

R&D Models: Introduction

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Issues

- ▶ We study models where **intentional innovation** drives productivity growth.
- ▶ We start by describing the demand block (common to essentially all models).
- ▶ Later we embed it into a GE model.

Background

- ▶ Historians often view innovation as the result of research that is not profit driven.
- ▶ Economists treat innovation as producing goods that are sold in markets ("blueprints").
- ▶ There are historical examples of both types of innovation.
- ▶ How important are the 2 cases? – An open question.

How to model innovation

- ▶ Current models are somewhat reduced form.
- ▶ The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- ▶ Knowledge is treated as a scalar - like capital.
- ▶ In fact, the only difference between blueprints and machines is **non-rivalry**:
 - ▶ blueprints can be used simultaneously in the production of several goods.

How to model innovation

There are N consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- ▶ Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- ▶ Each good can be made by many firms.
- ▶ Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- ▶ Each firm can invest to create a new variety ($N \rightarrow N + 1$)
- ▶ Then it becomes the monopolist for that variety

The Demand Block

Modeling the Demand Side

- ▶ The trick in all R&D models:
a demand side that generates a **constant price elasticity**
- ▶ This makes the monopoly price essentially exogenous
 $p_M = MC / (1 - 1/\varepsilon_D)$

Dixit Stiglitz Model

- ▶ The world is static.
- ▶ There are N consumption goods c_i with prices p_i .
- ▶ Household income is m .

Preferences

- ▶ Households aggregate the varieties using a CES aggregator

$$C = \left(\sum_{i=1}^N c_i^\theta \right)^{1/\theta} \quad (1)$$

- ▶ Utility is $u(C)$
- ▶ Elasticity of substitution $\varepsilon = 1/(1 - \theta) > 1$.
- ▶ Then $\theta = (\varepsilon - 1)/\varepsilon > 0$.
- ▶ The trick: constant substitution elasticity implies constant price elasticity.

Demand functions

The household solves:

$$\max u(C)$$

subject to

$$\sum_{i=1}^N p_i c_i = m \quad (2)$$

Given m , this is just a CES cost minimization problem.

Demand functions

One way of thinking about the household problem:

For any given C , find the cost minimizing c_i :

$$\min_{c_i} \sum_{i=1}^N p_i c_i + \lambda \left[C - \left(\sum_i c_i^\theta \right)^{1/\theta} \right] \quad (3)$$

FOC:

$$p_i = \lambda \left(\sum_i c_i^\theta \right)^{1/\theta-1} c_i^{\theta-1} \quad (4)$$

The implied demand function is of the form

$$c_i = X p_i^{1/(1-\theta)} = X p_i^{-\varepsilon} \quad (5)$$

with **constant price elasticity** ε .

Ideal price index

Define the minimized cost of C as

$$PC = \sum p_i c_i \quad (6)$$

The cost minimizing price index is

$$P = (\sum p_i^{1-\epsilon})^{1/(1-\epsilon)} \quad (7)$$

► Details

Love for variety

A key implication: simply having more varieties increases welfare.

Assume you have \bar{C} units of “stuff” that can be made (1-for-1) into any variety:

$$\sum_{i=1}^N c_i = \bar{C}.$$

Consider the symmetric case: $c_i = \bar{C}/N$.

Then

$$\begin{aligned} C &= \left(\sum_{i=1}^N [\bar{C}/N]^\theta \right)^{1/\theta} \\ &= \left(N [\bar{C}/N]^\theta \right)^{1/\theta} \end{aligned} \tag{8}$$

$$= N^{(1-\theta)/\theta} \bar{C} \tag{9}$$

Spreading \bar{C} over more varieties (N) increases utility.

Household summary

- ▶ Assume a Dixit-Stiglitz composite consumption good in preferences.
- ▶ Then demand is isoelastic.
 - ▶ the elasticity is determined by the elasticity of substitution across varieties in C .
- ▶ The cost of the optimal bundle C is given by the CES minimized cost P .
- ▶ More varieties increase utility.

Firms

- ▶ Each firm has a monopoly over a variety i .
- ▶ The demand elasticity is ϵ .
- ▶ Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\epsilon} \quad (10)$$

- ▶ Assumption: The firm is small enough to neglect its effect on C and P .

Equilibrium

Assume symmetry.

Price index:

$$\begin{aligned} P &= \left(\sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \\ &= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1 - 1/\varepsilon} \end{aligned}$$

Recall that the elasticity ε is > 1 .

Then increasing variety N decreases the cost of C (love for variety again).

Equilibrium: Profits

$$\begin{aligned}\pi_i &= c_i(p_i - \psi) \\ &= C P^\varepsilon p_i^{-\varepsilon} (p_i - \psi) \\ &= C N^{\varepsilon/(1-\varepsilon)} \frac{\varepsilon}{\varepsilon - 1} \psi\end{aligned}\tag{11}$$

More varieties can increase profits:

- ▶ Direct effect: P falls - more competitors erode profits.
- ▶ "Aggregate demand externality": C may rise (depends on preferences)
 - ▶ Higher N raises marginal utility for a given variety.
 - ▶ Innovators impose pecuniary externality on competitors.

Continuum of varieties

- ▶ Nothing changes when i is continuous.
- ▶ Replace all Σ with \int .

Reading

- ▶ Acemoglu (2009), ch. 12.
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

Ideal price index I

Proof:

$$\min \sum_i p_i c_i + \lambda \left[\left(\sum_j c_j^\theta \right)^{1/\theta} - C \right] \quad (12)$$

FOC:

$$p_i = \lambda \left(\sum_j c_j^\theta \right)^{(1/\theta)-1} c_i^{\theta-1} \quad (13)$$

$$= \lambda C^{1-\theta} c_i^{\theta-1} \quad (14)$$

Solve for λ :

$$c_i = (\lambda/p_i)^{1/(1-\theta)} C \quad (15)$$

Ideal price index II

$$\left(\sum c_i^\theta\right)^{1/\theta} = C\lambda^{1/(1-\theta)} \left(\sum p_i^{\theta/(1-\theta)}\right)^{1/\theta} \quad (16)$$

$$\lambda = \left(\sum p_i^{\theta/(1-\theta)}\right)^{(1-\theta)/\theta} \quad (17)$$

Substitute and simplify.

The demand functions $c_i/C = (p_i/P)^{-\varepsilon}$ emerge.

QED

Digression: An Alternative Derivation

By definition:

$$PC = \sum p_i c_i \quad (18)$$

We need to express C and $\sum p_i c_i$ as functions of prices to solve for P .

First-order conditions determine relative demands:

$$c_i/c_1 = p_i^{-\varepsilon}/p_1^{-\varepsilon} \quad (19)$$

Sub into expression for

$$\begin{aligned} \sum p_i c_i &= c_1 \sum p_i (c_i/c_1) \\ &= c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \end{aligned}$$

Alternative Derivation

Sub the same into expression for

$$\begin{aligned}C &= c_1 \left(\sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 \left(\sum (p_i/p_1)^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\&= c_1 p_1^\varepsilon \left(\sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}\end{aligned}$$

Take the ratio:

$$P = \frac{PC}{C} = \frac{c_1 p_1^\varepsilon}{c_1 p_1^\varepsilon} \frac{\sum p_i^{1-\varepsilon}}{(\sum p_i^{1-\varepsilon})^{\varepsilon/(\varepsilon-1)}}$$

Simplify to get the solution for P .

Alternative Derivation

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \quad (20)$$

Proof:

$$p_i c_i = p_i c_1 (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^\varepsilon \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^\varepsilon P^{1-\varepsilon} \end{aligned}$$

$$PC P^{\varepsilon-1} = c_1 p_1^\varepsilon$$

Rearrange. QED.

References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.