Matrix Computations in R

EPsy 8264

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2018-05-10

Enter a Matrix

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

To enter a matrix in R, use the function. The elements of the matrix will be filled-in by columns.

Fill Elements By Row

The argument will fill the elements by rows rather than columns.

Your Turn

Enter the matrix **B** into R.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Your Turn

Enter the matrix **B** into R.

$$
\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}
$$

Vectors

A matrix with a single column is referred to as a **column vector**.

$$\mathbf{a} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

A matrix with a single row is referred to as a **row vector**.

$$\mathbf{b} = \begin{bmatrix} -1 & 6 & 0 & 9 \end{bmatrix}$$

Dimensions of a Matrix

$$\underset{4\times 3}{\mathbf{X}} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\underset{4\times 1}{\mathbf{a}} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\underset{1\times 4}{\mathbf{b}} = \begin{bmatrix} -1 & 6 & 0 & 9 \end{bmatrix}$$

The function will return the dimensions of a matrix.

Your Turn

What are the dimensions of **B**?

$$
\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}
$$

Use R to verify the dimensions.

Your Turn

What are the dimensions of **B**?

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify the dimensions.

Transpose

The function will produce the transpose of a matrix.

$$\mathbf{b} = \begin{bmatrix} -1 & 6 & 0 & 9 \end{bmatrix}$$

$$\mathbf{b}' = \begin{bmatrix} -1 \\ 6 \\ 0 \\ 9 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} 1 & 4 & 7 & 0 \\ -2 & -5 & 8 & 0 \\ 3 & -6 & 9 & 0 \end{bmatrix}$$

Your Turn

Find the transpose of $\mathbf{B}$ and the dimensions of $\mathbf{B}'$.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the transpose of $\mathbf{B}$ and the dimensions of $\mathbf{B}'$.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Adding/Subtracting Matrices

Matrices that have the same dimensions can be added/subtracted.

$$\underset{2\times 3}{\mathbf{D}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\underset{2\times 3}{\mathbf{E}} = \begin{bmatrix} -5 & 1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$$

The resulting matrix has the same dimensions as the originals.

We add/subtract elements in the same position.

$$\mathbf{D} + \mathbf{E} = \begin{bmatrix} 1 + -5 & 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 0 & 6 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 & 5 \\ 7 & 5 & 10 \end{bmatrix}$$

$$\mathbf{D} - \mathbf{E} = \begin{bmatrix} 1 - -5 & 2 - 1 & 3 - 2 \\ 4 - 3 & 5 - 0 & 6 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & 1 \\ 1 & 5 & 2 \end{bmatrix}$$

Multiplication by a Scalar

A scalar is a 1×1 matrix (a number). A matrix and a scalar can be multiplied together by multiplying each element in the matrix by the scalar.

$$
\begin{aligned}
3 \times \mathbf{D} &= 3 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\
&= \begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times 4 & 3 \times 5 & 3 \times 6 \end{bmatrix} \\
&= \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix}
\end{aligned}
$$

Diagonal Elements

The function will return the diagonal elements of a square matrix.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Your Turn

Find the diagonal for $\mathbf{B}'$ (the transpose of $\mathbf{B}$).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the diagonal for $\mathbf{B}'$ (the transpose of $\mathbf{B}$).

$$
\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}
$$

Use R to verify.

Diagonal of Non-Square Matrices

The function also works on non-square matrices. However, it returns the elements on the diagonal starting with the element in the $[1, 1]$ position.

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

Matrix Trace

The trace of a matrix is the sum of its diagonal elements

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

You can also use the function from the **psych** library to compute the trace.

Your Turn

Find the trace for $\mathbf{B}'$ (the transpose of $\mathbf{B}$).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the trace for $\mathbf{B}'$ (the transpose of $\mathbf{B}$).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Identity Matrix

The function can also be used to create an *identity matrix*. The argument is the number of rows and columns.

$$\underset{3\times 3}{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$