Columbia University

Math Methods for Political Science
Fall 2017

Exercise Set 0

Due: no due date.

Exercise 1. Let $S = \{x, y, z\}$ and $T = \{a, b, y, z\}$ and $U = \{1, 2\}$.

(1) What is $T \setminus S$? $S \setminus T$? $S \setminus (T \cup U)$?
(2) Give all proper subsets of S, T, U.
(3) Give all proper subsets of $S \cap T$.
(4) Give all proper subsets of $S \cup U$.
(5) Are sets S, T disjoint? What about sets S, U? What about sets T, U?

Exercise 2. Let $A \subseteq B, B \subseteq C, C \subseteq A$. Prove $A = B = C$.

Exercise 3. Consider now the first associative law. The proof that $x \in (A \cup B) \cup C \Rightarrow x \in A \cup (B \cup C)$ is as follows:

$$\begin{aligned} x \in (A \cup B) \cup C &\Rightarrow x \in A \cup B \textit{ or } x \in C && |definition\ of\ union; \\ &\Rightarrow x \in A \textit{ or } x \in B \textit{ or } x \in C && |definition\ of\ union; \\ &\Rightarrow x \in A \textit{ or } x \in B \cup C && |definition\ of\ union; \\ &\Rightarrow x \in A \cup (B \cup C) && |definition\ of\ union. \end{aligned}$$

Prove the other direction, namely that $x \in A \cup (B \cup C) \Rightarrow x \in (A \cup B) \cup C$.

Exercise 4. Prove $(A \cap B) \cap C = A \cap (B \cap C)$. **Hint**: similar to the other associative law.

Exercise 5. Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **Hint**: prove that the left side is a subset of the right side; then prove that the right side is a subset of the left side.

Exercise 6. Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **Hint**: prove that the left side is a subset of the right side; then prove that the right side is a subset of the left side.