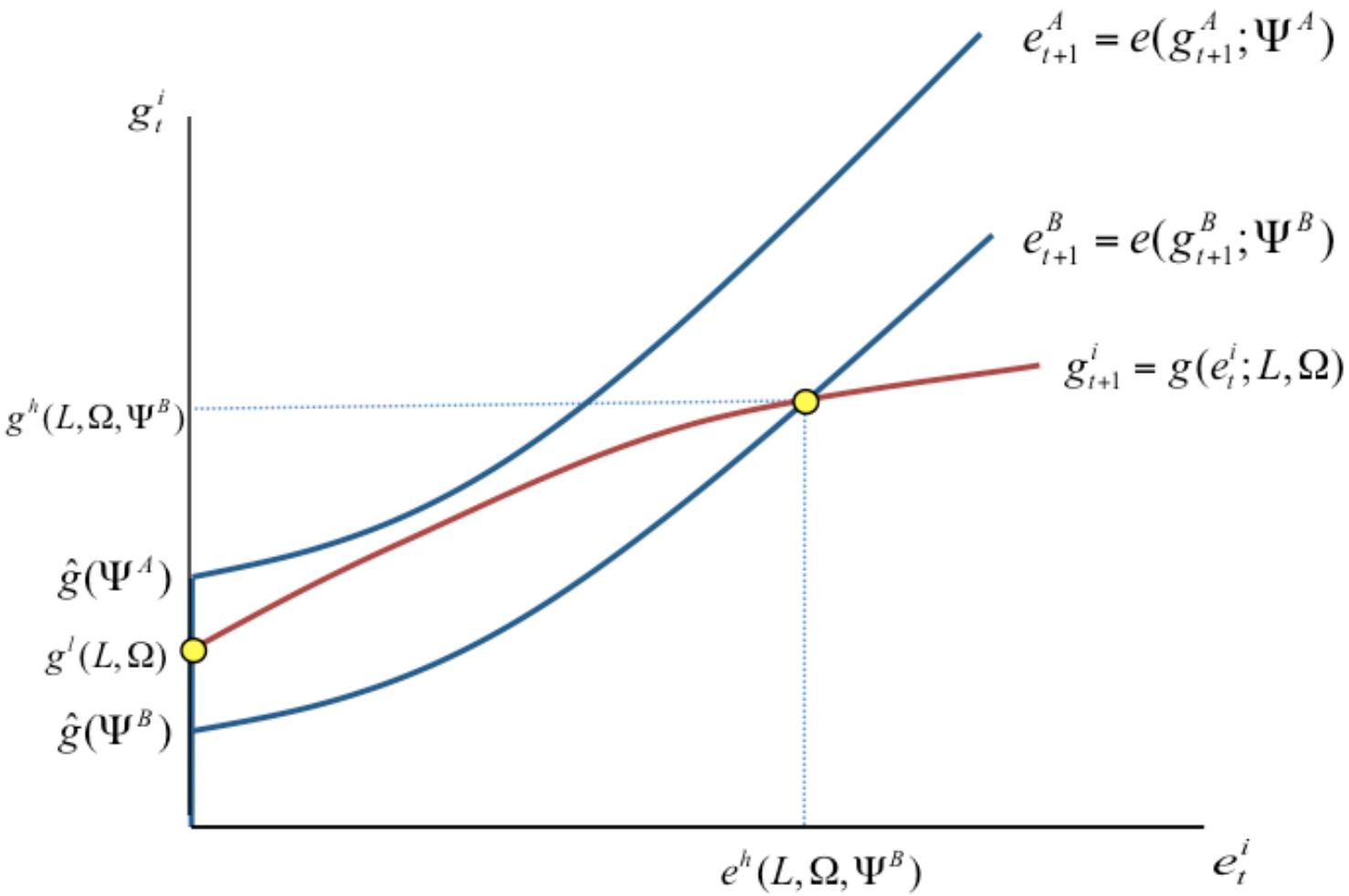
Variation in Characteristics Conducive for Human Capital Formation



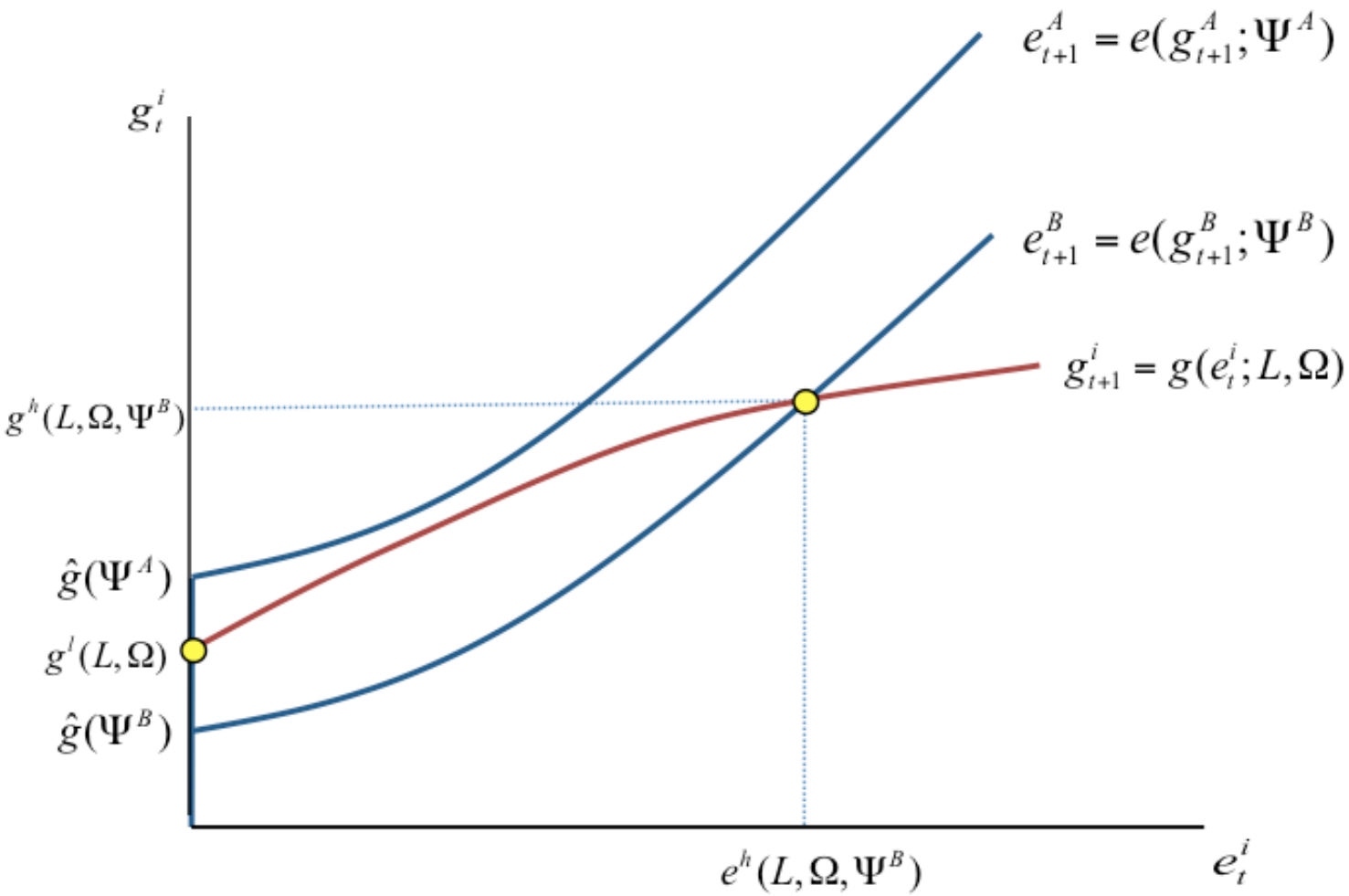
Concluding Remarks



Concluding Remarks



Concluding Remarks



Concluding Remarks