Homework #6

Due Wednesday, December 6

1. Solve

$$\max_{x,y} \quad xy \quad s.t. \qquad 5x + 3y \quad = 15.$$

   (a) Solve the problem by substitution.
   (b) Write the Lagrangian.
   (c) Using the first-order conditions, find the stationary point.
   (d) Use the bordered Hessian to prove that this is indeed a maximum.

2. Solve

$$\max_{x,y} \quad ln(x) + ln(y) \quad s.t. \qquad 3x + 6y \quad = 36.$$

   (a) Write the Lagrangian.
   (b) Using the first-order conditions, find the stationary point.
   (c) Use the bordered Hessian to prove that this is indeed a maximum.

3. Consider the following optimization problem.

$$\begin{aligned} \max_{x,y,z} \quad ln(x) + ln(y) + ln(z) \quad s.t. \qquad x + y + 2z &\leq 1 \\ x &\geq 0 \\ y &\geq 0 \\ z &\geq 0 \end{aligned}$$

   (a) Prove that at most one of the four constraints is binding with equality at the maximum.
   (b) Write down the Lagrangian function.
   (c) Without reference to the Lagrangian function, prove that this constraint is binding with equality at the maximum.
   (d) Write down the first-order conditions.
   (e) Solve the problem.
   (f) How does the value of the maximum change if the constraint $x + y + 2z \leq 1$ is replaced with $x + y + 2z \leq 1 + \epsilon$, where $\epsilon \to 0$?