Who wins, who loses?
Identification of the welfare impact of changing wages.*

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Abstract

The incidence of tax and other policy changes depends on their impact on equilibrium wages. The impact of wage changes on a worker's welfare equals current labor supply times the induced wage change, in a standard model of labor supply. Worker heterogeneity implies that wage changes vary across workers. In this context, in order to identify welfare effects one needs to identify the conditional causal effect of policy changes on wages given baseline labor supply and wages.

This paper characterizes identification of such conditional causal effects for general vectors of endogenous outcomes. Even with exogenous policy variation, conditional causal effects are only partially identified for outcome vectors of dimension larger than one. We provide assumptions restricting heterogeneity of effects just enough for point-identification and propose corresponding estimators.

This paper then applies the proposed methods to analyze the distributional welfare impact (i) of the expansion of the Earned Income Tax Credit (EITC) in the 1990s, using variation in state supplements in order to identify causal effects, and (ii) of historical changes of the wage distribution in the US in the 1990s. For the EITC, we find negative welfare effects of depressed wages as a consequence of increased labor supply, in particular for individuals earning around 20.000$ per year. Looking at historical changes, we find modest welfare gains rising linearly with earnings.

JEL Codes: C21, H22, J3
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1 Introduction

Reforms of the tax system have a direct impact on the welfare of tax payers and transfer recipients. They also affect incentives and thereby possibly labor supply and demand, so that they might shift the equilibrium in the labor market, and the wage distribution. This paper proposes methods for an ex-post evaluation of the welfare impact of such equilibrium effects in the labor market.

Two features make this a hard problem. First, welfare is not directly observable, in contrast to wages or earnings. By the envelope theorem, however, behavioral responses to marginal wage changes do not affect welfare, so that the welfare effect of a marginal wage change is equal to the wage change times a worker's baseline labor supply. Second, workers are heterogeneous, and thus so is the impact of policy changes on their wages. Suppose we are interested in the average welfare impact for workers of a given level of initial earnings; this is a key ingredient for a social welfare evaluation when welfare weights might vary across earnings levels. Then the combination of these two features implies that one needs to identify the average causal effect of the policy change on wages conditional on the initial wage and labor supply of a worker, both of which are endogenous to the policy.

Abstractly, one needs to identify the causal effect of a marginal policy change on a vector of endogenous outcomes, such as wage and labor supply, conditional on the initial value of this vector. Without conditioning, this is the standard problem of causal inference, to be solved with (quasi-)experimental variation of the policy. The case with conditioning on a one-dimensional endogenous outcome was solved by Hoderlein and Mammen (2007). The key technical contribution of this paper is a characterization of the case with conditioning on a vector of dimension larger than one, as needed for welfare evaluations. Our characterization is based on insights from continuum mechanics (fluid dynamics), relating changes of a density to the underlying flows.

Contributions This paper contributes to the literature in several ways. First, the paper discusses a static model of labor supply with wages endogenous to policy. In this model, it is shown that one needs to identify average causal effects conditional on a vector of endogenous outcomes in order to make welfare statements. Second, in the paper's main theoretical contribution, we characterize identification of conditional causal effects in a series of results drawing on insights from continuum mechanics. Third, these results are applied in an empirical analysis of the distributional impact of the earned income tax credit (EITC) expansion of the 1990s and of historical changes in the wage distribution during the same period.

Economic setting In Section 2, a simple static model of labor supply with income taxation and unrestricted heterogeneity across workers is introduced. Both taxes and wages may depend on a policy parameter x; the same holds for labor supply and consumption decisions. We will consider reduced form effects and will not model equilibrium explicitly. We are interested in the impact of marginal changes of x on social welfare, where social welfare is given by some aggregation of individual realized utility using generalized social welfare weights as in Saez and Stantcheva (2016).

The model allows the welfare weights corresponding to our aggregation to depend on income. This implies that we need to identify the welfare impact of the change of x conditional on income. Equivalently, we need to identify the impact on wages conditional on initial wage and labor supply, both of which are endogenous to the policy.

This paper focuses on a simple static model of labor supply, but the underlying logic based on the envelope theorem extends to more general settings. The envelope theorem has been shown to hold under quite general conditions by Milgrom and Segal (2002); it allows for discrete choices, dynamic considerations, etc.

Identification In Section 3 we consider an abstract setup with a vector of endogenous outcomes y that might be affected by an exogenous policy x; this abstract setup is motivated by the labor market setting just discussed. Our analysis starts with the baseline case where x is randomly assigned, so that the standard problem of causal inference poses no challenge: Under random assignment, the average effect of a marginal change of x on y, conditional on x, is identified by the slope of $E[y|x]$ with respect to x. We are, however, interested in the average effect of a marginal change of x on y, conditional not only on x but also on y. When the dimension of y is one, this effect is identified by the slope with respect to x of the conditional quantile $Q^{y|x}(v|x)$ of y given x, where $v = F(y|x)$, as shown in the elegant result of Hoderlein and Mammen (2007); we focus on the case where the dimension of y is bigger than one.

Assuming exogenous variation of x, the first result of Section 3 shows that the data identify the divergence of the conditional average effect of interest $h(y, x)$. The difference between any pair of functions in the identified set therefore has divergence 0. We provide an explicitly constructed function h^0 in the identified set, and describe the space $\mathscr{H}$ of functions $\tilde{h}$ of divergence 0.

The second result of Section 3 characterizes the space $\mathscr{H}$. When $\dim(y) = 1$ we get $\mathscr{H} = \{0\}$, and we recover the point identification result of Hoderlein and Mammen (2007). When $\dim(y) > 1$, as in our application to welfare effects, then $\mathscr{H}$ is a non-trivial space of functions (corresponding to incompressible flows in continuum mechanics), and point identification fails.

The third result of Section 3 provides assumptions sufficient for just-identification of h. These assumptions restrict the heterogeneity of causal effects. We argue that these assumptions are plausible when conditioning on a rich set of covariates. Under these assumptions, the effects of interest are identified by quantile regressions with control functions. The section concludes by considering extensions to the non-randomized case.

Application to the EITC expansion In Section 4, these identification results are applied to evaluate (i) historical changes of the wage distribution in the US and (ii) the welfare impact of the EITC expansion in the 1990s. The EITC is a refundable tax credit for low to moderate income working individuals and couples. A large literature documents that the EITC expansion increased labor supply, and these increases in labor supply likely depressed wages in the labor markets affected. Following Leigh (2010), we leverage the considerable variation across states and time in state-level supplements to the federal EITC in order to identify causal effects of the EITC, using a quantile difference-in-differences approach.

We find that the EITC expansion indeed decreased wages and thereby welfare, in particular for individuals with earnings around 20,000-30,000 US$ per year. These estimates suggest that the incidence of the EITC expansion was primarily on the employers' side. As for the welfare impact of historical wage changes over the period 1989-2002, we find modest welfare increases rising linearly with earnings, on the order of an average annual welfare increase of 0.6% of earnings.

Literature This paper draws on several literatures in economics that aim to empirically evaluate the distributional impact of policies or historical changes. This includes the empirical optimal tax literature in public finance (eg. Mirrlees, 1971; Saez, 2001; Chetty, 2009; Hendren, 2016; Saez and Stantcheva, 2016), the labor economics literature on determinants of the wage distribution (eg. Autor et al., 2008; Card, 2009), and the distributional decomposition literature (eg. DiNardo et al., 1996; Firpo et al., 2009). Our approach differs from these literatures as follows. (i) This paper allows for endogenous wages, in contrast to much of the empirical (income) taxation literature (with some exceptions, such as Kubik 2004 and Rothstein 2010). (ii) This paper is interested in (unobserved) realized utility rather than observed wages or incomes, in contrast to the wage distribution and decomposition literature.

Our empirical application builds on a large literature suggesting that the EITC expansion increased labor supply, including Meyer and Rosenbaum (2001); Chetty et al. (2013), and possibly depressed wages, see Rothstein (2010); Leigh (2010). The EITC expansion might have had impacts along other dimensions, too; Heckman et al. (2002) discuss its impact on skill formation.

Our identification results for multi-dimensional endogenous outcomes generalize and build on the characterization of Hoderlein and Mammen (2007), who consider the one-dimensional outcome case. Our results draw on ideas from continuum mechanics (fluid dynamics) and the theory of differential forms (cf. Rudin, 1991, chapter 10). The estimators proposed build on the well established literatures on quantile regression and nonparametric regression; see in particular Koenker (2005). The application uses panel data and a quantile difference-in-differences approach; for recent contributions and alternative approaches to identification using panel data see also Athey and Imbens (2006), Graham and Powell (2012), Chernozhukov et al. (2013), and Chernozhukov et al. (2015).

Roadmap The rest of this paper is structured as follows. Section 2 introduces a static model of labor supply with income taxation. This model motivates our empirical objects of interest. Section 3 discusses identification of conditional causal effects in a general setting. Section 4 applies these identification results to evaluate the expansion of EITC transfers in the 1990s, and historical wage changes over the same period. Section 5 concludes. Appendix A contains all proofs.

2 Taxes, wages, and welfare

Consider a population of individuals i making labor supply and consumption decisions. We are interested in the effect that marginal policy changes have on the welfare of these individuals and on social welfare. We consider policy changes that affect individuals both directly, through their impact on the income tax schedule $t(\cdot, x)$, and indirectly, through their impact on market wages w. We do not model labor market equilibrium explicitly, but rather allow for a reduced-form mechanism linking policy to wages. Policies are indexed by a parameter x. Individual welfare is given by realized utility υ_i. Social welfare SWF is a weighted sum of individuals' welfare, as in Saez and Stantcheva (2016). We aim to identify the impact of policy changes on social welfare, in particular the impact mediated through general equilibrium effects in the labor market.

The individual's problem Consider an individual i who maximizes her utility u_i. She chooses her labor supply l_i and consumption vector c_i, given prices p and her market wage w_i, subject to an income tax $t_i = t(l_i \cdot w_i, x)$, so that

$$(c_i, l_i) = \operatorname*{argmax}_{c,l}\ u_i(c, l) \quad s.t. \quad c_i \cdot p \leq l_i \cdot w_i - t(l_i \cdot w_i, x). \tag{1}$$

Her realized utility is given by $\upsilon_i = u_i(c_i, l_i)$. Denote pre-tax earnings by $a_i = l_i \cdot w_i$, and after-tax income by $z_i = a_i - t(a_i, x)$.

If there are general equilibrium effects of policy changes on the labor market, the wages w_i may depend on x, $w_i = w_i(x)$. By utility maximization, the same is true for the choice variables l_i and c_i. We assume, however, that prices are not affected by the policy changes under consideration, and neither are preferences. Assuming differentiability, denote the equilibrium impact of a marginal change of policy x on the wage w_i of worker i by

$$\dot{w}_i = \partial_x w_i(x),$$

the effect on taxes (holding earnings constant) by $\dot{t}_i = \partial_x t(a_i, x)$, and similarly for other variables.

The impact of marginal policy changes Consider now the impact of a marginal policy change (i) on after-tax income z_i, and (ii) on realized money-metric utility. The impact on after-tax income follows from simple differentiation. The impact on money-metric utility e_i

follows from the envelope theorem (Roy's identity).[1] The impact on money-metric utility is also equal to the impact of the policy change on utility υ_i, normalized by the impact of a lump sum transfer of 1 US$ on utility υ_i.

$$
\begin{aligned}
\dot{z}_i &= (\dot{l}_i \cdot w_i + l_i \cdot \dot{w}_i) \cdot (1 - \partial_a t_i) - \dot{t}_i \\
\dot{e}_i &= \qquad\qquad l_i \cdot \dot{w}_i \;\; \cdot (1 - \partial_a t_i) - \dot{t}_i.
\end{aligned} \tag{2}
$$

Both expressions include (i) the mechanical effect of the tax change on income, $\dot{t}_i$, and (ii) the wage effect (or general equilibrium effect), $l_i \cdot \dot{w}_i \;\cdot (1 - \partial_a t_i)$. The first expression additionally includes a term $\dot{l}_i \cdot w_i \;\cdot (1 - \partial_a t_i)$, corresponding to the effect of behavioral responses on earnings. Behavioral responses do not affect individual welfare to first order; this is the envelope theorem.

Social welfare Consider now an aggregation of individual utilities into social welfare as in Saez and Stantcheva (2016). This aggregation is based on generalized welfare weights ω_i. These welfare weights measure the relative marginal value attributed to an additional lump-sum dollar for person i. The impact of a marginal policy change on social welfare in the framework of Saez and Stantcheva (2016) is given by

$$
S\dot{W}F = E[\omega_i \cdot \dot{e}_i | x].
$$

In this expression, the expectation averages over the population of individuals i; we will omit the subscript i for simplicity of notation below. We condition on x to emphasize the dependence of social welfare on the policy choice x. Welfare weights might be informed by various considerations. In many public finance models the welfare weights are a function of earnings $a_i = w_i \cdot l_i$, see for instance Mirrlees (1971); Saez (2001). More generally, we might let them depend on w_i and l_i separately, as well as on observed covariates W_i, $\omega_i = \omega(w_i, l_i, W_i)$. If welfare weights take this form, the law of iterated expectations allows us to write

$$
S\dot{W}F = E[\omega \cdot E[\dot{e}|w, l, x, W]|x], \tag{3}
$$

where, by equation (2),

$$
E[\dot{e}|w, l, x, W] = E[\dot{w}|w, l, x, W] \cdot l \cdot (1 - \partial_a t(a, x)) - \partial_x t(a, x).
$$

In this expression, labor supply l is potentially directly observable, and $\partial_a t(a, x)$ as well as $\partial_x t(a, x)$ can be calculated mechanically. The term which poses an empirical challenge is $E[\dot{w}|w, l, x, W]$. This term corresponds to the general equilibrium effect of the policy change on wages.

[1] Since we consider marginal changes, the welfare effect $\dot{e}_i$ is equal to both equivalent and compensating variation. General conditions for the envelope theorem to hold can be found in Milgrom and Segal (2002); a review of expenditure functions and equivalent variation is provided by Mas-Colell et al. (1995), chapter 3.

The choice of welfare weights ω is a normative choice on which there might be reasonable disagreement. We propose below to report disaggregated estimates, plotting the conditional expected welfare effect of a policy change as a function of income, possibly conditional on other demographic covariates. The reader can then aggregate these welfare effects based on their own choice of welfare weights, trading off winners and losers, to evaluate the overall policy effect.

The identification problem For a social welfare evaluation of the policy change as in equation (3) it is necessary to identify the average causal (equilibrium) effect of a marginal policy change on wages conditional on the vector of endogenous variables w and l, current policy x, as well as exogenous covariates W. Denoting the vector of endogenous variables by $y = (w, l)$ this effect can be written as $E[\dot{y}|y, x, W]$. Dropping covariates for simplicity of notation, the general problem is to identify the average causal effect of x on y conditional on x and y for a vector of outcomes y,

$$g(y, x) = E[\dot{y}|y, x].$$

Suppose that there is random variation of x across labor markets observed in the data; this is the baseline case that will be considered first in Section 3. In that case the standard problem of causal inference poses no challenge, and the average causal effect of a marginal policy change on y, $E[\dot{y}|x]$, is equal to the slope of the regression function, $\partial_x E[y|x]$. It is the additional conditioning on y which makes identification of $g(y, x)$ a more challenging problem relative to identification of $E[\dot{y}|x]$.

Welfare versus other outcome measures In this paper we are interested in evaluating the effects $\dot{e}$ of a policy change on individual welfare, which in turn map into the effect $S\dot{W}F$ on social welfare. It is useful to contrast the welfare effects $\dot{e}_i = l_i \cdot \dot{w}_i \cdot (1 - \partial_a t_i) - \dot{t}_i$ with the effects on other outcome measures considered in the literature.

The empirical literature on optimal income taxation tends to focus on mechanical effects $\dot{t}$; general equilibrium effects on wages are assumed to be absent. This can be justified by models with perfectly substitutable workers. In such models, endogenous responses of behavior need to be taken into account only insofar as they affect government revenues; see for instance Saez (2001). There are of course exceptions; papers discussing endogenous wages and the incidence of income taxes include Kubik 2004 and Rothstein 2010.

The empirical literature on labor demand, in contrast, does consider equilibrium effects of labor supply on wages, but focuses on observed outcomes including wages w_i, labor supply l_i, and earnings $a_i = l_i \cdot w_i$; see for instance Autor et al. (2008); Card (2009).

The empirical literature on the distributional impact of price changes does consider welfare effects of the form $c_i \cdot \dot{p}$, where consumption behavior is held constant; see for instance Deaton (1989). In Deaton (1989) prices are assumed to be homogenous, so that $\dot{p}$ is directly observable. By contrast, our setting is complicated by the fact that wage changes $\dot{w}_i$ are not directly observable in the presence of worker heterogeneity.

3 Identification of conditional causal effects

We now turn to our abstract identification results for conditional causal effects of the form $E[\dot{y}|y, x]$. For most of this section we consider the setting of Assumption 1 below, where y is an endogenous outcome vector that is determined by a policy x and unobserved heterogeneity ϵ. The dimension of ϵ is left unrestricted in order to allow for arbitrary heterogeneity. We assume that x varies randomly across observations, so that the standard problem of causal inference poses no problem - the distribution of y given x is equal to the distribution of potential outcomes.

What makes our problem non-standard is the assumption that $\dim(y) > 1$, and the fact that we are interested in marginal causal effects conditional on y and x. We analyze identification of these conditional causal effects in a series of results, with Theorems 1 and 2 characterizing the identified set without further assumptions, and Theorem 3 providing sufficient additional conditions for point-identification. The section then concludes by discussing how our results extend to settings without random variation of x.

Assumption 1 (Abstract setup)
We observe a random sample from the joint distribution of (x, y)*, where*

- $y = y(x, \epsilon)$
- $y \in \mathbb{R}^k$*,* $x \in \mathbb{R}$*,* ϵ *is a random element in a space of unrestricted dimension.*
- x *is statistically independent of* ϵ*,* $x \perp \epsilon$*.*
- *The support of* x *contains an open neighborhood of* 0*, so that the observed data identify* $f(y|x)$ *for* $x \in (-\delta, \delta)$*.*
- y *is continuously distributed given* x*.*
- $y(x, \epsilon)$ *is differentiable in* x*.*
- *The function* $h(y, x) = E[\dot{y}|y, x] \cdot f(y|x)$ *is continuously differentiable in* y*, and equal to* 0 *outside the compact and convex set* $\mathbf{Y}$*.*

Example: labor market application In the context of our application, x is a tax policy parameter, $y = (w, l)$ are the wage and labor supply of a worker, and ϵ captures all unobserved worker characteristics. The structural function $y(x, \epsilon)$ traces out counterfactual equilibrium outcomes for a given worker as the policy varies. Independence of x and ϵ then means that policy varies randomly across labor markets and workers. The density $f(y|x)$ describes the joint distribution of wages and labor supply under the counterfactual policy x. The remaining assumptions are smoothness conditions.

Notation $f(y|x)$ is the conditional density of y given x. The letter Q denotes (conditional) quantiles. Superscripts j index components of y, while subscripts i index observations. Derivatives with respect to y^j are denoted ∂_{y^j}, and the gradient with respect to y is denoted $\nabla = (\partial_{y^1}, \dots, \partial_{y^k})$. Derivatives with respect to the policy parameter x are written $\dot{f} = \partial_x f(y|x)$, $\dot{y} = \partial_x y(x, \epsilon)$ etc. We define

$$
\begin{aligned}
g(y, x) &:= E[\dot{y}|y, x], \\
h(y, x) &:= g(y, x) \cdot f(y|x),
\end{aligned}
$$

and denote the divergence of h by

$$
\nabla \cdot h := \sum_{j=1}^{k} \partial_{y^j} h^j.
$$

Note that the "flow" g is identified (on the support of f) if and only if the "flow density" h is identified, since $h = g \cdot f$ and the density f is known.

Roadmap We will now develop a series of results characterizing the problem of identifying g (equivalently, h) based on knowledge of f. Theorem 1 first shows that the divergence of h is identified given f via the identity $\dot{f} = -\nabla \cdot h$. Theorem 1 then shows that the reverse is also true: any flow density h that satisfies this equation is in the identified set, absent further restrictions. Theorem 2 characterizes the identified set when the dimension of y is equal to 1, 2, or 3. Theorem 3 imposes the additional restriction $\partial_{y^j} E[\dot{y}^{j'}|y, x] = 0$ for $j > j'$ (a restriction on the heterogeneity of conditional causal effects), and shows that under this restriction h and g are just-identified by nonparametric quantile regressions with control functions. This section concludes by discussing the extension of Theorem 3 to quasiexperimental settings, including settings with conditional independence, instrumental variables, and panel data.

Partial identification The following theorem shows that knowledge of f identifies the divergence of h under Assumption 1. It also shows that the data *only* identify the divergence of h: Any h such that $\dot{f} = -\nabla \cdot h$ is consistent with the observed data and Assumption 1. Equation (5) explicitly provides one particular function h^0 which satisfies the equation $\dot{f} = -\nabla \cdot h$. The theorem further shows that the difference $\widetilde{h}$ between any other function h in the identified set and h^0 is in the set (linear subspace) $\mathscr{H} = \{\widetilde{h} : \nabla \cdot \widetilde{h} \equiv 0\}$. In the statement of the Theorem, $Q^{y^j|v^1,\dots,v^{j-1},x}(v^j|v^1, \dots, v^{j-1}, x)$ denotes the conditional v^j-quantile of y^j given $v^1, \dots, v^{j-1}, x$, so that in particular $Q^{y^j|v^1,\dots,v^{j-1},x}(v^j|v^1, \dots, v^{j-1}, x) = y^j$ when $v^j = F^{y^j|y^1,\dots,y^{j-1},x}(y^j|y^1, \dots, y^{j-1}, x)$.

Theorem 1 (Partial identification, general case)
Suppose Assumption 1 holds. Then

$$
\dot{f} = -\nabla \cdot h. \tag{4}
$$

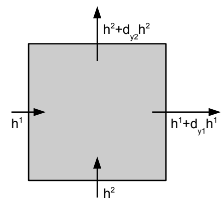

Figure 1: This figure illustrates Theorem 1. It relates the change of density f (mass in the square) to the divergence of h (difference in flow on different sides).

Let v^j be the random variable $v^j = F^{y^j|y^1,\ldots,y^{j-1},x}(y^j|y^1,\ldots,y^{j-1},x)$, define

$$h^{0j}(y,x) = f(y|x) \cdot \partial_x Q^{y^j|v^1,\ldots,v^{j-1},x}(v^j|v^1,\ldots,v^{j-1},x), \tag{5}$$

and let

$$\mathscr{H} = \{\widetilde{h} :\ \nabla \cdot \widetilde{h} \equiv 0,\ \widetilde{h}(y,x) = 0 \textit{ for } y \notin \mathbf{Y}\}. \tag{6}$$

Then the identified set for h is given by

$$h^0 + \mathscr{H}. \tag{7}$$

Intuition and analogy to fluid dynamics Figure 1 provides some intuition for the first result of Theorem 1, that $\dot{f} = -\nabla \cdot h$. Consider the density of observations in the shaded square. This density changes, as x changes, by (i) the difference between the outflow to the right and the inflow to the left, $\partial_{y^1} h^1 \cdot dy^1$, and (ii) the difference between the outflow on the top and the inflow on the bottom, $\partial_{y^2} h^2 \cdot dy^2$. The sum of these changes is equal to $-\sum_{j=1}^k \partial_{y^j} h^j \cdot dy^j$. The divergence $\nabla \cdot h$ thus measures the net outflow at a point corresponding to a flow density h. If the density does not change, or equivalently the net outflow equals 0, then the divergence of h has to be 0.

The setting of Assumption 1 has various analogies in physics, for instance in fluid dynamics. We can think of x as time, ϵ indexing individual particles, and y the position of a particle in space. The function $y(x,\epsilon)$ describes the trajectory of a particle over time. Then $f(y|x)$ is the density of the gas or liquid at location y and time x. As shown in Theorem 1, the change of this density over time is given by the divergence of the flow density (net flow) h. The case $\nabla \cdot h \equiv 0$ corresponds to the flow of an incompressible fluid, the density of which is constant over time, which is for instance approximately true for water. The source of the identification problem we face is thus accurately illustrated by the following analogy: By stirring your coffee (or other beverage of choice), you can create a variety of different flows $h(y,x)$ which are all consistent with the same constant density $f(y|x)$ of the beverage being stirred.

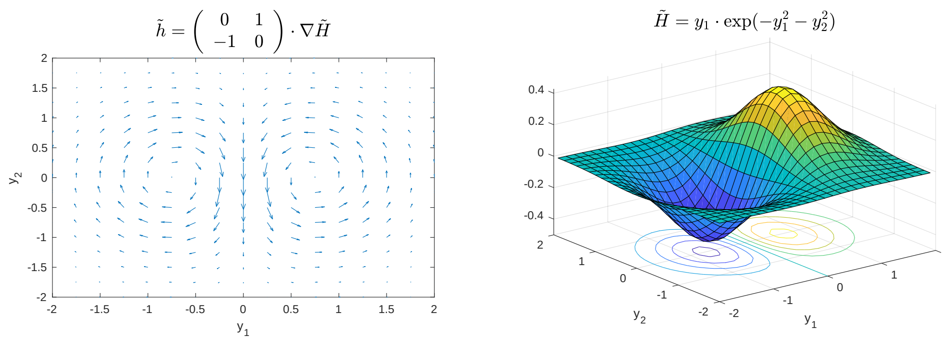

Figure 2: This figure illustrates the characterization of $\mathscr{H}$ in Theorem 2 for the case $k = 2$. The vector field $\widetilde{h}$ (first graph) is given by a 90 degree rotation of the gradient of some function H (second graph), and thus points along the lines of equal level of the function H (bottom of the second graph).

Characterizing the identified set Since the identified set for h is equal to $h^0 + \mathscr{H}$, point identification fails if $\mathscr{H}$ has more than one element. Our next result, Theorem 2, characterizes the nature of non-identification if this is the case. This theorem provides alternative representations of the "kernel" of the identified set which is given by $\mathscr{H} = \{\widetilde{h} : \nabla \cdot \widetilde{h} \equiv 0\}$. Theorem 2 uses Poincaré's Lemma to characterize the set $\mathscr{H}$ for dimensions $k = 1, 2,$ and 3.[2] For dimension 1 we recover the point identification result of Hoderlein and Mammen (2007) as a special case (and thus provide an alternative proof for this result), while for higher dimensions point-identification breaks down; see also Hoderlein and Mammen (2009).

The case $k = 2$ is of special interest in the context of this paper − recall that $y = (w, l)$ in the labor market setting of Section 2. For the case $k = 2$, the characterization takes on a particularly elegant form; the functions $\widetilde{h}$ in the kernel are exactly those functions which can be written as the gradient of some function H, rotated by 90 degrees. $\widetilde{h}$ is thus a vector field pointing along the lines of constant height of H. Figure 2 illustrates. This Figure plots a function $\tilde{H}$ with a local maximum and a local minimum, and the corresponding $\tilde{h}$, a vector field that points along the lines of constant $\widetilde{H}$ circling these extrema.

Theorem 2 (Partial identification, dimensions 1, 2, and 3)
Let $\mathscr{H} = \{\widetilde{h} : \nabla \cdot \widetilde{h} \equiv 0,\ \widetilde{h}(y, x) = 0 \text{ for } y \notin \mathbf{Y}\}$.

1. *Suppose* $k = 1$. *Then* $\mathscr{H} = \{\widetilde{h} \equiv 0\}$.

[2]Similar results can be stated for higher dimensions, but require increasingly cumbersome notation.

2. Suppose $k = 2$ and let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Then

$$\mathscr{H} = \{\widetilde{h} : \widetilde{h} = A \cdot \nabla H, \\ H : \mathbb{R}^2 \to \mathbb{R},\ H(y, x) = 0 \text{ for } y \notin \mathbf{Y}\}.$$

3. Suppose $k = 3$. Then

$$\mathscr{H} = \{\widetilde{h} : \widetilde{h} = \nabla \times G, \\ G : \mathbb{R}^3 \to \mathbb{R}^3,\ G(y, x) = 0 \text{ for } y \notin \mathbf{Y}\}.$$

where $\nabla \times G = \left(\partial_{y^2} G^3 - \partial_{y^3} G^2, \partial_{y^3} G^1 - \partial_{y^1} G^3, \partial_{y^1} G^2 - \partial_{y^2} G^1\right)$.

Just-identification Theorems 1 and 2 characterize the identified set for h absent any further identifying assumptions, that is if only Assumption 1 is imposed. The following theorem shows that the additional assumption of a "triangular" structure for $\nabla E[\dot{y}^j|y, x]$ (derivatives above the diagonal are 0) yields just-identification of h. Note that the ordering of the components of y matters if we assume such a triangular structure.

Theorem 3 (Point-identification)
Suppose Assumption 1 holds. Assume additionally that

$$\partial_{y^j} E[\dot{y}^{j'}|y, x] = 0 \text{ for } j > j'. \tag{8}$$

Then g and h are point identified, and

$$g(y, x) = \partial_x Q^{y^j|v^1,\ldots,v^{j-1},x}(v^j|v^1, \ldots, v^{j-1}, x), \tag{9}$$

where $v^j = F(y^j|y^1, \ldots, y^{j-1}, x)$. The flow density h is equal to h^0 as defined in Theorem 1.

There are no over-identifying restrictions implied by equation (8).

Discussion of the identifying assumption The key assumption that yields point identification is given by Equation (8). This assumption restricts the heterogeneity of causal effects. Consider the case $k = 2$ as in our application, where $y = (w, l)$. Equation (8) then reads $\partial_l E[\dot{w}|w, l, x] = 0$. This assumption implies that the average effect of a policy change on wages, conditional on labor supply and wages, does not depend on labor supply. The effect of a policy change on wages is still allowed to vary across values of the baseline wage, and to be heterogenous given baseline wage and labor supply. In our application this assumption is imposed conditional on observed covariates, so that the policy effect may also vary across observed demographic groups. In Section 4 we discuss empirical results based on this assumption and results based on the analogous assumption with the reverse ordering $y = (l, w)$, where the restriction

$\partial_w E[\dot{l}|w, l, x] = 0$ is imposed.

Equation (8) restricts heterogeneity less than popular assumptions imposed in the labor economics literature. Papers in the literature on the effect of labor supply on wages often estimate parametric demand systems, see for instance Card (2009) or Autor et al. (2008). For such demand systems, shifts of labor supply only affect the relative wages of a finite number of types of workers. Relative wages of different workers belonging to the same type remain unchanged as labor supply varies. Types of workers are defined for instance in terms of education. Put differently, and using our notation, these demand systems imply that there is no systematic heterogeneity of causal effects on log wages conditional on worker types W, $E[\dot{w}/w|l, w, W, x] = E[\dot{w}/w|W, x]$.

The literature on the distribution of treatment effects for binary treatments also needs to impose stronger restrictions than Equation (8) to achieve point identification. Assumptions which achieve point identification include (conditional) independence of potential outcomes for different treatment values, or (conditional) perfect dependence; see for instance Abbring and Heckman (2007). Relative to the binary treatment case, we can impose less stringent assumptions because we have a continuous treatment x and assume smoothness of the response functions $y(., \epsilon)$, and because we do not aim to identify the full distribution of treatment effects. Our setting finally allows for more heterogeneity than models designed to identify structural functions themselves; such models need to restrict the dimension of heterogeneity to be no larger than the dimension of observables, see e.g. Altonji and Matzkin (2005).

Relaxing random variation of x Thus far, we have considered the problem of identifying $E[\dot{y}|y, x]$ under the assumption that x is randomly assigned. We now discuss how to extend our results to quasi-experimental settings where this assumption does not hold, including (i) settings with conditional independence, (ii) instrumental variables, and (iii) panel data.. Throughout we continue to assume that Assumption 1 holds, except for exogeneity of x.

Suppose, first, that $\epsilon \perp x$ does not hold, but that instead x is *conditionally exogenous* given a vector of observed covariates W,

$$x \perp \epsilon | W.$$

In this case, Theorems 1, 2, and 3 continue to hold verbatim after conditioning all expressions on W.

Suppose, next, that x itself is not exogenous, but an *instrument* Z is. Suppose additionally, as in Imbens and Newey (2009), that x is determined by a first-stage relationship of the form $x = k(Z, \eta)$, where k is monotonic in the one-dimensional source of heterogeneity η and

$$Z \perp (\epsilon, \eta).$$

Then conditional independence of x and ϵ holds given the control function $v^z := F(x|z)$. Again, Theorems 1, 2, and 3 hold verbatim after conditioning all expressions on v^z.

Suppose, finally, that x is not exogenous, but we have *panel data* at our disposition, where τ_i is the time period of observation i, and s_i is the cross-sectional unit (e.g., labor market) of observation i. Assume that the policy x is a function of s and τ, $x = x(s, \tau)$, and that the distribution of heterogeneity ϵ does not vary over time within units s,

$$\epsilon|\tau, s \sim \epsilon|s.$$

This assumption is known as "marginal stationarity," cf. Graham and Powell (2012), Chernozhukov et al. (2013). Under this assumption, we can think of time τ as an instrument for the policy x, conditional on s. Assume the restrictions of Equation (8) apply given s, and define v accordingly. Then we get the following variation on the result of Theorem 3,

$$\frac{\partial_\tau Q^{y^j|v^1,\ldots,v^{j-1},s,\tau}(v^j|v^1,\ldots,v^{j-1},s,\tau)}{\partial_\tau x(s,\tau)} = E[\dot{y}^j|y,x,s].$$

In panel data settings it is often assumed that other factors affecting outcomes change over time, in a manner that is shared across units s. To maintain identification of causal effects, it is necessary to restrict the way that time enters the outcome equation. The most common approach, the "difference in differences" approach, is to assume that time fixed effects enter additively. We impose a variant of this assumption in Section 4 below, where we implement a *quantile difference in differences* estimator, allowing for fixed effects that vary across quantiles.

4 Application

This section applies the theoretical results of Sections 3 and 4 to study the welfare impact of changing wages in the United States over the 14-year period 1989-2002. Both the welfare impact of the EITC expansion and the welfare impact of historical changes of the wage distribution are discussed.

To identify the impact of the EITC expansion, variation across states and time in state-level supplements to the federal EITC is used. This identification approach and the data preparation closely follow Leigh (2010), who estimated the impact of the EITC expansion on average log wages and labor supply by educational group. The results presented here differ from those of Leigh (2010) in that we focus on (i) disaggregated, distributional effects rather than averages, and (ii) on welfare rather than observable outcomes.

This section is structured as follows. It starts with a description of our data and the Earned Income Tax Credit, followed by a replication of some of the results of Leigh (2010) and a description of our estimation procedure. Then the empirical findings for both the impact of the EITC and of historical changes of the wage distribution are discussed, as is the robustness of our results to alternative specifications.

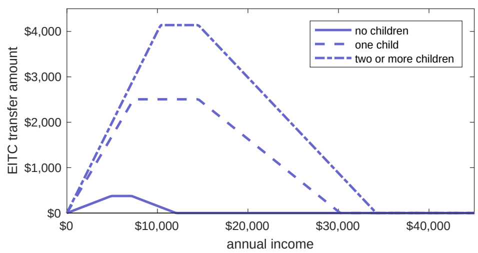

Figure 3: This figure plots federal EITC payments in 2002 as a function of household income and number of children.

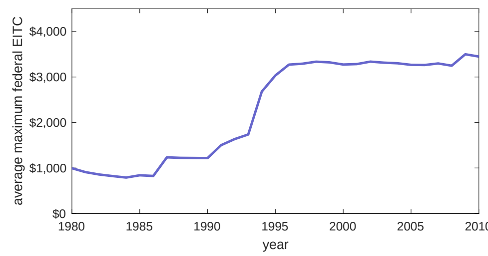

Figure 4: This figure plots weighted average maximum federal EITC payments, measured in 2000 US$.

Data Following Leigh (2010), we use a subsample of the Current Population Survey Merged Outgoing Rotation Group. We restrict the sample to the 14-year period 1989-2002, and to individuals aged 25-55 and not self-employed. Extreme observations with reported earnings less than half the federal minimum wage, or more than 100 times the federal minimum wage are excluded. This leaves us with 1,346,058 observations. Welfare effects are imputed and described for the baseline population corresponding to the subsample of 157,737 observations from the year 2002.

Hourly wage, for those not reporting it directly, is calculated by dividing weekly earnings by usual weekly hours. Wages are converted to wages in 2000 dollars using the CPI. Labor supply is set to "usual hours worked" for those working, and zero for those not employed. The individual-level controls we use include age as well as dummies for gender, whether the respondent identified as black or as hispanic, educational attainment, and number of children, and the same variables for a spouse if present. Unless otherwise noted, all earnings and welfare effects reported are on an annual scale.

The earned income tax credit The EITC is a refundable tax credit for low to moderate income working individuals and couples. The amount of EITC benefits depends on a recipient's income and number of children. Figure 3 plots the schedule of federal EITC payments in 2002 as a function of earnings and of the number of children in a household. For most of our analysis, we pool EITC schedules across numbers of children, and consider the weighted average maximum EITC payment (corresponding to the high-plateaus in Figure 3), with weights 0.4 for 1 child, 0.4 for 2 children, and 0.2 for 3 children. These weights correspond roughly to population proportions. Figure 4 plots the evolution of this weighted average of federal maximum EITC payments, measured in 2000 US$, based on the CPI. As this figure shows, maximum payments increased about threefold in the 1990s. The literature documents that this EITC expansion increased labor supply, cf. Meyer and Rosenbaum (2001); Chetty et al. (2013), and possibly depressed wages, cf. Rothstein (2010); Leigh (2010).

State	CO	DC	IA	IL	KS	MA	MD	ME	MN	MN	NJ	NY	OK	OR	RI	VT	WI	WI	WI
# of children							1+		0	1+		1+					1	2	3+
1984																	30	30	30
1985																	30	30	30
1986															22.21				
1987															23.46				
1988															22.96	23			
1989															22.96	25	5	25	75
1990			5												22.96	28	5	25	75
1991			6.5						10	10					27.5	28	5	25	75
1992			6.5						10	10					27.5	28	5	25	75
1993			6.5						15	15					27.5	28	5	25	75
1994			6.5						15	15		7.5			27.5	25	4.4	20.8	62.5
1995			6.5						15	15		10			27.5	25	4	16	50
1996			6.5						15	15		20			27.5	25	4	14	43
1997			6.5			10			15	15		20		5	27.5	25	4	14	43
1998			6.5		10	10	10		15	25		20		5	27	25	4	14	43
1999	8.5		6.5		10	10	10		25	25		20		5	26.5	25	4	14	43
2000	10	10	6.5	5	10	10	15	5	25	25	10	22.5		5	26	32	4	14	43
2001	10	25	6.5	5	10	15	16	5	33	33	15	25		5	25.5	32	4	14	43
2002	0	25	6.5	5	15	15	16	5	33	33	17.5	27.5	5	5	25	32	4	14	43

Table 1: This table shows the percentage amounts of state supplements to the federal EITC for the years 1984-2002. Based on Table 2 from Leigh (2010).

Some states supplement federal payments using proportional top-ups. Table 1 shows the variation of state supplements to federal EITC payments across states and time, for those states that do provide supplements. Effective EITC payments to any given household are equal to federal EITC payments to this household times (1+state EITC supplement). We use variation of these supplements, interacted with the federal expansion of EITC payments over the period considered, in order to identify the impact of the EITC expansion. Both the expansion of federal payments and state supplements take essentially the form of a proportional increase of payments, rather than varying the other parameters of the EITC schedule that govern the phase-in and phase-out of payments. This setting is therefore well described by a one-dimensional policy parameter x. Our definition of treatment x, indexing the generosity of EITC payments, is

$$x := \log(\text{maximum attainable EITC payments in a state and year}),$$

where payments are measured in 2000 US$. If, for example, a state provides a supplement of 10% to the federal EITC, this implies that the value of treatment x is increased by $0.095 \approx 10\%$ relative to what it would be in the absence of a state supplement.

Replicating Leigh (2010) Table 2 reproduces the main estimates from table 4 and 5 of Leigh (2010). These estimates imply that the expansion of the EITC increased the labor supply and depressed the wages of high school dropouts and of those with a high school diploma only, while

	All adults	High school dropouts	High school diploma only	College graduates
	dependent variable: Log real hourly wage			
Log max EITC	-0.121	-0.488	-0.221	0.008
	(0.064)	(0.128)	(0.073)	(0.056)
Fraction EITC-eligible	9%	25%	12%	3%
	dependent variable: whether employed			
Log max EITC	0.033	0.09	0.042	0.008
	(0.012)	(0.046)	(0.019)	(0.022)
Fraction EITC-eligible	14%	34%	17%	4%
	dependent variable: Log hours per week			
Log max EITC	0.037	0.042	0.011	0.095
	(0.019)	(0.040)	(0.014)	(0.027)
Fraction EITC-eligible	9%	25%	12%	3%

Table 2: Effect of EITC expansion on wages and labor supply. Estimates from Table 4 and 5 of Leigh (2010), for workers with and without children.

	All adults	High school dropouts	High school diploma only	College graduates
	dependent variable: Log real hourly wage			
Log max EITC	-0.105	-0.376	-0.223	0.0137
	(0.0768)	(0.134)	(0.0884)	(0.0548)
	dependent variable: whether employed			
Log max EITC	0.0182	0.0807	0.0342	-0.00531
	(0.0156)	(0.0413)	(0.0224)	(0.0255)
	dependent variable: Log hours per week			
Log max EITC	0.0307	-0.0289	0.0124	0.0699
	(0.0178)	(0.0393)	(0.0120)	(0.0203)

Table 3: Effect of EITC expansion without policy controls. This table replicates the results of Table 2, using our definition of treatment x, without state-level controls for other policies, and without weighting observations.

only having a small effect on the rest of the population. Notice the large magnitude of these effects. The reported coefficient for wages suggests that a 10% expansion of EITC payments results in a 5% drop of wages for high school dropouts, and in a 2% drop of wages for those with a high school diploma only. Wage effects of this size might more than cancel the increase in EITC payments. While large, these magnitudes are not necessarily out of line with estimates of labor demand using macro variation, given the estimated labor supply effects of the EITC. What is somewhat surprising – but is in line with other studies estimating the effects of the EITC expansion – is the magnitude of labor supply effects. This magnitude is surprising given that the reduction in wages appears to effectively cancel the subsidy of work provided by the EITC.

For our main estimates we slightly modify Leigh's specifications as follows. We recalculate the definition of treatment x, which yields slightly different values in some cases. For parsimony, we also drop state-level policy controls and drop weighting for the regressions. None of these modifications has a large effect; Table 3 reports the resulting estimates. The conclusions remain qualitatively the same, as can be seen from comparison of Tables 2 and 3. Our main results build on quantile regression analogs of the specifications in Table 3.

Estimation approach Theorem 3 shows that conditional average causal effects are just-identified under some restrictions on effect heterogeneity, given exogeneity of treatment. Our estimators build on this result. We implement the following three-step estimation procedure using quantile difference-in-difference regressions.[3] A justification of this procedure in the context of our theoretical results is discussed below.

In the first step we estimate quantile regressions of labor supply l on log EITC generosity x and controls,

$$l = (W_1 \times x) \cdot \beta_1 + W_2 \cdot \beta_2 + \delta_s + \delta_{C,\tau} + \epsilon, \tag{10}$$

where $P(\epsilon < 0|W_1, W_2, E, s, \tau, x) = v_1$, W_1 includes a constant, age, and a gender dummy, and W_2 includes gender by race dummies, age and age squared, and the same variables for the individual's spouse (if present). δ_s denotes state dummies, and $\delta_{C,\tau}$ year by number of children dummies. These regressions are estimated separately for each vingtile $v_1 = 0.05, 0.1, \ldots$, and for each educational group E (high school dropout, high school, and some college or more), as solutions of

$$(\widehat{\beta}, \widehat{\delta}) = \underset{\beta,\delta}{\operatorname{argmin}} \sum_i \rho_{v_1} \left(l_i - [(W_{1i} \times x_i) \cdot \beta_1 + W_{2i} \cdot \beta_2 + \delta_{s_i} + \delta_{C_i,\tau_i}] \right),$$

where $\rho_{v_1}(y) = y \cdot (v_1 - \mathbf{1}(y < 0))$. We then impute a conditional quantile v_1 to each observation in our sample based upon the closest predicted value between these regressions.

In the second step we again estimate regressions of the form of Equation (10), with log wages

[3] For the quantile regressions, the code provided by Roger Koenker at `http://www.econ.uiuc.edu/~roger/research/rq/rq.html` is used (accessed May 18 2017).

w instead of labor supply l as the dependent variable. For these regressions, we additionally include v_1 among the controls W_1 and W_2, so that the effect of x on w may vary with v_1.

In the third step we restrict attention to a baseline sample of observations from the year 2002. For each observation in this sample we impute a conditional quantile v_2 based upon the closest predicted value of the wage regressions from the second step. We then impute a wage effect of the form $(W_1 \times x) \cdot \beta_1$ to each observation, using the coefficients β_1 corresponding to the imputed v_2. Heterogeneity in these imputed wage effects is driven by age, gender, education, as well as wage and labor supply via the conditional quantiles v_1 and v_2. We next impute welfare effects of the form $l \cdot (W_1 \times x) \cdot \beta_1$ using these same coefficients. We finally run kernel regressions of imputed wage effects and welfare effects on actual earnings, using an Epanechnikov kernel and a bandwidth based on Silverman's rule of thumb.

Justification of the estimation approach Along the lines of the identification approach discussed at the end of Section 3, let the following assumptions hold: (i) marginal stationarity of unobserved heterogeneity, (ii) additive time effects, and (iii) the exclusion restriction $\partial_w E[\dot{l}|l, w, W, s, x] = 0$. The first two of these assumptions are justifications for the standard difference-in-differences approach, while the third assumption restricts effect heterogeneity to allow identification of conditional causal effects. We will explore robustness with respect to the third assumption below by considering the alternative restriction $\partial_l E[\dot{w}|l, w, W, s, x] = 0$. Under assumptions (i), (ii), and (iii), the conditional average causal effect of a policy change on wages is identified from the relationship

$$\partial_\tau Q^{w|v^1,W,s,\tau}(v^2|v^1, W, s, \tau) = E[\dot{w}|l, w, W, x] \cdot \partial_\tau x(s, \tau) + \partial_\tau \delta_\tau, \tag{11}$$

where we take W to include W_1 and W_2 as well as number of children C and education E. Our three-step procedure implements a sample analog of this result. In the first step $v_1 = F^{l|W,s,\tau,x}(l|W, s, \tau, x)$ is estimated using linear quantile regressions. In the second step $Q^{w|v^1,W,s,\tau,x}(v^2|v^1, W, s, \tau, x)$ is estimated, using linear quantile regressions again. In the third step, the conditional average wage effect $E[\dot{w}|l, w, W, s, \tau]$ is imputed to observations in the baseline sample, implicitly using the relationship (11). The kernel regressions then yield estimates of the conditional average welfare effect $E[\dot{w} \cdot l|w \cdot l, \tau]$.

Inference For inference, we use the Bayesian bootstrap, as introduced by Rubin (1981) and discussed by Chamberlain and Imbens (2003). It is implemented as follows: For each iteration, draw n i.i.d. exponentially distributed random variables B_i. Re-weight each observation by $B_i / \sum_{i'} B_{i'}$, and estimate the object of interest for the re-weighted distribution. Iterate the entire procedure, to obtain a set of R replicate estimates for each object of interest. The estimates obtained by this procedure are draws from the posterior corresponding to an (improper) Dirichlet process prior (with parameter 0) over the joint distribution of all observables. This procedure allows, in particular, to construct Bayesian credible sets for the object of interest, using quantiles of the re-sampling distribution. Additionally, and similarly to the standard bootstrap, the re-sampling distribution also provides an asymptotically valid estimate of the frequentist sampling distribution for objects such as sample moments and smooth functions thereof. This allows to interpret the Bayesian credible sets as frequentist confidence sets.

Results: The effect of the EITC expansion We now turn to our first set of empirical results, summarized by the plots in Figure 5. These plots show estimated effects on the 2002 baseline population of a 10% expansion of the EITC. Estimates are based on the three step estimation procedure outlined above, and control in particular for the conditional quantile of labor supply.

The key plot, shown in the top left of Figure 5, plots the estimated pre-tax conditional average welfare effect $E[\dot{w} \cdot l | w \cdot l]$ as a function of earnings.[4] Subject to our identifying assumptions, this plot suggests that the general equilibrium effect of the EITC expansion led to a decrease the welfare of low income individuals, with the most negative effect for individuals earning between 20,000 and 30,000 US$ per year. Note how this finding is not something that could have been deduced from the more conventional regression estimates of Table 2 and 3, since these regression estimates of the effect on observables do not allow to obtain welfare effects, nor do they allow to disentangle heterogeneity across income levels.

The next plot shows the mechanical effect of an EITC expansion by 10%. This plot corresponds to Figure 3, averaging over number of children given earnings. A comparison between estimated equilibrium effects and mechanical effects shows that, strikingly, the two effects are of opposite sign but similar magnitude and shape. It seems that across the income distribution, the incidence of the EITC expansion was primarily on the employers' side, with low income recipients' welfare essentially unaffected on net.

To better understand the shape of equilibrium welfare effects, it is useful to separately plot kernel regressions of imputed wage effects $\dot{w}$ and of labor supply (usual weekly hours) l on earnings z. If the conditional correlation of $\dot{w}$ and l given $w \cdot l$ were equal to zero, then the product of these two plotted functions would equal the conditional average welfare effects. These plots reveal that wages were depressed throughout the earnings distribution, but negative effects fade as earnings increase. Labor supply, on the other hand, is larger for individuals with higher earnings, but essentially flat and equal to around 40 hours per week for individuals earning more than 20,000 US$ per year. The combination of these two shapes – rising hours per week for low income individuals, declining wage effects for higher income individuals – explains the U-shape of welfare effects.

The kernel regressions themselves obscure considerable conditional heterogeneity of welfare effects given earnings. This is revealed by the bottom left scatter plot, which shows earnings and imputed welfare effects for a random subsample of 1000 observations from the baseline population. The final plot shows a histogram of the distribution of earnings in the baseline

[4]We focus on pre-tax effects to avoid the need to impute taxes based on the limited information available in the CPS.

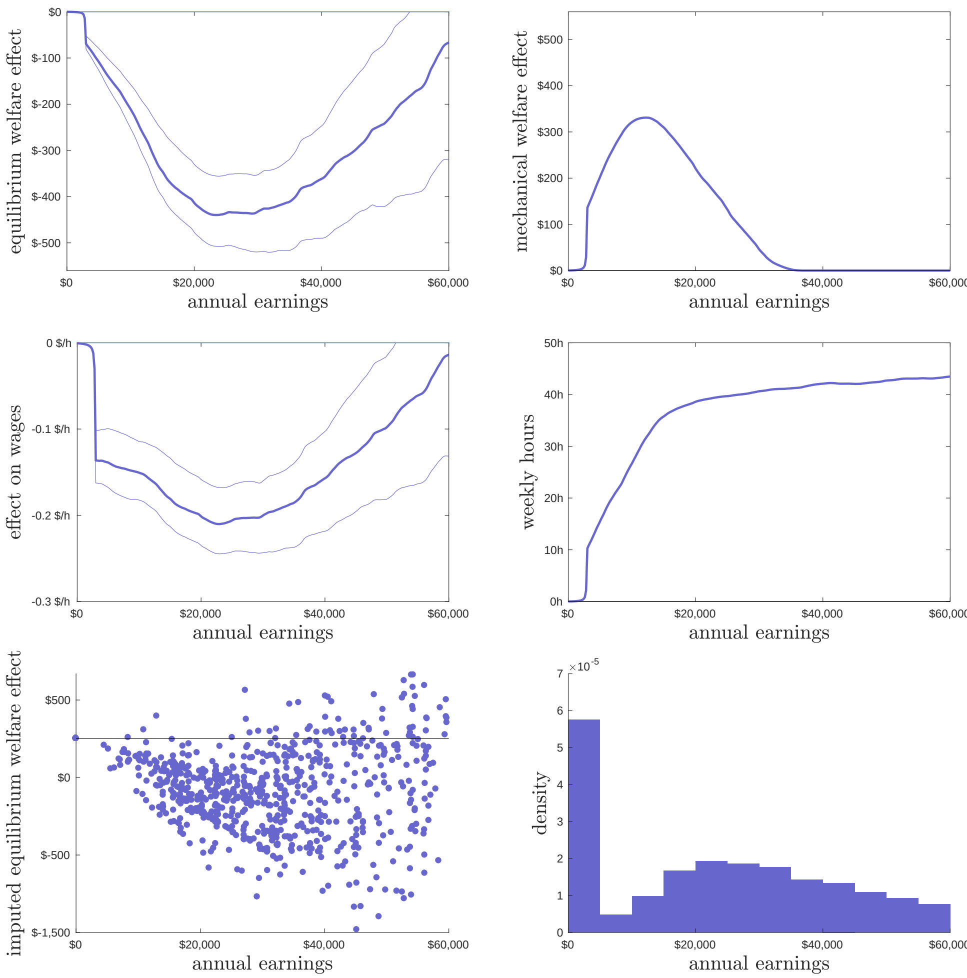

Figure 5: These figures plot the following for our 2002 baseline sample: Kernel regressions on earnings of (i) the estimated pre-tax welfare effect $\dot{w} \cdot l$ for a 10% EITC expansion, (ii) the imputed mechanical effect $\dot{t}$, (iii) the imputed wage effect $\dot{w}$, and (iv) usual weekly hours worked l. Furthermore (v) a scatter plot of imputed welfare effects for a random sample of 1000 observations from the baseline, and (vi) a histogram of the distribution of earnings. Imputations are based on the three step approach controlling for v_1 as described in Section 4. Figures (i) and (iii) additionally show pointwise 95% credible bands based on the Bayesian bootstrap.

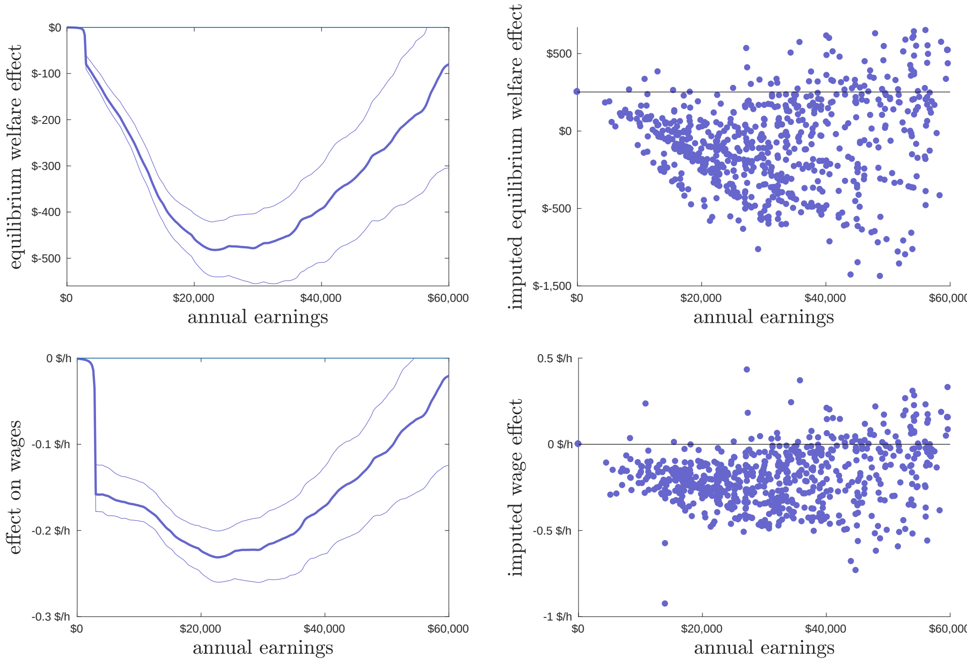

Figure 6: These figures show plots analogous to Figure 5, where imputations are based on a two-step approach and do not control for v_1.

sample; a significant number of individuals report zero earnings, while the modal non-zero level of earnings is slightly above 20,000 US$ per year.

Alternative restrictions on heterogeneity The imputed welfare and wage effects reported thus far rely on the restriction of heterogeneity implied by the assumption $\partial_w E[\dot{l}|w, l, W, s, \tau] = 0$, as in Theorem 3. We can also construct an estimator based on the alternative assumption $\partial_l E[\dot{w}|w, l, W, s, \tau] = 0$. This assumption suggests a two-step procedure which proceeds exactly like the one discussed above, except that (i) the first step is omitted, and (ii) in the second step we do not control for v_1 in the quantile regressions predicting w. Figure 6 shows the resulting estimates from this two-step procedure. All the qualitative conclusions remain unchanged. This empirical finding provides comfort that our results appear to be robust to alternative assumptions about heterogeneity.

Historical changes Thus far we have considered the causal impact of the EITC expansion on welfare and wages, under difference-in-difference type assumptions. We can also study the actual historical evolution of market wages, and the welfare impact of changing wages. Note that in this case, the problem of causal inference via exogenous treatment variation (which we solved using a difference-in-differences approach in the case of the EITC) disappears since we just consider changes over time, but we still face the problem of identifying conditional average effects. To do so, consider the same estimation approach as before, but drop all time fixed effects and replace the definition of treatment by $x = \tau/13$. The resulting estimates correspond to the welfare impact of historical changes over the period 1989-2002. The results of the three-step procedure are shown in Figure 7. The conditional average welfare effects, shown in the top left, reveal welfare gains that are essentially proportional to earnings. This picture corresponds to modest annual welfare gains of about 0.6% of earnings throughout the earnings distribution. The corresponding regression of wage changes on earnings (bottom left) reveals some relative catch-up of wages for very low earnings. This, in conjunction with labor supply rising over the earnings distribution, generates the observed linear pattern of welfare effects. We again consider the results of a two step procedure based on the alternative restriction on heterogeneity in Figure 8. As before, the results are similar to those using the three-step procedure, except that welfare gains appear both smaller and more heterogeneous given earnings, in this case.

5 Conclusion

In Section 2, a simple static model of labor supply with nonlinear income taxes was discussed. If wages are allowed to be endogenous in such a model, a welfare evaluation of tax changes needs to take into account both mechanical and equilibrium effects. If equilibrium wage effects are additionally allowed to be heterogeneous across workers, one faces a difficult identification problem – in this case, an evaluation of welfare effects requires identification of the average causal effect of the policy change on wages, conditional on both endogenous labor supply and wages. Similar considerations apply in other settings with heterogeneous goods. Consider for instance the housing market and the distributional welfare impact of urban policies that lead to shifts in housing prices. In this case, too, the housing supply and house prices are endogenous, and houses and effects on house prices are heterogeneous. We are thus led to the problem of identifying conditional average causal effects. In Section 3, this problem was analyzed in an abstract setting with multiple endogenous outcomes. Without restrictions on heterogeneity, conditional average causal effects are not point-identified for outcome vectors of dimension bigger than one, even in the presence of exogenous policy variation. We provide minimal restrictions on heterogeneity, however, that allow for just-identification of conditional average causal effects. Estimators based on such minimal sufficient restrictions, combined with a difference-in-differences identification strategy, were implemented in Section 4. Both the effect of the EITC expansion in the 1990s, and the effect of historical wage changes over the same time period were considered. Large negative impacts of the EITC expansion on wages were found that essentially offset the increased transfers, suggesting that the incidence of this expansion was primarily on the employers' side, who face increased labor supply and lower wages. This seems to be a rather extreme empirical finding to the extent that we wouldn't a-priori believe labor demand to be completely inelastic. It would be interesting to see if our findings could be replicated using alternative identification strategies.

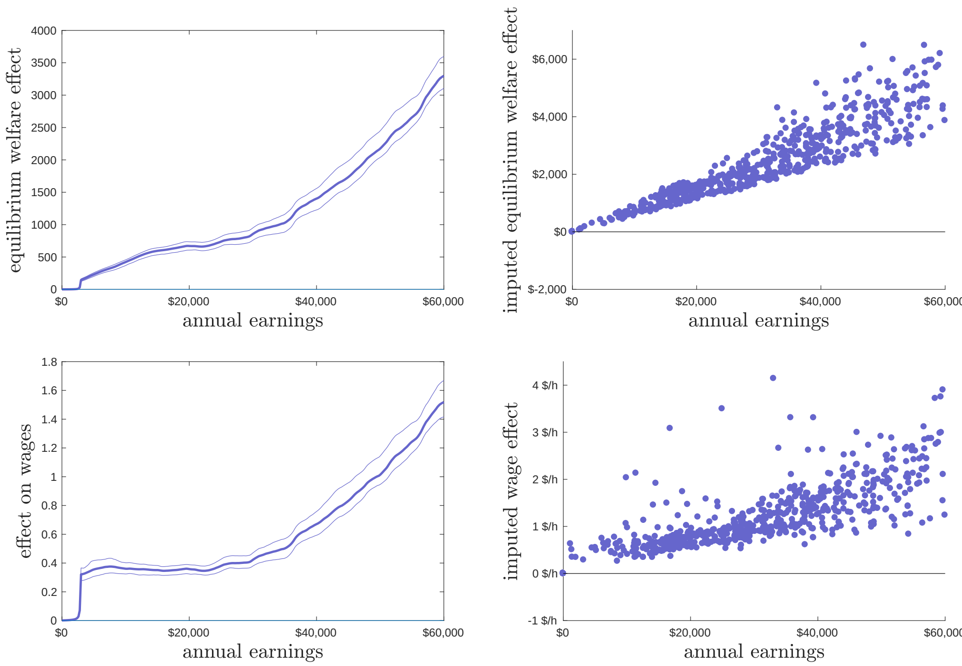

Figure 7: These figures show plots analogous to Figure 5, where now treatment x has been replaced by time τ, and time fixed effects have been dropped from the controls. Effects correspond to actual historical changes over the time period 1989-2002.

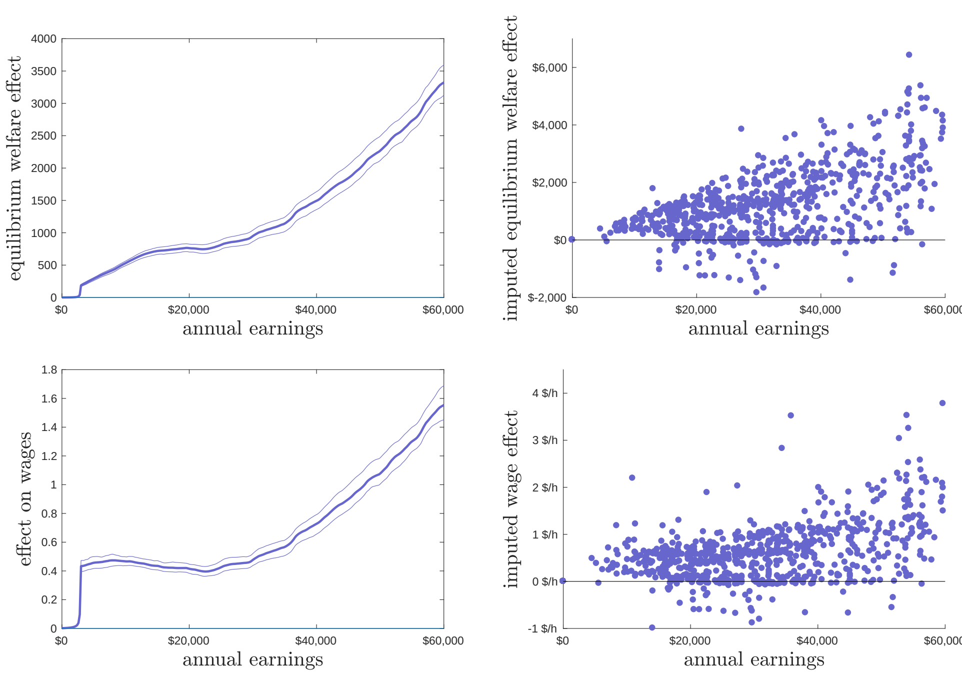

Figure 8: These figures show plots analogous to Figure 7, where imputations are based on a two-step approach and do not control for v_1.

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A Proofs

Proof of Theorem 1: Let b be an arbitrary differentiable function with bounded support $\mathscr{B}$, and define

$$
\begin{aligned}
B(x) := E[b(y(x,\epsilon))|x] &= \int b(y(x,\epsilon))dP(\epsilon) \\
&= \int b(y)f(y|x)dy.
\end{aligned}
$$

Corresponding to the two integral representations of $B(x)$, there are two representations for $\dot{B}(x)$. Using the first representation we get

$$
\begin{aligned}
\dot{B}(x) &= \int \partial_x b(y(x,\epsilon))dP(\epsilon) = \int \partial_y b(y(x,\epsilon)) \cdot \dot{y}(x,\epsilon)dP(\epsilon) \\
&= E[\partial_y b \cdot \dot{y}|x] = \sum_{j=1}^{k} E[\partial_{y^j} b \cdot \dot{y}^j|x] \\
&= \sum_{j=1}^{k} E[\partial_{y^j} b \cdot h^j/f|x] = \sum_{j=1}^{k} \int \partial_{y^j} b \cdot h^j dy \\
&= -\int b \cdot \sum_{j=1}^{k} \partial_{y^j} h^j dy = -\int b \cdot (\nabla \cdot h)dy.
\end{aligned}
$$

The two key steps of this derivation are (i) exchange of the order of differentiation and integration in the first equality, and (ii) partial integration, from the third to the fourth line. (i) is justified since by assumption the functions involved are differentiable and have compact support $\mathscr{B}$. (ii) does not involve any boundary terms, again by the compact support assumption for b.

Using the second integral representation for $B(x)$, we can alternatively write

$$
\begin{aligned}
\dot{B}(x) &= \partial_x \int b(y)f(y|x)dy \\
&= \int b(y)\dot{f}(y|x)dy.
\end{aligned}
$$

We get that

$$
\int b(y) \cdot \dot{f}(y|x)dy = -\int b(y) \cdot (\nabla \cdot h(y,x))dy
$$

for any differentiable b with bounded support. Since $\nabla \cdot h$ is continuous by assumption, it follows that $\dot{f} = -\nabla \cdot h$.

We next show that the identified set for h is equal to $h^0 + \mathscr{H}$.

1. $\underline{h \text{ satisfies } \dot{f} = -\nabla \cdot h \text{ if and only if it is in the identified set:}}$
   The "if" part follows from the argument above. To show the "only if" part, taking h as given we need to construct a distribution of ϵ and structural functions y consistent with (i) h, (ii) the observed data distribution, and (iii) Assumption 1. Such a data generating process can be constructed as follows: Let $y(0,\epsilon) = \epsilon$. This implies $f(\epsilon) = f(y|x = 0)$. Let $y(.,\epsilon)$ be a solution to the ordinary differential equation

   $$\dot{y} = h(y,x),\ y(0,\epsilon) = \epsilon.$$

   Such a solution exists by Peano's theorem. It is easy to check that this solution satisfies all required conditions.

2. $\underline{h^0 \text{ satisfies } \dot{f} = -\nabla \cdot h^0:}$
   Consider the model $y^j(x,\epsilon) = Q^{y^j|v^1,\ldots,v^{j-1},x}(v^j|v^1,\ldots,v^{j-1},x)$ where $\epsilon = (v^1,\ldots,v^k)$. Then this model implies $E[\dot{y}|y,x] \cdot f(y) = h^0(y)$, where h^0 is defined as in the statement of the theorem. This model is furthermore consistent with the observed data distribution, and thus in particular satisfies $\dot{f} = -\nabla \cdot h^0$ by Theorem 1.

3. $\underline{h \text{ satisfies } \dot{f} = -\nabla \cdot h \text{ if and only if } h \in h^0 + \mathscr{H}:}$
   For any h in $h^0 + \mathscr{H}$, we have $\nabla \cdot h = \nabla \cdot h^0 + \nabla \cdot \widetilde{h} = -\dot{f} + 0$.
   Reversely, for any h such that $\dot{f} = -\nabla \cdot h$, let $\widetilde{h} := h - h^0$. Then $\widetilde{h} \in \mathscr{H}$.

□

Proof of Theorem 2:

1. $k = 1$: In this case, $\nabla \cdot h = \partial_y h = 0$. Since h has its support contained in $\mathbf{Y}$, integration immediately yields $h \equiv 0$.

2. $k = 2$: This result is a special case of Poincaré's Lemma, which states that on convex domains differential forms are closed if and only if they are exact; cf. Theorem 10.39 in Rudin (1991). Apply this lemma to

   $$\omega = h^1 dy^2 - h^2 dy^1.$$

   Then

   $$d\omega = (\partial_{y^1} h^1 + \partial_{y^2} h^2) dy^1 \wedge dy^2 = 0$$

   if and only if

   $$\omega = dH = (\partial_{y^1} H) dy^1 + (\partial_{y^2} H) dy^2$$

   for some H.

3. This follows again from Poincaré's Lemma, applied to

$$\omega = h^1 dy^2 \wedge dy^3 + h^2 dy^3 \wedge dy^1 + h^3 dy^1 \wedge dy^2$$

and

$$\lambda = \sum_j G^j dy^j.$$

$\square$

Proof of Theorem 3:

1. <u>h^0 is consistent with this assumption:</u>
Consider again the model $y^j(x, \epsilon) = Q^{y^j|v^1,\ldots,v^{j-1},x}(v^j|v^1, \ldots, v^{j-1}, x)$ where $\epsilon = (v^1, \ldots, v^k)$, as in the proof of Theorem 1. Then this model implies $\partial_{y^j} E[\dot{y}^{j'}|y, x] = 0$ for $j > j'$. This model is furthermore consistent with the observed data distribution and satisfies $E[\dot{y}|y, x] \cdot f(y) = h^0(y)$.

2. <u>The only $\widetilde{h} \in \mathscr{H}$ consistent with this assumption is $\widetilde{h} \equiv 0$:</u>
As we have already shown h^0 to be consistent with this assumption, it is enough to show that $\nabla \cdot \widetilde{h} = 0$ implies $\widetilde{h} \equiv 0$ if $\widetilde{h}$ is consistent with this assumption. We proceed by induction in j.

   Consider the model where we only observe $y^1, \ldots, y^j$, and define $\widetilde{h}$ accordingly. Suppose we have shown $(\widetilde{h}^1, \ldots, \widetilde{h}^{j-1}) = (0, \ldots, 0)$. Applying Theorem 1 to the j dimensional model immediately implies $\partial_{y^j} \widetilde{h}^j = 0$. Integrating with respect to y^j, and using the fact that the support of $\widetilde{h}$ is contained in the support $\mathbf{Y}$ of y implies $\widetilde{h}^j \equiv 0$.

   Equation (8) implies

$$E[\dot{y}^{j'}|y^1, \ldots, y^k, x] = E[\dot{y}^{j'}|y^1, \ldots, y^j, x]$$

   for $j \geq j'$. As a consequence, $\widetilde{h}^{j'} = 0$ in the j dimensional model immediately implies $\widetilde{h}^{j'} = 0$ in the $j + 1$ dimensional model. The claim now follows by induction.

$\square$