Homework #4

Due Monday, November 6

1 Limits and continuity of functions of n variables

1. Let $(x_0, y_0) \in \mathbb{R}^2$ and use the $\delta - \epsilon$ method to show that:

   (a) $f(x, y) = x - y \implies \lim_{(x,y)\to(x_0,y_0)} f(x, y) = x_0 - y_0$ **without** using the theorem about limits and arithmetic operations.

   (b) $f(x, y) = yx^2 - y^3 \implies \lim_{(x,y)\to(x_0,y_0)} f(x, y) = y_0 x_0^2 - y_0^3$ using the theorem about limits and arithmetic operations.

2. Let $f(x, y) = \frac{x^2 y}{x^4 + y^2}$ and show that $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.
   Hint: consider what happens when the origin is approached along a line (e.g., $y = x$) and along a parabola (e.g., $y = x^2$).

3. Let $f(x, y) = \begin{cases} x + y & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y, z) = (0, 0, 0) \end{cases}$

   (a) Show that f is **not** continuous at $(0, 0)$.

   (b) Show that f is continuous on $S = \mathbb{R}^2 \setminus \{0\}$.

2 Differentiability of functions of n variables