Homework #3

Due Monday, October 23

1. Evaluate the following limits.

   (a) $\lim\limits_{x\to 2} \quad 2x^2 - 8x + 5$

   (b) $\lim\limits_{x\to -4} \quad x^4 - \frac{x+2}{2-x} - 17 + 9x^2$

   (c) $\lim\limits_{x\to \infty} \quad \frac{2x^4+3x^2+2}{x^3+2x+3}$

   (d) $\lim\limits_{x\to 4} \quad \frac{x^2+16}{x-4}$

   Hint: think about l'Hospital's rule whenever necessary.

2. Prove $\lim\limits_{x\to 1} \quad x^2 + 3 = 4$ using the $\delta - \epsilon$ method.

3. Find the limit of the sequence $x_n = \frac{1}{n} + \frac{2}{7n} + \frac{n+1}{n+2}$ and prove convergence.

4. Does $\lim\limits_{x\to 2} \frac{x^2-1}{x^2-3x+2}$ exist? If so, what is it? Prove your answer.

5. Prove that the function $g : \mathbb{R} \to \mathbb{R}$, defined as

$$g(x) = \begin{cases} -1 & | \, x < 0 \\ 1 & | \, x > 0 \\ 0 & | \, x = 0 \end{cases},$$

   does not have a limit at $x = 0$. **Hint**: you can use proof by contradiction.

6. Determine whether f is continuous or discontinuous from the right or left at $x_0 = 0$.

   (a) $f(x) = x$;

   (b) $f(x) = \sqrt{x}$;

   (c) $f(x) = 1/x$.

7. Differentiate the following functions with respect to x.

   (a) $x^2 - \frac{x^3+1}{2-x} - x^2 e^{2x}$;

   (b) $\frac{1-x+\sqrt{x}+2x}{e^{2x}+x^2}$;

8. Let $f(x) = 2x^2 - x + 7$ and $g(x) = x^2 + 2x$.

   (a) Write $f(g(x))$ and $f'(g(x))$.

   (b) Write $g(f(x))$ and $g'(f(x))$.

   (c) Let $F(x) = f(g(x)) + g(f(x))$. Write $F'(x)$.

9. Find all local maxima and minima (if any) of the following functions. Graph the functions to find out if they are also global maxima and minima.

   (a) $f(x) = 2x^2 + 3x - 72$;

   (b) $f(x) = log(x) + x^2$.

10. Is the following function continuous at $x = 0$? Prove your answer.

$$f(x) = \begin{cases} \frac{x-6}{x-3}, & x < 0 \\ 2 & x = 0, \\ \sqrt{4+x^2} & x > 0 \end{cases}$$

11. Let $F(x) = f(x) - g(x)$. Prove that $F'(x) = f'(x) - g'(x)$.