Example: Optimal Taxation

Econ720

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Model

Demographics:

- A single representative consumer who lives forever.

Endowments:

- k_0 units of the c/k good at $t = 0$.

Preferences:

$$\sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(g_t)\}$$

Model

Technology:

$$F(K_t, L_t) + (1-\delta) K_t = c_t + \varphi g_t + G_t + K_{t+1} \tag{1}$$

- $\varphi > 0$. F has constant returns to scale.

Government:

- Consumption taxes at rates τ_{ct} and τ_{gt}, respectively.
- Tax revenues are used to purchase G_t.

Markets:

- labor: w_t, capital rental: q_t, c/k purchases: 1, g: p_t.

Household

Budget constraint:

Bellman equation:

First-order conditions:

Household Solution

Sequences (c_t, g_t, k_t) that solve

$$\frac{v'(g)}{u'(c)} = p\,\frac{1+\tau_g}{1+\tau_c} \tag{2}$$

$$u'(c) = \beta\, R'\, u'(c')\,\frac{1+\tau_c}{1+\tau_c'}$$

and

- budget constraint
- k_0 given
- $\lim_{t\to\infty} \beta^t\, u'(c_t)\, k_t = 0$.

Observations

- Taxes do not always hit what you would think.
- Static FOC: $\frac{v'(g)}{u'(c)} = p\,\frac{1+\tau_g}{1+\tau_c}$
  - if $\tau_g = \tau_c$: no distortion
- Euler: $u'(c) = \beta\, R'\, u'(c')\,\frac{1+\tau_c}{1+\tau'_c}$
  - if $\tau_c = \tau'_c$: no distortion
- What happens when $\tau_g = \tau_c = \tau'_c$?

Equilibrium

A competitive equilibrium is an allocation
and a price system
that satisfy:

Steady State

- The Euler equation fixes the interest rate at $R_{ss} = 1/\beta$.
- The capital stock is then determined by $R_{ss} = 1 - \delta + f'(k_{ss})$.
- The static first-order condition together with goods market clearing,

$$y \equiv f(k_{ss}) - \delta k_{ss} - G = c_{ss} + \varphi g_{ss} \tag{3}$$

then determine c_{ss} and g_{ss}.

Optimal Taxation

What are the optimal tax rates in steady state?

The government solves:

Government Problem

$$\max_{g,\tau_g} u(y-\varphi g)+v(g)+\lambda\left\{v'(g)\left[1+\frac{G-\tau_g\varphi g}{y-\varphi g}\right]-\varphi(1+\tau_g)u'(y-\varphi g)\right\}$$

The c's have been substituted out using $c = y - \varphi g$.

The constraint in the braces is the static FOC.

The government budget constraint has been used to replace τ_c by $[G - \tau_g \varphi g]/c$.

First-order conditions

g: $u'(c)\,\varphi = v'(g) + \lambda \times \text{stuff}$

τ_g: $\lambda \left\{ v'(g) \frac{\varphi g}{y - \varphi g} + \varphi\, u'(y - \varphi g) \right\} = 0$

The term in the $\{\}$ must be strictly positive.

But then λ must be 0!

How is this possible?

Why Does This Happen?

Solution

Proceed mechanically.

Taking the first-order condition for g and imposing $\lambda = 0$ yields

$$\varphi u'(c) = v'(g)$$

The tax rates that implement this can be backed out from the static condition: $\tau_g = \tau_c$.

Why are the tax rates the same?

A fundamental principle of optimal taxation indicates to tax goods with lower demand elasticities more heavily.

But this does not apply here.

The two consumption taxes together are equivalent to a lump-sum tax and therefore first-best.