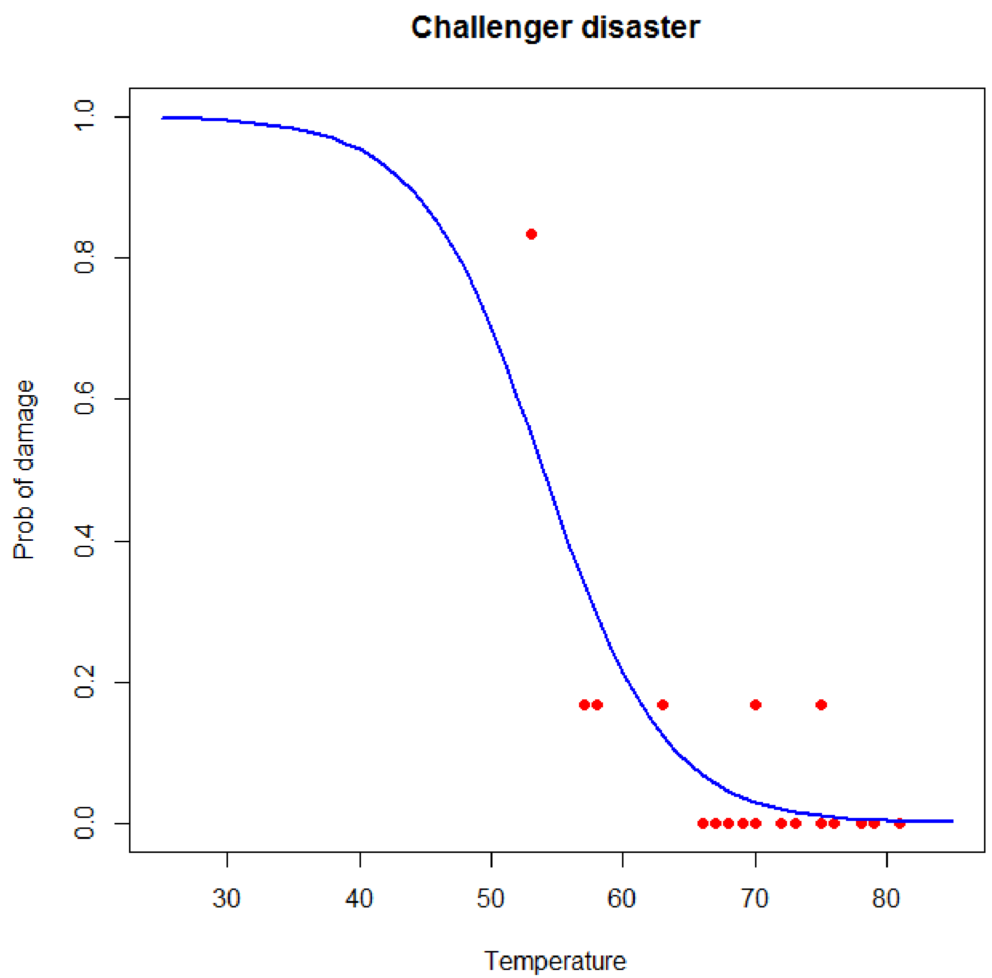
Generalized linear modeling with R



Source: Faraway (2006).

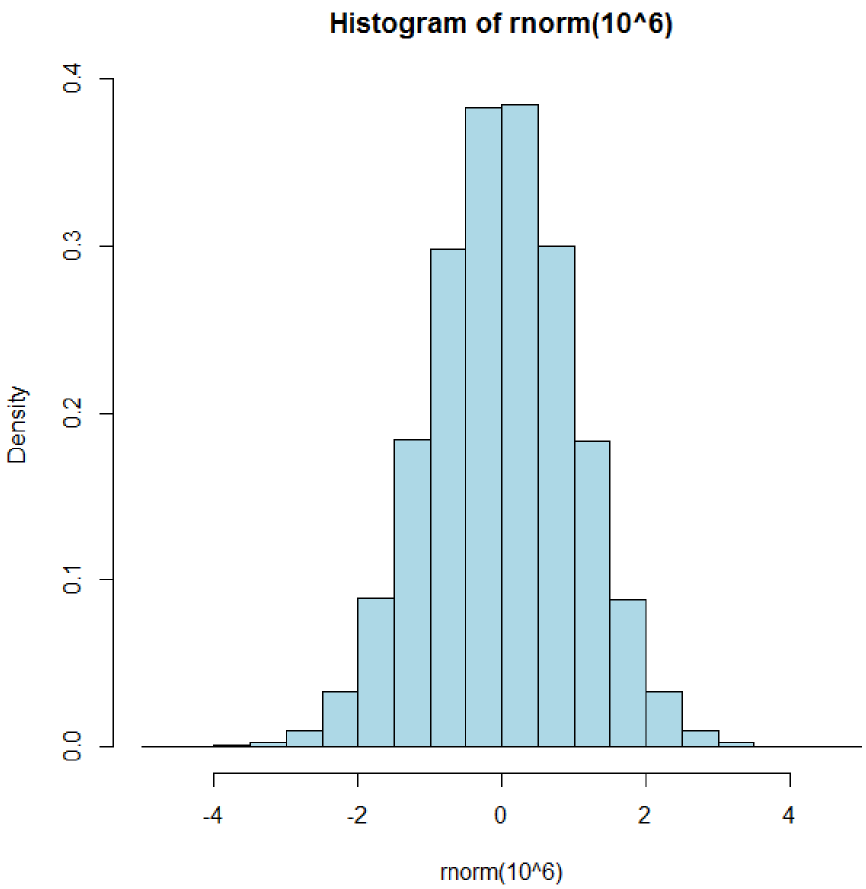
Why not linear regression?

- 0/1 data (activity, infection, presence, sex)
- proportional data (0 – 100 %)
- number of successes vs. number of failures
- count data (species richness)

Before getting started, we should think about

1. distribution of the **response**
2. systematic part (covariates)
3. link-function between expected value of the response and the systematic part

Normal distribution



$$f(y_i; \boldsymbol{\mu}, \boldsymbol{\sigma}) = \frac{1}{\sigma\sqrt{2\mu}} * e^{\frac{(y_i \; - \mu)^2}{2\sigma^2}}$$

defined by mean

and standard deviation

Poisson distribution

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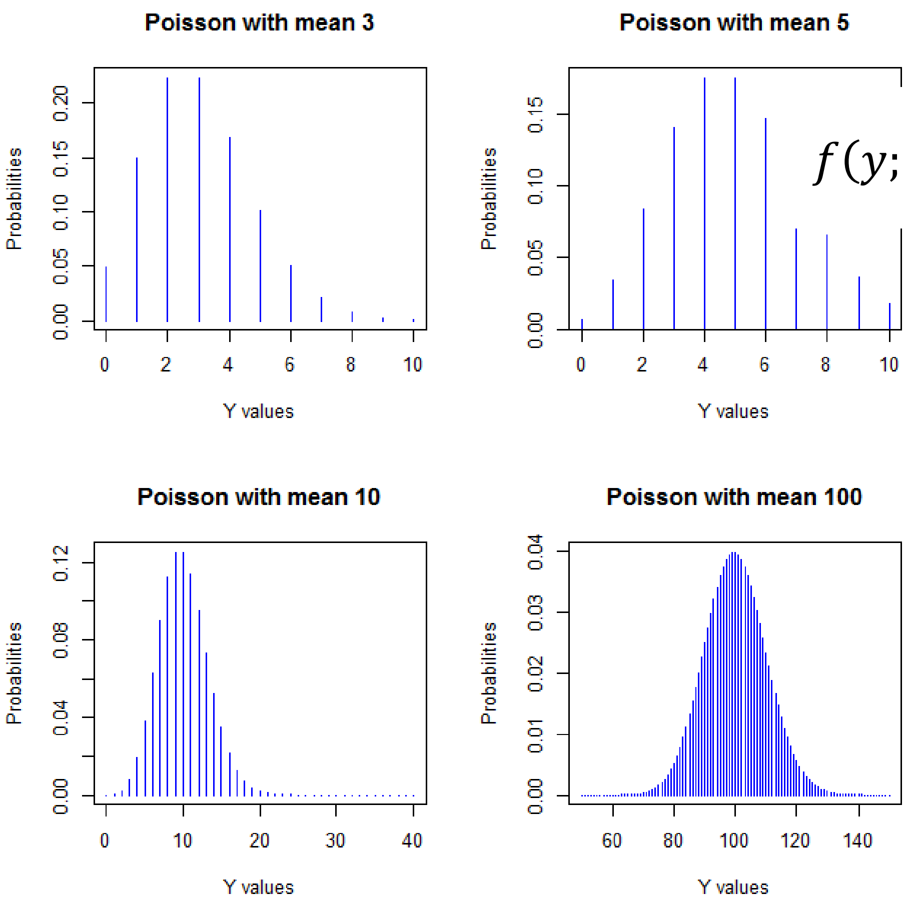

Modified after: Zuur et al. (2009).

$$f(y;\, \boldsymbol{\mu}) = \frac{\mu^{y} * e^{-\mu}}{y!} \qquad y \geq 0, y \text{ integer}$$

mean = variance

-> larger mean values have also a larger variance

overdispersion: if the variance is larger than the mean

Bernoulli and binomial distribution

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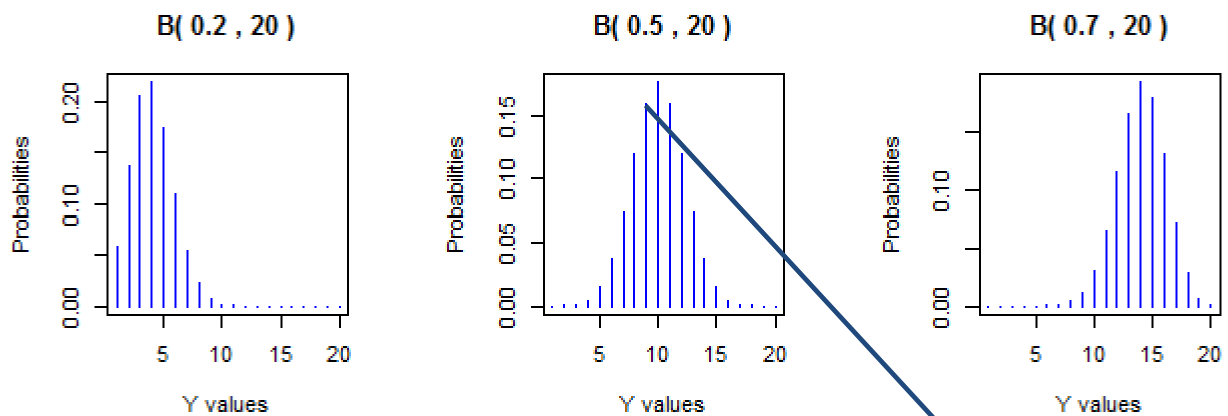
- Tossing a coin (head or tail)
- Bernoulli distribution = binomial distribution with N = 1

$$f(y;\,\pi) = \binom{N}{y} * \pi^{y} * (1-\pi)^{N-y}$$



Modified after: Zuur et al. (2009).

(20!/(9! * 11!))*0.5^9*(1-0.5)^11 = 0.16

Further common distributions

1. **Negative binomial distribution** -> quick and dirty solution for overdispersion
2. **Gamma distribution** -> can be used for a continous response variable that has positive values (> 0)

Predictor function

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$$g(x_i) = \boldsymbol{\alpha} + \boldsymbol{\beta_1 X_{1i}} + \cdots + \boldsymbol{\beta_n X_{ni}}$$

Poisson predictor function

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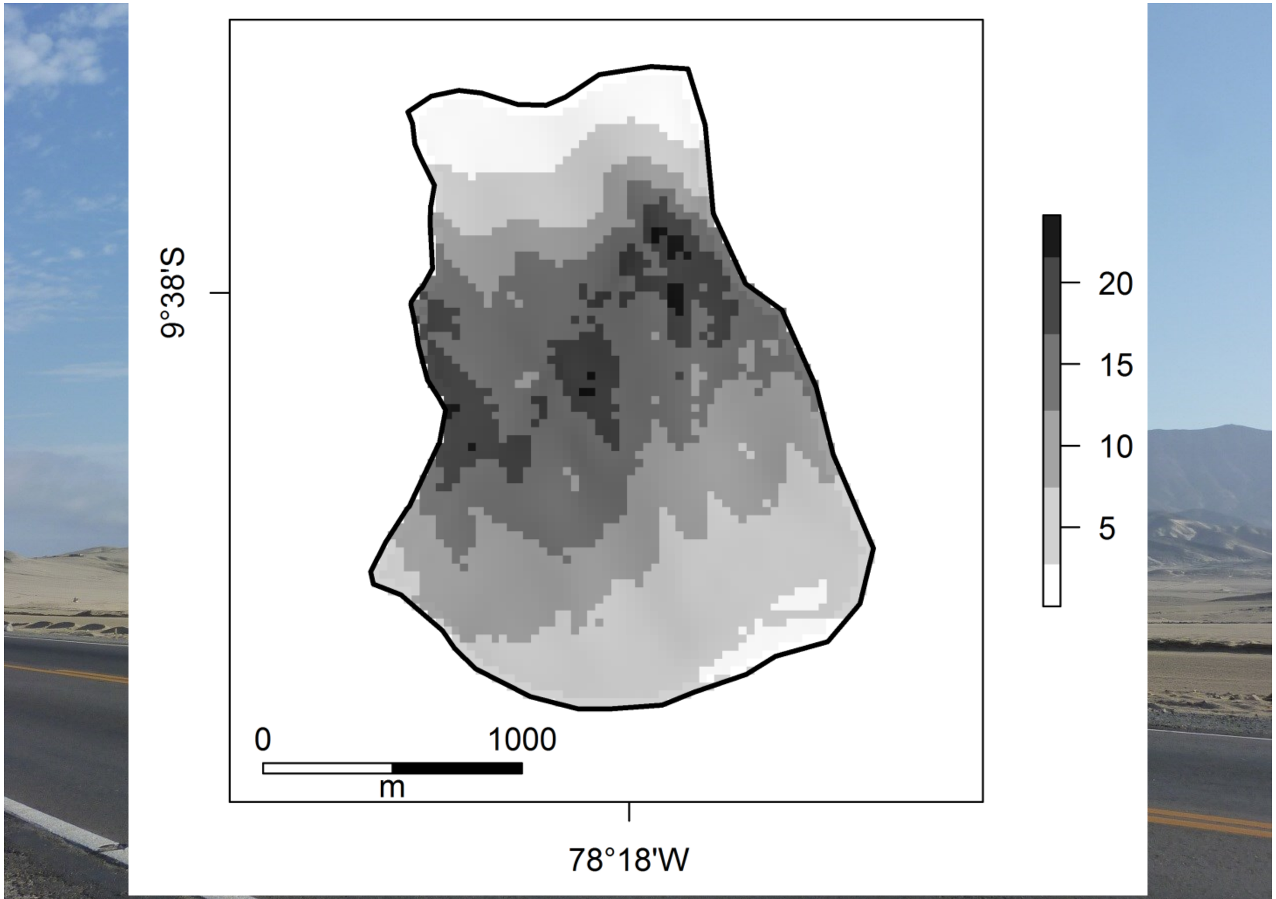
1. distribution -> increase in spread
2. density curves avoid negative realizations
3. exponential link -> no negative fitted values
4. Maximum likelihood algorithm (see Zuur 2009: 213)

$$Y_i \sim P(\mu_i) \text{ and } E[Y_i] = \mu_i = e^{\boldsymbol{\alpha}+\boldsymbol{\beta}_i \boldsymbol{X}_{1i}+\cdots+\boldsymbol{\beta}_n \boldsymbol{X}_{ni}}$$

$$\log(E[Y_i]) = \boldsymbol{\alpha} + \boldsymbol{\beta}_i \boldsymbol{X}_{1i} + \cdots + \boldsymbol{\beta}_n \boldsymbol{X}_{ni}$$

log-link

Modeling species richness

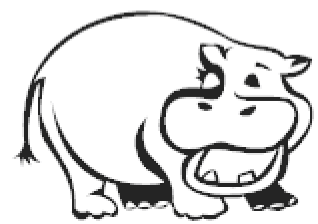


Source: Muenchow et al. (2013).

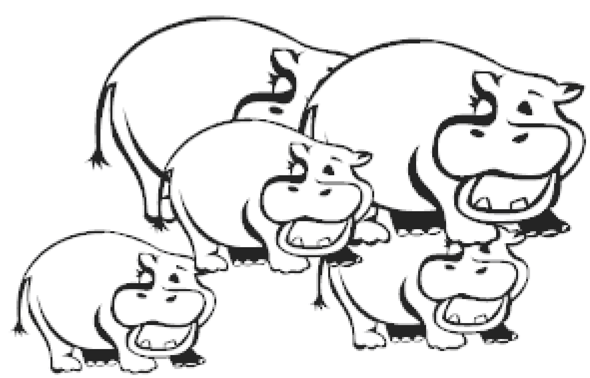
Binomial GLM

1. Express 0 and 1 as probability P
2. Apply a series of transformations on P
3. back-transform to 0-1 intervall

Absence (= 0)



I am not here, because the habitat is not good!



Here we are!

Presence (= 1)

modified after Zuur et al. (2009).

Concept of Odds

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$$O_i = \frac{\pi_i}{1 - \pi_i}$$

with O_i= Odds for the *i*th observation

π_i = Probability of the *i*th observation

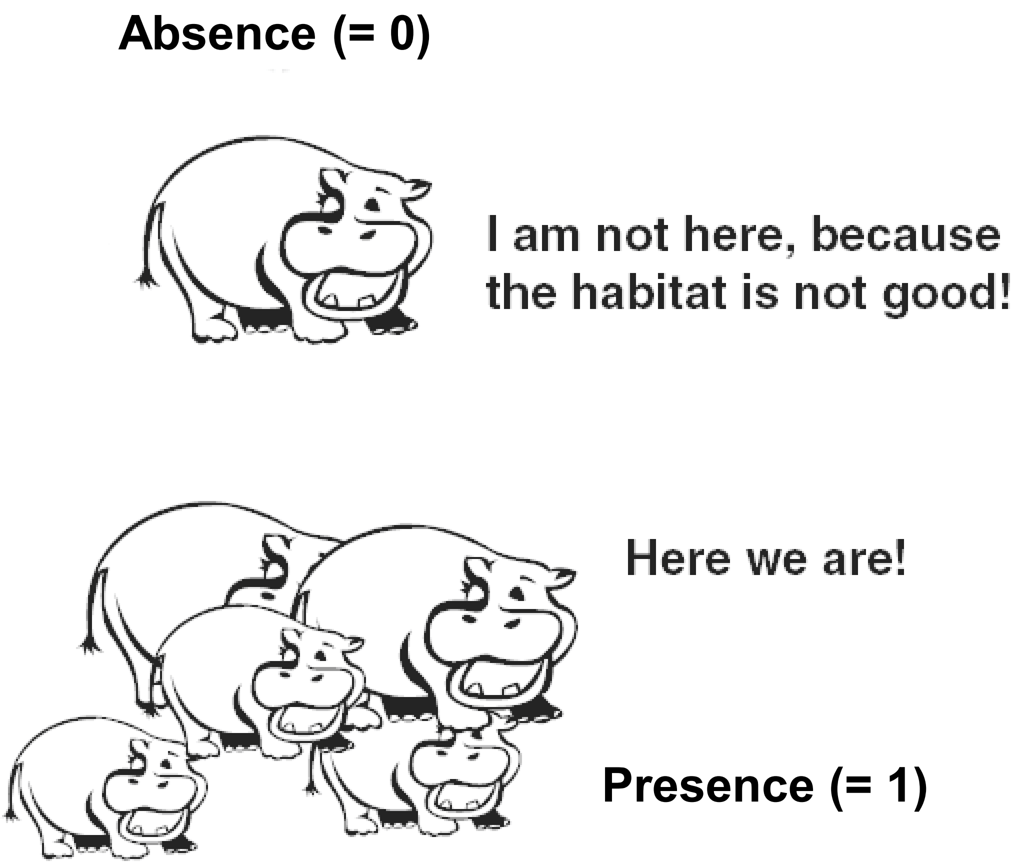
-> Value not any longer restricted to 0 and 1!!!

Concept of Odds

$$O_i = \frac{0.9}{(1-0.9)} = \mathbf{9}$$

Odds of 9 -> it is **9 times more likely** to record a hippo than not to record one.

Or: In **9 from 10** plots you will find a hippo.



modified after Zuur et al. (2009).

log-Odds as a function of predictor values

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log-link

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \eta$$

with η = predictor function

-> Negative values are possible

Solving for π_i

$$\pi_i = \frac{e^{\eta}}{1 + e^{\eta}}$$

-> term lies always between 0 and 1 (back-transformation)!!!

Logistic regression prediction formula

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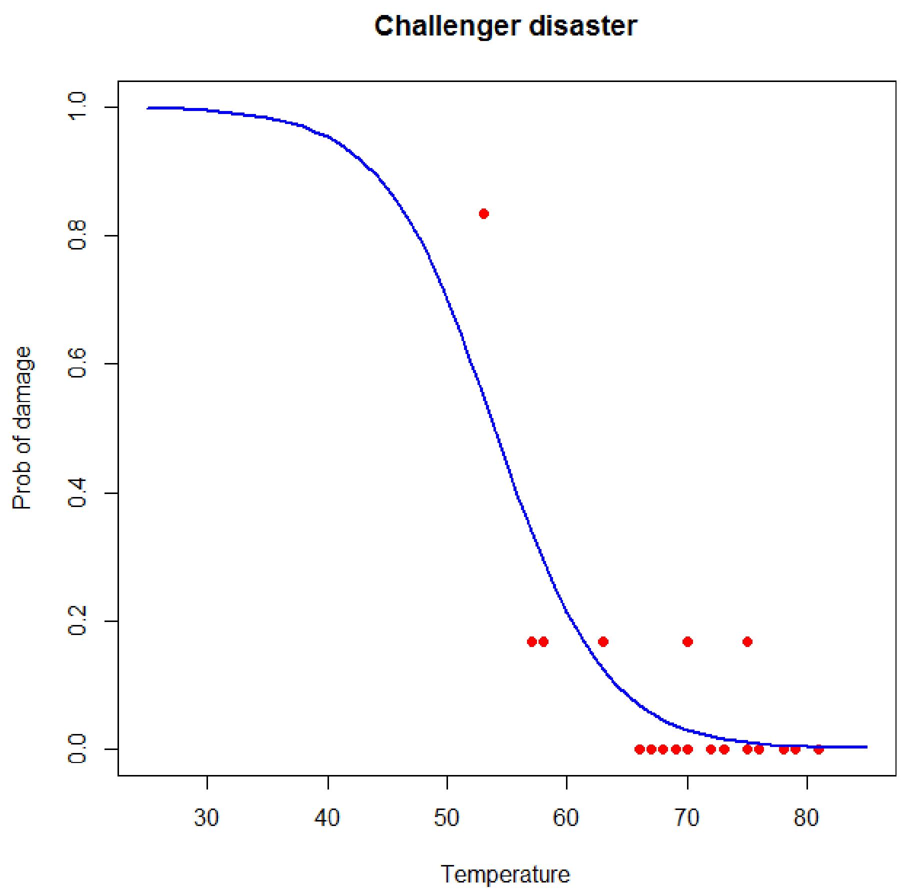
$$Y_i \sim B(1; P_i) \text{ and } E[Y_i] = P_i = \pi_i = \frac{e^{\boldsymbol{\alpha+\beta_i X_{1i}+\cdots+\beta_n X_{ni}}}}{1 + e^{\boldsymbol{\alpha+\beta_i X_{1i}+\cdots+\beta_n X_{ni}}}}$$

- appropriate distribution (Bernoulli)
- Maximum likelihood algorithm (see Zuur et al., 2007: 93)

Example: Challenger disaster



Source: Faraway (2006).

Modeling landslide susceptibility

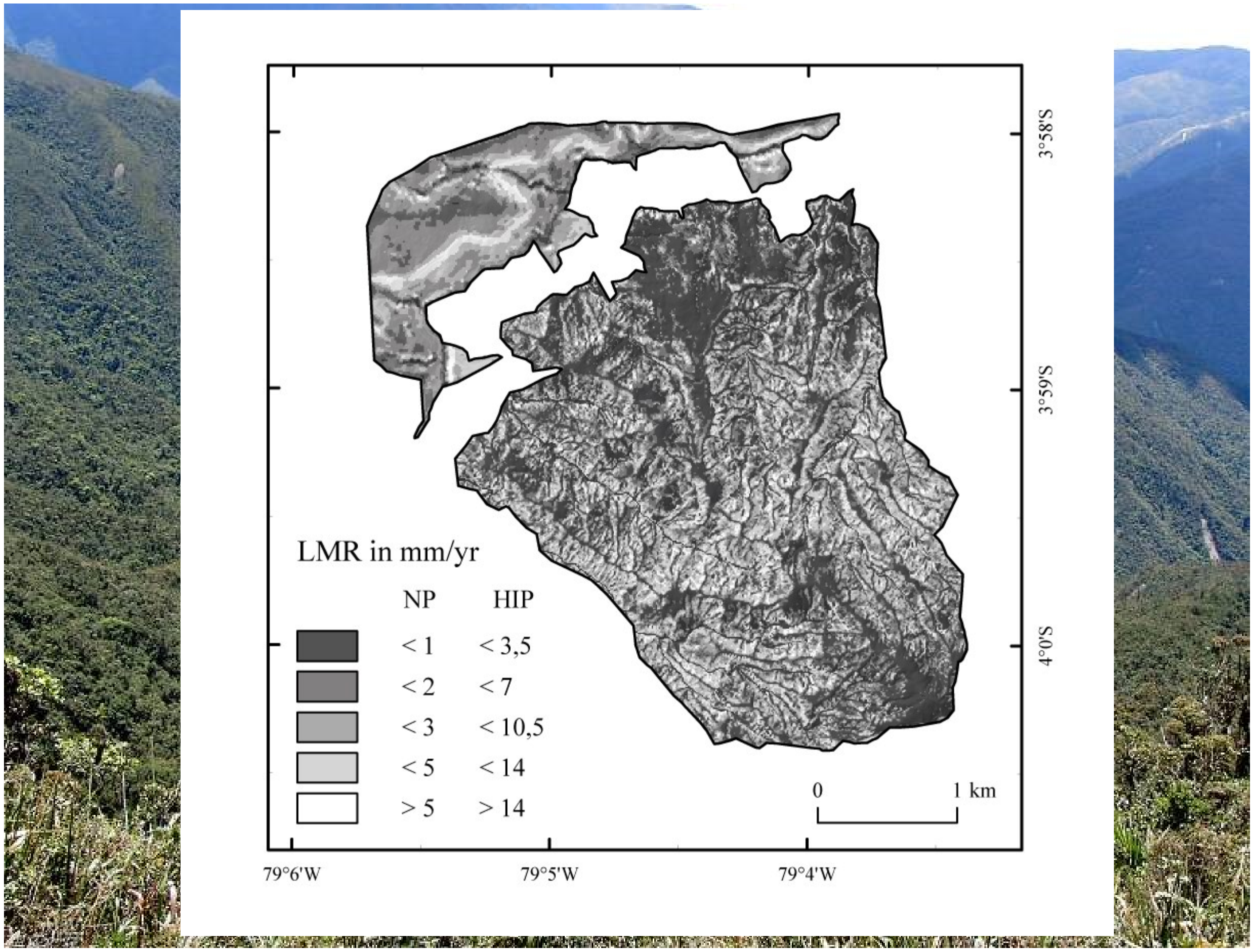


Foto: Michael Richter.

Source: Muenchow et al. (2011).

Literature

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