Models of Creative Destruction
Firm Dynamics

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Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \tag{1}$$

where

$$X(t) = \int_0^1 \psi x(v,t)\,dv \tag{2}$$

$$Z(t) = \int_0^1 [z(v,t) + \hat{z}(v,t)]\,q(v,t)\,dv \tag{3}$$

z and $\hat{z}$ are innovation inputs by incumbents and their rivals.

Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \int_0^1 q(v,t)^{\beta} x(v,t|q)^{1-\beta} dv \tag{4}$$

- the only change: quality is taken to power β
- implies: sales vary with quality (so the model has firm size implications)

Intermediate goods technology

- constant marginal cost ψ

Innovation technology for incumbents

- let $q(v,s)$ be the quality at the time the incumbent invented it
- investing zq implies a flow probability of innovation of ϕz
- the quality step is λ

Innovation technology for entrants

- investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})$ (decreasing)
- the quality step is $\kappa > \lambda$ (leapfrogging)
- innovators take η as given (an externality)

Solving each agent's problem

Solving each agents' problem

Household:

$$g(C) = \frac{r - \rho}{\theta} \tag{5}$$

Final goods producer:

$$x(v, t|q) = p^x (v, t|q)^{-1/\beta} q(v, t) L \tag{6}$$

$$w(t) = \beta Y(t) / L(t) \tag{7}$$

Intermediate goods producer

Assume drastic innovation

$$p^x(v,t|q) = \frac{\psi}{1-\beta} = 1 \tag{8}$$

Innovation by entrants

Free entry:

$$\eta\left(\hat{z}(v,t|q)\right)V(v,t|\kappa q)=q(v,t) \tag{9}$$

This assumes an equilibrium with entry.

Innovation by incumbents

Again assuming positive innovation.

Increase z until the marginal value equals marginal cost:

$$\phi z(v,t|q)\left[V(v,t|\lambda q) - V(v,t|q)\right] = q(v,t)\, z(v,t|q) \qquad (10)$$

Value of the firm

Expected discounted value or profits

$$\begin{aligned} rV(v,t|q) = \pi(v,t|q) - \dot{V}(v,t|q) - z(v,t|q)\,q(v,t) \qquad (11) \\ + \phi z(v,t|q)\left[V(v,t|\lambda q) - V(v,t|q)\right] \qquad (12) \\ - \hat{z}(v,t|q)\,\eta(\hat{z}(v,t|q))\,V(v,t|q) \qquad (13) \end{aligned}$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

Profit

$$\begin{aligned} \pi(v,t|q) &= \left[p^x(v,t|q) - \psi\right] x(v,t|q) \qquad (14) \\ &= \beta q L \qquad (15) \end{aligned}$$

because $p^x = 1$ and $x = qL$.

Equilibrium

Allocation
$\{C(t), X(t), Z(t), Y(t), L(t), z(v,t), \hat{z}(v,t), x(v,t), \pi(v,t), V(v,t)\}$
Prices $\{p^x(v,t), w(t), r(t)\}$
that satisfy:

- household: Euler (and TVC)
- final goods firm: 3
- intermediate goods firm: 1
- free entry of incumbents and entrants: 2
- market clearing: goods, labor (2)
- definitions of X, Z, π (3)
- definition of V (differential equation) (1)

Balanced Growth Path

Assert $\dot{V} = 0$, $z(q)$, and $\hat{z}(q)$ constant over time (verify later)

Law of motion for V implies: $V(q) = vq$.

- so that rV and $\pi = \beta Lq$ can grow at the same rate

Free entry for entrants:

- $\eta(\hat{z}(v,t|q))\, V(v,t|\kappa q) = q(v,t)$
- implies $\hat{z}$ is the same for all q
- but z for incumbents may vary with q

Innovation for incumbents

$\phi\left[V\left(v,t|\lambda q\right)-V\left(v,t|q\right)\right]=q\left(v,t\right)$ implies

$$V\left(q\right)=\frac{q}{\phi\left(\lambda-1\right)} \tag{16}$$

Law of motion for V:

$$rV\left(q\right)=\beta Lq-\hat{z}\eta\left(\hat{z}\right)V\left(q\right) \tag{17}$$

- the term reflecting incumbent innovation drops out (by its FOC)

Combine the two:

$$\eta\left(\hat{z}\right)=\frac{\phi\left(\lambda-1\right)}{\kappa} \tag{18}$$

This solves for $\hat{z}$.

Equilibrium growth rate

Substitute back into free entry

$$r = \phi(\lambda - 1)\beta L - \hat{z}\eta(\hat{z}) \tag{19}$$

Together with the Euler equation, this solves for the growth rate.

Implications for firm dynamics

Since $x(v,t|q) = qL$, firm size (sales) are governed by the evolution of q

For a given firm: x

- increases by factor λ with probability $\phi z \Delta t$
- stays the same with probability $\hat{z}\eta\left(\hat{z}\right)\Delta t$
- drops to 0 with complementary probability

Reading

- Acemoglu (2009), ch. 14.3.

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.