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Instrumental variables I, origins and binary treatment case

Maximilian Kasy

Department of Economics, Harvard University

Agenda instrumental variables part I

- Origins of instrumental variables: Systems of linear structural equations
- Strong restriction: Constant causal effects.
- Modern perspective: Potential outcomes, allow for heterogeneity of causal effects
- Binary case:
  1. Keep IV estimand, reinterpret it in more general setting: Local Average Treatment Effect (LATE)
  2. Keep object of interest average treatment effect (ATE): Partial identification (Bounds)

Agenda instrumental variables part II

- Continuous treatment case:
  1. Restricting heterogeneity in the structural equation:
     Nonparametric IV (conditional moment equalities)
  2. Restricting heterogeneity in the first stage:
     Control functions
  3. Linear IV:
     Continuous version of LATE

Takeaways for this part of class

- Instrumental variables methods were invented jointly with the idea of economic equilibrium.
- Classic assumptions impose strong restrictions on heterogeneity: same causal effect for every unit.
- Modern formulations based on potential outcomes relax this assumption.
- With effect heterogeneity, average treatment effects are not point-identified any more.
- Two solutions:
  1. Re-interpret the classic IV-coefficient in more general setting.
  2. Derive bounds on the average treatment effect.

Origins of IV: systems of structural equations

- econometrics pioneered by “Cowles commission” starting in the 1930s
- they were interested in demand (elasticities) for agricultural goods
- introduced systems of simultaneous equations
  - outcomes as equilibria of some structural relationships
  - goal: recover the slopes of structural relationships
  - from observations of equilibrium outcomes and exogenous shifters

System of structural equations

$$Y = A \cdot Y + B \cdot Z + \varepsilon,$$

- Y: k-dimensional vector of equilibrium outcomes
- Z: l-dimensional vector of exogenous variables
- A: unknown $k \times k$ matrix of coefficients of interest
- B: unknown $k \times l$ matrix
- ε: further unobserved factors affecting outcomes

Example: supply and demand

$$
\begin{aligned}
Y &= (P, Q) \\
P &= A_{12} \cdot Q + B_1 \cdot Z + \varepsilon_1 \text{ demand} \\
Q &= A_{21} \cdot P + B_2 \cdot Z + \varepsilon_2 \text{ supply}
\end{aligned}
$$

- demand function: relates prices to quantity supplied and shifters Z and ε_1 of demand
- supply function relates quantities supplied to prices and shifters Z and ε_2 of supply.
- does not really matter which of the equations puts prices on the “left hand side.’
- price and quantity in market equilibrium: solution of this system of equations.

Reduced form

- solve equation $Y = A \cdot Y + B \cdot Z + \varepsilon$
  for Y as a function of Z and ε
- bring $A \cdot Y$ to the left hand side,
  pre-multiply by $(I - A)^{-1} \Rightarrow$

$$\begin{aligned} Y &= C \cdot Z + \eta \text{ “reduced form”} \\ C &:= (I - A)^{-1} \cdot B \text{ reduced form coefficients} \\ \eta &:= (I - A)^{-1} \cdot \varepsilon \end{aligned}$$

- suppose $E[\varepsilon | Z] = 0$ (ie., Z is randomly assigned)
- then we can **identify** C from

$$E[Y|Z] = C \cdot Z.$$

Exclusion restrictions

- suppose we know C
- what we want is A, possibly B
- problem: $k \times l$ coefficients in $C = (I - A)^{-1} \cdot B$
  $k \times (k + l)$ coefficients in A and B
- $\Rightarrow$ further assumptions needed
- exclusion restrictions: assume that some of the coefficients in B or A are $= 0$.
- Example: rainfall affects grain supply but not grain demand

Supply and demand continued

- suppose Z is (i) random, $E[\varepsilon|Z] = 0$
- and (ii) “excluded” from the demand equation
  $\Rightarrow B_{11} = 0$
- by construction, $\text{diag}(A) = 0$
- therefore

$$\text{Cov}(Z,P) = \text{Cov}(Z, A_{12} \cdot Q + B_1 \cdot Z + \varepsilon_1) = A_{12} \cdot \text{Cov}(Z,Q),$$

- $\Rightarrow$ the slope of demand is identified by

$$A_{12} = \frac{\text{Cov}(Z,P)}{\text{Cov}(Z,Q)}.$$

- Z is an **instrumental variable**

Remarks

- historically, applied researchers have not been very careful about choosing Z for which
  (i) randomization and (ii) exclusion restriction are well justified.
- since the 1980s, more emphasis on credibility of identifying assumptions
- some additional problematic restrictions we imposed:
  1. linearity
  2. constant (non-random) slopes
  3. heterogeneity ε is k dimensional and enters additively
- $\Rightarrow$ causal effects assumed to be the same for everyone
- next section: framework which does not impose this

Modern perspective: Treatment effects and potential outcomes

- coming from biostatistics / medical trials
- potential outcome framework: answer to “what if” questions
- two “treatments:” $D = 0$ or $D = 1$
- eg. placebo vs. actual treatment in a medical trial
- Y_i person i’s outcome
  eg. survival after 2 years
- potential outcome Y_i^0:
  what if person i would have gotten treatment 0
- potential outcome Y_i^1:
  what if person i would have gotten treatment 1
- question to you: is this even meaningful?

- causal effect / treatment effect for person i :
  $Y_i^1 - Y_i^0$.
- average causal effect / average treatment effect:

$$ATE = E[Y^1 - Y^0],$$

- expectation averages over the population of interest

The fundamental problem of causal inference

- **we never observe both Y^0 and Y^1 at the same time**
- one of the potential outcomes is always missing from the data
- treatment D determines which of the two we observe
- formally:

$$Y = D \cdot Y^1 + (1 - D) \cdot Y^0.$$

Selection problem

- distribution of Y^1 among those with $D = 1$
  need not be the same as the distribution of Y^1 among everyone.
- in particular

$$E[Y|D=1] = E[Y^1|D=1] \neq E[Y^1]$$
$$E[Y|D=0] = E[Y^0|D=0] \neq E[Y^0]$$
$$E[Y|D=1] - E[Y|D=0] \neq E[Y^1 - Y^0] = ATE.$$

Randomization

- no selection $\Leftrightarrow$ D is random

$$(Y^0, Y^1) \perp D.$$

- in this case,

$$\begin{aligned}
E[Y|D=1] &= E[Y^1|D=1] = E[Y^1] \\
E[Y|D=0] &= E[Y^0|D=0] = E[Y^0] \\
E[Y|D=1] - E[Y|D=0] &= E[Y^1 - Y^0] = ATE.
\end{aligned}$$

- can ensure this by actually randomly assigning D
- independence $\Rightarrow$ comparing treatment and control actually compares “apples with apples”
- this gives **empirical content to** the “metaphysical” notion of **potential outcomes**!

Instrumental variables

- recall: simultaneous equations models with exclusion restrictions
- $\Rightarrow$ instrumental variables

$$\beta = \frac{\mathrm{Cov}(Z, Y)}{\mathrm{Cov}(Z, D)}.$$

- we will now give a new interpretation to β
- using the potential outcomes framework, allowing for heterogeneity of treatment effects
- "Local Average Treatment Effect" (LATE)

6 assumptions

1. $Z \in \{0,1\}$, $D \in \{0,1\}$
2. $Y = D \cdot Y^1 + (1-D) \cdot Y^0$
3. $D = Z \cdot D^1 + (1-Z) \cdot D^0$
4. $D^1 \geq D^0$
5. $Z \perp (Y^0, Y^1, D^0, D^1)$
6. $\text{Cov}(Z,D) \neq 0$

Discussion of assumptions

Generalization of randomized experiment

- D is "partially randomized"
- instrument Z is randomized
- D depends on Z, but is not fully determined by it

1. **Binary treatment and instrument:**
   both D and Z can only take two values
   results generalize, but things get messier without this
2. **Potential outcome equation for Y:** $Y = D \cdot Y^1 + (1 - D) \cdot Y^0$
   - *exclusion restriction*: Z does not show up in the equation determining the outcome.
   - *"stable unit treatment values assumption"* (SUTVA): outcomes are not affected by the treatment received by other units.
     excludes general equilibrium effects or externalities.

3. **Potential outcome equation for D:** $D = Z \cdot D^1 + (1 - Z) \cdot D^0$
   SUTVA; treatment is not affected by the instrument values of other units
4. **No defiers:** $D^1 \geq D^0$
   - four possible combinations for the potential treatments (D^0, D^1) in the binary setting
   - $D^1 = 0, D^0 = 1$, is excluded
   - $\Leftrightarrow$ monotonicity

Table: No defiers

	D^0	D^1
Never takers (NT)	0	0
Compliers (C)	0	1
Always takers (AT)	1	1
~~Defiers~~	1	0

5. **Randomization:** $Z \perp (Y^0, Y^1, D^0, D^1)$
    - Z is (as if) randomized.
    - in applications, have to justify both exclusion and randomization
    - no reverse causality, common cause!
6. **Instrument relevance:** $\mathrm{Cov}(Z, D) \neq 0$
    - guarantees that the IV estimand is well defined
    - there are at least some compliers
    - testable
    - near-violation: weak instruments

Graphical illustration

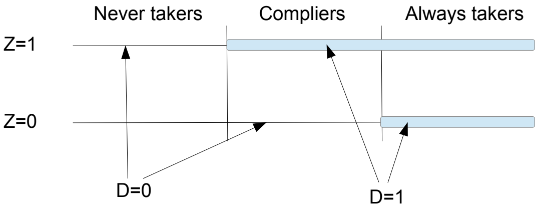


Illustration explained

- 3 groups, never takers, compliers, and always takers
- by randomization of Z:
  each group represented equally given $Z = 0$ / $Z = 1$
- depending on group:
  observe different treatment values and potential outcomes.
- will now take the IV estimand

$$\frac{\mathrm{Cov}(Z, Y)}{\mathrm{Cov}(Z, D)}$$

- interpret it in terms of potential outcomes:
  average causal effects for the subgroup of compliers
- idea of proof:
  take the “top part” of figure 28, and subtract the “bottom part.”

Preliminary result:

If Z is binary, then

$$\frac{\text{Cov}(Z,Y)}{\text{Cov}(Z,D)} = \frac{E[Y|Z=1]-E[Y|Z=0]}{E[D|Z=1]-E[D|Z=0]}.$$

Practice problem

Prove this.

Proof

- Consider the covariance in the numerator:

$$\begin{aligned} \text{Cov}(Z,Y) &= E[YZ] - E[Y]\cdot E[Z] \\ &= E[Y|Z=1]\cdot E[Z] - (E[Y|Z=1]\cdot E[Z] + E[Y|Z=0]\cdot E[1-Z])\cdot E[Z] \\ &= (E[Y|Z=1] - E[Y|Z=0])\cdot E[Z]\cdot E[1-Z]. \end{aligned}$$

- Similarly for the denominator:

$$Cov(Z,D) = (E[D|Z=1] - E[D|Z=0])\cdot E[Z]\cdot E[1-Z].$$

- The $E[Z]\cdot E[1-Z]$ terms cancel when taking a ratio

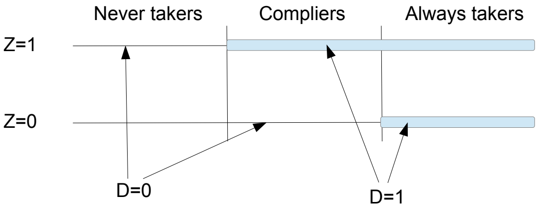
The "LATE" result

$$\frac{E[Y|Z=1]-E[Y|Z=0]}{E[D|Z=1]-E[D|Z=0]} = E[Y^1 - Y^0 | D^1 > D^0]$$

Practice problem

Prove this.

Hint: decompose $E[Y|Z=1]-E[Y|Z=0]$ in 3 parts corresponding to our illustration


Never takers
Compliers
Always takers
Z=1
Z=0
D=0
D=1

Proof

- "top part" of figure:

$$
\begin{aligned}
E[Y|Z=1] = E[Y|Z=1,NT]\cdot P(NT|Z=1) \\
+E[Y|Z=1,C]\cdot P(C|Z=1) \\
+E[Y|Z=1,AT]\cdot P(AT|Z=1) \\
= E[Y^0|NT]\cdot P(NT) + E[Y^1|C]\cdot P(C) + E[Y^1|AT]\cdot P(AT).
\end{aligned}
$$

  - first equation relies on the no defiers assumption
  - second equation uses the exclusion restriction and randomization assumptions.

- Similarly

$$
\begin{aligned}
E[Y|Z=0] = E[Y^0|NT]\cdot P(NT) + \\
E[Y^0|C]\cdot P(C) + E[Y^1|AT]\cdot P(AT).
\end{aligned}
$$

proof continued:

- Taking the difference, only the complier terms remain, the others drop out:

$$E[Y|Z=1]-E[Y|Z=0]=\left(E[Y^1|C]-E[Y^0|C]\right)\cdot P(C).$$

- denominator:

$$\begin{aligned}E[D|Z=1]-E[D|Z=0]&=E[D^1]-E[D^0]\\&=(P(C)+P(AT))-P(AT)=P(C).\end{aligned}$$

- taking the ratio, the claim follows. $\square$

Recap

LATE result:

- take the **same statistical object** economists estimate a lot
- which used to be interpreted as average treatment effect
- **new interpretation** in a more general framework
- allowing for heterogeneity of treatment effects
- $\Rightarrow$ treatment effect for a subgroup

Practice problem

Is the LATE, $E[Y^1 - Y^0 | D^1 > D^0]$, a structural object?

An alternative approach: Bounds

- keep the **old structural object** of interest: average treatment effect
- but analyze its identification in the more general framework with heterogeneous treatment effects
- in general: we can learn something, not everything
- $\Rightarrow$ bounds / "**partial identification**"

Same assumptions as before

1. $Z \in \{0,1\}$, $D \in \{0,1\}$
2. $Y = D \cdot Y^1 + (1-D) \cdot Y^0$
3. $D = Z \cdot D^1 + (1-Z) \cdot D^0$
4. $D^1 \geq D^0$
5. $Z \perp (Y^0, Y^1, D^0, D^1)$
6. $\text{Cov}(Z,D) \neq 0$

   additionally:

7. Y is bounded, $Y \in [0,1]$

Decomposing ATE in known and unknown components

- decompose $E[Y^1]$:

$$E[Y^1] = E[Y^1|NT] \cdot P(NT) + E[Y^1|C \vee AT] \cdot P(C \vee AT).$$

- terms that are identified:

$$\begin{aligned} E[Y^1|C \vee AT] &= E[Y|Z=1, D=1] \\ P(C \vee AT) &= E[D|Z=1] \\ P(NT) &= E[1-D|Z=1] \end{aligned}$$

and thus

$$E[Y^1|C \vee AT] \cdot P(C \vee AT) = E[YD|Z=1].$$

- Data tell us nothing about $E[Y^1|NT]$.
  Y^1 is never observed for never takers.
- but we know, since Y is bounded, that

$$E[Y^1|NT] \in [0, 1]$$

- Combining these pieces, get upper and lower bounds on $E[Y^1]$:

$$\begin{aligned} E[Y^1] \in [&E[YD|Z=1], \\ &E[YD|Z=1] + E[1-D|Z=1]]. \end{aligned}$$

- For Y^0, similarly

$$E[Y^0] \in [E[Y(1-D)|Z=0], \\ E[Y(1-D)|Z=0] + E[D|Z=0]].$$

- Data are uninformative about $E[Y^0|AT]$.

Practice problem

Show this.

Combining to get bounds on ATE

- lower bound for $E[Y^1]$, upper bound for $E[Y^0]$ $\Rightarrow$ lower bound on $E[Y^1 - Y^0]$

$$E[Y^1 - Y^0] \geq E[YD|Z=1] - E[Y(1-D)|Z=0] - E[D|Z=0]$$

- upper bound for $E[Y^1]$, lower bound for $E[Y^0]$ $\Rightarrow$ upper bound on $E[Y^1 - Y^0]$

$$E[Y^1 - Y^0] \leq E[YD|Z=1] - E[Y(1-D)|Z=0] + E[1-D|Z=1]$$

Between randomized experiments and nothing

- bounds on ATE:

$$E[Y^1 - Y^0] \in [E[YD|Z=1] - E[Y(1-D)|Z=0] - E[D|Z=0],$$
$$E[YD|Z=1] - E[Y(1-D)|Z=0] + E[1-D|Z=1]].$$

- length of this interval:

$$E[1-D|Z=1] + E[D|Z=0] = P(NT) + P(AT) = 1 - P(C)$$

- Share of compliers $\to 1$
  - interval ("identified set") shrinks to a point
  - In the limit, $D = Z$
  - thus $(Y^1, Y^0) \perp D$ – randomized experiment
- Share of compliers $\to 0$
  - length of the interval goes to 1
  - In the limit the identified set is the same as without instrument

References

- Local average treatment effect:

  Angrist, J., Imbens, G., and Rubin, D. (1996). Identification of causal effects using instrumental variables. Journal of the American Statistical Association, *91(434):444–455.*

- Bounds on the average treatment effect:

  Manski, C. (2003). Partial identification of probability distributions. *Springer Verlag, chapter 2 and 7.*