Assignment 1

EPsy 8282

Fall 2017

Matrix Algebra Operations

Let $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ be matrices defined as following:

$$\mathbf{X} = \begin{pmatrix} 1 & 52 & 1 \\ 1 & 34 & 0 \\ 1 & 35 & 1 \\ 1 & 57 & 1 \end{pmatrix} \qquad \mathbf{Y} = \begin{pmatrix} 4.0 \\ 3.1 \\ 3.6 \\ 3.6 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \mathbf{D} = \begin{pmatrix} 2 & 7 & 8 \\ 2 & 5 & 2 \end{pmatrix}$$

Report the results of the following operations.

1. $dim\,(\mathbf{X})$
2. $\mathbf{X}\mathbf{D}^T$
3. $|\mathbf{C}|$
4. $tr\,(\mathbf{C})$
5. $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}$

Working with Variances

Let $\mathbf{X}$ and $\mathbf{Y}$ be random variables with means μ_x and μ_y, variances σ_x^2 and σ_y^2, and covariance σ_{xy}, respectively. Let c_1 and c_2 be constants. Using the variance definitions from the matrix algebra handout (beginning at the end of p. 9), we can show that

$$\text{var}\big(c_1\mathbf{X} + c_2\mathbf{Y}\big) = c_1^2\sigma_x^2 + c_2^2\sigma_y^2 + 2c_1c_2\sigma_{xy}$$

In the next set of questions, you will verify this numerically.

6. Create and record two 5×1 vectors: $\mathbf{X}$ and $\mathbf{Y}$, and two constants c_1 and c_2.
7. Compute and record: μ_x, μ_y, σ_x^2, σ_y^2, and σ_{xy}.
8. Using the vectors and constants you created in Question 6, compute $var\big(c_1\mathbf{X} + c_2\mathbf{Y}\big)$. Show your work/syntax for full credit.
9. Using the vectors and constants you created in Question 6, compute $c_1^2\sigma_x^2 + c_2^2\sigma_y^2 + 2c_1c_2\sigma_{xy}$. Show your work/syntax for full credit.

Covariance Matrices

10. Using the vectors you created in Question 6, write out the covariance matrix ($\mathbf{\Sigma}$) for $\underset{5\times 2}{\mathbf{d}}$, where $\mathbf{d} = \begin{pmatrix} X_1 & Y_1 \\ X_2 & Y_2 \\ X_3 & Y_3 \\ X_4 & Y_4 \\ X_5 & Y_5 \end{pmatrix}$.

11. Using the constants and vectors you created in Question 6, verify that

$$var\left(\mathbf{d} \times \mathbf{c}\right) = \mathbf{c}^T \times \mathbf{\Sigma} \times \mathbf{c}$$

for $\underset{2\times 1}{\mathbf{c}} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$. Show your work/syntax for full credit.