Multivariate analysis of variance

Session 13

MATH 80667A: Experimental Design and Statistical Methods
for Quantitative Research in Management
HEC Montréal

Multivariate analysis of variance

Motivational example

From Anandarajan et al. (2002), Canadian Accounting Perspective

> This study questions whether the current or proposed Canadian standard of disclosing a going-concern contingency is viewed as equivalent to the standard adopted in the United States by financial statement users. We examined loan officers' perceptions across three different formats

Alternative going-concern reporting formats

Bank loan officers were selected as the appropriate financial statement users for this study.

Experiment was conducted on the user's interpretation of a going-concern contingency when it is provided in one of three disclosure formats:

1. Integrated note (Canadian standard)
2. Stand-alone note (Proposed standard)
3. Stand-alone note plus modified report with explanatory paragraph (standard adopted in US and other countries)

Multivariate response

4. Please circle the pricing you would charge on borrowings under a line of credit *as a spread over your bank's base lending rate ("Prime rate")*.

0.25	0.50	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
3.25		3.50		3.75		4.00		Other ___________		

5. Please circle on the scale shown below your perception of *the ability of the company to service debt*.

LOW ABILITY					**HIGH ABILITY**
	1	2	3	4	5

6. Please circle on the scale shown below your perception of the *likelihood that the company can improve its profitability*.

VERY UNLIKELY					**VERY LIKELY**
	1	2	3	4	5

Why use MANOVA?

1. Control experimentwise error
    - do a single test, reduces type I error
2. Detect differences in combination that would not be found with univariate tests
3. Increase power (context dependent)

Multivariate model

Postulate the following model:

$$\boldsymbol{Y}_{ij} \sim \mathsf{No}_p(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}), \qquad j = 1, \ldots J$$

Each response $\boldsymbol{Y}_{ij}$ is p-dimensional.

We assume multivariate measurements are independent of one another, with

- the same distribution
- same covariance matrix $\boldsymbol{\Sigma}$
- same mean vector $\boldsymbol{\mu}_j$ within each $j = 1, \ldots, J$ experimental groups.
    - (randomization)

The model is fitted using multivariate linear regression.

Model assumptions

The more complex the model, the more assumptions...

Same as ANOVA, with in addition

- The data follow a multivariate normal distribution
  - Shapiro–Wilk test, univariate QQ-plots
- The covariance matrix is the same for all subjects
  - Box's M test is often used, but highly sensitive to departures from the null (other assumptions impact the test)

Normality matters more in small samples.

When to use MANOVA?

In addition, for this model to make sense, you need just enough correlation (Goldilock principle)

- if correlation is weak, use univariate analyses
  - (no gain from multivariate approach)
  - less power due to additional covariance parameter estimation
- if correlation is too strong, redundancy
  - don't use Likert scales that measure a similar dimension

Only combine elements that theoretically or conceptually make sense together.

Testing equality of mean vectors

The null hypothesis is $\mathscr{H}_0 : \boldsymbol{\mu}_1 = \cdots = \boldsymbol{\mu}_J$ against the alternative that at least one vector is different from the rest. The null imposes $(J-1) \times p$ restrictions on the parameters.

With $J=2$ (bivariate), the MANOVA test finds the best composite score with weights for Y_{i1} and Y_{i2} that maximizes the value of the t-test.

The null distribution is Hotelling's T^2, but a modification of the test statistic can be approximated by a F distribution.

Choice of test statistic

In higher dimensions, with $J \geq 3$, there are many statistics that can be used to test equality of mean.

The statistics are constructed from within/between sum covariance matrices.

These are

- Roy's largest root (most powerful provided all assumptions hold)
- Wilk's Λ: most powerful, most commonly used
- **Pillai's trace**: most robust choice for departures from normality or equality of covariance matrices

Most give similar conclusion, and they are all equivalent with $J = 2$.

Sample size for MANOVA

The number of observations must be sufficiently large to estimate all covariance parameters and mean parameters.

To achieve a power of 80%, need the following number of replicates **per group**.

	3 groups				4 groups				5 groups			
effect size \ p	2	4	6	8	2	4	6	8	2	4	6	8
very large	13	16	18	21	14	18	21	23	16	21	24	27
large	26	33	38	42	29	37	44	48	34	44	52	58
medium	44	56	66	72	50	64	74	84	60	76	90	100
small	98	125	145	160	115	145	165	185	135	170	200	230

Laüter, J. (1978), Sample size requirements for the T^2 test of MANOVA (tables for one-way classification). *Biometrical Journal*, **20**, 389-406.

Post hoc testing

Researchers often conduct *post hoc* univariate tests using univariate ANOVA.

This is suboptimal, rather proceed with descriptive discriminant analysis.

This method tries to find the combinations of the mean that provides the best difference between the difference groups.