normal linear models

multilevel / hierarchical
linear mixed effects

Data = | y_1 | y_2 | ... | y_n |

change distribution

model

for each i in 1 ... n

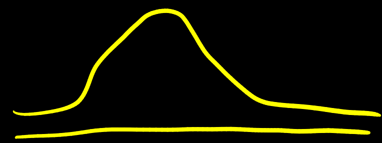
$y_i \sim N(\mu_i, \sigma^2)$

generalized linear models
link function

$\mu_i = \beta_0 + \beta_1 x_i$

- classical
- Bayesian

$\theta_i = f(\underline{x}_i)$ nonlinear population

sample Y_1 

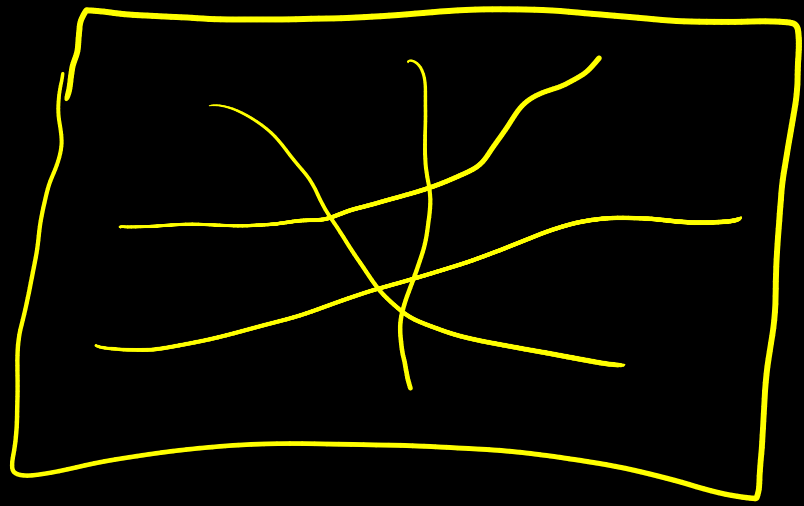
data: $\underline{y_1}$ $\underline{y_2}$ .... $\underline{y_n}$ sample

realizations of $\underline{\underline{n}}$ random variables

$\left[Y_1 \quad Y_2 \quad Y_3 \quad \ldots \quad Y_n \right]$ probability distribution

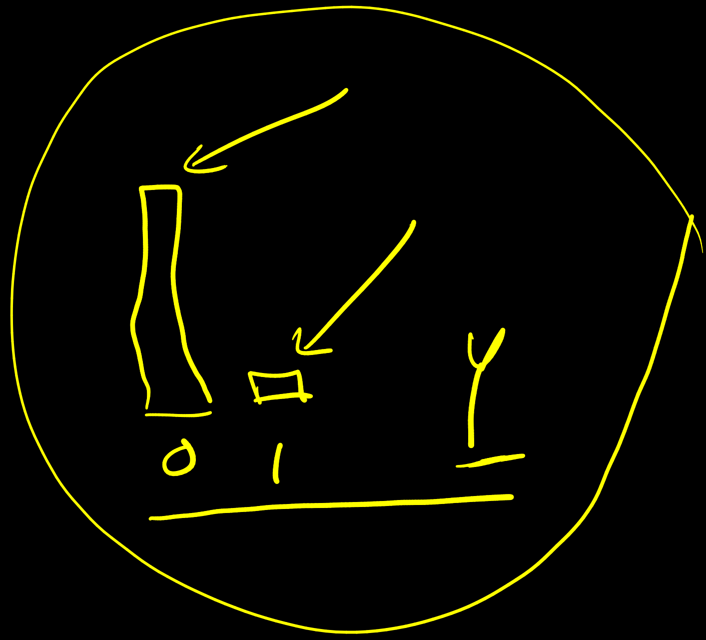
- population <=> probability
- sample
- model

Ex:
all models are wrong
but
some are useful



$y_1 \; y_2 \; \ldots\ldots \; y_{5000}$

$Y_1 = Y_2 = Y_3 \ldots\ldots Y_n$



$y_1 \quad y_2 \quad \ldots \quad y_n$

age $\quad X_{11}$

sex $\quad X_{12}$

ethnicity $\quad X_{13}$

$Y_1 \quad Y_2 \quad \ldots \quad Y_n$

data

$y_i \sim N(\mu, \sigma^2)$

model

inference
- classical
- Bayesian

fitted model

model evaluation

$y_1 \quad y_2 \quad \cdots\cdots \quad y_n \qquad (n = 250)$

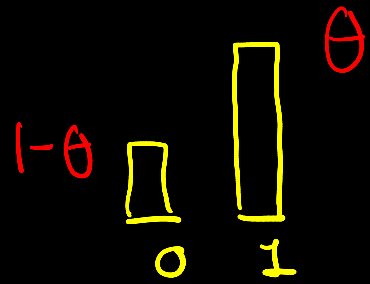
$y_i \in \{T, H\}$

$\in \{0, 1\}$

$m = 139$

model

each $\left[y_i \sim \text{Bernoulli}(\theta) \right]$



$y_1 \quad y_2 \quad \ldots \quad y_n$

iid

$Y_1 \quad Y_2 \quad \ldots \quad Y_n$

$Y_1 = Y_2 = Y_3 \ldots Y_n$

Data: $y_1 \; y_2 \; \dots \; y_n$

model: $y_i \sim \text{bernoulli}(\theta)$ for i in $1 \dots n$

probability of data given θ

$m = 139$

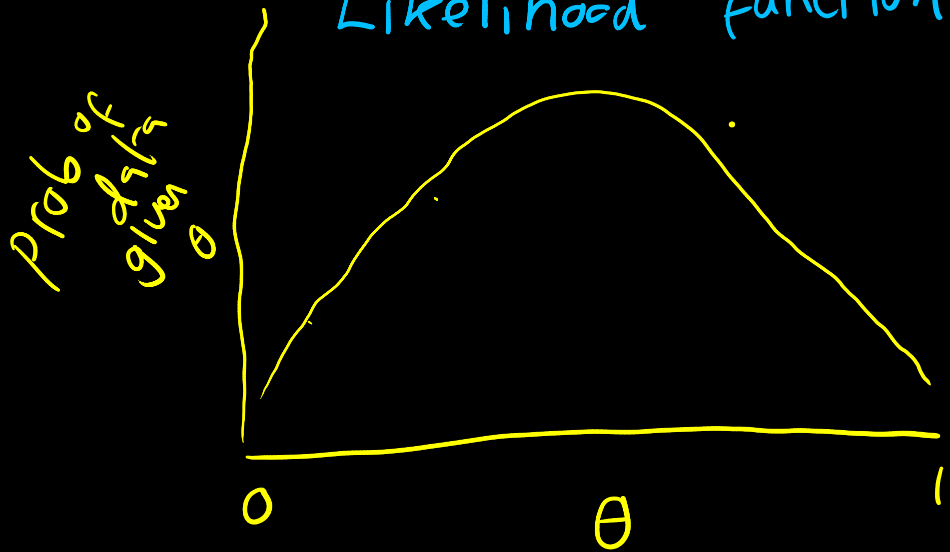
$n = 250$

$p(\text{Data} \mid \theta)$

$$p(\text{Data} \mid \theta) = \binom{m}{n} \theta^{m} \; 1-\theta^{n-m}$$

$$\frac{m!}{n!(n-m)!}$$

Likelihood function



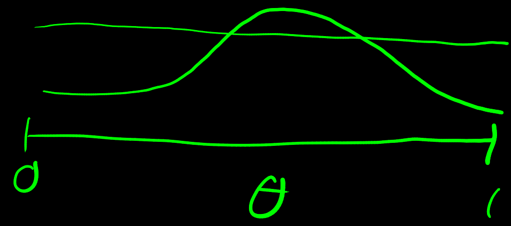
Classical inference

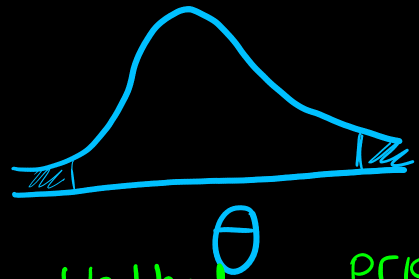
- estimator
- sampling distribution of estimator
- test hypotheses
- confidence intervals

Bayes's theorem

Data : D

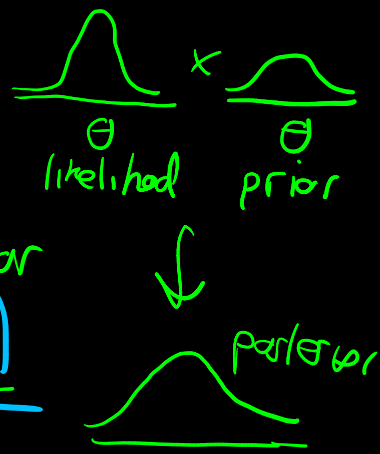
unknown : θ

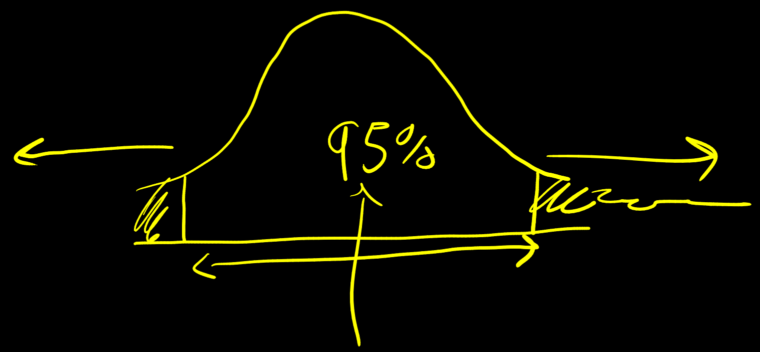




posterior distribution

$$p(\theta \mid D) = \frac{\overset{\text{likelihood}}{p(D \mid \theta)}\;\overset{\text{prior}}{p(\theta)}}{\int p(D \mid \theta)\, p(\theta)}$$




95%