

Within-subjects and mixed factorial designs

Session 11

MATH 80667A: Experimental Design and Statistical Methods
for Quantitative Research in Management
HEC Montréal

Outline

Hasse diagrams

Two factors within-subjects designs

Hasse diagrams

To-do list

Need to determine

1. relation between factors (nested or crossed)
2. nature of factors (fixed or random effects)
3. which interactions should be in the model (maximal structure supported by experimental design)

The statistical model must match the experiment design.

Building tests require considering variability of effects (mean squared errors and degrees of freedom of denominators).

What is a Hasse diagram?

A graph in which every effect is a node

- Node labels indicate fixed vs (random)
- superscript is number of instances
- subscript degrees of freedom
- lines joining variables show crossing/nesting relations and hierarchy

Degrees of freedom for a term U are found by starting with the superscript for U and subtracting out the degrees of freedom for all terms above U .

This rule only applies for balanced designs.

Examples of Hasse diagrams

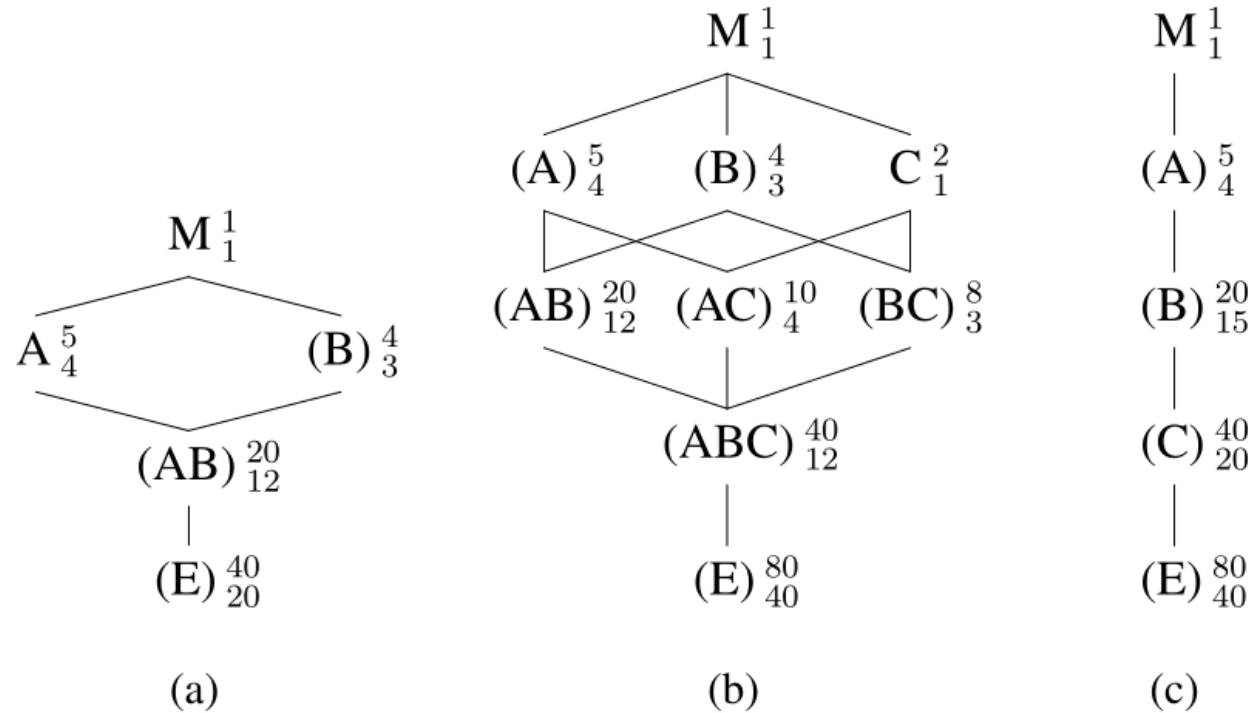


Figure 12.1: Hasse diagrams: (a) two-way factorial with A fixed and B random, A and B crossed; (b) three-way factorial with A and B random, C fixed, all factors crossed; (c) fully nested, with B fixed, A and C random. In all cases, A has 5 levels, B has 4 levels, and C has 2 levels.

Rules for testing (denominators)

From Oehlert (2001), Display 12.3

1. The denominator for testing a term U is the leading eligible random term below U in the Hasse diagram.
2. An eligible random term V below U is leading if there is no eligible random term that is above V and below U .
3. If there are two or more leading eligible random terms, then we must use an approximate test.
4. In the restricted model, all random terms below U are eligible except those that contain a fixed factor not found in U .

Example: chocolate rating

Example from L. Meier, adapted from Oehlert (2010)

A group of 10 rural and 10 urban raters rated 4 different chocolate types. Every rater got to eat two samples from the same chocolate type in random order.

Within-subjects and mixed designs

Why use within-subject factors?

- (+) can decompose variance (more power)
- (+) fewer participants needed
- (-) crossover effects
- (-) more complicated
- (-) assumptions about correlation structure

Two-factor within design

We observe $A \times B \times S$: each subject S_i gets assigned to every treatment pair $a_j \times b_k$.

- A and B are crossed (interaction)
- Hasse diagram (board work)

Testing in within-designs

Main-effect statistics are

- $A: MS_A / MS_{AS}$
- $B: MS_B / MS_{BS}$
- $AB: MS_{AB} / MS_{ABS}$

where MS is the mean squared error.

The term MS_{ABS} is equivalent to residuals if there are no replications.

Analyzing contrasts in within-designs

From Table 18.5 of Keppel and Wickens (2004)

Any effect involving a contrast on a within-subject factor is tested against an error term that includes the interaction of that contrast with subjects. To calculate this error term, create a contrast variable by applying the contrast to each subject's data, then analyse these computed data as if they come from a simpler design.

Any within-subject effect involving a portion of the data (e.g., a pairwise comparison between two levels of one factor), is tested against an error term derived exclusively from those data. To calculate this error term, simply extract these data and analyze them as if they come from a simpler design.

Two-factor mixed design

We observe $A \times (B \times S)$: each subject S_i is nested in A and we see each of b treatments for B

- Hasse diagram (board work)