Proposing before Match: A New Way to Look at Marriage Matching (Final Proposal)

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1. Introduction

Marriage is a crucial social phenomenon associated with almost every human's life. It has been an important function in human society for thousands of years. It also has been always an interesting economic problem. From last century on, there are many models built on labor market, and there are several similarities between the marriage market and the labor market: both of the markets has two sides, and both needs to divide the surplus between two sides. As a result, the market of marriage has been widely discussed and studied, and there have been plenty of matching models developed through the years.

However, these papers still do not look carefully at how the matching happens: the process of proposing remains in a black box. Matching does not happen without a decision to be made. By experiences, we know there would be a match if and only if one side approaches to the other side and proposes, and the other side says yes. This is especially true for the marriage market. If a boy likes a girl and think she would be a good match for him, he needs to at least walk to the girl and ask the girl out. Or with advanced technology today, he can text the girl or send her a Facebook massage. There would be nothing happened if the boy does not say anything to the girl.

The goal for this paper is to consider proposing in the matching problem, and to determine how this proposing process would affect matching in the "love" marriage. Based on experience, there could be a cost of proposing and cost of delaying. This paper would try to unveil how the cost of delaying may affect the dynamic of matching.

There are two approaches I would use in order to find out the effect of the cost of delaying to the matching dynamic. First approach is to develop a theoretical model having this proposing process incorporated. Second approach is to conduct an empirical study to see if the result of the theory would hold. In this proposal, I will mainly discuss the modelling of proposing in the marriage matching.

The rest of the proposal is organized as follows: Section 2 gives a literature review about the relating matching models. Section 3 provides some assumptions and explanation of the model. Section 4 talks about the model with proposing process. Section 5 discuss how cost of delaying (discounting) would affect the dynamic based on the model and the implications of the model. Section 6 states conclusion and possible real world application of the model.

2. Literature review

Early literatures such as Becker's "A Theory of Marriage Part I" (1973) consider that matching would happen, somehow magically, when the two side of market has properties sufficient to each other. In his paper, he assumes each man and woman have a universally accepted rank, and everyone tries to find the best to match. The result he found is best assortative matching (PAM), which the best man matches to the best woman; the next best man matches to the next best woman; and so on. However, he only provides a stable and optimal strategy of matching, but there is no concern of timing in his model. There is also no discussion about how the matching happens.

An extension of the Becker's PAM is Burdett and Coles' 1997 paper "Marriage and Class". In their paper, they uses the idea of pizzazz instead of ranking, which is a non-transferable utility flow. When a man and a woman marry, the flow utility to a man is simply the woman's pizzazz, and the flow to a woman is the man's pizzazz. In their paper, they concluded that there are classes of pizzazz, such that one would only marry to a person in the same class. It is less restrictive than that only the top man can marry top woman in PAM because it takes time to meet someone. Their literature, however, also regards the matching as an automatic process.

In the later discussions on the marriage market, there are many researchers focus on the searching and matching. Their idea is that people need to search the market in order to find a match, and there is generally a cost associated with the searching process. For example, Axell (1977) discusses in his paper how the information cost would affect the wage dispersion. However, I would look at the matching in a different way. In this paper, I would regard meeting as a natural process. We meet people on normal base, and what makes the difference is whether or not we choose to propose to a person in order to form a match.

A more recent approach of marriage matching that somewhat include the marriage proposal in the matching is Batabyal's 2001 paper. Although, he actually studies about arranged marriage, his idea of proposing may still offer some insight about the matching in "love" marriage. The proposals in Batabyal's setting are received in accordance with a Poisson process with a fixed rate β. Also, in his model, the proposal would be brought to the clients naturally. His conclusion is that people will wait and then act. His result is interesting, but would be different in the context of "love" marriage.

Proposing process can also be considered as a signaling device of marriage. In his 2012 paper, Hopkins combines Becker's theory of PAM and Spence's signaling model. Although he talks about the signaling in the content of job market, it can easily apply to the marriage market. Based on his idea, one side of the market has private information about their quality, and they signal the other side. Proposal can be regarded as a signal of type of individual, namely level of romantic. He actually considers both the non-transferable utility and transferable utility. With the non-transferable case, he concludes that in the separating equilibrium, a more romantic person would signal a higher level proposal. It results like the PAM that the position of the person in the population distribution would define the partner she would get. His idea of signaling, however, relies on the assumption that the two side of the market as a whole can meet each time, and one side and propose to anyone of the other side. This does not really hold, especially in the content of marriage market: there are physical limitations because one can meet only a few people at a time.

3. Assumptions and Explanations

3.1 Pizzazz and Utility Maximization

Consider there are two sides of the population (can be seen as male and female). The basic assumption from Becker's paper (1973) is made, that all people in the same group are homogeneous in a sense that they would have the same taste and strategy to select mate. For each person i in group A, any j from B matches with i would result in the same utility flow (Pizzazz of i) to j. I use $u_j(i)$ to illustrate the utility flow from i to j. Vice versa. This assumption of non-transferable utility is from Burdett and Coles' paper (1997). In this model, each individual would try to maximize their utility by proposing/matching.

3.2 Asymmetric Information and random acceptance

There is asymmetric information in this model, individuals from group B have perfect information, but those from A don't. In this model, I establish that each j would have a random idiosyncratic preference ε in addition to the pizzazz of i, $u_j(i)$. It could be the case that i really has a low pizzazz, but a high pizzazz *j* just likes him. The distribution of ε is described by function $F(\varepsilon)$ ($F(\varepsilon) > 0$ for all values of ε). This idiosyncratic preference would cause uncertainty whether *j* will accept *i* or not. Let's call this acceptance rate P.

This idiosyncratic preference assumption is very important in this model. Without it, there would be perfect information and there would remain no uncertainty of proposal being accepted. If *i* propose, *j* would just accept, and there would be no need to include the proposing process in the model.

Other than the idiosyncratic preference ε, information would be all available to both *i* and *j*. They both know the pizzazz $u_j(i)$ and $u_i(j)$. They would also know each other's expectation about future utility flow if not matched, $\overline{u_j}$ and $\overline{u_i}$. Because the idiosyncratic preference ε is random, by knowing all other information, *i* can estimate the acceptance rate *P*. Also, it is obvious that *j* can observe whether *i* proposes or not.

3.3 Matching Process and cost of delaying

At each time t, individual *i* from *A* and *j* from *B* would meet by random. This setting is different from the work of Becker (1973) or Burdett and Coles (1997). They have the setting that two groups would meet all together. However, there are physical limitations that how many people can an individual meet. I use the assumption that for a given period, a person would meet with just one person by random. I also consider this meeting is random and effortless. This paper is about proposing and matching in contrast to the traditional searching and matching studied exclusively by many researchers.

Then they try to somehow match with each other. First, i would decide, based on j's pizzazz, whether or not to propose. If *i* proposes, *j* would decide to accept or turn down the proposal. They would leave the market if they successfully match and earn each other's pizzazz as utility flow. If they fail, each would keep in the market and randomly meet another person in the next period. The process would start over again in the same fashion. The process is described below in the picture.

However, there is a cost of delaying or discounting cost C if one goes to the next period. This cost, however, will not affect the pizzazz of any individual. We can treat this cost as a deadweight loss of the utility.

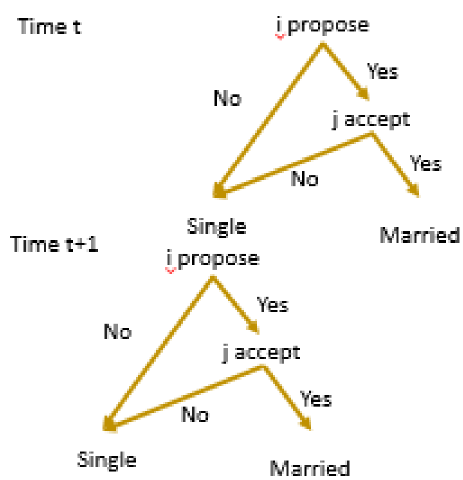

Figure 1. Proposing in marriage matching.

3.4 Demographics

The population cumulative distribution of pizzazz would be known to all individuals as $H(u_j(i))$ and $G(u_i(j))$. $H'(u_j(i)) = h(u_j(i))$ and $G'(u_i(j)) = g(u_i(j))$. In this paper, I assume that the population distribution of pizzazz would remain the same throughout the time horizon. In other words, there will be perfect replacement in each period.

3.5 Free Proposal

In this paper, I assume that there is a way to price this proposal cost into the pizzazz value and treat price of proposal as 0. It is possible that the cost of proposal will also change the dynamic of matching (implied in Hopkins' paper about signaling). This model only considers the effect cost of delaying, so for simplicity, I would treat the cost as 0.

3.5 Fixed pizzazz and fixed expectation about future utility flow

In the previous assumption, I assumed that the cost of delaying would not affect a person's pizzazz. Here, I would further assume that the pizzazz of an individual would hold constant over time.

Because of the perfect information other than j's idiosyncratic preference. Each individual would form their expectation of potential partner based on PAM. In PAM, the rank would be the determinant of matching. Here, for j, the rank of j in her group is $G(u_i(j))$ and the rank of a person with expected utility flow is $H(\overline{u_j})$. Therefore,

For j, $H(\overline{u_j}) = G(u_i(j)), \overline{u_j} = H^{-1}(G(u_i(j)))$

Similarly, for i, $G(\overline{u_i}) = H(u_j(i)), \overline{u_i} = G^{-1}H(u_j(i))$

Based on assumption of PAM, I assume that both $H^{-1}(G(u_i(j)))$ and $G^{-1}H(u_j(i))$ are increasing functions, i.e.

$$\frac{\partial \overline{u_j}}{\partial u_i(j)} > 0 \; and \; \frac{\partial \overline{u_i}}{\partial u_j(i)} > 0 \tag{1}$$

The previous assumption of pizzazz distribution do not change over time is important here. Because of that assumption, I conclude that the expectation about future utility flow is fixed for each individual across time. It does not depend on factors other than group A and B's distribution of pizzazz, and level of the individual's pizzazz.

4. Model

4.1 Acceptance

It is much easier to start with *j*'s decision of accepting the proposal or not. Although the idiosyncratic preference ε factor is randomly assigned, for *j*, she would already make the decision in her mind to accept *i* or not when they meet. *j* would choose to accept *i* when the pizzazz *i* would offer is higher than the utility flow from waiting plus the idiosyncratic preference ε:

$$u_j(i) \geq \overline{u_J} - C + \varepsilon$$

$$\varepsilon \leq u_j(i) - \overline{u_J} + C$$

As the distribution of ε, $F(\varepsilon)$, is known, we can calculate P and a function of C:

$$P(u_j(i), u_i(j), C) = \int_{-\infty}^{u_j(i) - \overline{u_J} + C} F(\varepsilon)\, d\varepsilon$$

Based on the inequality condition above, it is obvious that only when ε is smaller than $u_j(i) - \overline{u_J} + C$, *j* would accept a proposal from *i*. In other words, j does not want to demand too much at this time. The distribution of ε is $F(\varepsilon)$, so and integration would sufficiently provide the probability of accepting given values of $u_j(i), u_i(j), and\ C$.

In order to make the question easier to acess, I assume that

$$\int_{-\infty}^{t} F(\varepsilon)\, d\varepsilon = N(t), \text{ i.e. } \frac{\partial N}{\partial t} = F(t) > 0$$

$$P(u_j(i), u_i(j), C) = N(u_j(i) - \overline{u_J} + C) \qquad (2)$$

$N(\varepsilon)$ would be the cumulative distribution of ε.

4.2 Propose

i would know the distribution of ε. Therefore, he knows his possibility of getting a yes if he proposes. Then *i* would choose to propose if the expected utility gain would be larger than that of the future periods:

$$P * u_i(j) \geq \overline{u_\iota} - C \tag{3}$$

Whether *i* would propose actually depends on the probability of being accepted. Consider the reservation level of $u_j(i)$ = $\tilde{u}$. Any *i* with pizzazz below $\tilde{u}$ would propose to *j*.

$$P(\tilde{u}, u_i(j), C) * u_i(j) = \overline{u_\iota}(\tilde{u}) - C \tag{4}$$

The reservation pizzazz is the lowest possible *i* that would propose to a given *j*. Therefore, for each $u_i(j)$, there is a reservation rate associated with it. We can write $\tilde{u}$ as a function of $u_i(j)$ and C.

4.3 Matching

Now, we can consider the probability that a match would occur, when *i* proposes and *j* accepts. Considering the random meeting criteria, for a given *j* (i.e. $u_i(j)$ is determined), there is a probability that a random *i* would assigned to *j* that *i* would propose and at the same time *j* would agree to match:

$$Prob\big(matching|u_i(j)\big) = \int_{-\infty}^{\tilde{u}} h(u_j(i)) * P\big(u_j(i), u_i(j), C\big) d(u_j(i)) \tag{5}$$

In this equation, $h(u_j(i))$ is the distribution of *i*'s pizzazz. For a given *j*, $h(u_j(i))$ would indicate the probability that an *i* with pizzazz $u_j(i)$ would match with the *j*. With each $u_j(i)$, the probability of forming a match is $h * P$. Since we know that the *i*'s having pizzazz level below the reservation level $\tilde{u}$, an integration of $h * P$ from $-\infty$ to $\tilde{u}$ would give the possibilities of matching.

5. Discussion

5.1 Probability of accepting

We can now check the effect of various factors on the result of matching.

$$\frac{\partial P}{\partial u_j(i)}\left(u_j(i), u_i(j), C\right) = \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * \frac{\partial\left(u_j(i) - \overline{u_J} + C\right)}{\partial u_j(i)}$$

$$= \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * 1 > 0 \tag{6}$$

$$\frac{\partial P}{\partial u_i(j)}\left(u_j(i), u_i(j), C\right) = \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * \frac{\partial\left(u_j(i) - \overline{u_J} + C\right)}{\partial \overline{u_J}} * \frac{\partial \overline{u_J}}{\partial u_i(j)}$$

$$= \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * (-1) * (+) < 0 \tag{7}$$

$$\frac{\partial P}{\partial C}\left(u_j(i), u_i(j), C\right) = \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * \frac{\partial\left(u_j(i) - \overline{u_J} + C\right)}{\partial C}$$

$$= \frac{\partial N}{\partial\left(u_j(i) - \overline{u_J} + C\right)} * 1 > 0 \tag{8}$$

Note that the $\frac{\partial \overline{u_J}}{\partial u_i(j)}$>0 applied in inequality (7) is from inequality (1), which is a result of Fixed expected future utility flow assumption. These three inequalities fit well with our intuitions about the probability of accepting a proposal. Inequality (6), $\frac{\partial P}{\partial u_j(i)} > 0$, illustrates that as the pizzazz of *i* goes up, *j* would have a higher probability to accept *i* (if *i* proposes). This makes sense because if $u_j(i)$ increases, it means that *i* is more attractive, and of course, *j* would more likely to accept him. Inequality (7), $\frac{\partial P}{\partial u_i(j)} < 0$, illustrates that as the pizzazz of j goes up, *j* would have a lower probability to accept *i*. This also works because that the as $u_i(j)$ goes up, it means that j is more attractive, so she would have a higher standard, so for each level of i's pizzazz, j would less likely to accept i.

The focus of this paper is on how the cost of delaying, C, would affect this dynamic. Based on inequality (8), as the cost of delaying goes down, there is a decrease in probability that j would accept a given i. This is because that as the cost of delaying goes down, people may want to wait longer to find a better partner. And since there is a better chance to find a good partner, j may raise her standard of choosing i.

5.2 Class effect

There is a sense of class in this model: for a given j, only the i's with pizzazz lower than the threshold, $\tilde{u}(u_i(j))$, would try to propose to her. However, this is dramatically different from Burdett and Coles' result of classes (1997), that people would only match with the people with in the classes. Although low j would only match to low i's, the structure in my model is asymmetric. There is still a chance, although really low, for a low pizzazz i match with a high pizzazz j, "shooting for the stars". It is made possible by the randomness generated form j's idiosyncratic preference ε, and i's power to propose.

5.3 Marriage Market dynamic

Finally, let's consider how the cost of delaying would affect the whole marriage market's dynamic as a whole. I would approach this problem on the side of j to see how the change in C would affect the probability of matching for a given individual j.

With the individual j's pizzazz level $u_i(j)$ fixed, the probability of matching is given in the equation (5). As C decreases, there are two variables changing in this integration: P and $\tilde{u}$.

Based on the previous discussion, P would decrease as C decreases. Since $h>0$ and $P>0$ for all values, the decrease of P would cause $h*P$ decrease. As the $0<h*P'<h*P$ on each level of $u_j(i)$, the integration would decrease while holding $\tilde{u}$ constant. Therefore, controlling the effect of $\tilde{u}$, we would find that the probability of matching would decrease.

How about C's effect on probability of matching through the change of reservation pizzazz of i? In order to find this out, we need to first consider change of $\tilde{u}$ due to the change of C. Before the change, for the i with reservation pizzazz,

$$P(\tilde{u}, u_i(j), C) * u_i(j) = \overline{u_i}(\tilde{u}) - C.$$

However, after the decrease of C to C',

$$P(\tilde{u}, u_i(j), C') < P(\tilde{u}, u_i(j), C)$$

$$\overline{u_i}(\tilde{u}) - C < \overline{u_i}(\tilde{u}) - C'.$$

As a result, $P(\tilde{u}, u_i(j), C') * u_i(j) < \overline{u_i}(\tilde{u}) - C'$ because the decrease on left hand side and increase on right hand side.

The old threshold violates the inequality (4), so now the i with $\tilde{u}$ would not propose to the given j anymore. Because any i with pizzazz level below the reservation would propose, it has to be the case that the reservation pizzazz decreases. $\tilde{u}' < \tilde{u}$. As reservation pizzazz decreases, the integration in equation (5) would further decrease. Probability of matching would decrease.

Intuitively, as delaying cost decreases, the number of proposers to a given j and j's probability of accepting a i would both drop. They would have the same direction effect, hindering the process of matching. Because of the delay cost decreases, people would like to wait longer in order to find a better partner.

This finding is interesting because the literature normally regards the cost of delaying as friction to the market. However, as the friction reduces, we can see a delaying of the matching process: the market become less efficient with less friction.

6. Conclusion

My project may help to explain the marriage market dynamics in a different angle, in which we have a process of proposing building in. I look at the problem of matching as proposing and matching instead of searching and matching. It could potentially to answer some questions about the change of the dynamic of marriage market. There was a report made by D'Vera Cohn (2011), "…the age at which men and women marry for the first time continues to rise to record levels." As the technology advances, there are more and more outside sources of utility boosting other than marriage. It could be the case that the delay of the first marriage was due to the delay cost of marriage is decreasing.

In the book *The Changing Transitions to Adulthood in Developing Countries: Selected Studies*, Mensch et al (2005) writes explicitly that "with the exception of Becker's work, we have few theories that explicitly address age at marriage." The theory of proposing and matching could potentially provide some more understanding this this area of timing through the change in probability of matching in each period.

In South's longitudinal study, *Variable Effects of Family Background on the Timing of First Marriage (2001)*, he mentions that people from a higher level of socio-economic back ground tend to delay their marriage. It could be possible that this delay is a result of a lower delay cost: people from a wealthier household could enjoy life even not married.

There are many possible contributions to the literature: adding proposing process into the matching model, having acceptance as a random process, the result of lower friction resulting in delayed matching, and providing more economic insight to the timing of first marriage. However, there are also many limitations of this theory. Further researchers can relax some strict assumptions, such as fixed pizzazz distribution, to see how the different pizzazz distribution would have different structures of matching. It can also be done to include a cost of proposal into the model, while in this model, I assumed that it could be included in the pizzazz. It could be the case that the cost of proposal will change the dynamic a lot. I would also suggest that there could be a possible additional cost of being rejected. This may also have an impact on the dynamic of matching. Much more aspects are worth to explore in the content of proposing.

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