# Problem Set 3: Infinite Horizon Model in Discrete Time

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## 1 Ben-Porath Model

We study the decision problem of an infinitely lived agent in discrete time. At $t = 0$, the agent is endowed with $h_0$ units of human capital. In each period, he can invest $l_t$ units of time, so that human capital evolves according to

$$h_{t+1} = (1-\delta)\,h_t + F\left(h_t l_t\right) \tag{1}$$

$$F\left(hl\right) = \left(hl\right)^{\alpha} \tag{2}$$

with $0 < \alpha, \delta < 1$. The objective is to maximize the present value of lifetime earnings, given by

$$Y = \sum_{t=0}^{\infty} R^{-t} w_t h_t \left(1 - l_t\right) \tag{3}$$

where $R > 0$ is taken as given.

**Questions:**

1. Write down the agent's Dynamic Program.
2. Derive and interpret the first-order condition for $l$.
3. Derive $V'(h) = w + (1-\delta)\,R^{-1} V'(h')$.
4. Derive and interpret $V'(h) = w\frac{R}{r+\delta}$ where $R = 1 + r$.
5. How do the wage and the interest rate affect steady state $h$ and $l$?

## 2 Education Costs

Consider the following version of a standard growth model with human capital. The planner solves

$$\max \sum_{t=1}^{\infty} \beta^t u\left(c_t\right) \tag{4}$$

s.t.

$$k_{t+1} = (1-\delta)\,k_t + x_{kt} \tag{5}$$

$$h_{t+1} = (1-\delta)\,h_t + x_{ht} \tag{6}$$

$$c_t + x_{kt} + \eta x_{ht} = f\left(k_t, h_t\right) \tag{7}$$

with $k_1$ and $h_1$ given. Here $c$ is consumption, $k$ is physical capital, $h$ is human capital, and $\eta$ is a constant representing education costs. Assume that the production function is Cobb-Douglas:

$$f(k, h) = zk^{\alpha}h^{\varepsilon} \tag{8}$$

where $z$ is a constant technology parameter and $\alpha + \varepsilon < 1$.

Questions:

1. Derive the first-order condition for the planner's problem using Dynamic Programming. Define a solution in sequence language and in functional language.

2. Solve for the steady state levels of $k/h$ and $k$.

3. Characterize the impact of cross-country differences in education costs ($\eta$) on output per worker in steady state. In particular, calculate the ratio of outputs per worker for two countries that only differ in their $\eta$'s.