

Matrix Computations in R

EPsy 8264

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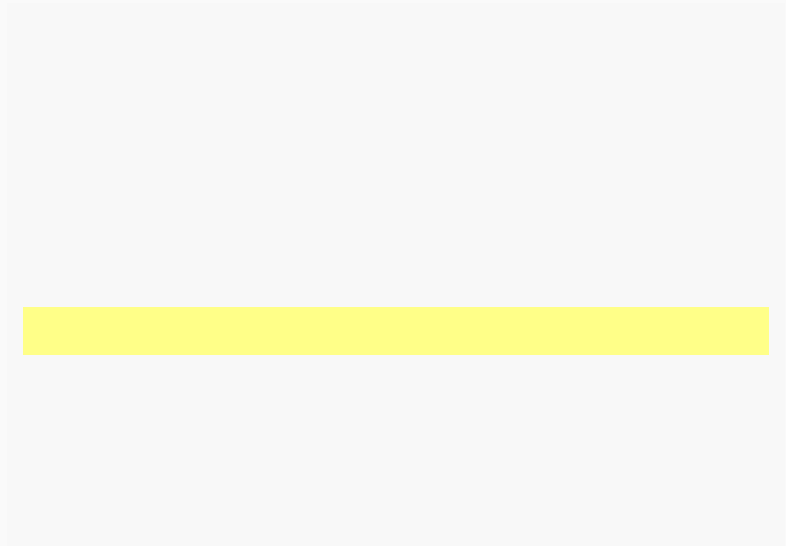
Enter a Matrix

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

To enter a matrix in R, use the `matrix()` function. The elements of the matrix will be filled-in by columns.

Fill Elements By Row

The `byrow` argument will fill the elements by rows rather than columns.



Your Turn

Enter the matrix **B** into R.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

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Vectors

A matrix with a single column is referred to as a **column vector**.

$$\mathbf{a} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

A matrix with a single row is referred to as a **row vector**.

$$\mathbf{b} = [-1 \quad 6 \quad 0 \quad 9]$$

Dimensions of a Matrix

$$\mathbf{X}_{4 \times 3} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\mathbf{a}_{4 \times 1} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\mathbf{b}_{1 \times 4} = [-1 \quad 6 \quad 0 \quad 9]$$

The `size` function will return the dimensions of a matrix.

Your Turn

What are the dimensions of **B**?

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify the dimensions.

Your Turn

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$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify the dimensions.

Transpose

The `np.transpose` function will produce the transpose of a matrix.

$$\mathbf{b} = [-1 \quad 6 \quad 0 \quad 9]$$

$$\mathbf{b}' = \begin{bmatrix} -1 \\ 6 \\ 0 \\ 9 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} 1 & 4 & 7 & 0 \\ -2 & -5 & 8 & 0 \\ 3 & -6 & 9 & 0 \end{bmatrix}$$

Your Turn

Find the transpose of **B** and the dimensions of **B'**.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the transpose of **B** and the dimensions of **B'**.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Adding/Subtracting Matrices

Matrices that have the same dimensions can be added/subtracted.

$$\mathbf{D}_{2 \times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

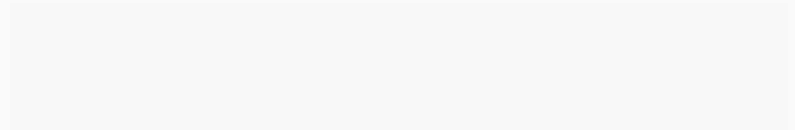
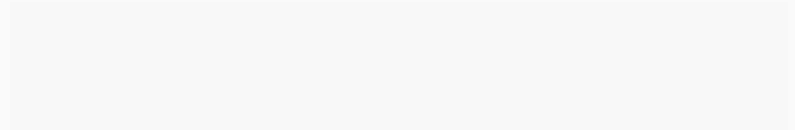
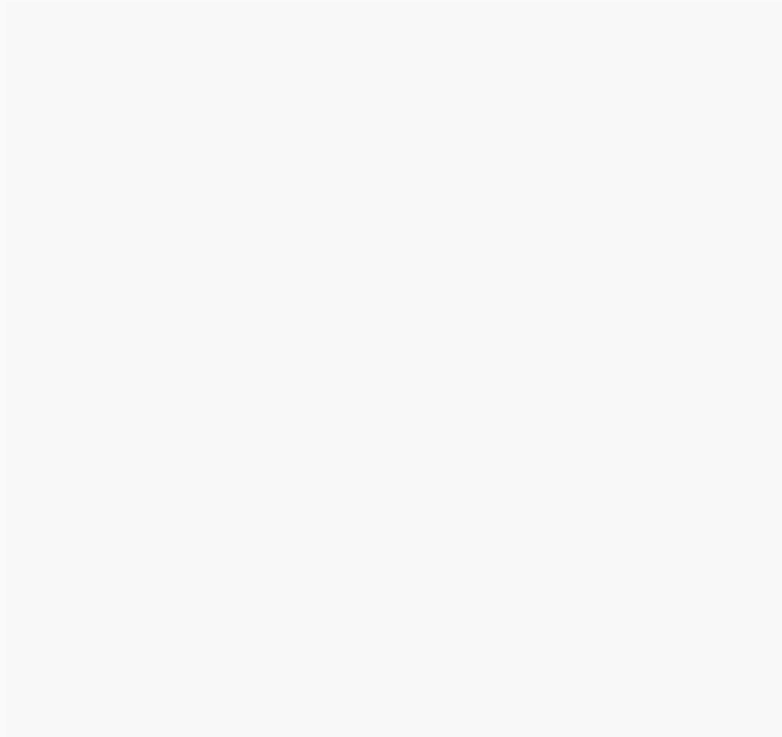
$$\mathbf{E}_{2 \times 3} = \begin{bmatrix} -5 & 1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$$

The resulting matrix has the same dimensions as the originals.

We add/subtract elements in the same position.

$$\begin{aligned} \mathbf{D} + \mathbf{E} &= \begin{bmatrix} 1 + -5 & 2 + 1 & 3 + 2 \\ 4 + 3 & 5 + 0 & 6 + 4 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 3 & 5 \\ 7 & 5 & 10 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{D} - \mathbf{E} &= \begin{bmatrix} 1 - -5 & 2 - 1 & 3 - 2 \\ 4 - 3 & 5 - 0 & 6 - 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 1 & 1 \\ 1 & 5 & 2 \end{bmatrix} \end{aligned}$$



Multiplication by a Scalar

A scalar is a 1×1 matrix (a number). A matrix and a scalar can be multiplied together by multiplying each element in the matrix by the scalar.

$$\begin{aligned} 3 \times \mathbf{D} &= 3 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times 4 & 3 \times 5 & 3 \times 6 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix} \end{aligned}$$

Diagonal Elements

The `diag` function will return the diagonal elements of a square matrix.

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Your Turn

Find the diagonal for \mathbf{B}' (the transpose of \mathbf{B}).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the diagonal for \mathbf{B}' (the transpose of \mathbf{B}).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Diagonal of Non-Square Matrices

The `diag` function also works on non-square matrices. However, it returns the elements on the diagonal starting with the element in the `[1,1]` position.

$$\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -5 & -6 \\ 7 & 8 & 9 \\ 0 & 0 & 10 \end{bmatrix}$$

Matrix Trace

The trace of a matrix is the sum of its diagonal elements

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

You can also use the `trace` function from the **psych** library to compute the trace.

Your Turn

Find the trace for \mathbf{B}' (the transpose of \mathbf{B}).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Your Turn

Find the trace for \mathbf{B}' (the transpose of \mathbf{B}).

$$\mathbf{B} = \begin{bmatrix} -5 & 1 & 3 \\ 2 & 2 & 6 \\ 7 & 3 & -4 \end{bmatrix}$$

Use R to verify.

Identity Matrix

The `identity` function can also be used to create an *identity matrix*. The argument is the number of rows and columns.

$$\mathbf{I}_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$