

Assignment 03

Matrix Algebra

This goal of this assignment is to give you experience using matrix algebra that will be useful for the remainder of the course. Turn in a printed version of your responses to each of the questions on this assignment. Please adhere to the following guidelines for further formatting your assignment:

- All graphics should be set to an appropriate aspect ratio and sized so that they do not take up more room than necessary. They should also have an appropriate caption.
- Any typed mathematics (equations, matrices, vectors, etc.) should be appropriately typeset within the document.
- Syntax or computer output should not be included in your assignment unless it is specifically asked for.

This assignment is worth 18 points.

Use for Questions 1–6

Consider the following two matrices:

$$\mathbf{X} = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

1. Compute \mathbf{YX} .
2. Show how to compute element $\mathbf{YX}_{3,1}$ (i.e., the element in the 3rd row and 1st column).
3. Show how to compute the trace of \mathbf{X} .
4. Compute the $\left((3\mathbf{Y})(0.5\mathbf{X}) \right)^\top$.
5. Show how to compute the determinant of \mathbf{X} .
6. Show how to compute \mathbf{X}^{-1} . (1.5pts)

Use for Questions 7–9

Consider the following column vector \mathbf{E} with n elements:

$$\mathbf{E} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix}$$

7. Compute $\mathbf{E}^\top \mathbf{E}$ and simplify the result..
8. In general, if we have a column vector (say \mathbf{X}), what is the result of computing $\mathbf{X}^\top \mathbf{X}$ in as simplified a form as possible.

9. Use your result from Question #8 to indicate what the result of computing $\mathbf{X}^\top \mathbf{X}$ is, if \mathbf{X} is a column of n ones:

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Use for Questions 10–11

Consider the following matrix \mathbf{X} :

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \\ 1 & x_n \end{bmatrix}$$

10. Compute $\mathbf{X}^\top \mathbf{X}$ and simplify the result. **(1.5pts)**
11. Compute the determinant of $\mathbf{X}^\top \mathbf{X}$.

Use for Questions 12–14

Consider the following system of equations:

$$\begin{aligned} 4X + 3Y &= -2 \\ 8X - 2Y &= 12 \end{aligned}$$

12. Write this system of equations using matrices.
13. Explain how to solve this systems of equations for X and Y using matrix algebra.
14. Use matrix algebra to solve the systems of equations for X and Y .

Use for Questions 15–16

Consider the following system of equations:

$$\begin{aligned} 3X + 2Y &= 10 \\ 1.2X - 0.8Y &= 4 \end{aligned}$$

15. Write this system of equations using matrices.
16. Try to solve this systems of equations for X and Y using matrix algebra. If you were to compute the inverse of the left-side matrix, you get an error indicating that the system is **computationally singular** (the inverse cannot be computed because the determinant is indistinguishable from zero). Explain why this occurs with this system of equations. **(1.5pts)**

Use for Questions 17–18

Consider the following system of equations:

$$\begin{aligned}3X - Y &= -2 \\ -12X + 4Y &= -4\end{aligned}$$

17. Write this system of equations using matrices.
18. Try to solve this systems of equations for X and Y using matrix algebra. If you were to compute the inverse of the left-side matrix, you get an error indicating that the system is **exactly singular** (the inverse cannot be computed because the determinant is zero). Explain why this occurs with this system of equations. **(1.5pts)**