Multilevel Bernoulli & Binomial models

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Inferring the bias on a single coin

- If we observe n flip of a single coin, our data is $y_1, y_2 \ldots y_n$, where each $y_i \in \{0, 1\}$.
- We model this as

$$y_i \sim \mathrm{Bernoulli}(\theta) \quad \text{for } i \in 1 \ldots n,$$

  where θ is the coin's *bias* (i.e., the probability it will come up heads on any flip).
- The value of θ is unknown.
- If we assume $\theta \sim P(\Omega)$, for some specified (known) value of Ω, then we can use Bayes rule to infer the probable value of θ.

Inferring the bias on J coins

- Let's say we observe n flips of each of J coins.
- Our model of the data is as follows.

  For $j \in 1 \ldots J$,

$$
\begin{aligned}
\theta_j &\sim P(\Omega), \\
y_i &\sim \text{Bernoulli}(\theta_j) \quad \text{for } i \in 1 \ldots n,
\end{aligned}
$$

- In this case, if Ω is specified (known), and we calculate the posterior distribution over each θ_j then this is the equivalent of J *independent* Bayesian inferences.
- If we put a (hyper)prior on Ω, we now have *multilevel* or *hierarchical* Bayesian model.