# Assignment 03

*Matrix Algebra*

This goal of this assignment is to give you experience using matrix algebra that will be useful for the remainder of the course. Turn in a printed version of your responses to each of the questions on this assignment. Please adhere to the following guidelines for further formatting your assignment:

- All graphics should be set to an appropriate aspect ratio and sized so that they do not take up more room than necessary. They should also have an appropriate caption.
- Any typed mathematics (equations, matrices, vectors, etc.) should be appropriately typeset within the document.
- Syntax or computer output should not be included in your assignment unless it is specifically asked for.

This assignment is worth 18 points.

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**Use for Questions 1–6**

Consider the following two matrices:

$$\mathbf{X} = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

1. Compute $\mathbf{YX}$.
2. Show how to compute element $\mathbf{YX}_{3,1}$ (i.e., the element in the 3rd row and 1st column).
3. Show how to compute the trace of $\mathbf{X}$.
4. Compute the $\Big((3\mathbf{Y})(0.5\mathbf{X})\Big)^{\intercal}$.
5. Show how to compute the determinant of $\mathbf{X}$.
6. Show how to compute $\mathbf{X}^{-1}$. **(1.5pts)**

**Use for Questions 7–9**

Consider the following column vector $\mathbf{E}$ with $n$ elements:

$$\mathbf{E} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix}$$

7. Compute $\mathbf{E}^{\intercal}\mathbf{E}$ and simplify the result..
8. In general, if we have a column vector (say $\mathbf{X}$), what is the result of computing $\mathbf{X}^{\intercal}\mathbf{X}$ in as simplified a form as possible.

9. Use your result from Question #8 to indicate what the result of computing $\mathbf{X}^\intercal\mathbf{X}$ is, if $\mathbf{X}$ is a column of $n$ ones:

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

**Use for Questions 10–11**

Consider the following matrix $\mathbf{X}$:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \\ 1 & x_n \end{bmatrix}$$

10. Compute $\mathbf{X}^\intercal\mathbf{X}$ and simplify the result. **(1.5pts)**
11. Compute the determinant of $\mathbf{X}^\intercal\mathbf{X}$.

**Use for Questions 12–14**

Consider the following system of equations:

$$\begin{aligned} 4X + 3Y &= -2 \\ 8X - 2Y &= 12 \end{aligned}$$

12. Write this system of equations using matrices.
13. Explain how to solve this systems of equations for $X$ and $Y$ using matrix algebra.
14. Use matrix algebra to solve the systems of equations for $X$ and $Y$.

**Use for Questions 15–16**

Consider the following system of equations:

$$\begin{aligned} 3X + 2Y &= 10 \\ 1.2X + 0.8Y &= 4 \end{aligned}$$

15. Write this system of equations using matrices.
16. Try to solve this systems of equations for $X$ and $Y$ using matrix algebra. If you were to compute the inverse of the left-side matrix, you get an error indicating that the system is `computationally singular` (the inverse cannot be computed because the determinant is indistinguishable from zero). Explain why this occurs with this system of equations. **(1.5pts)**

**Use for Questions 17–18**

Consider the following system of equations:

$$\begin{aligned} 3X - Y &= -2 \\ -12X + 4Y &= -4 \end{aligned}$$

17. Write this system of equations using matrices.

18. Try to solve this systems of equations for $X$ and $Y$ using matrix algebra. If you were to compute the inverse of the left-side matrix, you get an error indicating that the system is `exactly singular` (the inverse cannot be computed because the determinant is zero). Explain why this occurs with this system of equations. **(1.5pts)**