# Which findings should be published?

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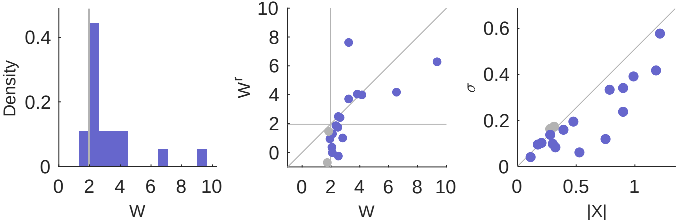
# Introduction

- Not all empirical findings get published (prominently).
  - Selection for publication might depend on findings.
  - Selection criteria: Statistical significance, surprisingness, or confirmation of prior beliefs.
- This might be a problem.
  - Selective publication distorts statistical inference.
  - If only positive significant estimates get published, then published estimates are systematically upward-biased.
  - Explanation of "replication crisis?"
  - Ioannidis (2005), Christensen and Miguel (2016).

# Evidence on selective publication




- Data from Camerer et al. (2016), replications of 18 lab experiments in QJE and AER, 2011-2014.

  left Histogram: Jump in density of z-stats at critical value.

  middle Original and replication estimates: More cases where original estimate is significant and replication not, than reversely.

  right Original estimate and standard error: Larger estimates for larger standard errors.

- Andrews and Kasy (2017): Can use replications (middle) or meta-studies (right) to identify selective publication.

# Reforming scientific publishing

- Publication bias motivates calls for reform:
  Publication should not select on findings.
  - De-emphasize statistical significance, ban "stars."
  - Pre-analysis plans to avoid selective reporting of findings.
  - Registered reports reviewed and accepted prior to data collection.
- But: Is eliminating bias the right objective?
  How does it relate to informing decision makers?
- We characterize optimal publication rules from an instrumental perspective:
  - Study might inform the public about some state of the world.
  - Then the public chooses a policy action.
  - Take as given that not all findings get published (prominently).

# Key results

1. Optimal rules selectively publish surprising findings.
   In leading examples: Similar to two-sided or one sided tests.
2. But: Selective publication always distorts inference.
   There is a trade-off policy relevance vs. statistical credibility.
3. With dynamics: Additionally publish precise null results.
4. With incentives: Modify publication rule to encourage more precise studies.

# Example of relevance-credibility trade-off

- Suppose that there are many potential medical treatments tested in clinical trials.
- Most of them are ineffective.
- Doctors don't have the time to read about all of them.
- Two possible publication policies:
  1. Publish only the most successful trials.
     - The published effects are systematically upward biased.
     - But doctors learn about the most promising treatments.
  2. Publish based on sample sizes and prior knowledge, but independent of findings.
     - Then the published effects are unbiased.
     - But doctors don't learn about the most promising treatments.

# Roadmap

1. **Baseline model.**
2. Optimal publication rules in the baseline model.
3. Selective publication and statistical inference.
4. Extension 1: Dynamic model.
5. Extension 2: Researcher incentives.
6. Conclusion.

# Baseline model – timeline and notation

| | |
|---|---|
| State of the world | $\theta$ |
| Common prior | $\theta \sim \pi_0$ |
| **Study might be submitted** | |
| Exogenous submission probability | $q$ |
| Design (e.g., standard error) | $S \perp \theta$ |
| Findings | $X \sim f_{X\mid\theta,S}$ |
| **Journal decides whether to publish** | $D \in \{0,1\}$ |
| Publication probability | $p(X,S)$ |
| Publication cost | $c$ |
| Public updates beliefs | $\pi_1 = \pi_1^{(X,S)}$ if $D = 1$ |
| | $\pi_1 = \pi_1^0$ if $D = 0$ |
| **Public chooses policy action** | $a = a^*(\pi_1) \in \mathbb{R}$ |
| Utility | $U(a,\theta)$ |
| Social welfare | $U(a,\theta) - Dc.$ |

# Belief updating and policy decision

- Public belief when study is published: $\pi_1^{(X,S)}$.
  - Bayes posterior after observing $(X,S)$
  - Same as journal's belief when study is submitted.
- Public belief when no study is published: $\pi_1^0$.
  Two alternative scenarios:
  1. Naive updating: $\pi_1^0 = \pi_0$.
  2. Bayes updating: $\pi_1^0$ is Bayes posterior given no publication.
- Public action $a = a^*(\pi_1)$
  maximizes posterior expected welfare, $\mathbb{E}_{\theta \sim \pi_1}[U(a,\theta)]$.
  Default action $a^0 = a^*(\pi_1^0)$.

# Optimal publication rules

- **Interim gross benefit** $\Delta(\pi, a^0)$ of publishing equals
  - Expected welfare given publication, $\mathbb{E}_{\theta\sim\pi}[U(a^*(\pi), \theta)]$,
  - minus expected welfare of default action, $\mathbb{E}_{\theta\sim\pi}[U(a^0, \theta)]$.
- **Interim optimal publication rule**:
  Publish if interim benefit exceeds cost $c$.
- Want to maximize **ex-ante expected welfare**:

$$\begin{aligned} EW(p, a^0) =& \mathbb{E}[U(a^0, \theta)] \\ & + q \cdot \mathbb{E}\Big[p(X, S) \cdot (\Delta(\pi_1^{(X,S)}, a^0) - c)\Big]. \end{aligned}$$

- Immediate consequence:
  **Optimal policy is interim optimal** given $a^0$.

# Optimality and interim optimality

- Under **naive updating**:
  - Default action $a^0 = a^*(\pi_0)$ does not depend on $p$.
  - **Interim optimal** rule given $a^0$ **is optimal**.
- Under **Bayes updating**:
  - $a^0$ maximizes $EW(p, a^0)$ given $p$.
  - $p$ maximizes $EW(p, a^0)$ given $a^0$, when interim optimal.
  - These conditions are **necessary but not sufficient** for joint optimality.
- **Commitment does not matter** in our model.
  - Ex-ante optimal is interim optimal.
  - This changes once we consider researcher incentives (endogenous study submission).

# Leading examples

- **Normal prior and signal**, normal posterior:

$$\theta \sim \pi_0 = \mathcal{N}(\mu_0, \sigma_0^2)$$
$$X|\theta, S \sim \mathcal{N}(\theta, S^2)$$

- **Canonical utility functions**:
  1. Quadratic loss utility, $\mathscr{A} = \mathbb{R}$:

  $$U(a,\theta) = -(a-\theta)^2$$

  Optimal policy action: $a =$ posterior mean.

  2. Binary action utility, $\mathscr{A} = \{0,1\}$:

  $$U(a,\theta) = a \cdot \theta$$

  Optimal policy action: $a = 1$ iff posterior mean is positive.

# Interim optimal rules for leading examples

- Quadratic loss utility: "**Two-sided test**." Publish if

$$\left|\mu_1^{(X,S)} - a^0\right| \geq \sqrt{c}.$$

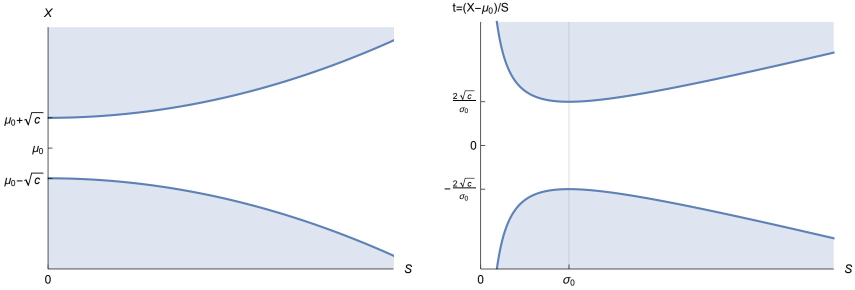
- Binary action utility: "**One-sided test**." Publish if

$$a^0 = 0 \text{ and } \mu_1^{(X,S)} \geq c, \quad \text{or}$$
$$a^0 = 1 \text{ and } \mu_1^{(X,S)} \leq -c.$$

- Normal prior and signals:

$$\mu_1^{(X,S)} = \tfrac{\sigma_0^2}{S^2+\sigma_0^2} X + \tfrac{S^2}{S^2+\sigma_0^2} \mu_0.$$

# Quadratic loss utility, normal prior, normal signals



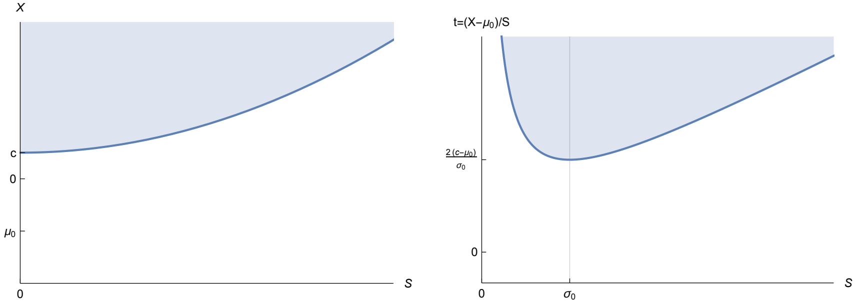

- Optimal publication region (shaded).

  left Axes are standard error $S$, estimate $X$.

  right Axes are standard error $S$, "t-statistic" $(X-\mu_0)/S$.

- Note:
  - Given $S$, publish outside symmetric interval around $\mu_0$.
  - Critical value for t-statistic is non-monotonic in $S$.

# Binary action utility, normal prior, normal signals




- Optimal publication region (shaded).
  - left Axes are standard error $S$, estimate $X$.
  - right Axes are standard error $S$, "t-statistic" $(X - \mu_0)/S$.
- Note:
  - When prior mean is negative, optimal rule publishes for large enough positive $X$.

# Generalizing beyond these examples

Two key results that generalize:

- **Don't publish null results:**
  A finding that induces $a^*(\pi^I) = a^0 = a^*(\pi_1^0)$ always has 0 interim benefit and should never get published.

- **Publish findings outside interval:**
  Suppose
  - $U$ is supermodular.
  - $f_{X|\theta,S}$ satisfies monotone likelihood ratio property given $S = s$.
  - Updating is either naive or Bayes.

  Then there exists an interval $I^s \subseteq \mathbb{R}$ such that $(X, s)$ is published under the optimal rule if and only if $X \notin I^s$.

# Roadmap

1. Baseline model.
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4. Extension 1: Dynamic model.
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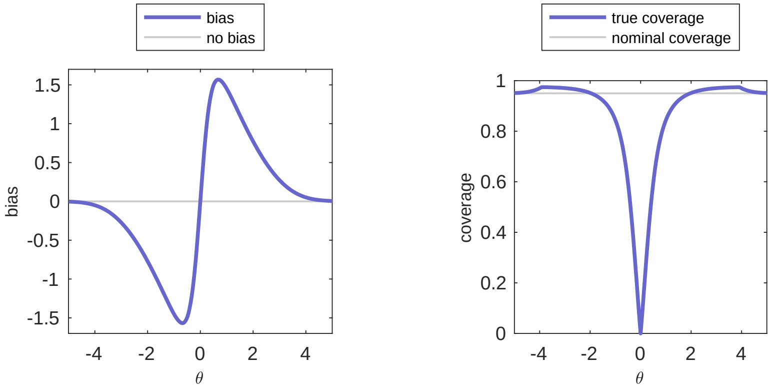
# Selective publication and inference

- Just showed:
  **Optimal publication rules select on findings.**
- But: Selective publication rules can distort inference.
- We show a stronger result:
  Any selective publication rule distorts inference.
- Put differently:
  **If we desire that standard inference be valid, then the publication rule must not select on findings at all.**
- Next two slides:
  1. Bias and size distortions,
  2. distortions of likelihood and of naive posterior,

  when publication is based on statistical significance.

# Distortions of frequentist inference.

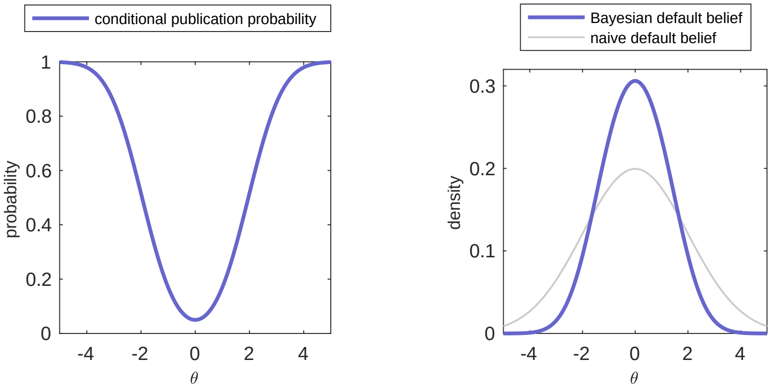



- $X|\theta \sim \mathcal{N}(\theta, 1)$; **publish iff** $X > 1.96$.

  left **Bias** of $X$ as an estimator of $\theta$, conditional on publication.

  right **Coverage** probability of $[X - 1.96, X + 1.96]$ as a confidence set for $\theta$, conditional on publication.

# Distortions of likelihood and Bayesian inference.




- Same model.

  left **Probability of publication** conditional on $\theta$.

  right **Bayesian default belief** and naive default belief, for prior $\theta \sim \mathcal{N}(0,4)$.

# Validity of inference is equivalent to no selection

For normal signals and prior support with non-empty interior, **the following statements are equivalent**:

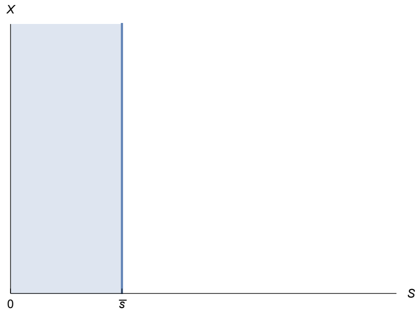
1. Non-selective publication.
   $p(x, s)$ is constant in $x$ for each $s$.
2. Publication probability constant in state.
   $\mathbb{E}[p(X, S)|\theta, S = s]$ is constant over $\theta \in \Theta_0$ for each $s$.
3. Frequentist unbiasedness.
   $\mathbb{E}[X|\theta, S = s, D = 1] = \theta$ for $\theta \in \Theta_0$ and for all $s$.
4. Bayesian validity of naive updating.
   For all distributions $F_S$, the Bayesian default belief $\pi_1^0$ is equal to the prior $\pi_0$.

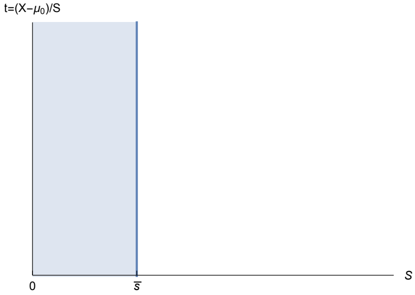
# Intuition and implications

- Sketch of proof:
  - Non-selective publication $\Rightarrow$ the other conditions: immediate.
  - Constant publication probability $\Rightarrow$ non-selective publication: Completeness of the normal location family.
  - Unbiasedness $\Rightarrow$ constant publication probability: "Tweedie's formula" and integration.
- **Optimal publication if we require non-selectivity**?
- Suppose
  - There are normal signals.
  - Updating is either naive or Bayesian.
  - The publication rule is restricted to not select on $X$.

  Then there exists $\bar{s} \geq 0$ for which the optimal rule **publishes** a study **if and only if** $S \leq \bar{s}$.

# Optimal non-selective publication region




- For quadratic loss utility, normal prior, normal signals.
- Subject to the constraint that $p(x, s)$ is restricted to not depend on $x$.

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# A dynamic two-period model

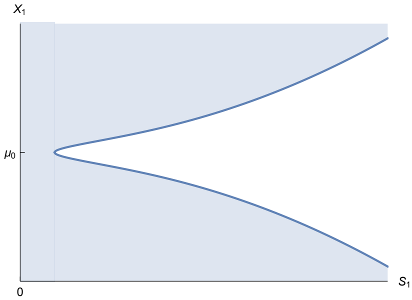
- Period 1 as before, with study $(X_1, S_1)$, action $a_1 = a^*(\pi_1)$.
- Now additionally: Period 2 study, always published.
- Independent estimate

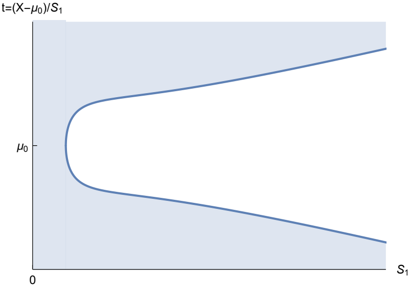
$$X_2|\theta, X_1, S_1 \sim F_{X_2|\theta}.$$

- Period 2 action $a_2 = a^*(\pi_2)$.
- Social welfare

$$\alpha U(a_1, \theta) - Dc + (1-\alpha) U(a_2, \theta).$$

# Quadratic loss utility, normal prior, normal signals, dynamic model with naive updating




- Optimal publication region (shaded).
- Note:
  - For $S$ small enough, publish even when $X = \mu_0$.

# General implications of dynamic extension

- Publishing a precise (null) result in period 1 can help reduce mistakes in period 2.
- Holds under more general conditions, for normal signals:
  1. The benefit of publication is strictly positive whenever $\pi_1^I \neq \pi_1^0$.
  2. The benefit goes to 0 as either $s_2 \to 0$ or $s_2 \to \infty$.
- Put differently:
  1. Even **null results that improve precision** are valuable to **prevent future mistakes.**
  2. This value disappears for
     a) very precise future information
     (won't make any mistakes either way), and
     b) very imprecise future information
     (effectively back to one-period case).

# Researcher Incentives

- Thus far: study submission and design exogenous, random.
- Assume now that a researcher
  1. decides whether or not to submit a study,
  2. and picks a design $S$.
- Normal signals with standard error $S$.
- Researcher utility:
  1. Utility 1 from getting published,
  2. cost $\kappa(S)$ depending on design $S$.
- Expected researcher utility

$$E_{\theta\sim\pi_0, X\sim N(\theta,S^2)}[p(X,S)] - \kappa(S).$$

- Outside option with utility 0.
- Journal faces
  1. participation constraint (PC) and
  2. incentive compatibility constraint (ICC).

# Constrained optimal rule

- Journal objective as before, $U(a,\theta) - Dc$.
- Journal commits to publication rule $p(x,s)$ ex-ante. Commitment matters in this extension!
- Optimal publication rule subject to (PC) and (ICC)?
- Solution: Relative to baseline model, journal **distorts publication rule** in two ways
    - Reject imprecise studies (large $S$) – even if valuable ex post.
    - For low enough $S$, set interim benefit threshold for acceptance below $c$.

# Summary and outlook

- Eliminating selection on findings has costs as well as benefits. Important for reform debates!
- Key results:
  1. Optimal rules selectively publish surprising findings. In leading examples: Similar to two-sided or one sided tests.
  2. But: Selective publication always distorts inference. There is a trade-off policy relevance vs. statistical credibility.
  3. With dynamics: Additionally publish precise null results.
  4. With incentives: Modify publication rule to encourage more precise studies.
- Agenda and open questions:
  - Statistics as (optimal) communication.
    - Not just “you and the data.”
    - What do we communicate to whom?
    - Subject to what costs and benefits?
  - Why not publish everything?
    - Publish all papers, report full datasets in papers,...
    - Attention cost? How much information can readers process?

Thank you!