# Which findings should be published?*

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**Abstract**

Given a scarcity of journal space, what is the optimal rule for whether an empirical finding should be published? Suppose publications inform the public about a policy-relevant state. Then journals should publish extreme results, meaning ones that move beliefs sufficiently. This optimal rule may take the form of a one- or a two-sided test comparing a point estimate to the prior mean, with critical values determined by a cost-benefit analysis. Consideration of future studies may additionally justify the publication of precise null results. If one insists that standard inference remain valid, however, publication must not select on the study's findings.

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## 1 Introduction

Not all empirical findings get published. Journals may be more likely to publish findings that are statistically significant, as documented for instance by Franco et al. (2014), Brodeur et al. (2016), and Andrews and Kasy (2019). They may also be more likely to publish findings that are surprising, or conversely ones that confirm some prior belief. Whatever its form, selective publication distorts statistical inference. If only estimates with large effect sizes were to be written up and published, say, then

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published studies would systematically overstate true effects. Such publication bias has been offered as one explanation for the perceived replication crisis in the social and life sciences.[1]

In response to these concerns, there have been calls for reforms in the direction of non-selective publication. One proposal is to promote statistical practices that de-emphasize statistical significance, for instance by banning "stars" in regression tables. Another proposal is for journals to adopt Registered Reports, in which pre-registered analysis plans are reviewed and accepted prior to data collection (see Nosek and Lakens (2014) or Chambers et al. (2014)). This has been implemented, for instance, at the Journal of Development Economics. Registered Reports guarantee that publication will not select at all on findings – after a plan is accepted, the journal is committed to publishing the study and the researcher has no flexibility over which results to write up.

This paper seeks the optimal rule for determining whether a study should be published, given both its design (which determines a study's standard error) and its findings (the point estimate). Our analysis is from an instrumental perspective: the value of a study is that it informs the public about some policy-relevant state of the world before the public chooses a policy action. In this framework, we will show that non-selective publication is not in fact optimal. Some findings are more valuable to publish than others. Since selective publication distorts statistical inference, this implies a trade-off between policy relevance and statistical credibility.

In a world without constraints, the first-best rule would be for all results – or even better, all raw data – to be published. This paper solves for a second-best publication rule. In particular, we take as given that there is some cost of publication. One interpretation is that the cost is an opportunity cost of shifting a public's attention away from other studies. In that case, we can think of our paper as asking which findings should be published in top journals, i.e., ones where the results are more likely to be noticed.

The basics of our model are as follows. If a submitted study is published, the public

[1] Worries about selective publication go back at least to Sterling (1959). Discussions of publication bias and other threats to the credibility and reproducibility of scientific output can be found in Ioannidis (2005), Ioannidis (2008), and in reviews including Simmons et al. (2011), Gelman and Loken (2014), and Christensen and Miguel (2016). Open Science Collaboration (2015) and Camerer et al. (2016) conduct large-scale replications of experimental studies in psychology and economics, giving insight into the extent to which published results are in fact reproducible.

observes its results and takes the optimal policy action given its updated belief. If a study is not published, the public never observes the results, and does not necessarily know that a study was conducted; the public then takes a default action. This default action in the absence of publication is based on a default belief. We allow for either a Bayesian public whose default belief correctly accounts for publication bias or for a naive public whose default belief always remains at its prior. The optimal publication rule is the one that maximizes the public's expected payoff from the eventual policy choice, minus the publication cost.

The optimal publication rule will select on a study's findings. To understand why, observe that there is no instrumental value from publishing a study with a "null result" that doesn't move the policy away from the default action. Publishing a null result incurs a cost without a benefit: the same policy would have been chosen even if the study weren't published. The studies that are worth publishing are the ones that show a payoff gain from taking an action other than the default.

In order to generally characterize optimal publication rules, consider "supermodular" policy decisions in which the public's preferred policy action is monotonic in the state of the world. For example, the preferred investment in a public good increases in its expected return. In any supermodular environment, Theorem 1 establishes that it is more valuable to publish studies with more extreme results. These are the ones that move beliefs and actions further from the defaults.

For two canonical special cases, we give a more explicit description of optimal publication rules. In the first case, the public makes a continuous policy decision, such as the choice of a tax rate, and has quadratic losses in matching the policy to its ideal point. The optimal publication rule then takes the form of a "two-sided test:" the journal publishes estimates that are sufficiently far above or below the prior mean. In the second case, the public makes a binary policy choice, such as whether to implement a job training program. Here, the optimal rule is a "one-sided test." For instance, in the absence of publication, suppose that the public by default does not implement the program. The journal then publishes positive estimates of the program's value, ones that convince the public to implement it, but not negative estimates. For these two-sided and one-sided tests, the critical values that determine publication come from a cost-benefit calculation. They do not correspond to a conventional significance level such as p = .05 against a null hypothesis of zero.

After characterizing optimal publication rules, we return to the distortions caused by selective publication. In Theorem 2, we show that under any rule in which the publication probability depends on the point estimate, common forms of frequentist inference will be invalid after conditioning on publication. Point estimates are no longer unbiased, for instance, and uncorrected likelihood-based estimation will be flawed. Moreover, when a study is not published, a naive public that maintains its prior will have a distorted belief relative to a Bayesian public that accounts for publication bias. If we desire that standard inference or naive updating be valid, we must impose a non-selective publication rule that does not depend at all on the point estimate (although journals may still publish studies with small standard errors over studies with large standard errors).

Putting these results together, we see that selectively publishing extreme results is better for policy-relevance but leads to distorted inference. Therefore, a move away from the current (selective) publication regime towards non-selective publication in order to improve statistical credibility might have costs as well as benefits.

An abstraction in the model as described is that it considers a "static" environment with a single paper to be published and a single action to be taken. One may also be interested in the longer-term implications of publication rules, as in McElreath and Smaldino (2015) and Nissen et al. (2016).[2] To get some insight into these issues, we consider a dynamic extension to our model that appends a second period in which exogenous information arrives before another action is taken. The publication decision in the first period now affects the actions in both periods. Theorem 3 characterizes the optimal publication rule for this two-period model. Here, we find a new benefit of publishing null results that don't change the current action. Publishing a null result today helps avoid future mistakes arising from the noise in the information that has yet to arrive.

In addition to the broad conclusions described above, the paper derives a number of comparative statics on optimal publication rules. Some are straightforward: for example, it is more valuable to publish a given point estimate when the standard error is smaller, and correspondingly in the two-period model it is more valuable to publish a precise null result than a noisy null result. We believe that some of these comparative statics, however, would have been difficult to intuit without a

[2]McElreath and Smaldino (2015) and Nissen et al. (2016) provide dynamic models to study whether an academic publication process with publication bias will eventually converge to truthful estimates. Akerlof and Michaillat (2017) performs a similar exercise for a more evolutionary form of the accumulation of academic knowledge.

formal model. For instance, consider the canonical quadratic loss policy environment. Suppose that we fix the statistical significance of some result (the t-statistic of the point estimate relative to the prior mean) while varying the point estimate. Point estimates close to the prior move the posterior mean very little, and thus, naturally, publishing those results is not valuable. But point estimates too far from the prior also barely move the posterior. So at any specified statistical significance level, we only publish point estimates that are an intermediate distance from the prior mean. A similar nonmonotonicity occurs in the two-period model: publishing null results today is most valuable when the future information is expected to be neither too precise nor too noisy.

We wish to stress that the nature of our exercise is to solve for the socially optimal rule regarding whether to publish a study that has some given set of results. That is, our model does not consider the incentives of researchers or journals, and we are not attempting to characterize the equilibrium publication rule arising from a strategic interaction of these agents. As discussed in Glaeser (2006), researcher incentives play an important role in the publication process. Researchers make choices over the topics they study and their study designs, and then may selectively submit or possibly even manipulate their findings.[3] We do explore one way in which researcher study design choices may respond to journal publication rules in Appendix A.3.

Our derivations of optimal publication rules rely on characterizing the value of information for specified decision problems. Most theoretical treatments of the value of information study the ex-ante value of an experiment, i.e., the expected value prior to the realization; see classic treatments in Blackwell (1953), Lehmann (1988), or Persico (2000). These ex-ante comparisons are relevant for a characterization of non-selective publication rules, as we examine in Proposition 3. However, we generally allow for publication to select on a study's findings. We are thus predominantly concerned with the ex-post value of information given an experiment's realization, as studied in Frankel and Kamenica (2019).

The decision to reveal a signal, at a cost, based on its realization is also related to the analysis of the discretionary disclosure of product quality in Jovanovic (1982) or

[3]Furukawa (2018) looks at a model (without journals) in which researcher decisions to publish papers interact with a public policy choice, where in equilibrium researchers choose to publish papers with extreme results. Müller-Itten (2017) looks at a competition between journals for prestige, in which journals choose whether to publish a submitted study based on a signal of its quality while researchers choose which journal to submit to.

of accounting news in Verrecchia (1983), and in follow-up work. As in those papers, we find that information is disclosed only if it is sufficiently valuable ex post. Those papers, however, focus on a private value of disclosing information that may contain positive or negative news about one's type. We instead consider a social value of information in making better decisions.

The rest of the paper is structured as follows. Section 2 introduces our basic model of publication. Section 3 shows how to solve for the optimal publication rules and provides some characterizations of the solution. Section 4 addresses the distortions that arise from selective rather than non-selective publication. Section 5 presents a two-period version of the model. Section 6 concludes and presents some extensions that we explore further in the Appendix. We consider publication objectives that do not arise from policy-based welfare, and instead derive from an inherent value of learning the state or of publishing point estimates close to the truth; we show how a journal might adjust its publication rule if submitted study designs are endogenous to the publication rule; and we discuss the possibility that a study's findings may be informative about the quality of its design. Proofs are in the Appendix.

# 2 The model of publication

## 2.1 Set-up

There is an uncertain state of the world whose value is relevant to some public policy decision. A study that reveals information about this state may or may not arrive, i.e., may or may not be submitted to a journal. If a study arrives, the journal decides whether to publish it. If it is published, the results of the study are observed by the public. Finally, the public chooses a policy.

Let $\theta \in \Theta \subseteq \mathbb{R}$ denote the state of the world, and suppose there is a common prior $\pi_0$ on $\theta$ shared by the public and the journal. The probability that a study arrives is $q \in (0, 1]$, independent of $\theta$. A study is summarized by the random variables $(X, S)$, where $S$ is drawn from some distribution $F_S$ on $\mathcal{S} = \mathbb{R}_{++}$, and $X$ is normally distributed on $\mathcal{X} = \mathbb{R}$ according to $X|\theta, S \sim \mathcal{N}(\theta, S^2)$. Call $X$ the *point estimate* and $S$ the *standard error*. In particular, $S$ reflects the study's design, containing no direct information about the state $\theta$. By contrast, $X$ is the study's finding, which is informative about $\theta$; its information content depends on $S$.

If a study arrives, it will be evaluated by a journal that observes $(X, S)$ and then decides whether to publish the study. The journal uses a publication rule $p : \mathcal{X} \times \mathcal{S} \to [0, 1]$ where $p(X, S)$ describes the probability that a study $(X, S)$ is published. We say that the journal publishes a study when $p(X, S) = 1$ and does not publish a study when $p(X, S) = 0$. Let $D = 0$ denote the event that no study is published (because no study arrived, or because one arrived but was not published) and $D = 1$ the event that a study arrives and is published.

After a study is published or not, the public's belief on $\theta$ updates to a posterior $\pi_1$. When no study has been published ($D = 0$), $\pi_1$ is equal to some *default belief* $\pi_1^0$. When a study has been published ($D = 1$), $S$ and $X$ are publicly observed, and $\pi_1$ is instead equal to the belief $\pi_1^{(X,S)}$. See Section 2.2 below for the description of the belief updating process that determines $\pi_1^0$ and $\pi_1^{(X,S)}$.

Given updated beliefs $\pi_1$, the public takes a policy action $a \in \mathcal{A} \subseteq \mathbb{R}$ to maximize its expectation of a utility function $U : \mathcal{A} \times \Theta \to \mathbb{R}$. Let $a^*(\pi_1) \in \arg\max_a \mathbb{E}_{\theta \sim \pi_1}[U(a, \theta)]$ indicate the chosen action when the public holds beliefs $\pi_1$. We assume existence of this argmax for any relevant utility functions and posterior distributions, and we confirm existence for all of our examples. Let $a^0 = a^*(\pi_1^0)$ be the *default action*, i.e., the action taken under the default belief, whereas $a^*\left(\pi_1^{(X,S)}\right)$ is the action taken if a study $(X, S)$ is published.

Social welfare, corresponding to the shared objective of both the journal and public, is the action payoff net of a publication cost. Let $c > 0$ indicate the social cost of publication. The welfare $W(D, a, \theta)$ induced by publication $D$, chosen action $a$, and state of the world $\theta$, is

$$W(D, a, \theta) = U(a, \theta) - Dc. \tag{1}$$

We will search for the publication rule $p$ that maximizes the ex-ante expectation of welfare, which we call the *optimal* publication rule.

## 2.2 Belief updating

If a study $(X, S)$ is published, the public's posterior belief is $\pi_1 = \pi_1^{(X,S)}$. We assume that $\pi_1^{(X,S)}$ is derived according to Bayes' Rule given the signal $X|\theta, S \sim \mathcal{N}(\theta, S^2)$. Denote the pdf of a standard normal distribution by $\varphi$. By Bayes' Rule, recalling that $S$ is independent of $\theta$, the density of $\pi_1^{(X,S)}$ relative to the prior $\pi_0$ is given by

$$\frac{d\pi_1^{(X,S)}}{d\pi_0}(\theta) = \frac{\varphi((X-\theta)/S)}{\int \varphi((X-\theta')/S)d\pi_0(\theta')}. \tag{2}$$

Since the journal and the public share a common prior, we see that $\pi_1^{(X,S)}$ also represents the journal's Bayesian belief after it observes a submitted study $(X,S)$. As such, we often refer to $\pi_1^{(X,S)}$ as the *interim belief* that the journal holds when evaluating a paper for publication.

If a study is not published then the public's posterior belief is $\pi_1 = \pi_1^0$, the default belief. We consider the possibility of two distinct updating rules to determine $\pi_1^0$. *Bayesian updating* is the sophisticated rule that correctly accounts for selection induced by the publication process. *Naive updating* is the unsophisticated rule which ignores the possibility of unpublished studies, and fails to account for selection.

**Bayesian updating:** When no study is published, the public understands that this event could have occurred because no study arrived (probability $1-q$) or because a study arrived (probability $q$) and was unpublished (probability $1-p(X,S)$, with $\theta \sim \pi_0$, $S \sim F_S$, and $X|\theta,S \sim \mathcal{N}(\theta,S^2)$). The public then updates beliefs on $\theta$ to $\pi_1^0$ according to Bayes rule.[4] Denote this Bayesian default belief under publication rule $p$ by $\pi_1^{0,p}$; its density relative to the prior $\pi_0$ is given by

$$\frac{d\pi_1^{0,p}}{d\pi_0}(\theta) = \frac{1-q\mathbb{E}[p(X,S)|\theta]}{1-q\mathbb{E}[p(X,S)]}. \tag{3}$$

**Naive updating:** The public's default belief, $\pi_1^0$, is equal to its prior: $\pi_1^0 = \pi_0$.

While Bayesian updating is "correct" in the fully specified model, we consider naive updating to be, in many cases, a realistic description of updating. One can interpret a naive public as having an incorrect model of the world: in the absence of seeing a publication, the public is unaware of the possibility that a study might have been submitted and rejected. Alternatively, naive beliefs arise as the limiting Bayesian beliefs when $q \to 0$, i.e., a rational public that did not expect a study to be submitted on this topic.

[4] If a publication rule publishes with probability one and the probability of study arrival is $q = 1$, then nonpublication is a zero probability event and beliefs are not pinned down by Bayes' rule. As a convention, we then let the Bayesian default belief be equal to the prior $\pi_0$.

## 2.3 Leading examples of priors and utility functions

A typical state of the world $\theta$ estimated in an empirical economics study might be a demand or supply elasticity, the magnitude of a treatment effect, or the net benefit of implementing a program. Our leading example for a prior distribution will be the *normal prior*, in which case $\Theta = \mathbb{R}$ and $\pi_0$ is $\mathcal{N}(\mu_0, \sigma_0^2)$, with $\mu_0 \in \mathbb{R}$ and $\sigma_0 \in \mathbb{R}_{++}$. With a normal prior, the posterior belief after observing a study $(X, S)$ is given by

$$\pi_1^{(X,S)} = \mathcal{N}\left(\tfrac{\sigma_0^2}{S^2+\sigma_0^2}X + \tfrac{S^2}{S^2+\sigma_0^2}\mu_0, \tfrac{S^2\sigma_0^2}{S^2+\sigma_0^2}\right). \tag{4}$$

Two utility functions we consider are quadratic loss and binary action utility. The *quadratic loss* utility function has $\mathcal{A} = \mathbb{R}$ and $U(a, \theta) = -(a-\theta)^2$. This is a canonical utility function for a public that makes a continuous policy decision $a$, with the state $\theta$ representing the public's ideal point. Under quadratic loss utility, the maximizing action choice given belief $\pi_1$ is $a^*(\pi_1) = \mathbb{E}_{\theta\sim\pi_1}[\theta]$.

The *binary action* utility function has $\mathcal{A} = \{0, 1\}$ and $U(a, \theta) = a\cdot\theta$. Here there is a binary decision, such as implementing a program ($a = 1$) or not ($a = 0$). The state $\theta$ represents the net benefit of implementation. The chosen action is then $a^*(\pi_1) = \mathbf{1}(\mathbb{E}_{\theta\sim\pi_1}[\theta] > 0)$, where $\mathbf{1}$ is the indicator function (taking $a = 0$ at indifference).

## 2.4 Interpretations of the model

As mentioned in the introduction, not all research findings get published. The acceptance rate at top economics journals is now below 6% (Card and DellaVigna, 2013). This paper can be viewed as solving for the optimal publication rule conditional on any fixed share of studies to be published. The cost $c$ would then be a shadow cost on this publication constraint. Alternatively, there may be a genuine opportunity cost of the public's attention: if the public has a limited ability to process information, publishing one study pulls attention from others. To the extent that papers in high-ranking, selective journals receive disproportionate attention and influence, one can interpret our analysis as characterizing which papers should be published in these top journals.

In this model, the decision to publish is based on the objective of maximizing policy-based welfare. We discuss alternative social objectives in Section 6 and Appendices A.1 and A.2. The model also assumes that information arrives only once, abstracting from the possibility that later studies would further change the public's beliefs and policies. Section 5 extends the model to explore the decision to publish a study today when the journal expects more information to arrive in the future.

Throughout the paper, we make a simplifying assumption that a study is summarized by a normal signal with a point estimate $X$ and standard error $S$. Note that one might interpret the variable $S$ to in fact be larger than a study's reported standard error, which only captures uncertainty due to sampling variation. That could occur if the study has limited external validity, meaning that the estimated parameter is only partially informative about the policy parameter of interest. Violations of the identifying assumptions required for internal validity could also add noise to the estimate. We discuss some considerations that may arise when $S$ is not fully observed by the journal in Appendix A.4.

# 3 Optimal publication rules

## 3.1 Characterizing the optimum

Write out the ex ante expected welfare given some publication rule $p$ and default action $a^0$ as $EW(p, a^0)$:

$$EW(p, a^0) = \mathbb{E}\Big[qp(X,S)(U(a^*(\pi_1^{(X,S)}), \theta) - c) + \big(1 - qp(X,S)\big)U(a^0, \theta)\Big], \tag{5}$$

where the expectation is taken with respect to $\theta \sim \pi_0$, $S \sim F_S$, and $X|\theta, S \sim \mathcal{N}(\theta, S^2)$. Given a specified updating rule, the optimal publication rule $p$ maximizes expected welfare $EW(p, a^0)$ for the appropriately determined default action $a^0$. Under naive updating, $p$ is chosen to maximize $EW(p, a^*(\pi_0))$. Under Bayesian updating, $p$ is chosen to maximize $EW(p, a^*(\pi_1^{0,p}))$.

While we seek to maximize ex ante welfare, it is helpful to consider the journal's interim problem after a study has been submitted. The journal observes $(X, S)$ and has interim belief $\pi$ given by $\pi = \pi_1^{(X,S)}$. At this interim belief, the journal can evaluate the expected payoff from publication (leading to public belief $\pi_1 = \pi$ and action $a^*(\pi)$) and from nonpublication (leading to public belief $\pi_1 = \pi_1^0$ and action $a^0$). Denote by $\Delta(\pi, a^0)$ the gross interim benefit – not including publication costs –

of publishing a study that induces interim belief $\pi$ given default action $a^0$:[5]

$$\Delta(\pi, a^0) = \mathbb{E}_{\theta \sim \pi}[U(a^*(\pi), \theta) - U(a^0, \theta)]. \tag{6}$$

Say that publication rule $p$ is *interim optimal* given default action $a^0$ if it (almost surely) publishes a study when $\Delta(\pi_1^{(X,S)}, a^0) > c$ and does not publish when $\Delta(\pi_1^{(X,S)}, a^0) < c$.

Under naive updating, expected welfare is straightforwardly maximized by choosing the interim optimal publication rule given default action $a^0 = a^*(\pi_0)$. Under Bayesian updating, the default action $a^0$ depends on the publication rule. However, we find that the Bayesian optimal publication rule is also interim optimal against the default action that it induces.

**Lemma 1.** *Under either naive or Bayesian updating, let $p$ be an optimal publication rule and let $\pi_1^0$ be the induced default belief. The publication rule $p$ is interim optimal given default action $a^*(\pi_1^0)$.*

In other words, even under Bayesian updating, the journal's publication rule is a best response to the public's default action.[6] To show this result, we first establish that for any fixed publication rule $p$, the Bayesian default action $a^*\left(\pi_1^{0,p}\right)$ maximizes expected welfare $EW(p, a^0)$ over choice of $a^0$. Under Bayesian updating, then, the journal and public can be thought of as engaging in a sequential game of common interest. The value of such a game is unchanged when the journal moves first (as we posit) or moves second: $\max_p \max_{a^0} EW(p, a^0) = \max_{a^0} \max_p EW(p, a^0)$.

Lemma 1 is a key result for characterizing optimal policies under Bayesian updating. First, the lemma implies that – as with naive updating – any characterization of interim optimal publication rules extends to optimal publication rules. Second, it simplifies the maximization program we use to solve for the optimal Bayesian policy: instead of maximizing over all publication rules, we can restrict attention to

[5] Frankel and Kamenica (2019) provide general characterizations of these gross interim benefit functions – the value of information, in an ex-post sense – across decision problems.

[6] In a game with different timing in which the journal could not commit to a publication rule, one might define a publication rule $p$ and default belief $\pi_1^0$ as constituting a Bayes Nash equilibrium if they jointly satisfy (i) the publication rule $p$ is interim optimal given default action $a^0 = a^*(\pi_1^0)$, and (ii) the default belief is Bayesian, i.e., $\pi_1^0 = \pi_1^{0,p}$. Our notion of optimality under Bayesian updating does not impose (i); nevertheless, Lemma 1 below clarifies that (i) will in fact be satisfied for an optimal publication rule. Hence, any optimal publication rule would induce a Bayes Nash equilibrium, but there may exist Bayes Nash equilibria that are not optimal.

publication rules that are interim optimal given some default action.

One immediate corollary of Lemma 1 is as follows. Define a study $(X, S)$ to be a *null result* if publishing the study does not change the optimal action from the default action, i.e., if $a^*(\pi_1^{(X,S)}) = a^0$.[7]

**Observation 1** (Do not publish null results)**.** The gross interim benefit of publishing a null result is zero. Therefore, by Lemma 1, the optimal publication rule does not publish null results.

Hence, it is only ever optimal publish studies that move the public's beliefs – and its corresponding action – away from the default. Intuitively, we would expect that "extreme" findings – ones which move beliefs and actions further from the defaults – are more valuable to publish than "moderate" findings. The following condition on utility functions allows us to formalize this result.

Say that a utility function $U : \mathcal{A} \times \Theta \to \mathbb{R}$ is *supermodular* if for all $\underline{a} < \overline{a}$ and $\underline{\theta} < \overline{\theta}$, it holds that $U(\overline{a}, \overline{\theta}) + U(\underline{a}, \underline{\theta}) \geq U(\underline{a}, \overline{\theta}) + U(\overline{a}, \underline{\theta})$. Supermodular utilities guarantee that the public takes higher actions when it believes that the state is higher. Quadratic loss and binary action utilities are both supermodular.

**Theorem 1.** *Fix either updating rule. Let the utility function $U$ be supermodular. Under an optimal publication rule, for every standard error $S = s$ there exists an interval $I \subseteq \mathbb{R}$ such that $(X, s)$ is published if and only if $X \notin I$.*[8]

So with supermodular utilities, the journal publishes point estimates that are sufficiently high, sufficiently low, or both. Putting together Observation 1 and Theorem 1, we see that under these utility functions, the journal indeed publishes extreme findings – ones that lead to extreme beliefs and actions relative to the default.

The logic behind Theorem 1 is straightforward. Higher point estimates $X$ yield higher interim beliefs on the state (in the sense of first order stochastic dominance), because point estimates at a fixed standard error are ordered by the monotone likelihood ratio property. Under supermodularity, higher beliefs increase the benefit of taking higher actions. So for any low point estimate that yields an optimal action

[7]Whether a study is a "null result" in this sense depends on the default belief. Our definition differs from its common usage, which often refers to a point estimate that is not statistically significantly different from 0.

[8]It is possible that $I = \emptyset$, in which case all studies with $S = s$ are published; or that $I = \mathbb{R}$, in which no studies at $S = s$ are published.

below the default, there would be a greater interim benefit of publishing an even lower point estimate. Likewise, for any high point estimate that yields an action above the default, there would be a greater interim benefit of publishing an even higher point estimate. Hence, it is interim optimal to publish point estimates that are sufficiently high or sufficiently low. Finally, Lemma 1 implies that this characterization of interim optimal rules extends to the optimal publication rule as well.

The next subsection derives explicit solutions for optimal publication rules for our leading examples of normal priors and quadratic loss or binary action utility functions.

## 3.2 Examples of optimal publication rules

In this section we present the optimal publication rules for our two leading example utility functions, quadratic loss and binary action, under the assumption of normal priors. Appendix B.1 provides the formal derivations of these policies for the case of Bayesian updating; for naive updating, we can simply look for the interim optimal policy against the prior belief. It turns out that for these example utility functions and priors, Bayesian updating and naive updating yield identical optimal policies. Indeed, for these two utility functions, Appendix B.1 presents more general conditions on priors under which Bayesian and naive updating yield identical publication rules.

### 3.2.1 Quadratic loss utility with normal priors

Under quadratic loss utility, welfare is $W(D, a, \theta) = -(a-\theta)^2 - Dc$ for $a \in \mathcal{A} = \mathbb{R}$. The public chooses an action equal to its posterior mean belief about the state. So when the default action is $a^0$, the gross interim benefit of publishing a study $(X, S)$ that induces a belief with mean $\mu_1^{(X,S)}$ evaluates to $(\mu_1^{(X,S)} - a^0)^2$. The interim optimal publication rule is therefore to publish if $\left|\mu_1^{(X,S)} - a^0\right| \geq \sqrt{c}$.

Under normal priors (for which $\mu_1^{(X,S)}$ is given by (4)), Proposition 1 part 1 establishes that the above rule is optimal – for Bayesian as well as naive updating – with the default action $a^0 = \mu_0$. Parts 2 and 3 provide comparative statics.

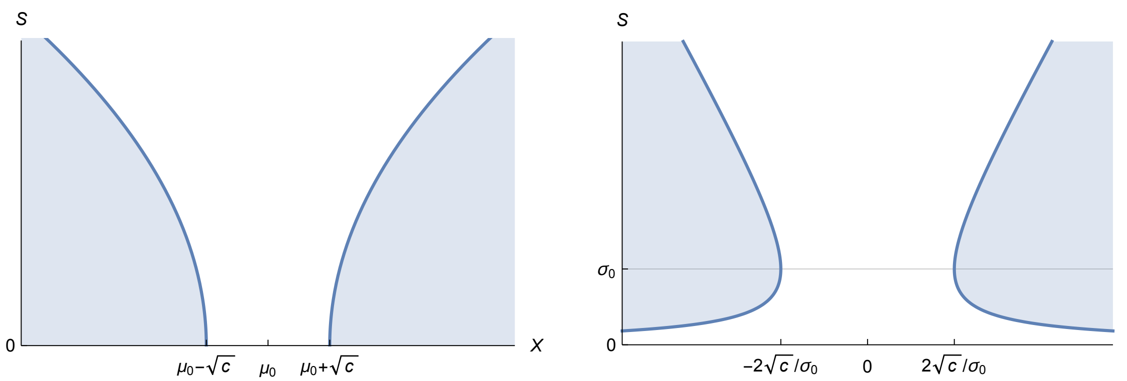
**Proposition 1.** *Suppose there is quadratic loss utility and a normal prior.*

1. *Under either Bayesian or naive updating, it is optimal to publish a study* $(X, S)$ *if and only if* $|X - \mu_0| \geq \left(1 + \frac{S^2}{\sigma_0^2}\right)\sqrt{c}$, *i.e.,* $\frac{|X-\mu_0|}{S} \geq \left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$.

2. *The publication cutoff* $\left(1 + \frac{S^2}{\sigma_0^2}\right)\sqrt{c}$ *in terms of the difference of the point estimate from the prior mean is independent of the study arrival probability* $q$ *and the mean* $\mu_0$*. It is larger when the standard error* $S$ *is larger, the prior variance* $\sigma_0^2$ *is smaller, or the cost of publication* $c$ *is larger.*
3. *The publication cutoff* $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$ *in terms of the magnitude of the t-statistic is nonmonotonic and convex in the standard error* $S$*: it has a minimum at* $S = \sigma_0$ *and goes to infinity as* $S \to 0$ *or* $S \to \infty$*.*

The publication rule described in Proposition 1 part 1 corresponds to a "two-sided test" in which the journal publishes if the point estimate is sufficiently high or sufficiently low. Equivalently, we can restate the publication rule in terms of a two-sided test for the t-statistic $(X - \mu_0)/S$. See Figure 1.

Figure 1: Optimal publication region shaded for quadratic loss utility, normal prior.




(a) In terms of the point estimate $X$ and the standard error $S$.

(b) In terms of the t-statistic $t = (X - \mu_0)/S$ and the standard error $S$.

The form of a two-sided test is of course familiar from the null-hypothesis significance testing paradigm. However, we wish to highlight two ways in which our policy is distinct from two-sided tests as they are traditionally applied. First, we compare the point estimate $X$ to the prior mean, not to some other point, e.g., a null hypothesis of $\theta = 0$. Second, the cutoff for publication is not given by a conventional value, such as a t-statistic of 1.96 corresponding to a p-value of .05. The cutoff is instead determined by a cost-benefit analysis.

Proposition 1 part 2 finds that a given point estimate of $X$ moves beliefs more, and thus makes publication more likely – in the sense of a smaller cutoff value for $|X - \mu_0|$ – when the standard error $S$ is smaller or when the prior uncertainty $\sigma_0$ is larger. Likewise, publication is more likely when the cost of publication $c$ is lower.

When deciding to publish at a given t-statistic, rather than point estimate, Proposition 1 part 3 finds a non-monotonic publication threshold as a function of $S$. (For other parameters, the comparative statics in terms of the t-statistic would be identical to those on the point estimate.) For a precise study with a low standard error or an imprecise one with a high standard error, the journal requires a high t-statistic to be willing to publish; for a study of intermediate precision, the journal publishes at a lower t-statistic. The journal is most willing to publish a study at a given t-statistic when the standard error $S$ is equal to $\sigma_0$, the standard deviation of the prior.

To gain intuition on this nonmonotonic comparative static, recall that here the journal publishes studies that move the interim mean sufficiently far from the prior mean. Fix a prior mean and standard deviation of $\mu_0 = 0$ and $\sigma_0 = 1$, and consider different studies that might arrive with a given t-statistic $t = X/S$, say, $t = 4$. If a very precise study ($S \simeq 0$) arrives with $t = 4$, the point estimate must have been close to 0, so it moves the mean very little. As we scale up the point estimate and standard error while keeping $t = 4$, the posterior mean moves higher, peaking at a mean of 2 when $X = 4$ and $S = 1$. Increasing the point estimate and standard error further, the mean falls back towards 0, because the study becomes too noisy (relative to the prior) to move beliefs much.[9] In other words, fixing the "statistical significance" as measured by the t-statistic, the change in mean first grows and then declines in the "practical significance" as measured by the magnitude of the point estimate.

#### 3.2.2 Binary action utility with normal priors

Under binary action utility, welfare is given by $W(D, a, \theta) = a\theta - Dc$ for $a \in \mathcal{A} = \{0, 1\}$. The public chooses action $a = 0$ if its posterior mean belief about the state is weakly less than 0, and action $a = 1$ if the posterior mean is positive. So for the default action $a^0 = 0$, the gross interim benefit of publishing a study $(X, S)$ inducing belief $\pi_1^{(X,S)}$ with mean $\mu_1^{(X,S)}$ evaluates to $\max\{0, \mu_1^{(X,S)}\}$. For the default action $a^0 = 1$, the gross interim benefit is instead $\max\{0, -\mu_1^{(X,S)}\}$.

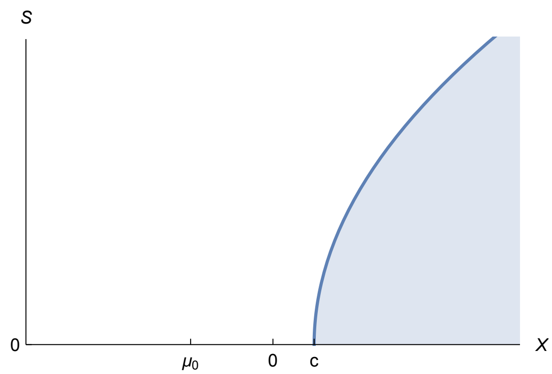
Under normal priors with prior mean normalized to $\mu_0 \leq 0$, Proposition 2 establishes that the optimal rule – for Bayesian as well as naive updating – yields

[9] The general formula for the change in mean given a t-statistic $t = (X - \mu_0)/S$ and standard error $S$ – with a corresponding point estimate of $X = \mu_0 + tS$ – is $t\frac{\sigma_0^2 S}{\sigma_0^2 + S^2}$. To understand why this change in mean falls to zero when we fix $t$ and take $S \to \infty$, recall that the interim mean is a weighted average of the prior mean $\mu_0$ and the point estimate $X$. The point estimate scales linearly with $S$, but the weight on the point estimate is proportional to $1/S^2$.

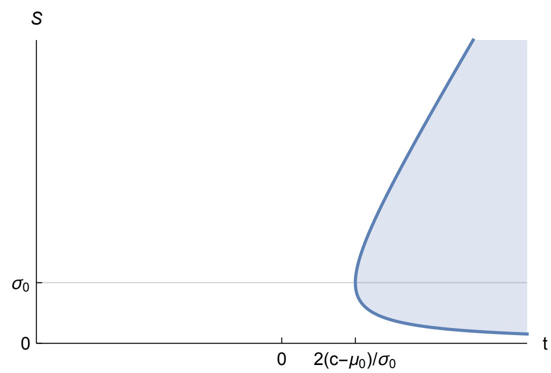
$a^0 = 0$. The corresponding (interim) optimal publication rule is therefore to publish if $\mu_1^{(X,S)} \geq c$. This publication rule corresponds to a "one-sided test." At any given standard error, a study is published only if the point estimate is sufficiently high.

**Proposition 2.** *Suppose there is binary action utility and a normal prior with $\mu_0 \leq 0$. Then under either Bayesian or naive updating, it is optimal to publish a study if and only if $X \geq \left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2}\mu_0$, i.e., $\frac{X-\mu_0}{S} \geq \left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)(c - \mu_0)$.*

Figure 2: Optimal publication region shaded for binary action utility, normal prior.




(a) In terms of the point estimate $X$ and the standard error $S$.




(b) In terms of the t-statistic $t = (X - \mu_0)/S$ and the standard error $S$.

Proposition 9 in Appendix B.2 provides comparative statics for how the publication cutoff varies with parameters. Most of the comparative statics are analogous to those from the quadratic loss publication rule in Proposition 1. However, the two policies depend differently on the prior mean. Suppose we fix a point estimate $X > 0$ and we consider prior means $\mu_0 < 0$. With quadratic loss utility, increasing $\mu_0$ towards 0 would make the journal less willing to publish: there will be a smaller difference $X - \mu_0$, and therefore the posterior mean will be closer to the prior mean. With binary actions, increasing $\mu_0$ towards 0 makes the journal more willing to publish: the posterior mean will be higher in absolute terms, indicating that the benefit of switching from $a = 0$ to $a = 1$ is higher.

# 4 Selective publication and statistical inference

The key conclusion from the previous section was that welfare-maximizing publication rules should selectively publish extreme findings over moderate findings. This conclusion, of course, contrasts with calls for reform aimed at eliminating selection. Selective publication is understood to distort inference and harm replicability. See, for instance, Rosenthal (1979) and Ioannidis (2005) on how standard inference from published results can be inaccurate when publication is based on statistical significance.

To study these issues in our model, say that a publication rule $p$ is *non-selective* if $p(x, s)$ is constant in the point estimate $x$ for each standard error $s$,[10] and otherwise is *selective*. That is, non-selective publication rules do not condition publication on the study's findings. Non-selective publication rules may still condition on the standard error (i.e., on the study's design), which is independent of the state.

We can now see how selective publication may distort inference. Recall that the point estimate $X$ is drawn from the distribution $X|\theta, S \sim \mathcal{N}(\theta, S^2)$ with density $f_{X|\theta,S}(x|\theta, s) = \varphi((x - \theta)/s)/s$. Conventional statistical inference on $\theta$ from $X$ and $S$ is based on this density. Under publication rule $p$, however, the density of $X$ *conditional on publication* ($D = 1$) is instead

$$f_{X|\theta,S,D=1}(x|\theta, s) = \frac{p(x, s)}{\mathbb{E}[p(X, S)|\theta, S = s]} \cdot f_{X|\theta,S}(x|\theta, s). \tag{7}$$

If publication is non-selective, the density conditional on publication $f_{X|\theta,S,D=1}$ matches $f_{X|\theta,S}$. With selective publication, these densities may differ. In that case conventional inferences would be flawed. We present a set of novel results below showing that a non-selective publication rule is not just sufficient but also necessary for the validity of standard inference in a number of senses.

## 4.1 Distortions from selection

Let $\Phi$ denote the cdf of a standard normal distribution.

**Theorem 2.** *Suppose that there is an open set $\Theta_0 \subseteq \mathbb{R}$ contained in the support of the prior distribution of $\theta$. Fix some standard error $s > 0$. Each of the following conditions holds if and only if the publication rule $p$ is non-selective:*

1. Unbiasedness. *For each $\theta \in \Theta_0$, $\mathbb{E}[X|\theta, S = s, D = 1] = \theta$.*

[10] We mean by this statement that $p(x, s)$ is constant in $x$ almost surely over realizations of $X$, i.e., that $\text{Prob}\big(p(X, s) = \mathbb{E}[p(X, s)|S = s]|\theta, S = s\big) = 1$ for all $\theta$. Nothing changes if $p(x, s)$ may vary with $x$ on sets of $X$ that can only occur with zero probability given $\theta, S = s$.

2. Publication probability constant in state. *The publication probability conditional on standard error,* $\mathbb{E}[p(X,S)|\theta, S=s]$*, is constant over* $\theta \in \Theta_0$*.*
3. Undistorted naive updating. *For all distributions* $F_S$ *on* $\mathcal{S}$*, the Bayesian default belief* $\pi_1^{0,p}$ *is equal to the naive default belief* $\pi_0$*.*

*Suppose additionally that* $\Theta = \mathbb{R}$ *and that the publication rule takes the form of* $p(x,s) = \mathbf{1}(x \notin I(s))$ *for some interval* $I(s) \subsetneq \mathbb{R}$*, where non-selective publication corresponds to* $I(s) = \emptyset$*. Fix some critical value* $z > 0$*. Then the publication rule* $p$ *is non-selective if and only if:*

4. Size control of confidence intervals. *For each* $\theta' \in \Theta$*,* $\mathrm{Prob}\big(\theta' \in [X - zs, X + zs]|\theta = \theta', S = s, D = 1\big) \geq \Phi(z) - \Phi(-z)$*.*

As suggested above, when the publication rule is non-selective, the conclusions of parts 1 - 4 of Theorem 2 – the validity of conventional inferences – hold fairly straightforwardly. The novel results of the theorem are the converses, that each part in turn implies non-selective publication.[11] We will first go over the interpretation of each part, and we then discuss the intuition behind their proofs.

### Interpreting Theorem 2

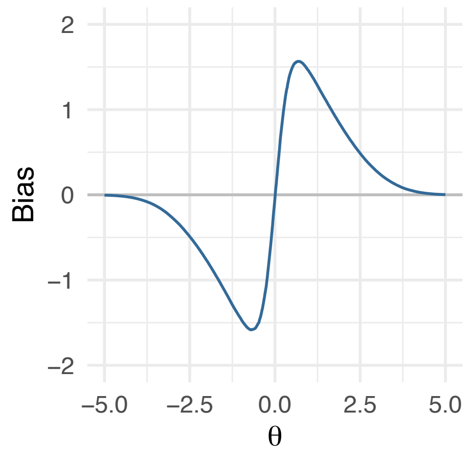
Part 1 of Theorem 2 establishes that after conditioning on publication, selective publication implies that the point estimate $X$ is a biased estimator for the state $\theta$. For instance, suppose that a study is only published when $|X| > 1.96 \cdot S$; this is the conventional cutoff for rejecting a null hypothesis of $\theta = 0$ at the 95% statistical significance level. In that case, the bias $\mathbb{E}[X|\theta, S=s, D=1] - \theta$ will be positive if the state $\theta$ is above 0, and will be negative if the state is below 0. Figure 3 panel (a) illustrates the bias conditional on publication when $S = 1$ and when a study is only published if $|X| > 1.96$.[12]

Part 2 establishes that, if publication is selective ($p(x,s)$ varies with $x$), then the publication probability $E[p(X,S)|\theta, S=s]$ conditional on any standard error $S = s$ and $\theta$ varies with the state $\theta$. From Equation (7), if the publication probability varies with $\theta$, then so too does the ratio $f_{X|\theta,S,D=1}(x|\theta,s)/f_{X|\theta,S}(x|\theta,s)$. Selective publication therefore renders invalid uncorrected likelihood-based inference, such as maximum likelihood estimation or likelihood ratio tests. For the example in which

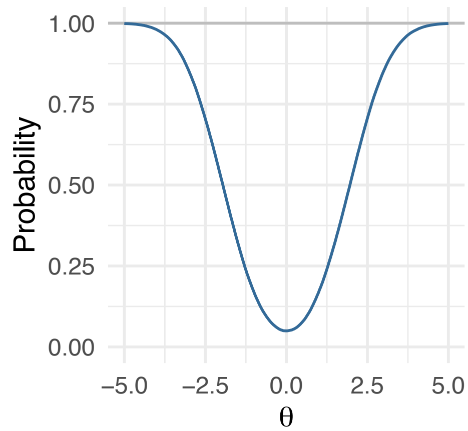
[11] For parts 1, 2, and 4 of the theorem, if publication is non-selective, then the results hold for each $s$ and/or $z$; and if the results hold for any $s$ and/or $z$, then publication is non-selective.

[12] Figures similar to Figure 3 panels (a) and (d) can be found in Andrews and Kasy (2019).

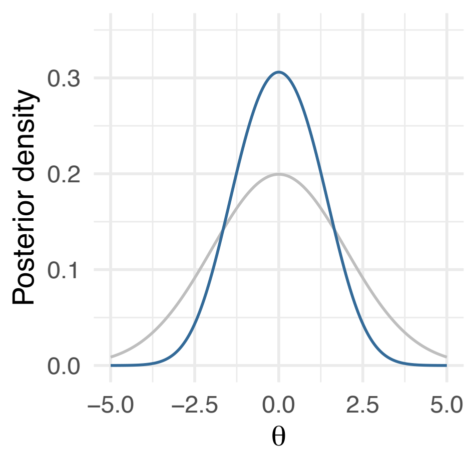
Figure 3: Distortions due to selectively publishing point estimates $|X| > 1.96$.



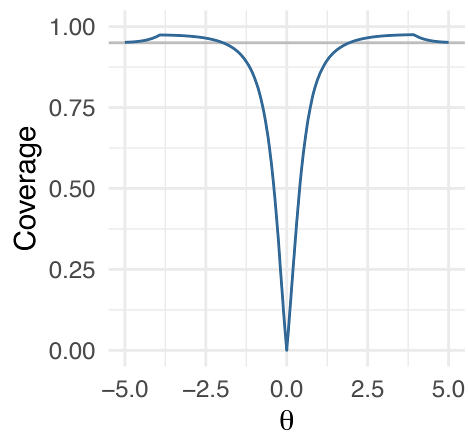

(a) Bias of $X$ as an estimator for $\theta$




(b) Publication probability given $\theta$




(c) Bayesian (blue) and naive (grey) default beliefs




(d) Nominal 95% confidence interval coverage

a study is published if and only if $|X| > 1.96 \cdot S$, Figure 3 panel (b) fixes $S = 1$ and then shows how the publication probability depends on $\theta$. Here we see that the publication probability falls as the state $\theta$ gets closer to 0, the center of the nonpublication interval.

Part 3 says that the Bayesian default belief is equal to the naive default belief (i.e., the prior) for every possible distribution of standard errors if and only if publication is non-selective. To interpret this result, think of a "partially sophisticated" public. This public is aware that studies may sometimes go unpublished, and thus that naive updating may lead to distorted beliefs. But it does not know the study arrival rate $q$ or the distribution of standard errors $F_S$, and may not even have a well-specified prior over these objects. Therefore it does not know how to correct these distortions. Under a non-selective publication rule, though, the partially sophisticated public can be confident in updating naively. For any $q$ and any $F_S$, the Bayesian default belief is guaranteed to be equal to the naive one.[13]

Figure 3 panel (c) illustrates how the Bayesian and naive default beliefs differ under the selective publication rule that publishes if $|X| > 1.96 \cdot S$. When no publication is observed, a Bayesian understands that there may have been a study with point estimate $X \simeq 0$ that was submitted but went unpublished. Hence the Bayesian default belief places a higher posterior probability on $\theta$ close to 0, and a lower probability on $\theta$ far from 0.[14] The figure assumes a prior of $\theta \sim \mathcal{N}(0, 4)$; study arrival rate $q = 1$; and that the standard error is drawn as $S = 1$ with certainty.

Finally, part 4 of Theorem 2 restricts attention to the form of publication rule found in Theorem 1, in which studies are published only if the point estimate falls outside of an interval. We find that selective publication of this form will fail to control the size of conventional frequentist confidence intervals. Given standard error $S = s$ and z-score $z$, there exists some realization of the state $\theta$ for which the probability that the interval $[X - zs, X + zs]$ contains $\theta$ is less than the nominal confidence level $\Phi(z) - \Phi(-z)$.[15] Figure 3 panel (d) fixes $S = 1$ and shows the coverage probability of the nominal 95% confidence interval $[X - 1.96, X + 1.96]$ as a function of the state when the journal publishes only if $|X| > 1.96$. If the true state is $\theta = 0$, there is in fact zero probability conditional on publication that this confidence interval contains the state. For some other values of $\theta$, the probability that the confidence interval contains the state is higher than 95%.

[13]The arrival rate $q$ does not affect whether naive beliefs are distorted. The Bayesian default belief is equal to the prior for some $q \in (0, 1]$ if and only if it is equal to the prior for all $q \in (0, 1]$.

[14]Abadie (2018) demonstrates how a failure to pass a standard statistical significance threshold can be extremely informative when studies are precise. In such cases, the Bayesian default belief diverges greatly from the naive one.

[15]In Appendix B.3 we show that, fixing $z > 0$, there do exist selective publication rules outside of the form of Theorem 1 for which the coverage probability of the confidence interval $[X - zS, X + zS]$ does in fact always equal $\Phi(z) - \Phi(-z)$.

**Intuition for proof of Theorem 2**

We now discuss the key intuitions behind the proof that parts 1 - 4 of Theorem 2 each imply non-selective publication.

That part 2 (a constant publication probability in the state, conditional on the standard error $S = s$) implies non-selective publication follows from the completeness of the normal location family of distributions; see, for instance, Theorem 6.22 in Lehmann and Casella (1998). Completeness is the continuous analog of a full rank condition, applied to the linear operator mapping $g$, a function of $x$, to $\mathbb{E}[g(X)|\theta, S = s]$,a function of $\theta$. Completeness implies that for any function $g(x)$ for which $\mathbb{E}[g(X)|\theta, S = s]$ is constant in $\theta$ over an open set, it holds that $g(x)$ must be almost everywhere constant. Plugging in the function $g(x) = p(x, s)$, we see that if the conditional publication probability at state $\theta$ is constant in $\theta$, then the publication probability $p(x, s)$ cannot vary with $x$.

To show that part 1 (unbiasedness) implies part 2, let $S = 1$ without loss. We leverage "Tweedie's formula," which holds because $\varphi'(x) = -x\varphi(x)$:

$$\int (x - \theta)\varphi(x - \theta)p(x, 1)dx = \partial_\theta \mathbb{E}[p(X, S)|\theta, S = 1].$$

Unbiasedness implies that the left hand side of this equality is 0, and thus so is the right hand side, yielding part 2.

Next, to see that part 3 (undistorted naive updating) implies part 2, observe from Equation (3) that the Bayesian default belief matches the prior – the naive default belief – only if the publication probability unconditional on the standard error, $\mathbb{E}[p(X, S)|\theta]$, is constant in $\theta$. If we desire that this hold for all possible distributions over the standard error $S$, then the publication probability conditional on any given $S = s$ must also be constant, which is exactly part 2.

Finally, we show that part 4 (size control of confidence intervals) implies part 2. Suppose for the sake of contradiction that the publication rule is selective: at $S = s$, it only publishes point estimates $X$ outside a non-empty interval $I(s)$. Then a straightforward calculation shows that there are states $\theta'$ in the interior of $I(s)$ for which the confidence interval contains $\theta'$ with probability less than the nominal level.

## 4.2 Optimal non-selective publication

The fact that selective publication distorts inference means that there is a trade-off between policy-relevance and credibility. The policy-relevance criterion pushes towards selectively publishing extreme results. But if we wish standard inference to remain valid, then we must restrict ourselves to non-selective publication rules. What is the *optimal non-selective* publication rule – the rule that maximizes utility subject to the constraint of being non-selective?

**Proposition 3.** *There exists $\bar{s} \in \mathbb{R}_+ \cup \{\infty\}$ for which an optimal non-selective publication rule is to publish a study $(X, S)$ if and only if $S < \bar{s}$.*[16] *The optimal non-selective rule is the same under naive and Bayesian updating.*

When the journal is not allowed to screen on the point estimate, the only remaining option is to screen on the standard error. In that case, the journal should publish studies with smaller standard errors over those with larger standard errors. The result follows immediately from the fact that $X \sim \mathcal{N}(\theta, s^2)$ is a Blackwell more informative signal of the state $\theta$, and is thus more valuable ex ante, when the standard error $s$ is smaller. Under a normal prior and quadratic loss utility, for instance, we can explicitly solve for the optimal non-selective publication rule. If $\sigma_0^2 \geq c$ (high prior uncertainty, low publication costs), then a study is published if $S \leq \bar{s}$, with $\bar{s} = \sigma_0 \sqrt{\frac{\sigma_0^2}{c} - 1}$; and if $\sigma_0^2 < c$ (low prior uncertainty, high costs) then no study is published.[17]

# 5 A two-period model

The model introduced in Section 2 assumes that a single study is published or not, and then a policy is chosen. If an additional study were to arrive, though, the public might want to switch to a new policy. This new policy would depend on the results of the later study as well as those of the original one (if published). So if a journal expects additional studies to arrive in the future, it faces new considerations when deciding whether to publish a study today.

[16] For $\bar{s} = 0$, no study would be published.

[17] If a non-selective publication rule is used and no publication is observed, then the default belief will be $\pi_0$ (under either updating rule) so the expected welfare will be $-\operatorname{Var}_{\theta \sim \pi_0}[\theta] = -\sigma_0^2$. Conditional on $S$ but not $X$, the expected welfare of publishing can be solved for as $\sigma_0^2 \cdot \frac{S^2}{S^2 + \sigma_0^2} - c$. An optimal non-selective publication rule publishes a study if $\sigma_0^2 \cdot \frac{S^2}{S^2 + \sigma_0^2} - c > -\sigma_0^2$.

In order to explore such issues, this section introduces a *two-period model.* As before, there is an unknown policy-relevant state of the world $\theta$, which we take to be persistent over time. The original model of publication and policy choice – which we now refer to as the one-period model – makes up the first period. The new second period captures, in reduced form, the impact of future studies: Additional exogenous information arrives and the public takes another action.

## 5.1 Set-up of the two-period model

At the start of the game, the common prior over $\theta$ is $\pi_0$. In the first period, a study is submitted to a journal with probability $q$. If the study arrives, it has point estimate and standard error $(X_1, S_1)$ with $S_1 \sim F_{S_1}$ and $X_1|\theta, S_1 \sim \mathcal{N}(\theta, S_1^2)$. The study is published with probability $p(X_1, S_1)$. The public updates to belief $\pi_1$ as before, with $\pi_1 = \pi_1^{(X_1,S_1)}$ following publication ($D = 1$) or to default belief $\pi_1 = \pi_1^0$ following non-publication ($D = 0$). The induced belief $\pi_1^{(X_1,S_1)}$ is given by Bayes' Rule and the default belief $\pi_1^0$ may be determined either by naive or Bayesian updating. Then the action $a_1$ is taken, with $a_1 = a^*(\pi_1) \in \arg\max_a \mathbb{E}_{\theta\sim\pi_1}[U(a, \theta)]$.

In the second period, an exogenous signal $X_2 \sim \mathcal{N}(\theta, s_2^2)$ – independent of $D$ and of $(X_1, S_1)$ given $\theta$ – is publicly observed. Beliefs update according to Bayes' Rule from prior $\pi_1$ to posterior $\pi_2$. Finally, the action $a_2$ is taken, with $a_2 = a^*(\pi_2) \in \arg\max_a \mathbb{E}_{\theta\sim\pi_2}[U(a, \theta)]$.

We assume that the standard error of the second-period signal, $s_2$, is a parameter that is known by the journal at the start of the game. Our interpretation is that $s_2$ would be low (i.e., precise) when the journal expects that other high quality studies on the topic in question will soon be performed. The parameter $s_2$ would be high (imprecise) when the journal expects future studies on the topic to be performed infrequently, or to be of low quality.

Let $\alpha \in [0, 1)$ describe the payoff weight on the first-period action, relative to $1 - \alpha$ on the second period action. Social welfare is the weighted sum of action payoffs, minus a cost of publication $c > 0$ incurred if a study is published:

$$W(D, a_1, a_2, \theta) = \alpha U(a_1, \theta) - Dc + (1 - \alpha)U(a_2, \theta). \tag{8}$$

A *dynamically optimal* publication rule maximizes the ex-ante expectation of welfare from (8). For a journal that has period-one interim belief $\pi_1^I$ after observing a study and faces default belief $\pi_1^0$, the *gross interim benefit* of publishing the study, denoted $\Delta(\pi_1^I, \pi_1^0)$, is the journal's subjective belief about the increase in weighted action payoffs $\alpha U(a_1, \theta) + (1-\alpha) U(a_2, \theta)$ if the study is published.[18] A publication rule is *dynamically interim optimal* given default belief $\pi_1^0$ if it (almost surely) publishes a study when $\Delta(\pi_1^{(X,S)}, \pi_1^0) > c$ and does not publish when $\Delta(\pi_1^{(X,S)}, \pi_1^0) < c$.

Going forward in this section, we will restrict attention to quadratic loss utility. We explore binary action utility in Appendix B.4.

## 5.2 General properties of dynamically optimal publication

We start our analysis by observing that the result of Lemma 1, that optimal publication rules are interim optimal, extends immediately to the two-period model.

**Lemma 1′.** *Consider the two-period model. Under either naive or Bayesian updating, let $p$ be a dynamically optimal publication rule and let $\pi_1^0$ be the induced default belief. Then the publication rule $p$ is dynamically interim optimal given default belief $\pi_1^0$.*

In other words, given the appropriate default belief, the journal will publish a submitted paper if the gross interim benefit is above the publication cost $c$.

Theorem 3, below, derives some key properties of gross interim benefit functions under quadratic loss utility. First, the theorem establishes that there is a positive benefit of publishing any study that changes the public's belief distribution from the default – even null results that don't change the mean belief. So, unlike in the one-period model, null results may now sometimes be published. Second, the theorem looks at how the benefit of publishing null results depends on the informativeness of the future study, parametrized by standard error $s_2$. It finds that the benefit of publishing null results goes to zero when the future study is either very precise or very imprecise. That is, it is more valuable to publish a null result when the precision of future information is neither too high nor too low.

[18] Unlike in the one-period model, the gross interim benefit here depends on the full default belief $\pi_1^0$, not just the first period default action $a^*(\pi_1^0)$. To expand out the formula for $\Delta(\pi_1^I, \pi_1^0)$, first denote by $\pi_2^{I,X_2}$ and $\pi_2^{0,X_2}$ the Bayesian updated beliefs after observing $X_2$, starting from respective priors $\pi_1^I$ and $\pi_1^0$. We then have

$$\Delta(\pi_1^I, \pi_1^0) = \mathbb{E}_{\theta \sim \pi_1^I, X_2 \sim \mathcal{N}(\theta, s_2^2)} \Big[ \alpha \Big( U\left(a^*\left(\pi_1^I\right), \theta\right) - U\left(a^*\left(\pi_1^0\right), \theta\right) \Big) \\ + (1-\alpha) \Big( U\left(a^*\left(\pi_2^{I,X_2}\right), \theta\right) - U\left(a^*\left(\pi_2^{0,X_2}\right), \theta\right) \Big) \Big].$$

To guarantee that the benefit of publication goes to zero as future information becomes imprecise, we introduce a mild sufficient condition on the prior distribution $\pi_0$. Say that a belief $\pi$ is *bounded by Pareto tails with finite variance* if there exist $K > 0$, $C > 0$, and $\gamma > 3$ such that for $\theta$ outside of the interval $[-K, K]$, $\pi$ admits a density, and this density is bounded above by $C|\theta|^{-\gamma}$.[19]

**Theorem 3.** *Consider the two-period model with quadratic loss utility. Given some prior $\pi_0$ with finite variance, let $\pi_1^0$ be the induced default belief either from naive updating or from Bayesian updating under some publication rule and some $q < 1$. Consider $\Delta(\pi_1^I, \pi_1^0)$, the gross interim benefit of publishing a study that induces period-1 interim belief $\pi_1^I$.*

1. *If $\pi_1^I \neq \pi_1^0$ then $\Delta(\pi_1^I, \pi_1^0)$ is strictly positive.*
2. *Suppose further that $\pi_1^0$ and $\pi_1^I$ have the same mean. Then:*
   (a) *$\Delta(\pi_1^I, \pi_1^0)$ goes to zero as $s_2$ goes to 0.*
   (b) *Under the additional assumption that $\pi_0$ is bounded by Pareto tails with finite variance, $\Delta(\pi_1^I, \pi_1^0)$ goes to zero as $s_2$ goes to infinity.*

As mentioned above, the novel takeaway from part 1 is that in the two-period model there is now a benefit from publishing a null result study, one for which the mean of the induced interim belief $\pi_1^I$ is equal to the mean of the default belief $\pi_1^0$. To prove this result, it suffices for us to show that whenever $\pi_1^I \neq \pi_1^0$, there exists a positive measure of realizations of $X_2$ for which the posterior means of the second-period beliefs updated from these priors would differ. At these realizations of $X_2$, the expected second-period utility $U(a_2, \theta)$ – evaluated by a journal with belief $\pi_1^I$ – is higher for a public that updated from $\pi_1^I$ than for a public that updated from $\pi_1^0$.

Restating the above logic, the benefit of publishing a null result study in period 1 is that it helps the public avoid mistakes when taking the period 2 action. Publishing a null result helps prevent the noisy signal $X_2$ from moving the public's mean belief in period 2 away from the truth. Theorem 3 part 2a observes that when the second-period signal is extremely precise ($s_2 \simeq 0$), there is actually no such benefit from publishing a null result. The signal $X_2$ will reveal the state very precisely, so to the extent that $X_2$ moves beliefs, it moves beliefs to the truth. Part 2b similarly points

[19] As indicated by the terminology, the Pareto distribution with pdf decaying at a rate of $\theta^{-\gamma}$ has finite variance if and only if $\gamma > 3$ (corresponding to a standard Pareto shape parameter, usually denoted $\alpha$, strictly greater than 2). Any distribution with compact support, with normal tails, or with exponentially decaying tails is bounded by Pareto tails with finite variance.

out that when the second-period signal is extremely imprecise ($s_2 \simeq \infty$), there is also no need to publish a null result: with high probability, observing $X_2$ will barely move beliefs. The period 2 studies that may cause mistakes by *moving the public's belief away from the truth* are those with an intermediate level of precision.

If the second period signal were fully informative or completely uninformative, then the game would reduce to a one-period problem (with respective payoff weights $\alpha$ and 1) in which null results had no benefit. So parts 2a and 2b can essentially be reinterpreted as stating that the value of information is "continuous" as the second period information approaches these limits via a normal signal.

To see how such continuity might fail in the absence of bounds on the tail behavior of the prior, consider the $s_2 \to \infty$ limit, and suppose that the prior and default belief are given by the improper uniform prior on the real line. Updating this improper prior with the second period signal yields the posterior $\mathcal{N}(X_2, s_2^2)$. If a first-period result $(X_1, S_1)$ is published, then the expected second-period action payoff is at worst $-S_1^2$, which is the expected payoff from taking $a_2 = X_1$. If the result is not published, though, then with posterior $\mathcal{N}(X_2, s_2^2)$ the public chooses $a_2 = X_2$ and gets expected payoff $-s_2^2$. So as $s_2 \to \infty$, the benefit of publishing the first-period result grows without bound. Similar behavior can arise for proper but heavy-tailed priors with well-defined means.

In order to prove continuity in the appropriate limit, we need to show that the second-period posterior mean after publication of a first-period null result converges in mean-square to the second-period posterior mean after non-publication (integrating over the realization of $X_2$). We are able to show this result for the $s_2 \to 0$ limit by assuming a finite variance for the default belief, in which case both posterior means converge to $X_2$; see Lemma 6 in Appendix C.3. We are able to show this result for the $s_2 \to \infty$ limit by imposing the stronger assumption that the default belief is bounded by Pareto tails with finite variance, in which case both posterior means converge to the prior mean; see Lemma 7 in Appendix C.3.

## 5.3 Example of dynamically optimal publication

The following proposition gives an explicit formula for the gross interim benefit function of the two-period model under quadratic loss utility, normal priors, and naive updating. (Naive updating guarantees that when evaluating this interim benefit, the default belief will be the prior $\pi_0$.) It is dynamically optimal to publish a study if the gross interim benefit is above the publication cost $c$.

**Proposition 4.** *In the two-period model with quadratic loss utility, normal priors, and naive updating, the gross interim benefit* $\Delta(\pi_1^{(X_1,S_1)}, \pi_0)$ *of publishing a study* $(X_1, S_1)$ *is given by* $\beta_0 + \beta_2 \cdot (X_1 - \mu_0)^2$*, where*

$$\beta_0 = (1-\alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2 + S_1^2)(\sigma_0^2 + s_2^2)^2(\sigma_0^2 S_1^2 + \sigma_0^2 s_2^2 + S_1^2 s_2^2)} > 0, \tag{9}$$

$$\beta_2 = \frac{\sigma_0^4(s_2^4 + 2\alpha\sigma_0^2 s_2^2 + \alpha\sigma_0^4)}{(\sigma_0^2 + S_1^2)^2(\sigma_0^2 + s_2^2)^2} > 0. \tag{10}$$

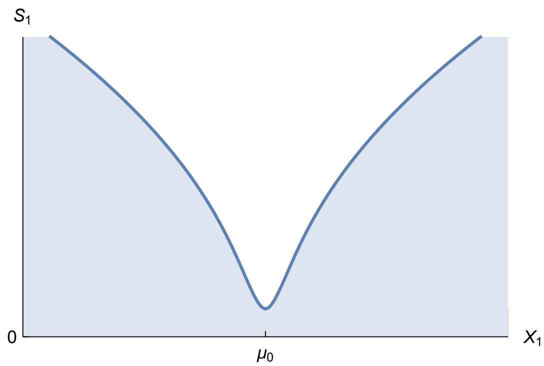
Proposition 4 finds that the interim benefit of publication can be broken up into two additively separable terms, $\beta_0 + \beta_2 \cdot (X_1 - \mu_0)^2$, where neither $\beta_0$ nor $\beta_2$ depends on the point estimate $X_1$. We can interpret $\beta_2 \cdot (X_1 - \mu_0)^2$ as a benefit of publishing extreme findings, as is familiar from the one-period model. This benefit is larger when $X_1$ is further from the prior mean $\mu_0$. Then $\beta_0$ represents the new benefit of publishing null results studies with $X_1 = \mu_0$.

One insight from the formula for $\beta_0$ in (9) is that the benefit of publishing a null result decreases in its standard error $S_1$: there is a bigger benefit of publishing *precise* null results. Indeed, as $S_1 \to \infty$, the benefit of publishing a null result goes to zero. Proposition 10 in Appendix B.2 provides some additional comparative statics on the benefit of publishing a null result. It is more valuable to publish a null result when the prior uncertainty $\sigma_0$ is larger and when the relative payoff weight on the first period $\alpha$ is smaller. Moreover, in line with Theorem 3 part 2, the benefit of publishing a null result increases then decreases in the precision of the second period signal, going to 0 in the fully precise and imprecise limits.

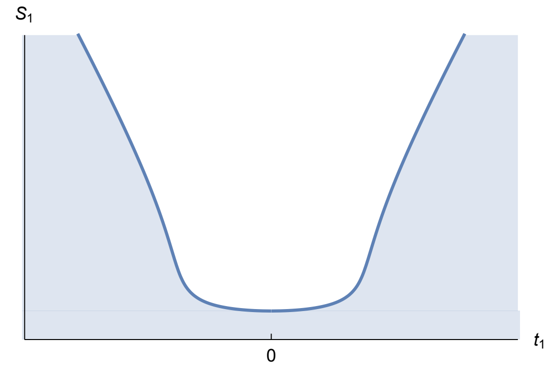
# 6 Conclusion

Sections 2 and 3 of this paper presented and analyzed our benchmark model of publication. A submitted paper is to be published or not, and the social value of publication is derived from its impact on a public policy decision. There is thus an instrumental value in publishing some new result only insofar as it changes public policies. Broadly speaking, we argued for the publication of extreme results over moderate

Figure 4: Dynamically optimal publication region (shaded) for quadratic loss utility, normal prior, naive updating.




(a) In terms of the point estimate $X_1$ and the standard error $S_1$.




(b) In terms of the t-statistic $t_1 = (X_1 - \mu_0)/S_1$ and the standard error $S_1$.

Parameter values $\sigma_0 = 1$, $\alpha = 1/2$, $s_2 = 1$, $c = .1$. Under different parameters such as a higher $c$, a study with a null result of $X_1 = \mu_0$ would not be published even for $S_1 \simeq 0$.

ones. It is more valuable to publish extreme results because they move public beliefs, and therefore public policies, further from the defaults.

As has been noted by many observers, there are reasons outside of this model to be concerned about selectively publishing only extreme results. Section 4 formalizes some of these concerns. Selective publication invalidates standard statistical inference, and causes problems for a public that updates naively in the absence of publication.

Putting these points together, we view the main contribution of this paper as highlighting an important trade-off between the statistical credibility and the policy-relevance of the publication process – a trade-off which has not been generally appreciated in some current debates focusing on replicability. Moreover, we believe that the simple model of publication we introduced can serve as a basis for further analysis. Section 5 explored one such direction, looking at a two-period model. We now conclude the paper by describing a series of additional extensions, some of which are covered in greater detail in the Appendix. Each of these illustrates how our results might change if we were to bring some additional consideration into the framework of our model.

**Alternative social objectives.** Consider publication rules that maximize social objectives other than policy-based welfare. Appendix A.1 presents a *learning* objective. When the social objective is to learn the true state of the world independently of any policy problem, we show that the form of the optimal publication rule may be essentially unchanged from our earlier analysis. The journal continues to publish extreme results. Next, Appendix A.2 presents an *accuracy* objective. When the social objective is to publish accurate results that are as close as possible to the truth, the publication rule can reverse: the journal now prefers to publish unsurprising results.

**Researcher incentives and endogenous study design.** One assumption maintained throughout the paper was that the arrival of studies submitted to journals is exogenous. Appendix A.3 considers an extension in which researchers may alter their study designs in response to the publication rule. Specifically, the researcher chooses whether to perform a study on a given question, and if so, at what level of precision. The researcher receives a benefit if the study is published. Her cost of performing the study depends on its precision, e.g., she faces a higher cost to run an experiment with a larger sample size. Taking into account the researcher's incentives, we find that the journal optimally adjusts the publication rule in two ways: the journal rejects imprecise studies regardless of their findings, and it becomes more willing to publish studies that are sufficiently precise. This modified publication rule induces the researcher to conduct studies that are more precise than she would otherwise choose. Nonetheless, extreme results are still published over moderate ones.

**Imperfectly observed study designs.** In Appendix A.4, we discuss the possibility that study designs may not be perfectly observed – a study may be a less reliable signal of the state than is indicated by its reported standard error. If that is the case, we will need to qualify our earlier claim about publishing extreme results: it would still be optimal to publish results that moved beliefs further, but those results might not be the ones with the most extreme point estimates. Extreme point estimates could be considered "implausible," suggesting problems with the study rather than an extreme state.

**Heterogeneous policymakers.** The audience for a study may consist of a number of heterogeneous policymakers, each with different beliefs about the state of the world or different preferences, as in Andrews and Shapiro (2019). Equivalently, there may be a single policymaker whose beliefs or preferences are uncertain to outsiders. In this case, we can get insight into the optimal publication rule by recalling the two-period model of Section 5: different policymaker types are like members of the public who take different actions in period 2 because they have observed different signal realizations $X_2$. (The map between these models can be made exact when the heterogeneity is driven by private information.) As we have seen, there can now be a positive benefit of publishing a "null result" that doesn't change the average belief, since it still moves the actions of some policymakers towards this mean. But there will be a larger benefit of more extreme findings.

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# Online Appendix



# A Extensions

## A.1 Alternative objective: learning

Separate from any decision problem, the public might value more precise knowledge of the state of the world for its own sake. One natural way of measuring the precision of beliefs is by looking at the variance. We formalize a *learning* objective by supposing that the public seeks a publication rule that minimizes the expected variance of the posterior beliefs $\pi_1$. Formally, under the learning objective we replace the earlier "relevance" welfare function $W(D, a, \theta)$ from Equation (1) with

$$W(D, \pi_1) = -\operatorname{Var}_{\theta \sim \pi_1}[\theta] - Dc, \tag{11}$$

where $c > 0$ continues to represent the social opportunity cost of publication. The *learning-optimal* publication rule $p$ is the one which maximizes the ex-ante expectation of (11).

Suppose that the public uses Bayesian updating. There is then a clear connection between learning and relevance. The posterior expectation of the relevance utility under a quadratic loss utility function $-(a-\theta)^2$ – with the public choosing an action equal to its expectation of the state – is minus the posterior variance. That is exactly the learning welfare. So the learning-optimal policy is identical to the policy that maximizes the quadratic loss relevance objective under Bayesian updating, regardless of assumptions about signals or priors. In order to maximize learning and minimize uncertainty over the state of the world, then, it remains optimal to publish only those studies which induce extreme posteriors. This gives an alternative interpretation of some previous results that were motivated by decision problems.

## A.2 Alternative objective: accuracy

Under an *accuracy* objective, a journal seeks to publish point estimates $X$ that are as close as possible to the true state of the world $\theta$. These estimates can be thought of as the ones that would be the most "replicable" by future studies. Letting $\Theta = \mathcal{X} = \mathbb{R}$, we formalize our accuracy objective by replacing welfare from (1) with

$$W(D, \theta, X) = D \cdot (-(X - \theta)^2 + b), \tag{12}$$

where $b > 0$ indicates the shadow benefit of publication; if no study arrives, welfare is normalized to zero. For simplicity, we now assume a quadratic loss from publishing values of $X$ further from $\theta$. (We consider a generalized loss function below.)

If the goal is to publish accurate results, a non-selective rule will do better than one that publishes only extreme findings. But, as we show, a different kind of selective rule can do even better. Let the *accuracy-optimal* publication rule be the one maximizing the ex-ante expectation of this welfare function.

Under the accuracy objective, publication depends only on the belief $\pi_1^{(X,S)}$. The accuracy-optimal rule publishes a study $(X, S) = (x, s)$ if the interim expected welfare from (12) is greater than 0, i.e., if

$$\mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[(x - \theta)^2] \leq b. \tag{13}$$

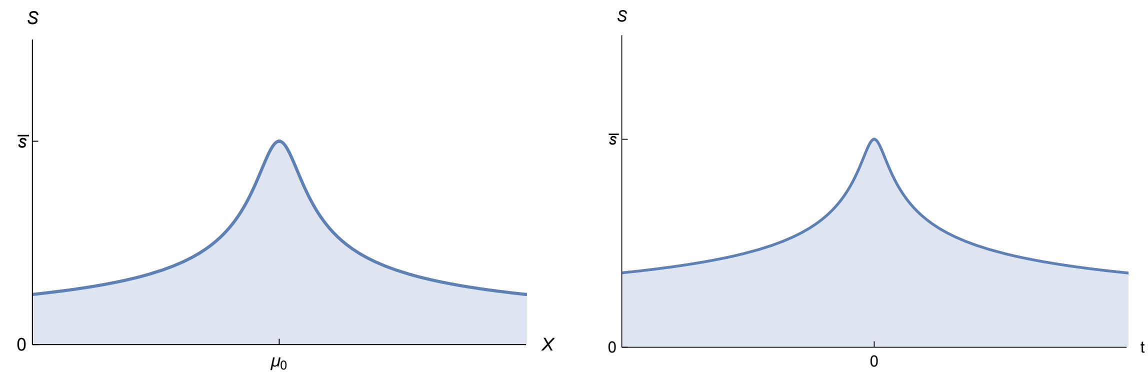
We can explicitly solve for this rule when there are normal priors: publish if $(X - \mu_0)^2 \leq \left(1 + \frac{\sigma_0^2}{S^2}\right)\left(b + b\frac{\sigma_0^2}{S^2} - \sigma_0^2\right)$.[20] At any standard error $S$, it is accuracy-optimal to publish studies with the point estimate $X$ in a symmetric interval about $\mu_0$; see Figure 5. (At sufficiently high standard errors, it may be the case that no studies are published.)

In other words, the accuracy-optimal publication rule has the opposite form as the publication rule maximizing quadratic loss relevance: at a given standard error, it publishes moderate findings and does not publish extreme ones. By the same token, publishing only extreme findings at a given standard error would minimize accuracy. This is because point estimates closer to the prior mean are thought (under the interim belief) to be closer to the true state. For intuition, recall that the distance of the point estimate from the interim mean, $X - \mu_1^{(X,S)}$, is linear in the distance of

[20] As a first step to deriving this expression, rewrite (13) as $\mathrm{Var}_{\theta \sim \pi_1^{(x,s)}}[\theta] + (x - \mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[\theta])^2 \leq b$. Then plug in the variance and expectation from (4) to derive the publication rule above.

the point estimate from the prior mean, $X - \mu_0$. Of course, the accuracy-optimal publication rule is still partially aligned with the earlier (relevance-)optimal rules in that it publishes a larger range of point estimates when standard errors are smaller.

Figure 5: Accuracy-optimal publication region (shaded) for quadratic distance, normal prior.




(a) In terms of the point estimate $X$ and the standard error $S$.

(b) In terms of the t-statistic $t = (X - \mu_0)/S$ and the standard error $S$.

If $b < \sigma_0^2$, as pictured, then no studies are published for $S > \bar{s}$, with $\bar{s} = \frac{\sigma_0 \sqrt{b}}{\sqrt{\sigma_0^2 - b}}$. If instead $b \geq \sigma_0^2$, then an interval of $X$ containing $[\mu_0 - (b - \sigma_0^2), \mu_0 + (b - \sigma_0^2)]$ would be published for any $S$.

Just as the relevance-optimal rule is bad for accuracy, so too is the accuracy-optimal rule bad for relevance. For a fixed standard error and a fixed share of studies to be published, the rule of publishing only moderate point estimates would actually minimize quadratic loss utility – and would therefore also be the worst for the learning objective.[21] A non-selective publication rule would be intermediate on both quadratic loss relevance and on accuracy.

Without giving an explicit characterization, the same qualitative result of publishing moderate results to maximize accuracy would hold if we were to generalize the accuracy objective (12) beyond a quadratic cost of distance. Consider a generalized accuracy objective of

$$W(D, \theta, X) = D \cdot (-\delta((X - \theta)^2) + b), \tag{12$'$}$$

[21] As described in Section B.1, we solved for the rule that maximized quadratic loss utility for Bayesian updating by first showing that the problem was equivalent to $\max_p \max_{a^0} EW(p, a^0)$; rearranging the order of maximization let us conclude that the globally optimal $p$ was also interim-optimal given $a^0$. To solve for the policy that minimizes quadratic loss utility (at a fixed and commonly known standard error), one solves $\min_p \max_{a^0} EW(p, a^0)$. By a minimax theorem, one can rearrange the order of minimization and maximization and conclude that the globally pessimal $p$ is also interim-pessimal given $a^0$, and the interim-pessimal policy is to publish moderate results.

for a strictly increasing function $\delta(\cdot)$. (An arbitrary increasing function of $(X-\theta)^2$ is equivalent to an arbitrary increasing function of $|X-\theta|$.) One can establish that under normal priors, the generalized accuracy-optimal policy maximizing (12′) takes the same qualitative form as that maximizing (12): at a given standard error, either point estimates in a symmetric interval around $\mu_0$ are published, or no point estimates are published.

**Proposition 5.** *Let there be normal priors. The publication rule maximizing the generalized accuracy objective* (12′) *takes the following form: at $S = s$, either no studies $(X, s)$ are published, or there exists $k$ such that a study $(X, s)$ is published if and only if $(X-\mu_0)^2 \leq k$.*

## A.3 A model with researcher incentives

Thus far, we have taken submissions to the journal to be exogenous. In reality submissions come about from a sequence of decisions by researchers: which topics to work on, what designs $S$ to choose, and which findings $X$ to actually write up and submit. In solving for an optimal journal publication rule, one ought to take into account the researchers' endogenous response to the incentives provided. To illustrate, this section presents a stylized *model with incentives* that explores a publication-motivated researcher's choices of whether to conduct a study and how to design that study.

Our analysis here complements some other recent theoretical investigations of how researcher or experimenter design choices may respond to incentives. In our example, the researcher's type will be commonly known and the design of a (submitted) study will be publicly observable, as in Henry and Ottaviani (2017) or the main analysis of McClellan (2017). Tetenov (2016) and Yoder (2018) study how a principal can screen across heterogeneous experimenters with privately known types. Libgober (2015) considers a setting in which study findings are observable, but the study design that led to a finding may be obscured.

**Set-up.** There is a single researcher who takes a research topic as given. There is a common prior $\theta \sim \pi_0$ shared by all parties: the researcher, the journal, and the public.

The timing of the game is as follows. First, the journal publicly commits to a publication rule $p$ for studies on this topic. Then the researcher chooses whether to conduct a study and, if so, what study design $S$ to use; the researcher will submit the results of any study to a journal. Then the game proceeds as in Section 2. If a study $(X, S)$ is submitted it is published with probability $p(X, S)$, and finally the public updates its belief and takes a policy action. The key distinction from the original model is that the study submission probability $q$ and the distribution of study designs $F_S$ are now endogenous to the publication rule $p$.

To keep the analysis simple, we will restrict attention to naive updating. We will also continue to focus on a normal signal structure, with $S \in \mathbb{R}_{++}$ and $X|\theta, S \sim \mathcal{N}(\theta, S^2)$.

**The researcher's problem.** The researcher observes the publication rule $p$ and then decides whether to conduct a study. If she does conduct a study then she chooses its standard error $S \in (0, \infty)$.

Normalize the researcher's outside option payoff from not conducting a study to 0. If a study is conducted, the researcher values its publication, but pays a cost that depends on the precision of the study. Specifically, the researcher gets a benefit of 1 for getting a study published, independently of the study's results. The researcher pays a cost $\kappa(S)$ for conducting a study with standard error $S$, with $\kappa : (0, \infty) \to \mathbb{R}_+$. (Assumptions such as $\kappa'(S) < 0$ would be natural – the researcher pays more for an experiment with a larger sample size, say – but we do not actually need to impose any conditions on the cost function for the results that follow.) So the researcher's ultimate payoff if she conducts a study with standard error $S$ and publication outcome $D$ is

$$D - \kappa(S).$$

Denote the researcher's expected payoff from conducting a study with standard error $S = s$, given journal publication rule $p$, by $V(s, p)$:

$$V(s, p) = \mathbb{E}_{\theta \sim \pi_0, X \sim \mathcal{N}(\theta, s^2)}[p(X, s)] - \kappa(s).$$

The researcher's participation constraint for being willing to conduct a study is

$$\max_{s \in (0,\infty)} V(s, p) \geq 0, \tag{P}$$

where we assume that the maximum is attained. Conditional on conducting a study, the researcher's choice of standard error $S$ is determined by the incentive compatibility condition

$$S \in \arg\max_{s \in (0,\infty)} V(s, p). \tag{IC}$$

As before, we will assume that an argmax exists for any relevant $p$, without giving explicit conditions on primitives to guarantee that this will be the case.

**The journal's problem.** Let the journal maximize the expectation of welfare $W$ given by the policy payoff minus any cost of publication:

$$W = U(a, \theta) - Dc.$$

That is, we suppose that the journal does not place any weight on the researcher's utility. Furthermore, assume that the public updates naively, so that the public's default action is fixed at $a^0 = a^*(\pi_0)$.

The journal's objective function takes the same form as in the original model, with the key distinction that the arrival of studies is no longer exogenous to the publication rule $p$. First, the study submission probability $q$ depends on $p$: $q = 1$ if the participation constraint (P) is satisfied, and $q = 0$ otherwise. Second, conditional on participation, the standard error $S$ depends on $p$ through the incentive compatibility condition (IC). As is standard, assume that the researcher resolves indifferences in favor of the journal's preferences. The journal's problem is to choose an *incentive-optimal* publication rule $p$ that maximizes expected welfare subject to these endogenous responses.

Observe that, conditional on the arrival of a study, the journal's gross interim benefit of publication is unchanged from its earlier definition in (6). A study that induces a journal interim belief of $\pi_1^{(X,S)}$ when the public's default action is $a^0 = a^*(\pi_0)$ yields gross interim benefit of $\Delta(\pi_1^{(X,S)}, a^*(\pi_0))$.

In the original model with exogenous study submission, the journal's optimal policy was given by the interim-optimal publication rule in which a study is published if and only if $\Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c$; indicate this interim-optimal publication rule by $p^{I(a^*(\pi_0))}$. Let us impose the assumption that the researcher would in fact be willing to participate if the journal were to use the publication rule $p^{I(a^*(\pi_0))}$ and would submit a study with $S = s^{\text{int}}$. This assumption will simplify both the solution and the exposition of our results.

**Assumption 1.** *The participation constraint* (P) *is satisfied under the interim-optimal publication rule $p = p^{I(a^*(\pi_0))}$. Let $s^{\text{int}} \in \arg\max_s V\left(s; p^{I(a^*(\pi_0))}\right)$ be the researcher's choice of study design in response to the interim-optimal publication rule.*

**Characterizing the optimal publication rule.**

**Proposition 6.** *Consider the model with incentives under naive updating, and suppose that Assumption 1 holds. Then there exist $\overline{s} \leq s^{\text{int}}$, $\lambda \geq 0$, and $\rho \in [0, 1]$ such that the following rule $p$ is incentive-optimal:*

$$p(X,S) = \begin{cases} 1 & \text{if } S = \overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c - \lambda, \\ & \quad \text{or if } S < \overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c \\ \rho & \text{if } S = \overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) = c - \lambda \\ 0 & \text{otherwise} \end{cases}.$$

*Given this rule, the researcher chooses to conduct a study with $S = \overline{s}$.*

The form of the optimal rule – at least at the chosen study design $S = \overline{s}$ – is very similar to the interim-optimal rule that was used in the model without incentives. A study is published if the gross interim benefit is sufficiently high.

However, the journal distorts publication from the interim-optimal rule in two ways. First, the journal does not publish any studies with standard error $S > \overline{s}$. The researcher is therefore induced to invest additional resources into the precision of studies and to reduce $S$ from $s^{\text{int}}$ to $\overline{s}$. Second, at $S = \overline{s}$ the journal relaxes the interim benefit threshold for publication from $c$ to $c - \lambda$ in order to encourage researcher participation. Without that relaxation, a researcher might decide that a study at $S = \overline{s}$ would be too costly to conduct given its low likelihood of being published. (While in equilibrium the researcher never chooses $S < \overline{s}$, the journal has no reason to distort the publication rule at those more precise designs.)

In the original model without incentives, a journal which internalized all costs and benefits of publication would not need commitment power: ex-ante payoffs were maximized by publishing according to what was interim-optimal after receiving a study. Having added researcher incentives, the two distortions now require two forms of journal commitment. The journal commits not to publish imprecise studies, even if such a study was conducted and turned out to have extremely striking results. This commitment is never actually tested on the equilibrium path, though – imprecise studies are not conducted. The journal also commits to publish studies with weak findings when they have the appropriate precision. This second form of commitment is tested, as these studies are submitted (and published) in equilibrium.

One key simplification of this model of incentives is the assumption that there is no heterogeneity across researchers. This fact guarantees that researchers would always choose to conduct a study with a single standard error, known in advance. In a richer model, we would expect publication rules to reward more precise studies with higher publication probabilities in a more continuous manner than what we found here.

## A.4 Imperfectly observed study designs

In determining whether to publish a study, a journal cares about the study's true information content. It may not be enough to treat the *reported* standard error as the variable $S$ in our model of normal signals. As previously discussed, one concern is external validity: the parameter being estimated in the study may only be a proxy for the policy parameter of interest. Another concern is that the study may be internally flawed: a study with a misspecified model or an unconvincing identification strategy may report a very small standard error without actually being close to the truth.

When the study design is imperfectly observed, the point estimate can itself be informative as to the study's precision. To be concrete, assume that there are normal priors with mean normalized to 0 and there are normal signals, so that $\theta \sim \mathcal{N}(0, \sigma_0^2)$ and $X \sim \mathcal{N}(\theta, S^2)$. But now assume that the realization of $S \sim F_S$ is *unobserved* by the journal and the public. As noted in Subramanyam (1996), observing a point estimate with a larger magnitude $|X|$ leads to higher beliefs on the unobserved noise $S$. In our application, a small point estimate would suggest that the study design was precise, while a large point estimate would be suggestive of some hidden noise. The extreme realization might be attributed to a violation of the identifying assumptions, to a coding error, or to some other unseen flaw.

Continuing the example with $X$ but not $S$ observed by the journal and public, and with $\theta \sim \mathcal{N}(0, \sigma_0^2)$ and $X \sim \mathcal{N}(\theta, S^2)$, suppose further that there is quadratic loss utility. The journal makes a publication decision based on the posterior mean of $\theta$, now conditional on $X$ but not $S$:

$$\mu_1^{(X)} = \mathbb{E}[\theta|X] = \mathbb{E}[\mathbb{E}[\theta|X,S]|X] = \mathbb{E}\left[\tfrac{\sigma_0^2}{S^2+\sigma_0^2}|X\right] \cdot X.$$

The journal wants to publish if the interim benefit $(\mu_1^{(X)} - \mu_1^0)^2$ exceeds the publication cost $c$. A higher belief on $S$ due to a larger point estimate $|X|$ translates into a lower weight $\mathbb{E}\left[\tfrac{\sigma_0^2}{S^2+\sigma_0^2}|X\right]$ on the point estimate. Indeed, when the prior on $S$ is sufficiently dispersed, $\mathbb{E}\left[\tfrac{\sigma_0^2}{S^2+\sigma_0^2}|X\right]$ can decrease fast enough that $\mathbb{E}[\theta|X]$ is nonmonotonic and falls to 0 as $X$ goes to infinity. (In addition to Subramanyam (1996), see discussion of this issue in Dawid (1973), O'Hagan (1979), and Harbaugh et al. (2016).) An intermediate point estimate would therefore move an observer's mean belief more than a very large, "implausible," point estimate would. Let us restate that our results in Section 3 support publishing "extreme results" in the sense of results that *lead to extreme beliefs*. If extreme signal realizations are written off as implausible, then they would not lead to extreme beliefs and thus should not be published.

A related possibility is that the study design $S$, capturing the true informational content of the study's findings, is better observed by the journal than by the public. After all, the journal editor and referees are experts who are charged with carefully evaluating the quality of a paper; a policymaker reading the study might not have this expertise. Consider a model where the journal observes $(X, S)$ when making a publication decision, while if a paper is published the public sees only $X$. In such a model, the public can make an inference on the quality of the study design from the fact that the study was published. Publication implies that the journal had chosen to certify the study as clearing the bar of peer review. Suppose additionally that even unpublished studies are publicly available as working papers or preprints. In this case the only role of "publication" by a journal is certification or signaling value. A formal analysis of optimal publication rules in such an environment is an interesting topic for future research.

# B Additional Results

## B.1 Solving for Bayesian optimal publication rules

### B.1.1 Technical results

Under Bayesian updating, the optimal publication rule solves $\max_p EW(p, a^0)$ subject to $a^0 = a^*(\pi_1^{0,p})$. Solving this program requires taking into account the fact that $a^0$ changes with $p$. However, one can simplify the problem by observing that for any fixed $p$, the induced Bayesian default action $a^0 = a^*(\pi_1^{0,p})$ maximizes expected welfare $EW(p, a^0)$ over choice of $a^0$. This is because

$$\begin{aligned}\arg\max_{a^0} EW(p, a^0) &= \arg\max_{a^0} \mathbb{E}[(1 - qp(X, S))U(a^0, \theta)] = \arg\max_{a^0} \mathbb{E}[U(a^0, \theta)|D = 0] \\ &= \arg\max_{a^0} \mathbb{E}_{\theta \sim \pi_1^{0,p}}[U(a^0, \theta)], \end{aligned} \tag{14}$$

where the last equality holds by Bayesian updating, because the conditional distribution of $\theta$ given $D = 0$ is equal to $\pi_1^{0,p}$. Therefore the Bayesian optimal publication rule $p$ equivalently solves $\max_p \max_{a^0} EW(p, a^0)$. Moreover, it holds that $\max_p \max_{a^0} EW(p, a^0) = \max_{a^0} \max_p EW(p, a^0) = \max_{p,a^0} EW(p, a^0)$. Put differently, in a sequential game of common interest, the value is the same regardless of which player moves first. The value is also equal to the "planner's solution" maximizing the objective over the joint choice of $p$ and $a^0$. Lemma 2 formally states this conclusion.

**Lemma 2.** *Under Bayesian updating, let $p$ be an optimal publication rule and let $a^0 = a^*(\pi_1^{0,p})$ be the induced default action. Then for any publication rule $p'$ and any action $a'$, it holds that $EW(p, a^0) \geq EW(p', a')$.*

The following lemma provides a recipe for solving for Bayesian optimal publication rules, summarizing some implications of Lemmas 1 and 2. In the statement of this lemma, the optimal publication rule is characterized in terms of an interim optimal publication rule given a particular default action. While all interim optimal publication rules would in fact yield the same payoff, for concreteness, let $p^{I(a^0)}$ be the interim optimal publication rule given $a^0$ that deterministically publishes at indifference:

$$p^{I(a^0)} = \begin{cases} 1 & \text{if } \Delta(\pi_1^{(X,S)}, a^0) \geq c. \\ 0 & \text{otherwise} \end{cases}$$

**Lemma 3.**

*Suppose that the updating rule is Bayesian, in which case $\pi_1^0 = \pi_1^{0,p}$ and $a^0 = a^*(\pi_1^{0,p})$ under publication rule $p$.*

1. *Let $\hat{a} \in \arg\max_{a \in \mathcal{A} \text{ s.t. } a = a^*\left(\pi_1^{0,p^{I(a)}}\right)} EW\left(p^{I(a)}, a\right)$. Then $p^{I(\hat{a})}$ is an optimal publication rule.*

*2. Let* $\hat{a} \in \arg\max_{a\in\mathcal{A}} EW\left(p^{I(a)}, a\right)$*. Then* $p^{I(\hat{a})}$ *is an optimal publication rule.*

Lemma 3 provides two alternative maximization programs that can be solved to find Bayesian optimal publication rules. (Depending on the setting, one or the other may be more straightforward to apply.) Rather than maximizing over the original function space of publication rules, the Lemma allows us to simplify the problem by maximizing over the action space, or a subset thereof.

To understand part 1 of Lemma 3, recall that just as each action induces an interim optimal publication rule, so too does each publication rule induce a Bayesian default action. Lemma 1 establishes that the optimal publication rule is interim optimal with respect to its induced default action. In other words, the default action is a "fixed point" of the mapping from actions to publication rules and back to actions. Therefore, when searching for an optimal publication rule, it is sufficient to maximize over interim optimal rules that are induced by some fixed point default action.

Unfortunately, solving for the set of fixed point default actions might not be straightforward. Part 2 of Lemma 3 gives us a version of the result that does not require solving for fixed points. Instead, we can maximize over the full action space. Moreover, while the payoff of the publication rule $p^{I(a)}$ that is interim optimal with respect to action $a$ is generally given by $EW(p^{I(a)}, a^*(\pi_1^{0,p^{I(a)}}))$ – requiring us to solve for the Bayesian default action induced by $p^{I(a)}$ – we need only evaluate the simpler expression $EW(p^{I(a)}, a)$. To see why it suffices to evaluate the simpler expression, observe that if $EW(p^{I(a)}, a)$ achieves $\max_{p,a^0} EW(p, a^0)$ – the welfare of the optimal policy, following Lemma 2 – then $p^{I(a)}$ is an optimal policy that does in fact induce welfare $EW(p^{I(a)}, a)$.[22]

### B.1.2 Applications to the leading example utility functions

We now apply the two parts of Lemma 3 to solve for Bayesian optimal publication rules for quadratic loss and binary action utility, under certain distributional conditions. In both cases, the distributional conditions we find imply that Bayesian optimal policy will be the same as the naive optimal policy. Propositions 1 and 2 in the body of the paper, assuming normal priors, follow as corollaries.

For quadratic loss utility, we impose the distributional condition that the interim mean is single-peaked and symmetric about the prior mean.

**Proposition 7.** *Suppose that there is quadratic loss utility, and that conditional on a study arriving the distribution of the interim mean* $\mu_1^{(X,S)}$ *is single-peaked and symmetric about the prior mean* $\mu_0$*.*[23] *Then the optimal publication rule under Bayesian updating is the same as under naive updating: publish if and only if* $\left|\mu_1^{(X,S)} - \mu_0\right| \geq \sqrt{c}$*.*

[22] By (14), for any action $a$, $EW(p^{I(a)}, a) \leq EW(p^{I(a)}, a^*(\pi_1^{0,p^{I(a)}}))$, with the right-hand side of the inequality being the ex ante welfare of publication rule $p^{I(a)}$. So if $EW(p^{I(a)}, a) = \max_{p,a^0} EW(p, a^0)$ then $EW(p^{I(a)}, a^*(\pi_1^{0,p^{I(a)}})) = \max_{p,a^0} EW(p, a^0)$ as well.

[23] To be precise, by single-peaked and symmetric, we mean that (i) the distribution of the random

To prove this result, we show that under single-peakedness and symmetry, the prior mean is the *only* fixed point default action under Bayesian updating. So by Lemma 3 part 1 it must be the default action for the optimal policy.

For binary action utility, normalize the prior mean of $\theta$ to be less than zero, meaning that the naive default action will be $a^0 = 0$. We then show that the Bayesian optimal publication rule is the same as the naive one when the ex-ante distribution of interim expectations on the state is sufficiently "left-leaning" relative to $\theta = 0$. (An analogous result would hold for $a^0 = 1$ and a sufficiently "right-leaning" distribution if the prior mean were above zero.)

**Proposition 8.** *Let $\mu_0 \leq 0$. Suppose that there is binary action utility, and that conditional on a study arriving the distribution of the interim mean satisfies* $\text{Prob}(\mu_1^{(X,S)} \leq -k) \geq \text{Prob}(\mu_1^{(X,S)} \geq k)$ *for all $k > 0$. Then the optimal publication rule under Bayesian updating is the same as under naive updating: publish if and only if $\mu_1^{(X,S)} \geq c$.*

The distributional assumption of Proposition 8 is strictly weaker than that of Proposition 7: given a prior mean $\mu_0 \leq 0$, any symmetric distribution of the interim mean $\mu_1^{(X,S)}$ is guaranteed to satisfy the condition of Proposition 8 even if it is not single-peaked.

To prove Proposition 8, we apply Lemma 3 part 2. There are two possible default actions, and we confirm that the interim optimal publication rule for default action $a^0 = 0$ gives a higher payoff than for default action $a^0 = 1$.

Note that in proving Proposition 7, with quadratic loss utility, we made use of the fact that there was a unique fixed point default action. For the binary action setting of Proposition 8, it is in fact possible that both default actions could be fixed points. That is, with a low default action, the journal would only publish high signals, and after nonpublication the public would take a low action in response; while with a high default action, the journal would only publish low signals, and the public would take a high action in response.

## B.2 Additional comparative statics

Proposition 9 presents additional comparative statics on the binary action publication rule solved for in Proposition 2. Recall that this result assumed a normal prior, and allowed for either Bayesian or naive updating.

**Proposition 9.** *Under the hypotheses and publication rule of Proposition 2,*

1. *The publication cutoff $\left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2}\mu_0$ in terms of the point estimate is independent of the study arrival probability $q$. It is decreasing in the mean $\mu_0$. It is*

variable $\mu_1^{(X,S)}$ has a pdf that is symmetric about $\mu_0$; and (ii) for any $\mu' < \mu'' \leq \mu_0$ it holds that if the pdf evaluated at $\mu'$ is strictly positive, then the pdf evaluated at $\mu''$ is strictly larger than at $\mu'$. (Symmetry implies the same result for $\mu_0 \leq \mu'' < \mu'$.)

larger when the standard error $S$ is larger, the prior variance $\sigma_0^2$ is smaller, or the cost of publication $c$ is larger.

2. The publication cutoff $\left(\frac{1}{S}+\frac{S}{\sigma_0^2}\right)(c-\mu_0)$ in terms of the t-statistic is nonmonotonic and convex in the standard error $S$: it has minimum at $S=\sigma_0$ and goes to infinity as $S\to 0$ or $S\to\infty$.

Next we present additional comparative statics for the gross interim benefit of publishing null results in two-period model. Here we maintain the assumptions of Proposition 4, looking at naive updating, normal priors, and quadratic loss utility.

**Proposition 10.** *Under the hypotheses of Proposition 4, the gross interim benefit of publishing a result $(X_1, S_1)$ with $X_1=\mu_0$, given by*

$$(1-\alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2+S_1^2)(\sigma_0^2+s_2^2)^2(\sigma_0^2 S_1^2+\sigma_0^2 s_2^2+S_1^2 s_2^2)},$$

*is:*

1. *decreasing in $\alpha$, going to 0 as $\alpha\to 1$;*
2. *increasing in $\sigma_0$, going to 0 as $\sigma_0\to 0$;*
3. *decreasing in $S_1$, going to 0 as $S_1\to\infty$;*
4. *nonmonotonic and quasiconcave in $s_2$, approaching 0 as $s_2\to 0$ or $s_2\to\infty$.*

## B.3 Size control for selective publication rules

Fix a z-score $z>0$. In this subsection we show how to construct selective publication rules for which the coverage probability of the confidence interval $[X-zS, X+zS]$ is equal to $\Phi[z]-\Phi[-z]$ for all $\theta$. (Of course, as established by Theorem 2 part 4, such publication rules can not take the form of those in Theorem 1.) This exercise demonstrates that while non-selectivity is sufficient for confidence intervals to control size, it is not necessary.

**Case of S=1:** Normalizing $S=1$, let the distribution of the finding $X$ be given by $X\sim\mathcal{N}(\theta,1)$ and the publication probability be given by $p(X)$. Then the coverage probability of a confidence interval of the form $[X-z, X+z]$ is given by

$$P(\theta\in[X-z,X+z])=\frac{\int p(\theta+\epsilon)\mathbf{1}(\epsilon\in[-z,z])\varphi(\epsilon)d\epsilon}{\int p(\theta+\epsilon)\varphi(\epsilon)d\epsilon}.$$

This coverage probability is equal to its nominal level, $\Phi(z)-\Phi(-z)$, for all $\theta$, if and only if

$$\int p(\theta+\epsilon)\left[\mathbf{1}(\epsilon\in[-z,z])-(\Phi(z)-\Phi(-z))\right]\varphi(\epsilon)d\epsilon=0 \text{ for all } \theta.$$

Taking the Fourier transform $\mathscr{F}$ of this expression, and recalling that the Fourier transform maps convolutions into products, the above expression is equivalent to the condition

$$\mathscr{F}(p(\cdot)) \cdot \mathscr{F}([\mathbf{1}(\cdot \in [-z,z]) - (\Phi(z) - \Phi(-z))]\, \varphi(\cdot)) \equiv 0.$$

If the coverage probability is equal to its nominal level, we thus get that $\mathscr{F}(p(\cdot))$ has to equal zero everywhere except possibly at points where $\mathscr{F}([\mathbf{1}(\cdot \in [-z,z]) - (\Phi(z) - \Phi(-z))]\, \varphi(\cdot)) = 0$. Reversely, by the Fourier inversion theorem,[24] this condition is also sufficient for the coverage probability to be equal to its nominal level.

The Fourier transform $\mathscr{F}([\mathbf{1}(\cdot \in [-z,z]) - (\Phi(z) - \Phi(-z))]\, \varphi(\cdot))$ is real-valued, even, and continuous. Let $t^*$ be any zero of this Fourier transform. Then for any publication rule of the form $p(x) = r_0 + r_1 \cdot \sin(t^* \cdot x) + r_2 \cdot \cos(t^* \cdot x)$ we get that nominal size control is satisfied. (Of course, one must ensure that the publication probability is bounded between 0 and 1.) We can also take linear combinations of these functions over different roots $t^*$. These are the only publication rules with nominal size control.

While we cannot obtain analytic solutions, at any $z$ we can numerically solve for such roots. For instance, for $z = 1.96$, solutions include $t^* \simeq 2.11045$, $3.49544$, etc. So under either of the publication rules $p(x) = .5 + .5\cos(2.11045x)$ or $p(x) = .5 + .5\cos(3.49544x)$, for example, the probability of $\theta \in [X-1.96, X+1.96]$ conditional on publication would be 95% at all $\theta$.

**General case:** Fixing $z$, suppose that $p(x)$ is some publication rule that satisfies nominal coverage for $S = 1$. Then $p(x,s) = p(x/s)$ achieves nominal coverage for $S = s$.

## B.4 Two-period model with binary actions

Consider the two-period model with normal priors and naive updating. Proposition 4 in Section 5 presented the gross interim benefit of publication – and therefore the optimal publication rule – for that setting under quadratic loss utility. Here, we will illustrate how some conclusions can change under binary action utility. We focus on characterizing how the interim benefit of publication varies as a function of the point estimate of the first-period study, $X_1$.

First, recall the quadratic loss analysis. With quadratic loss utility, the benefit of publishing towards the $t = 1$ action payoff – that is, the expected increase in $\alpha U(a_1, \theta)$ – is quadratic in $(X_1 - \mu_0)$, giving a symmetric benefit of publishing more extreme results in either direction. The benefit of publishing towards the $t = 2$ action payoff – the expected increase in $(1-\alpha)U(a_2, \theta)$ – has one term that is quadratic in $(X_1 - \mu_0)$ and another term that is positive and constant in $X_1$. There is a benefit of publishing any result, including a null result with $X_1 = \mu_0$, and an additional benefit

[24] https://en.wikipedia.org/wiki/Fourier_inversion_theorem

of publishing more extreme results. These disaggregated benefits are illustrated in panel (a) of Figure 6.

Now consider the model with binary action utility. The public's optimal action is $a = 0$ when its posterior mean is negative and $a = 1$ when its posterior mean is positive. Assume that $\mu_0 < 0$, and recall that we consider the case of naive updating, so the default action at $t = 1$ under nonpublication is $a = 0$. In that case the benefit towards the $t = 1$ payoff is $\alpha\mu_1^{(X_1,S_1)}$ if $\mu_1^{(X_1,S_1)} > 0$ and is 0 otherwise.[25] Since $\mu_1^{(X_1,S_1)}$ increases linearly with $X_1$, the benefit is zero at every $X_1$ from minus infinity through some positive number, and it increases linearly for larger $X_1$. See the blue curve in panel (b) of Figure 6.

Conditional on $(X_1, S_1)$ and on $X_2$, the realized benefit of publication towards the $t = 2$ payoff is $(1 - \alpha)|\mu_2^{(X_1,S_1),(X_2)}|$ if $\mu_2^{(X_1,S_1),(X_2)}$ and $\mu_2^{0,(X_2)}$ are of different signs, and is zero otherwise. The publication decision is made at $t = 1$, and so the benefit is evaluated by taking expectation over $X_2$ (under the $t = 1$ interim beliefs $\pi_1^{(X_1,S_1)}$). See the orange curve in panel (b) of Figure 6 for an illustration of this expected $t = 2$ benefit. As we see, this $t = 2$ benefit is somewhat subtle.

The first thing to note is that the expected $t = 2$ payoff is strictly positive everywhere except $X_1 = 0$. The $t = 2$ benefit of publishing a result with $X_1 = 0$ is zero (as is the $t = 1$ benefit) because a study reporting $X_1 = 0$ never changes the period 2 action. The action depends on the sign of the mean, and a study with $X_1 = 0$ moves the posterior mean closer to zero without changing the sign.

Moving away from $X_1 = 0$, there is a positive $t = 2$ benefit of publishing a result $X_1$ with an intermediate positive or negative value. Publishing a positive finding avoids the public's mistake of taking the action $a = 0$, in accord with its priors, when the unpublished period-1 study would actually indicate that the state is positive. Publishing a negative finding avoids the public's mistake of taking $a = 1$ after a positive finding in the second period, when the period-1 study would have indicated a negative state. Figure 6 shows that these costs are asymmetric (a conclusion we see in other numerical examples): there is a larger cost of failing to publish a study with a positive result, one that goes against the public's prior.

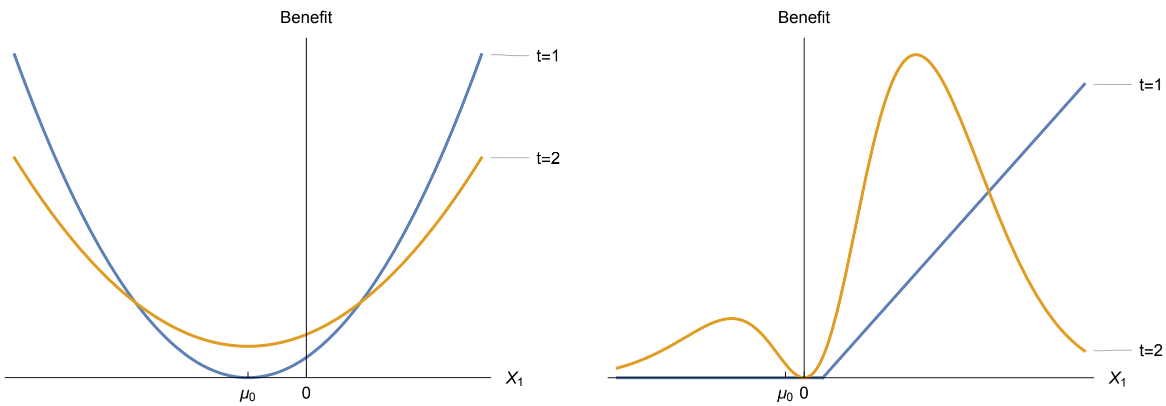
Finally, as $X_1$ gets more extreme in either direction, the $t = 2$ payoff benefit approaches zero. This is because "$t = 2$" is defined as the time after some additional information has arrived.[26] And an extreme $X_1$ is suggestive of an extreme state, meaning that the period 2 signal is very likely to reveal whether the state is positive or negative. For instance, if $X_1$ has a very large positive value, then we expect $X_2$ to have a very large positive value as well. So publishing this study would give a

[25] We follow the notational convention of the proof of Proposition 4 here, in which $\mu_1^{(X_1,S_1)}$ is the period 1 mean belief conditional on observing the period 1 study; $\mu_2^{(X_1,S_1),(X_2)}$ is the period 2 mean conditional on observing both studies; and $\mu_2^{0,(X_2)}$ is the period 2 mean after observing the second study if the first was not published.

[26] If there is a longer expected wait before new studies arrive and actions are updated, that corresponds in our model to a larger weight $\alpha$ on the $t = 1$ payoff.

$t = 1$ benefit by moving the first period action from $a_1 = 0$ to $a_1 = 1$. But the public will take $a_2 = 1$ in the second period regardless of whether the first period study is published.

Figure 6: Dynamic interim payoffs




(a) Quadratic loss utility (b) Binary action utility

For both examples, we set $S_1 = 2$, $\sigma_0 = 2$, $\mu_0 = -1$, and $s_2 = 2$. The relative weight coefficient on the first period, $\alpha$, is chosen to make the curves of similar scale as graphed; increasing $\alpha$ scales up the $t = 1$ benefit relative to that at $t = 2$. For quadratic loss utility, we have chosen $\alpha = .3$, with $X_1$ ranging from $-5$ to 3. For binary action utility, we have chosen $\alpha = .05$, with $X_1$ ranging from $-10$ to 15.

# C Proofs

## C.1 Proofs for Section 3

*Proof of Lemma 1.* For naive updating, the result follows from arguments in the text. For Bayesian updating, the result follows from Lemma 2 in Appendix B.1.1. In particular, if $p$ is the Bayesian optimal publication rule and $a^0$ is the induced Bayesian default action, then Lemma 2 establishes that $EW(p, a^0) \geq \max_{p'} EW(p', a^0)$. Hence, $EW(p, a^0) = \max_{p'} EW(p', a^0)$, establishing that $p$ is interim optimal given $a^0$. $\square$

*Proof of Theorem 1.* We begin by stating a Lemma, which we prove just below. The notation $\geq_{FOSD}$ indicates an ordering of distributions according to first order stochastic dominance.

**Lemma 4.** *Let $U$ be supermodular. Let beliefs $\pi', \pi''$, and $\pi'''$ satisfy $\pi''' \geq_{FOSD} \pi'' \geq_{FOSD} \pi'$. Then for any default action $a^0$, it holds that $\Delta(\cdot, a^0)$ is quasiconvex in the sense that $\Delta(\pi'', a^0) \leq \max\{\Delta(\pi', a^0), \Delta(\pi''', a^0)\}$.*

Fix standard error $S = s$ and consider ordered point estimates $x''' > x'' > x'$. To prove the theorem, it suffices to show that if study $(x'', s)$ is published, then at least one of $(x''', s)$ or $(x', s)$ is published as well. By Lemma 1, it is in turn sufficient to show that the gross interim benefit of publishing the middle study $(x'', s)$ cannot be strictly higher than that from both the lower study $(x', s)$ and the higher study $(x''', s)$.

To see why this is the case, recall that at any fixed standard error $S = s$, higher point estimates are more likely than lower point estimates at higher states in the sense of the monotone ratio likelihood property (MLRP).[27] MLRP implies that for any fixed prior, the corresponding posteriors are ranked by first order stochastic dominance according to their point estimates: $\pi_1^{(x''',s)} \geq_{FOSD} \pi_1^{(x'',s)} \geq_{FOSD} \pi_1^{(x',s)}$. Hence, Lemma 4 implies the result. □

*Proof of Lemma 4.* Let $a' = a^*(\pi')$, $a'' = a^*(\pi'')$, and $a''' = a^*(\pi''')$. Moreover, recall that for any actions $\underline{a} \leq \overline{a}$ and any distributions $\underline{\pi} \leq_{FOSD} \overline{\pi}$, supermodularity implies that

$$\mathbb{E}_{\theta\sim\overline{\pi}}[U(\underline{a}, \theta)] + \mathbb{E}_{\theta\sim\underline{\pi}}[U(\overline{a}, \theta)] \leq \mathbb{E}_{\theta\sim\underline{\pi}}[U(\underline{a}, \theta)] + \mathbb{E}_{\theta\sim\overline{\pi}}[U(\overline{a}, \theta)]. \tag{15}$$

Now consider the two exhaustive cases of $a^0 \leq a''$ and $a^0 \geq a''$.

If $a^0 \leq a''$, then

$$\begin{aligned}
\mathbb{E}_{\theta\sim\pi'''}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi''}[U(a'', \theta)] &\leq \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi'''}[U(a'', \theta)] \\
&\leq \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi'''}[U(a''', \theta)] \\
\Rightarrow \ \mathbb{E}_{\theta\sim\pi''}[U(a'', \theta)] - \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] &\leq \mathbb{E}_{\theta\sim\pi'''}[U(a''', \theta)] - \mathbb{E}_{\theta\sim\pi'''}[U(a^0, \theta)] \\
\Rightarrow \Delta(\pi'', a^0) &\leq \Delta(\pi''', a^0),
\end{aligned}$$

where, on the first line, the first inequality follows from (15) and the second inequality follows from the fact that $a''' = a^*(\pi''')$. The second line then rearranges terms from the left-hand side and the right-hand side of the first line.

Alternatively, if $a^0 \geq a''$, then by a similar argument

$$\begin{aligned}
\mathbb{E}_{\theta\sim\pi'}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi''}[U(a'', \theta)] &\leq \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi'}[U(a'', \theta)] \\
&\leq \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta\sim\pi'}[U(a', \theta)] \\
\Rightarrow \ \mathbb{E}_{\theta\sim\pi''}[U(a'', \theta)] - \mathbb{E}_{\theta\sim\pi''}[U(a^0, \theta)] &\leq \mathbb{E}_{\theta\sim\pi'}[U(a', \theta)] - \mathbb{E}_{\theta\sim\pi'}[U(a^0, \theta)] \\
\Rightarrow \Delta(\pi'', a^0) &\leq \Delta(\pi', a^0).
\end{aligned}$$
□

*Proof of Proposition 1.*

1. This prior and signal structure satisfy the hypotheses of Proposition 7 in Appendix B.1: the distribution of the interim mean is normally distributed, there-

[27] That is, for point estimates $x'' > x'$, the ratio $\frac{f_{X|\theta,S}(x''|\theta,s)}{f_{X|\theta,S}(x'|\theta,s)}$ is increasing in $\theta$.

fore symmetric and single-peaked. So the optimal policy, for either updating rule, is to publish if $|\mu_1^{(X,S)} - \mu_0| \geq \sqrt{c}$. By the normal updating formula (4), $|\mu_1^{(X,S)} - \mu_0| = \frac{\sigma_0^2}{S^2+\sigma_0^2}|X - \mu_0| = \left(1 + \frac{S^2}{\sigma_0^2}\right)^{-1} |X - \mu_0|$.

2. All comparative statics are immediate from the formula.
3. The only comparative static that is not immediate is that for the t-statistic cutoff, $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$, with respect to $S$. Taking straightforward limits confirms that the cutoff goes to infinity as $S \to 0$ and $S \to \infty$. The derivative of the cutoff with respect to $S$ is $\left(-\frac{1}{S^2} + \frac{1}{\sigma_0^2}\right)\sqrt{c}$, and the second derivative is $\frac{2\sqrt{c}}{S^3}$. Since the second derivative is positive, the cutoff is convex over $S \in \mathbb{R}_{++}$ and is minimized at the point where the first derivative is 0, which is $S = \sigma_0^2$.

□

*Proof of Proposition 2.* This prior and signal structure satisfy the hypotheses of Proposition 8 in Appendix B.1: the distribution of the interim mean is normally distributed, therefore symmetric. So the optimal policy, for either updating rule, is to publish if $\mu_1^{(X,S)} \geq c$. From the normal updating formula (4), that corresponds to $\frac{\sigma_0^2}{\S^2+\sigma_0^2}X + \frac{S^2}{S^2+\sigma_0^2}\mu_0 \geq c$, i.e., $X \geq \left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2}\mu_0$. □

## C.2 Proofs for Section 4

We begin this section with a Lemma establishing that non-selective publication implies the key properties of parts 1 - 4 of Theorem 2. For parts 1 and 4, we prove something stronger than what is in Theorem 2. Part 1 establishes the unbiasedness of more general estimators than the estimator $X$ for the state $\theta$. Part 4 establishes size control for arbitrary confidence sets.

**Lemma 5.** *Suppose that the publication rule is non-selective and that $P(D = 1) > 0$. Then $f_{X|\theta,S,D=1}(x|\theta, s) = f_{X|\theta,S}(x|\theta, s)$, and thus the following properties hold.*

1. Frequentist unbiasedness. *If the estimator $\widehat{g} : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$ for the estimand $g : \Theta \times \mathcal{S} \to \mathbb{R}$ satisfies $\mathbb{E}[\widehat{g}(X, S)|\theta, S = s] = g(\theta, s)$ for all $\theta$ and $s$, then $\mathbb{E}[\widehat{g}(X, S)|\theta, S = s, D = 1] = g(\theta, s)$ for all $\theta$ and $s$.*
2. Publication probability constant in state. *The publication probability $\mathbb{E}[p(X, S)|\theta, S = s]$ is constant in $\theta$ for all $s$.*
3. Bayesian validity of naive updating. *The Bayesian default belief $\pi_1^{0,p}$ is equal to the naive default belief, i.e., the prior $\pi_0$.*
4. Frequentist size control. *Fix a level $\alpha \in (0, 1)$ and consider a confidence set $C$ mapping from $\mathcal{X} \times \mathcal{S}$ to subsets of $\Theta$. If $\text{Prob}(\theta \in C(X, S)|\theta, S = s) \geq 1 - \alpha$ for all $\theta$ and $s$, then $\text{Prob}(\theta \in C(X, S)|\theta, S = s, D = 1) \geq 1 - \alpha$ for all $\theta$ and $s$.*

*Proof of Lemma 5.* As stated, when publication is non-selective, the distribution of $X|\theta, S = s, D = 1$ is identical to the distribution $X|\theta, S = s$ for every $s$. Parts 1 and 4 follow immediately from that observation. Part 2 follows from the definition of non-selective publication: $p(x,s)$ constant in $x$ implies that $\mathbb{E}[p(X,S)|\theta, S=s]$ is equal to that same constant. To show part 3, recall that the independence of $S$ and $\theta$ implies that if $\mathbb{E}[p(X,S)|\theta, S=s]$ is constant for each $s$, then it is constant in expectation across $S$, and so $\mathbb{E}[p(X,S)|\theta]$ is constant as well. The result then follows from (3). $\square$

*Proof of Theorem 2.* Non-selectivity implies the other parts by Lemma 5. Specifically, for part 1 of the Theorem, apply part 1 of Lemma 5 with in $\widehat{g}(X,S)=X$ and $g(\theta,S)=\theta$; and for part 4 of the Theorem, apply part 4 of Lemma 5 with $C=[X-zs, X+zs]$ and $1-\alpha=\Phi(z)-\Phi(-z)$.

We next show the reverse implications.

- Part 2 $\Rightarrow$ non-selective publication: Fixing $S=s$, recall that $X$ is a complete statistic for $\theta$ in the normal location model when $\Theta_0$ contains an open set in $\mathbb{R}$; see for instance Theorem 6.22 in Lehmann and Casella (1998). Completeness means that for any measurable function $g:\mathcal{X}\to\mathbb{R}$, if $\mathbb{E}[g(X)|\theta, S=s]=0$ for all $\theta\in\Theta_0$, then $P(g(X)=0|\theta, S=s)=1$ for all $\theta\in\Theta_0$. Apply this definition to $g(x)=p(x,s)-\mathbb{E}[p(X,s)|S=s]$. Assuming part 2, that the publication probability is constant over $\theta\in\Theta_0$, it holds that the expectation of $g(X)$ is 0 for all $\theta\in\Theta_0$, and hence that $p(X,s)=\mathbb{E}[p(X,s)|S=s]$ with probability 1 given $\theta$ and $S=s$, establishing non-selective publication.
- Part 1 $\Rightarrow$ part 2: To simplify notation, consider without loss of generality the case $s=1$. Then the unbiasedness condition $\mathbb{E}[X|\theta, S=1, D=1]$ can be written as
$$\frac{\int x\varphi(x-\theta)p(x,1)dx}{\int \varphi(x-\theta)p(x,1)dx}=\theta.$$
Equivalently, using the fact that $\varphi'(x)=-x\cdot\varphi(x)$,
$$\begin{aligned}0 &= \int (x-\theta)\varphi(x-\theta)p(x,1)dx\\ &= -\int \varphi'(x-\theta)p(x,1)dx\\ &= \partial_\theta\left[\int \varphi(x-\theta)p(x,1)dx\right]\\ &= \partial_\theta \mathbb{E}[p(X,S)|\theta, S=1].\end{aligned}$$
If the last line is equal to 0 then $E[p(X,S)|\theta, S=1]$ is constant over $\theta$ in any open set contained in the support. The same argument applies for all other values of $S$.
- Part 3 $\Rightarrow$ part 2: Restating (3), the relative density of the Bayesian default

belief to the prior is given by

$$\frac{d\pi_1^{0,p}}{d\pi_0}(\theta) = \frac{1 - q \cdot \mathbb{E}[p(X,S)|\theta]}{1 - q \cdot \mathbb{E}[p(X,S)]}.$$

The Bayesian default belief is equal to the prior when, under the prior $\theta \sim \pi_0$, this relative density is almost surely constant in $\theta$ (in which case the ratio is identically equal to 1). In other words, it holds when $\mathbb{E}[p(X,S)|\theta]$ is almost surely constant in $\theta$. Moreover, note that $\mathbb{E}[p(X,S)|\theta]$ must be continuous in $\theta$ since the signal density function $f_{X|\theta,S}(x|\theta,s)$ is a smooth function of $\theta$ for all $x, s$. Hence, if the Bayesian default belief is equal to the prior, then $\mathbb{E}[p(X,S)|\theta]$ must be constant in $\theta$ over the support of the prior.
Now, highlighting the dependence of this publication probability on the distribution $F_S$,

$$\mathbb{E}[p(X,S)|\theta] = \int_{s\in\mathcal{S}} \mathbb{E}[p(X,S)|\theta, S=s]dF_S(s).$$

We see that the LHS of this equation is constant over $\theta$ in the support of the prior for all distributions $F_S$ if and only if, for all $s$, $\mathbb{E}[p(X,S)|\theta, S=s]$ is constant over $\theta$ in the support. (If there exists $s'$ such that $\mathbb{E}[p(X,S)|\theta, S=s']$ varies in $\theta$, then the distribution $F_S$ placing all probability mass on $s'$ will have $\mathbb{E}[p(X,S)|\theta]$ vary in $\theta$.) So if the Bayesian default belief is equal to the prior for all $F_S$, then the publication probability is constant over $\theta$ in $\Theta_0$ for all $s$.

- Part 4 $\Rightarrow$ part non-selective publication: Without loss of generality, fix $s = 1$. We show that if $I(1) \neq \emptyset$, then there exists $\theta'$ for which $\text{Prob}\big(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, D = 1\big) < \Phi(z) - \Phi(-z)$.
First consider the case of a bounded interval $I(1)$. Then there exist $\theta'$ (the midpoint of the interval) and $y > 0$ (the radius) such that $I(1) = [\theta' - y, \theta' + y]$. If $y > z$,

$$\text{Prob}(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, D = 1) = 0,$$

and the result follows. If $y \leq z$, applying the law of iterated expectations and letting $R = 1$ denote the event of study submission,

$$\begin{aligned}\Phi(z) - \Phi(-z) &= \text{Prob}(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, R = 1) \\ &= \text{Prob}(D = 0|\theta = \theta', S = 1, R = 1)\cdot \\ &\qquad \text{Prob}(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, R = 1, D = 0) \\ &\quad + \text{Prob}(D = 1|\theta = \theta', S = 1, R = 1)\cdot \\ &\qquad \text{Prob}(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, R = 1, D = 1).\end{aligned}$$

Conditional on a study submitted but not published, it holds that $X \in [\theta' -$

$y, \theta' + y]$, and therefore since $y \le z$ that $\theta' \in [X - z, X + z]$:

$$\text{Prob}(\theta' \in [X - z, X + z] | \theta = \theta', S = 1, R = 1, D = 0) = 1.$$

Therefore $\Phi(z) - \Phi(-z)$ is equal to a weighted average of 1 and $\text{Prob}(\theta' \in [X - z, X + z] | \theta = \theta', S = 1, D = 1, R = 1)$ – with positive weights on both – and hence $\text{Prob}(\theta' \in [X - z, X + z] | \theta = \theta', S = 1, D = 1, R = 1) < \Phi(z) - \Phi(-z)$, yielding the desired result.

Consider finally the case of unbounded $I(1)$. If $I(1) = (-\infty, y]$ for some $y$, then for $\theta' < y - z$,

$$\text{Prob}(\theta' \in [X - z, X + z] | \theta = \theta', S = 1, D = 1) = 0 < \Phi(z) - \Phi(-z).$$

A symmetric argument holds for $I(1) = [y, \infty)$ and $\theta' > y + z$, concluding our proof. □

*Proof of Proposition 3.* As stated in the text, the result follows from the fact that the signal $X|S = s$ distributed according to $\mathcal{N}(\theta, s^2)$ is a Blackwell more informative signal of $\theta$ when $s$ is smaller. Blackwell more informative signals have higher expected value to a decisionmaker regardless of utility function $U$ or prior $\pi_0$. The updating rule is irrelevant because with non-selective publication, Bayesian updating and naive updating are identical. □

## C.3 Proofs for Section 5

*Proof of Lemma 1′.* The argument closely follows that for Lemma 1, with updated notation. First, given the welfare function (8), define $EW(p, \pi_1^0)$ for the two-period model analogously to the earlier definition of $EW(p, a^0)$ from (5) for the one-period model. For this definition, let $\pi_2^{0,X_2}$ and $\pi_2^{(X_1,S_1),X_2}$ be the Bayesian updated beliefs following observation of $X_2$ from the period-two priors of either $\pi_1^0$ (if no period-one study was published) or $\pi_1^{(X_1,S_1)}$ (if $(X_1, S_1)$ was published at period-one):

$$\begin{aligned} EW(p, \pi_1^0) = \mathbb{E}\Big[ & qp(X_1, S_1)\big(\alpha U(a^*(\pi_1^{(X_1,S_1)}), \theta) - c + (1 - \alpha)U(a^*(\pi_2^{(X_1,S_1),X_2}), \theta)\big) \\ & + (1 - qp(X_1, S_1))\big(\alpha U(a^*(\pi_1^0), \theta) + (1 - \alpha)U(a^*(\pi_2^{(0,X_2}), \theta)\big)\Big]. \end{aligned}$$

A publication rule $p$ is dynamically interim optimal given default belief $\pi_1^0$ if it solves $\max_{p'} EW(p', \pi_1^0)$.

For the case of naive updating, the publication rule doesn't affect the default belief. So, as in the one-period model, it is immediate that a dynamically optimal publication rule is dynamically interim optimal.

For the case of Bayesian updating, we can essentially follow the one-period argument of Appendix B.1.1. By the same argument as in that section (see Equation

(14)), for any fixed publication rule $p$, the Bayesian default belief $\pi_1^{0,p}$ maximizes $EW(p, \pi_1^0)$ over choice of default beliefs $\pi_1^0$. Hence, the analog of Lemma 2 extends to the two-period model: at the Bayesian dynamically optimal rule $p$ and the corresponding Bayesian default beliefs $\pi_1^{0,p}$, we have $EW(p, \pi_1^{0,p}) = \max_p' \max_{\pi_1'} EW(p', \pi_1') = \max_{p',\pi_1'} EW(p', \pi_1')$. In particular, $EW(p, \pi_1^{0,p}) = \max_{p'} EW(p', \pi_1^{0,p})$, and so $p$ is interim optimal. $\square$

*Proof of Theorem 3.* The proof of part 1 holds for any distributions $\pi_1^I \neq \pi_1^0$. For part 2, the proofs rely on the fact that both distributions arise from the same prior $\pi_0$ (implying, for instance, that they share a common support), and that $q < 1$ if updating is Bayesian.

1. Write mean beliefs at the first period when a study $(X_1, S_1)$ is published or not by $\mu_1^{(X_1,S_1)}$ and $\mu_1^0$, and in the second period conditional on $X_2$ by $\mu_2^{(X_1,S_1),X_2}$ and $\mu_2^{0,X_2}$. The gross interim benefit of publishing a study $(X_1, S_1)$ can be expressed as follows as the first-period action benefit plus the expected second-period action benefit:

$$\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2 + (1-\alpha)\mathbb{E}_{\theta\sim\pi_1^{(X_1,S_1)}, X_2\sim\mathcal{N}(\theta,s_2^2)}[(\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2})^2]$$

   The first term, the first-period action benefit, is nonnegative (and strictly positive when the means $\mu_1^{(X_1,S_1)}$ and $\mu_1^0$ differ). So it suffices to show that when $\pi_1^{(X_1,S_1)} \neq \pi_1^0$, the second term, the expected second-period action benefit, is strictly positive. In turn, it suffices to show that when $\pi_1^{(X_1,S_1)} \neq \pi_1^0$, there exists $X_2$ for which $\mu_2^{(X_1,S_1),X_2} \neq \mu_2^{0,X_2}$. The second-period action benefit is nonnegative and is continuous in $X_2$, and $X_2$ has full support given any first-period interim belief $\pi_1^{(X_1,S_1)}$. So if the second-period action benefit is strictly positive at some $X_2$, then it is strictly positive in expectation.
   The claim thus follows if we can show that, if $\mu_2^{(X_1,S_1),X_2} = \mu_2^{0,X_2}$ holds for all $X_2$, then $\pi_1^{(X_1,S_1)} = \pi_1^0$.
   Without loss of generality, normalize $s_2 = 1$, so that $X_2 \sim \mathcal{N}(\theta, 1)$. Define

$$m(x;\pi) = \mathbb{E}_{\theta\sim\pi}[\theta|X_2 = x]$$

   as the posterior mean of $\theta$ under $\pi$ when $X_2 = x$. We seek to show that if $m(x;\pi) = m(x;\tilde{\pi})$ for almost all $x \in \mathbb{R}$, then $\pi = \tilde{\pi}$.
   Taking $\varphi$ to be the PDF of the standard normal, define $\pi * \varphi$ to be the marginal density of $X_2$ given $\theta \sim \pi$, which always exists:

$$(\pi * \varphi)(x) = \int_{\mathbb{R}} \varphi(x-\theta)d\pi(\theta).$$

It then holds that

$$\frac{\partial \log((\pi * \varphi)(x))}{\partial x} = \frac{1}{(\pi * \varphi)(x)} \frac{\partial (\pi * \varphi)(x)}{\partial x} = \frac{\int_{\mathbb{R}} \varphi'(x-\theta) d\pi(\theta)}{\int_{\mathbb{R}} \varphi(x-\theta) d\pi(\theta)} = \frac{\int_{\mathbb{R}} (\theta - x)\varphi(x-\theta) d\pi(\theta)}{\int_{\mathbb{R}} \varphi(x-\theta) d\pi(\theta)}$$
$$= \mathbb{E}_{\theta \sim \pi}[\theta | X_2 = x] - x = m(x;\pi) - x \tag{16}$$

where the last equality on the first line follows from the identity $\varphi'(x) = -x\varphi(x)$. (This equation is also known as "Tweedie's formula.") Integrating the left- and right-hand sides yields

$$(\pi * \varphi)(x) = C \cdot \exp\left( \int_0^x (m(x;\pi) - x) dx \right)$$

for a constant of integration $C$ pinned down by the fact that $\pi * \varphi$ integrates to 1. The same formula holds for $\tilde{\pi} * \varphi$, replacing $\pi$ by $\tilde{\pi}$ on the right-hand side. We can therefore conclude that if $m(x;\pi) = m(x;\tilde{\pi})$ for all $x$, then $(\pi * \varphi)(x) = (\tilde{\pi} * \varphi)(x)$ for all $x$ as well.

So, suppose that $m(x,\pi) = m(x,\tilde{\pi})$ for all $x \in \mathbb{R}$, and hence that $(\pi * \varphi)(x) = (\tilde{\pi} * \varphi)(x)$. For any distribution $\pi$ of $\theta$, denote its characteristic function (Fourier transform) by $\psi_\pi(t) = E_{\theta \sim \pi}[\exp(it\theta)]$. The fact that $\pi * \varphi = \tilde{\pi} * \varphi$ implies

$$\psi_\pi(t) \cdot \exp(-t^2/2) = \psi_{\tilde{\pi}}(t) \cdot \exp(-t^2/2)$$

for all $t$, where $\exp(-t^2/2)$ is the characteristic function of the standard normal distribution. This holds because the Fourier transform maps convolutions of random variables into products of their characteristic functions. It immediately follows that $\psi_\pi(\cdot) = \psi_{\tilde{\pi}}(\cdot)$, since $\exp(-t^2/2)$ is different from 0 for all $t$, so that the characteristic function of $\pi$ is equal to the characteristic function of $\tilde{\pi}$. Equality of their characteristic functions implies equality of $\pi$ and $\tilde{\pi}$, by Lemma 2.15 in Van der Vaart (2000).

2. Let $\mu_1$ denote the shared mean of $\pi_1^0$ and of $\pi_1^I$. Throughout this proof, it will be convenient to highlight the dependence of the distribution of the signal $X_2$ on the standard error parameter $s_2$, and so we will write the signal as $X_2^{(s_2)}$. In particular, $X_2^{(s_2)} | \theta \sim \mathcal{N}(\theta, s_2^2)$. Furthermore, let

$$m(x;\pi,s_2) = \mathbb{E}_{\theta \sim \pi}[\theta | X_2^{(s_2)} = x]$$

be the public's period-2 expectation of $\theta$ given period-1 belief $\pi$ followed by period-2 observation $X_2^{(s_2)} = x$. As a final notational point, in this proof and the proofs of the corresponding Lemmas, any integral is to be interpreted as a definite integral over the domain $\mathbb{R}$ unless otherwise specified.

Since the two beliefs $\pi_1^0$ and of $\pi_1^I$ yield the same period 1 action of $a_1 = \mu_1$, the interim gross benefit of publishing the study is the expected benefit in the second-period, which can be written as

$$(1-\alpha)\mathbb{E}_{\theta\sim\pi_1^I}\left[\left(m(X_2^{(s_2)};\pi_1^I,s_2)-m(X_2^{(s_2)};\pi_1^0,s_2)\right)^2\right]. \tag{17}$$

We seek to show that, under the appropriate conditions, the expression (17) goes to zero as $s_2 \to 0$ (for part 2a) and as $s_2 \to \infty$ (for part 2b).

**Lemma 6.** *If distribution $\pi$ has a finite mean and variance, then*

$$\lim_{s_2\to 0}\mathbb{E}_{\theta\sim\pi}\left[\left(m(X_2^{(s_2)};\pi,s_2)-X_2^{(s_2)}\right)^2\right]=0.$$

**Lemma 7.** *If distribution $\pi$ has mean $\mu_1$ and is bounded by Pareto tails with finite variance, then*

$$\lim_{s_2\to\infty}\mathbb{E}_{\theta\sim\pi}\left[\left(m(X_2^{(s_2)};\pi,s_2)-\mu_1\right)^2\right]=0. \tag{18}$$

We will apply Lemma 6 to show part 2a of the Theorem and Lemma 7 to show part 2b.
Before proceeding, it is valuable to establish one other preliminary result.

**Lemma 8.** *Given any $\pi_1^0$ and $\pi_1^I$ as derived under the hypotheses of Theorem 3, there exists $C'>0$ such that for all $s_2>0$ and all functions $y:\mathbb{R}_+\to\mathbb{R}_+$, it holds that $\mathbb{E}_{\theta\sim\pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right]\le C'\mathbb{E}_{\theta\sim\pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right]$.*

We now proceed to the proofs of each part.

(a) First observe that the distributions $\pi_1^0$ and $\pi_1^I$ both have a finite variance. To see that this holds for $\pi_1^I$, recall that $\pi_1^I=\pi_1^{(x_1,s_1)}$ is a posterior distribution updated after observing a normal signal $(X_1,S_1)=(x_1,s_1)$. The posterior distribution (from any prior) after observing a normal signal has a finite variance. To see that this holds for $\pi_1^0$, recall that $\pi_1^0$ arises as a default belief from the prior $\pi_0$ with a finite variance. In the case of naive updating, $\pi_1^0=\pi_0$, so the result is immediate. In the case of Bayesian updating, observe from (3) that $\frac{d\pi_1^0}{d\pi_0}(\theta)\le\frac{1}{1-q}$ for all $\theta$, and therefore $\pi_0\ge(1-q)\pi_1^0$; so if $\pi_1^0$ had an infinite variance, then so too would $\pi_0$.
Plugging $\pi=\pi_1^I$ into Lemma 6, we have that

$$\lim_{s_2\to 0}\mathbb{E}_{\theta\sim\pi_1^I}\left[\left(m(X_2^{(s_2)};\pi_1^I,s_2)-X_2^{(s_2)}\right)^2\right]=0.$$

Applying Lemma 8, we also have that there exists a constant $C'>0$ such that

$$0 \leq \lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi_1^I} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)} \right)^2 \right] \leq \lim_{s_2 \to 0} C' \mathbb{E}_{\theta \sim \pi_1^0} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)} \right)^2 \right].$$

Plugging $\pi = \pi_1^0$ into Lemma 6, we have that the right-hand expression is equal to 0. Hence,

$$\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi_1^I} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)} \right)^2 \right] = 0.$$

In other words, both $m(X_2^{(s_2)}; \pi_1^I, s_2)$ and $m(X_2^{(s_2)}; \pi_1^0, s_2)$ converge to $X_2^{(s_2)}$ in mean-square as $s_2 \to 0$ under $\theta \sim \pi_1^I$. Therefore they converge to each other in mean-square, establishing the desired conclusion that the expression (17) goes to 0 as $s_2 \to 0$, as long as the three variables $m(X_2^{(s_2)}; \pi_1^I, s_2)$, $m(X_2^{(s_2)}; \pi_1^0, s_2)$, and $X_2^{(s_2)}$ are all square-integrable under $\theta \sim \pi_1^I$.
The three variables are indeed square-integrable, as they each have finite means and variance. To see that, observe that the posterior mean $m(X_2^{(s_2)}; \pi_1^I, s_2)$ has mean equal to $\mu_1$ and, by the Law of Total Variance, variance less than $\mathrm{Var}_{\theta \sim \pi_1^I}$: the variance of the posterior mean given some signal is bounded above by the variance of the prior. The other posterior mean variable $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite mean and variance under the distribution $\theta \sim \pi_1^0$ by the same arguments, and therefore finite mean and variance under the distribution $\theta \sim \pi_1^I$ by Lemma 8.[28] Finally, the mean of $X_2^{(s_2)}$ is $\mu_1$ and the variance is $\mathrm{Var}_{\theta \sim \pi_1^I}(\theta) + s_2^2$.

(b) First observe that the distributions $\pi_1^0$ and $\pi_1^I$ are both bounded by Pareto tails with finite variance since they arise from the prior $\pi_0$ that is bounded by Pareto tails with finite variance. To see that this holds for $\pi_1^I$, recall that $\pi_1^I = \pi_1^{(x_1, s_1)}$ is a posterior distribution updated after observing a normal signal $(X_1, S_1) = (x_1, s_1)$. It holds that $\frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$ is equal to a constant times $\varphi(\frac{x_1 - \theta}{s_1})$, and hence the tails of $\pi_1^I$ decay at a rate at least as fast as those of $\pi_0$. To see that this holds for $\pi_1^0$ in the case of naive updating, $\pi_1^0 = \pi_0$, and so the result is immediate. To see that this holds for $\pi_1^0$ in the case of Bayesian updating, observe from (3) that $\frac{d\pi_1^0}{d\pi_0}(\theta) \leq \frac{1}{1-q}$ for all $\theta$, and therefore $\pi_0 \geq (1-q)\pi_1^0$; so if $\pi_1^0$ were not bounded by Pareto tails with

[28] To see that $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite mean under $\theta \sim \pi_1^I$, recall $\mathbb{E}_{\theta \sim \pi_1^0}[m(X_2^{(s_2)}; \pi_1^0, s_2)]$ is finite if and only if $\mathbb{E}_{\theta \sim \pi_1^0}[\left| m(X_2^{(s_2)}; \pi_1^0, s_2) \right|]$ is finite; and the latter being finite implies by Lemma 8 that $\mathbb{E}_{\theta \sim \pi_1^I}[\left| m(X_2^{(s_2)}; \pi_1^0, s_2) \right|]$ and hence $\mathbb{E}_{\theta \sim \pi_1^I}[m(X_2^{(s_2)}; \pi_1^0, s_2)]$ are finite. Call $\tilde{\mu}$ the mean of $m(X_2^{(s_2)}; \pi_1^0, s_2)$ under $\theta \sim \pi_1^I$; the fact that $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite variance under $\tilde{\theta} \sim \pi_1^0$ means that $\mathbb{E}_{\theta \sim \pi_1^0}[m(X_2^{(s_2)}; \pi_1^0, s_2) - \tilde{\mu})^2]$ is finite, and thus by Lemma 8 $\mathbb{E}_{\theta \sim \pi_1^I}[m(X_2^{(s_2)}; \pi_1^0, s_2) - \tilde{\mu})^2]$ is also finite.

finite variance, then neither would $\pi_0$.
Plugging $\pi = \pi_1^I$ into Lemma 7, we have that

$$\lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I} \left[ \left( m(X_2^{(s_2)}; \pi_1^I, s_2) - \mu_1 \right)^2 \right] = 0.$$

Applying Lemma 8, we also have that there exists a constant $C' > 0$ such that

$$0 \leq \lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1 \right)^2 \right] \leq \lim_{s_2 \to \infty} C' \mathbb{E}_{\theta \sim \pi_1^0} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1 \right)^2 \right].$$

Plugging $\pi = \pi_1^0$ into Lemma 7, we have that the right-hand expression is equal to 0. Hence,

$$\lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I} \left[ \left( m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1 \right)^2 \right] = 0.$$

In other words, both $m(X_2^{(s_2)}; \pi_1^I, s_2)$ and $m(X_2^{(s_2)}; \pi_1^0, s_2)$ converge to $\mu_1$ in mean-square as $s_2 \to 0$ under $\theta \sim \pi_1^I$. Therefore they converge to each other in mean-square, establishing the desired conclusion that the expression (17) goes to 0 as $s_2 \to 0$, as long as they are both square-integrable under $\theta \sim \pi_1^I$; that was established in the proof of the previous part. ☐

*Proof of Lemma 6.* First observe that

$$\begin{aligned} &\mathbb{E}_{\theta \sim \pi}[(X_2^{(s_2)} - \theta)^2] = s_2^2 \\ \Rightarrow &\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi}[(X_2^{(s_2)} - \theta)^2] = 0. \end{aligned} \tag{19}$$

Next recall that for any $s_2$ and any realization $X_2^{(s_2)} = x$, the posterior mean of the updated belief, $m(x; \pi_1^I, s_2)$, minimizes the expected square distance to $\theta$:

$$\begin{aligned} m(x; \pi, s_2) &\in \arg \min_{g_{s_2}: \mathbb{R} \to \mathbb{R}} \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(x) - \theta)^2 | X_2^{(s_2)} = x] \\ \Rightarrow \mathbb{E}_{\theta \sim \pi}[(m(x; \pi, s_2) - \theta)^2 | X_2^{(s_2)} = x] \\ &\leq \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(x) - \theta)^2 | X_2^{(s_2)} = x] \ \forall g_{s_2}. \end{aligned}$$

Since this inequality holds for each realization $X_2^{(s_2)} = x$, it also holds in expectation:

$$\mathbb{E}_{\theta \sim \pi}[(m(X_2^{(s_2)}; \pi) - \theta)^2] \leq \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(X_2^{(s_2)}) - \theta)^2] \ \forall g_{s_2}.$$

Plugging in $g_{s_2}(x)$ equal to the identity function $x$,

$$0 \le \mathbb{E}_{\theta\sim\pi}[(m(X_2^{(s_2)};\pi,s_2)-\theta)^2] \le \mathbb{E}_{\theta\sim\pi}[(X_2^{(s_2)}-\theta)^2].$$

Taking the limit as $s_2 \to 0$ as in (19), the right-hand side of the above expression converges to 0, and hence

$$\lim_{s_2\to 0}\mathbb{E}_{\theta\sim\pi}[(m(X_2^{(s_2)};\pi,s_2)-\theta)^2] \to 0. \tag{20}$$

So we see that $m(X_2^{(s_2)};\pi,s_2)$ and $X_2^{(s_2)}$ both converge to $\theta$ in mean-square as $s_2 \to 0$. We can conclude that $m(X_2^{(s_2)};\pi,s_2)$ converges to $X_2^{(s_2)}$ in mean-square, and hence we have proven our result, if $m(X_2^{(s_2)};\pi,s_2)$, $X_2^{(s_2)}$, and $\theta$ are all square-integrable under $\theta\sim\pi$. In turn it suffices to show that these random variables all have a finite mean and a variance. By assumption, the mean and variance of $\theta$ under $\pi$ are finite. Then $X_2^{(s_2)}$ and $m(X_2^{(s_2)};\pi,s_2)$ also share the mean of $\theta$ under $\pi$ for all $s_2$. The variance of $X_2$ is given by $\mathrm{Var}_{\theta\sim\pi}(\theta)+s_2^2$. Finally, the variance of $m(X_2;\pi,s_2)$ is bounded above by $\mathrm{Var}_{\theta\sim\pi}(\theta)$ by the Law of Total Variance: the variance of the posterior mean given some signal is bounded above by the variance of the prior. $\square$

*Proof of Lemma 7.* Applying a transformation with $\lambda = 1/s_2$, let $\hat{X}_2^{(\lambda)} = \lambda X_2^{(1/\lambda)}$; $X_2^{(\lambda)}$ is equal to the t-statistic $X_2^{(s_2)}/s_2$. That is, $\hat{X}_2^{(\lambda)}|\theta \sim \mathcal{N}(\lambda\theta,1)$, where $\hat{X}_2^{(\lambda)}|\theta$ has pdf at $\hat{x}$ of $\varphi(\hat{x}-\lambda\theta)$. Correspondingly, let

$$\hat{m}(\hat{x};\pi,\lambda) = \mathbb{E}_{\theta\sim\pi}[\theta|\hat{X}_2^{(\lambda)}=\hat{x}]$$

be the public's period-2 expectation of $\theta$ given period-1 belief $\pi$ followed by period-2 observation $\hat{X}_2^{(\lambda)}=\hat{x}$, i.e., given $X_2^{(1/\lambda)}=\hat{x}/\lambda$. This transformation will be convenient because as $s_2\to\infty$ and $\lambda = 1/s_2 \to 0$, the variable $\hat{X}^{(\lambda)}|\theta$ approaches a standard normal, whereas $X_2^{(s_2)}|\theta$ approaches an improper distribution with infinite variance.

We seek to show that for any $\pi$ with mean $\mu_1$ that is bounded by Pareto tails with finite variance, it holds that

$$\lim_{\lambda\to 0}\mathbb{E}_{\theta\sim\pi}[(\hat{m}(\hat{X}^{(\lambda)};\pi,\lambda)-\mu_1)^2] = 0. \tag{21}$$

Writing the expectation from (21) out in integral form,

$$\mathbb{E}_{\theta\sim\pi}[(\hat{m}(\hat{X}^{(\lambda)};\pi,\lambda)-\mu_1)^2] = \int\int(\hat{m}(\hat{x};\pi,\lambda)-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)d\hat{x}.$$

By Lebesgue's dominated convergence theorem, to show (21), it suffices to show (i) for all $\hat{x}$, $\lim_{\lambda\to 0}\int(\hat{m}(\hat{x};\pi,\lambda)-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)=0$; and (ii) there exists a "dominating" function $g:\mathbb{R}_+\to\mathbb{R}_+$ that is Lebesgue-integrable, i.e., $\int g(\hat{x})d\hat{x}$ is finite, such that for $\lambda$ sufficiently small, $\int(\hat{m}(\hat{x};\pi,\lambda)-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)\le g(\hat{x})$ for all $\hat{x}$.

**Step 1:** *Show that for all $\hat{x}$, $\lim_{\lambda \to 0} \int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) = 0$.*

It holds that

$$\begin{aligned}
\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) &= (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&\leq (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(0) d\pi(\theta) \\
&= (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(0).
\end{aligned}$$

So to show the desired result that $\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)$ converges to 0 for all $\hat{x}$, it suffices to show that $(\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2$ converges to 0 for all $\hat{x}$. In turn, it suffices to show that $\hat{m}(\hat{x}; \pi, \lambda)$ converges to $\mu_1$ for any fixed $\hat{x}$. Writing $\hat{m}(\hat{x}; \pi, \lambda)$ in integral form,

$$\hat{m}(\hat{x}; \pi, \lambda) = \frac{\int \theta \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} \tag{22}$$

In the denominator of (22), for all $\theta$, $\varphi(\hat{x} - \lambda\theta) \to \varphi(\hat{x})$ as $\lambda \to 0$. Moreover, $\varphi(\hat{x} - \lambda\theta) \leq \varphi(0)$ for all $\theta$ and $\lambda$, and $\int \varphi(0) d\pi(\theta) = \varphi(0) < \infty$. So $\varphi(0)$ is a dominating function for $\varphi(\hat{x} - \lambda\theta)$ that is integrable with respect to $\pi_0$, and hence by the dominated convergence theorem the denominator approaches $\int \varphi(\hat{x}) d\pi(\theta) = \varphi(\hat{x})$.

In the numerator of (22), for all $\theta$, $\theta\varphi(\hat{x} - \lambda\theta) \to \theta\varphi(\hat{x})$ as $\lambda \to 0$. Moreover, $|\theta\varphi(\hat{x} - \lambda\theta)| \leq |\theta|\varphi(0)$ for all $\theta$ and $\lambda$, and $\int \theta\varphi(0) d\pi(\theta) = \varphi(0) \int |\theta| d\pi(\theta) < \infty$ because $\pi$ has a finite mean. So $|\theta|\varphi(0)$ is a dominating function for $\theta\varphi(\hat{x} - \lambda\theta)$ that is integrable with respect to $\pi$, and hence by the dominated convergence theorem the numerator approaches $\int \theta\varphi(\hat{x}) d\pi(\theta) = \mu_1 \varphi(\hat{x})$.

Taking the ratio, we have that $\hat{m}(\hat{x}; \pi, \lambda)$ converges to $\mu_1 \varphi(\hat{x}) / \varphi(\hat{x}) = \mu_1$ as $\lambda \to 0$, completing this step.

**Step 2:** *Show that there exists a dominating function $g : \mathbb{R}_+ \to \mathbb{R}_+$ that is Lebesgue-integrable, such that for $\lambda$ sufficiently small, $\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \leq g(\hat{x})$ for all $\hat{x}$.*

First, observe that

$$\begin{aligned}
\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) &= (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&= \left( \frac{\int \theta\varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} - \mu_1 \right)^2 \cdot \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&\leq \frac{\int (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} \cdot \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&= \int (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \qquad (23)
\end{aligned}$$

where the inequality in the third line follows from Jensen's inequality: $(\mathbb{E}[\theta|\hat{X}^{(\lambda)} = \hat{x}] - \mu_1)^2 = (\mathbb{E}[\theta - \mu_1|\hat{X}^{(\lambda)} = \hat{x}])^2 \le \mathbb{E}[(\theta-\mu_1)^2|\hat{X}^{(\lambda)} = \hat{x}]$.

So it suffices to find an integrable function $g$ for which $g(\hat{x})$ is everywhere larger than (23) for all $\lambda \in (0,1]$.

- *Constructing $g$ for $\hat{x} \in [-2K, 2K]$.*
  The expression (23) is uniformly bounded above by $\int (\theta-\mu_1)^2 \varphi(0) d\pi(\theta) = \varphi(0)\operatorname{Var}_{\theta\sim\pi}(\theta)$. So, let

$$g(\hat{x}) = \varphi(0)\operatorname{Var}_{\theta\sim\pi}(\theta) \text{ for } \hat{x} \in [-2K, 2K].$$

  It holds that $\int_{-2K}^{2K} g(\hat{x}) d\hat{x} = 4K\varphi(0)\operatorname{Var}_{\theta\sim\pi}(\theta) < \infty$.
- *Constructing $g$ for $\hat{x} > 2K$.*
  Expanding out (23), we have

$$\int (\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta) = \underbrace{\int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)}_{A} + \underbrace{\int_{\frac{\hat{x}}{2\lambda}}^{\infty} (\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)}_{B} \tag{24}$$

  First let us bound the term labeled $A$ in (24). For $\theta \le \frac{\hat{x}}{2\lambda}$, it holds that $\hat{x} - \lambda\theta \ge \hat{x}/2$. Therefore, assuming further that $\hat{x} \ge 2K$ – and in particular that $\hat{x} \ge 0$ – it holds that $\varphi(\hat{x}-\lambda\theta) \le \varphi(\hat{x}/2)$. Hence,

$$\begin{aligned}\underbrace{\int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)}_{A} &\le \int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta-\mu_1)^2\varphi(\hat{x}/2)d\pi(\theta) \\ &\le \int_{-\infty}^{\infty} (\theta-\mu_1)^2\varphi(\hat{x}/2)d\pi(\theta) \\ &= \varphi(\hat{x}/2)\operatorname{Var}_{\theta\sim\pi}(\theta).\end{aligned}$$

  Now we move to the term labeled $B$ in (24). By the fact that $\pi$ is bounded by

Pareto tails with finite variance,

$$\underbrace{\int_{\frac{\hat{x}}{2\lambda}}^{\infty}(\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\pi(\theta)}_{B} \leq \int_{\frac{\hat{x}}{2\lambda}}^{\infty} C\theta^{-\gamma}(\theta-\mu_1)^2\varphi(\hat{x}-\lambda\theta)d\theta$$

$$\begin{aligned}
&\leq \int_{\frac{\hat{x}}{2\lambda}}^{\infty} C\theta^{-\gamma}(\theta+|\mu_1|)^2\varphi(\hat{x}-\lambda\theta)d\theta \\
&\leq C\frac{(\frac{\hat{x}}{2\lambda}+|\mu_1|)^2}{(\frac{\hat{x}}{2\lambda})^\gamma}\int_{\frac{\hat{x}}{2\lambda}}^{\infty}\varphi(\hat{x}-\lambda\theta)d\theta \\
&= C\frac{(\frac{\hat{x}}{2\lambda}+|\mu_1|)^2}{(\frac{\hat{x}}{2\lambda})^\gamma}\frac{1}{\lambda}(1-\Phi(-\frac{\hat{x}}{2})) \\
&= 2^{\gamma-2}C\lambda^{\gamma-3}\frac{(\hat{x}+2\lambda|\mu_1|)^2}{\hat{x}^\gamma}\Phi(\frac{\hat{x}}{2}) \\
&\leq 2^{\gamma-2}C\frac{(\hat{x}+2|\mu_1|)^2}{\hat{x}^\gamma} \text{ for } \lambda \in (0,1]
\end{aligned}$$

The inequality in the third line follows because $\theta^{-\gamma}(\theta+|\mu_1|)^2$ is decreasing in $\theta$ over $\theta > 0$ for any $\gamma > 2$, so we increase the expression when we plug in the lowest value of $\theta$, i.e., $\theta = \hat{x}/(2\lambda)$. The inequality in the last line follows because $\lambda^{\gamma-3}(\hat{x}+2\lambda|\mu_1|)^2$ is increasing in $\lambda$ over $\lambda > 0$ for any $\gamma > 3$, so we increase the expression relative to $\lambda \leq 1$ when we plug in $\lambda = 1$; and we also increase the expression when we replace $\Phi(\frac{\hat{x}}{2})$ by 1. These two observations about increasing and decreasing functions can be straightforwardly confirmed by taking derivatives.[29]

Putting the bounds on terms A and B together, let

$$g(\hat{x}) = \varphi(\hat{x}/2)\operatorname{Var}_{\theta\sim\pi}(\theta) + 2^{\gamma-2}C\frac{(\hat{x}+2|\mu_1|)^2}{\hat{x}^\gamma} \text{ for } \hat{x} > 2K.$$

As established, $g(\hat{x})$ is larger than (23) for all $\lambda \leq 1$. Moreover, $\int_{2K}^{\infty} g(\hat{x})d\hat{x}$ is finite: the first term is an integral of a normal pdf, and the second term is an integral of an expression that decays to zero as $\hat{x}$ goes to infinity at a rate of $\hat{x}^{2-\gamma}$, with the exponent $2-\gamma < -1$.

- *Constructing $g$ for $\hat{x} < -2K$.*
  This case proceeds symmetrically to the construction for $\hat{x} > 2K$, now taking

$$g(\hat{x}) = \varphi(\hat{x}/2)\operatorname{Var}_{\theta\sim\pi}(\theta) + 2^{\gamma-2}C\frac{(|\hat{x}|+2|\mu_1|)^2}{|\hat{x}|^\gamma} \text{ for } \hat{x} < 2K.$$

[29] The derivative of $\theta^{-\gamma}(\theta+|\mu_1|)^2$ with respect to $\theta$ evaluates to $-\theta^{-(1+\gamma)}(\theta+|\mu_1|)(|\mu_1|\gamma+(\gamma-2)\theta) < 0$. The derivative of $\lambda^{\gamma-3}(\hat{x}+2\lambda|\mu_1|)^2$ with respect to $\lambda$ evaluates to $\lambda^{\gamma-4}(\hat{x}+2|\mu_1|\lambda)((\gamma-3)\hat{x}+2|\mu_1|(\gamma-1)\lambda) > 0$.

Just as with $\hat{x} > 2K$, when $\hat{x} < -2K$ we have that $g(\hat{x})$ is an upper bound for (23) when $\lambda \leq 1$, and $\int_{-\infty}^{-2K} g(\hat{x})d\hat{x}$ is finite.

We have now established that $g(\hat{x})$ is an upper bound for (23) for all $\lambda \leq 1$ and for all $\hat{x}$, and that $\int g(\hat{x})d\hat{x} < \infty$, concluding the proof. $\square$

*Proof of Lemma 8.* Define $f^I_{X_2^{(s_2)}}(x) = \frac{1}{s_2}\int \varphi(\frac{x-\theta}{s_2})d\pi_1^I(\theta)$ and $f^0_{X_2^{(s_2)}}(x) = \frac{1}{s_2}\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)$ to be the marginal densities of $X_2^{(s_2)}$ under the respective distributions on $\theta$ of $\pi_1^I$ and $\pi_1^0$.

**Step 1:** *Show that there exists $C' > 0$ such that $\frac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} \leq C'$ for all $s_2$.*

First observe that

$$\frac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} = \frac{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^I(\theta)}{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)} = \frac{\int \varphi(\frac{x-\theta}{s_2})\frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)}d\pi_1^0(\theta)}{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)} \leq \sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)}.$$

Next, recall that $\pi_1^I = \pi_1^{(x_1,s_1)}$, which is a posterior belief on $\theta$ given prior $\theta \sim \pi_0$ and some fixed signal realization $(X_1, S_1) = (x_1, s_1)$. Hence

$$\frac{d\pi_1^I(\theta)}{d\pi_0(\theta)} = \frac{\varphi(\frac{x_1-\theta}{s_1})}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$$
$$\Rightarrow \sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)} \leq \frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta').}$$

Under naive updating, $\pi_1^0 = \pi_0$, and thus $\sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)} = \sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$, bounded by the finite constant $C' = \frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$. (Recall that $x_1$ and $s_1$ are taken as constants here.) Under Bayesian updating with study arrival probability $q < 1$, (3) implies that $\frac{d\pi_0(\theta)}{d\pi_1^0(\theta)} \leq \frac{1}{1-q}$ for all $\theta$, and therefore that $\sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)} = \sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}\frac{d\pi_0(\theta)}{d\pi_1^0(\theta)} \leq \frac{1}{1-q}\sup_\theta \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$. Hence for Bayesian updating we have a bound $C' = \frac{1}{1-q}\frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$.

In either case $C'$ gives an upper bound on $\frac{f^I_{X_2^{(s_2)}{}_2}(x)}{f^0_{X_2^{(s_2)}}(x)}$.

**Step 2:** *Show that $\mathbb{E}_{\theta\sim\pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right] \leq C'\mathbb{E}_{\theta\sim\pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right]$.*

Rewriting expectations in integral form,

$$\begin{aligned}
\mathbb{E}_{\theta\sim\pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right] &= \int y\left(X_2^{(s_2)}\right) f^I_{X_2^{(s_2)}}(x)dx \\
&= \int y\left(X_2^{(s_2)}\right) \frac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} f^0_{X_2^{(s_2)}}(x)dx \\
&\leq \int y\left(X_2^{(s_2)}\right) C' f^0_{X_2^{(s_2)}}(x)dx \text{ (by Step 1)} \\
&= C'\mathbb{E}_{\theta\sim\pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right]. \qquad \square
\end{aligned}$$

*Proof of Proposition 4.* Suppose a study $(X_1, S_1)$ arrives at period 1. Let $\mu_1^0$ indicate the posterior mean at period 1 in the absence of publication, and $\mu_1^{(X_1,S_1)}$ the posterior mean at period 1 if the study is published. Let $\mu_2^{0,X_2}$ indicate the posterior mean at period 2 if the study had not been published and then the second period signal is observed to be $X_2$, and $\mu_2^{(X_1,S_1),X_2}$ the posterior mean at period 2 if the study had been published. We can calculate these posterior means as follows:

$$\begin{aligned}
\mu_1^0 &= \mu_0 \\
\mu_1^{(X_1,S_1)} &= \frac{1}{\frac{1}{\sigma_0^2}+\frac{1}{S_1^2}}\left(\frac{\mu_0}{\sigma_0^2}+\frac{X_1}{S_1^2}\right) \\
\mu_2^{0,X_2} &= \frac{1}{\frac{1}{\sigma_0^2}+\frac{1}{s_2^2}}\left(\frac{\mu_0}{\sigma_0^2}+\frac{X_2}{s_2^2}\right) \\
\mu_2^{(X_1,S_1),X_2} &= \frac{1}{\frac{1}{\sigma_0^2}+\frac{1}{S_1^2}+\frac{1}{s_2^2}}\left(\frac{\mu_0}{\sigma_0^2}+\frac{X_1}{S_1^2}+\frac{X_2}{s_2^2}\right)
\end{aligned}$$

Consider the interim stage, at which $(X_1, S_1)$ has been observed by the journal and not yet published, and hence at which the journal has interim belief $\pi_1^1(X_1, S_1)$. From this interim perspective, publication has a cost of $c$. It then delivers a benefit towards the first-period action payoff, and a benefit towards the second-period action payoff.

The benefit of publication towards the first-period payoff is $\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2$, which simplifies to

$$\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2 = \alpha\frac{\sigma_0^4}{(\sigma_0^2+S_1^2)^2}(X_1-\mu_0)^2 \tag{25}$$

The period 2 action if the study is published is $\mu_2^{(X_1,S_1),X_2}$, and is $\mu_2^{0,X_2}$ otherwise. Hence, conditional on $X_2$, the benefit of first-period publication towards the second-period payoff is $(1-\alpha)(\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2})^2$. At the interim stage, then, the expected second-period payoff is the expectation of that value over the random variable $X_2$, given beliefs $\theta \sim \pi_1^{(X_1,S_1)}$ and $X_2 \sim \mathcal{N}(\theta, s_2^2)$. Writing out this expectation and simplifying,

$$\begin{aligned}&\mathbb{E}\left[(1-\alpha)\left(\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2}\right)^2 |X_1, S_1\right] \\ &\quad = (1-\alpha)\Big(\mathbb{E}\left[\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2}|X_1,S_1\right]^2 + \mathrm{Var}\left[\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2}|X_1,S_1\right]\Big)\end{aligned} \tag{26}$$

Next observe that, given $X_1$ and $S_1$, the conditional distribution of $X_2$ is

$$X_2|(X_1,S_1) \sim \mathcal{N}\left(\mu_1^{(X_1,S_1)}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{S_1^2}} + s_2^2\right).$$

Plugging this conditional distribution into the various terms of (26) and simplifying,

$$(1-\alpha)\mathbb{E}\left[\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2}|X_1,S_1\right]^2 = (1-\alpha)\left(\frac{\sigma_0^2 s_2^2}{(\sigma_0^2+S_1^2)(\sigma_0^2+s_2^2)}\right)^2 (X_1-\mu_0)^2 \tag{27}$$

$$(1-\alpha)\,\mathrm{Var}\left[\mu_2^{(X_1,S_1),X_2} - \mu_2^{0,X_2}|X_1,S_1\right] = (1-\alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2+S_1^2)(\sigma_0^2+s_2^2)^2(\sigma_0^2S_1^2+\sigma_0^2s_2^2+S_1^2s_2^2)} \tag{28}$$

The gross interim payoff of publication is the sum of the right-hand sides of (25), (27), and (28). To get the form stated in the proposition, we add up the coefficients on $(X_1-\mu_0)^2$ in (25) and (27):

$$\alpha\frac{\sigma_0^4}{(\sigma_0^2+s_1^2)^2} + (1-\alpha)\left(\frac{s_2^2\sigma_0^2}{(\sigma_0^2+s_1^2)(\sigma_0^2+s_2^2)}\right)^2 = \frac{\sigma_0^4(s_2^4+2\alpha\sigma_0^2s_2^2+\alpha\sigma_0^4)}{(\sigma_0^2+S_1^2)^2(\sigma_0^2+s_2^2)^2}. \qquad \square$$

## C.4 Proofs for Appendix A

### C.4.1 Proofs for Appendix A.2

*Proof of Proposition 5.* Recall that under normal priors, the variance of $\pi_1^{(X,S)}$ is independent of $X$. So fix $S=s$, and without loss of generality normalize the variance of $\pi_1^{(X,s)}$ to 1. Then given $X=x$ and $\theta \sim \pi_1^{(x,s)}$, the distribution of a random variable $Y=(x-\theta)^2$ is a noncentral chi-squared distribution with noncentrality parameter $\lambda$ (equal to $(x-\mathbb{E}_{\theta\sim\pi_1^{(x,s)}}[\theta])^2$) that increases in $(x-\mu_0)^2$. The variable $Y$ has CDF over realizations $y$ given by $1 - Q_{1/2}(\sqrt{\lambda}, \sqrt{y})$ for $Q$ the Marcum $Q$-function.[30] By Sun et al. (2010) Theorem 1(a), $Q_{1/2}(\sqrt{\lambda}, \sqrt{y})$ strictly increases in its first term $\sqrt{\lambda}$, implying that the distribution of $(x-\theta)^2$ under $\pi_1^{(x,s)}$ increases in the sense of FOSD as $(x-\mu_0)^2$ increases. Hence $\mathbb{E}_{\theta\sim\pi_1^{(x,s)}}[\delta((x-\theta)^2)]$ increases in $(x-\mu_0)^2$. A study $(X,S)=(x,s)$ is published if and only if $\mathbb{E}_{\theta\sim\pi_1^{(x,s)}}[\delta((x-\theta)^2)] \leq b$, so at standard error $S=s$ studies are published only if $(X-\mu_0)^2$ is sufficiently small. $\square$

### C.4.2 Proofs for Appendix A.3

*Proof of Proposition 6.* We first state a lemma that does not depend on Assumption 1.

**Lemma 9.** *In searching for an incentive-optimal publication rule, it is without loss of generality to restrict to rules $p(X,S)$ satisfying*

$$p(X,S) = \begin{cases} 1 & \text{if } S=\overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c-\lambda, \\ & \quad \text{or if } S<\overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c \\ 0 & \text{if } S>\overline{s}, \\ & \quad \text{or if } S=\overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) < c-\lambda \\ & \quad \text{or if } S<\overline{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c \end{cases}$$

*for some $\overline{s} \in (0,\infty)$ and $\lambda$ in $\mathbb{R}\cup\{-\infty,\infty\}$ in which the researcher chooses $S=\overline{s}$ if she conducts a study.*

It remains only to show that in the incentive-optimal contract of the form in Lemma 9, the researcher chooses to conduct a study; that $\overline{s} \leq s^{\text{int}}$; and that $\lambda \geq 0$.

The facts that the researcher conducts a study and that $\overline{s} \leq s^{\text{int}}$ both follow from Assumption 1.

First, Assumption 1 guarantees that the journal prefers to follow the interim-optimal rule – at which the researcher conducts a study with $S = s^{\text{int}}$, and the journal only publishes studies with a nonnegative interim net benefit – than any rule that publishes nothing at all. (In the model without incentives in which $q=1$ and $S$ is deterministically equal to $s^{\text{int}}$, publishing no studies is feasible, but the interim-optimal rule is preferred.) So the incentive-optimal rule will induce the researcher to conduct a study, meaning that the researcher must be choosing $S=\overline{s}$.

Second, fix any publication rule of the form in Lemma 9 with $\overline{s}=s^h$ and $\lambda=\lambda^h$, for $s^h > s^{\text{int}}$. We claim that the publication rule of the same form with $\overline{s}=s^{\text{int}}$ and $\lambda=0$ weakly improves payoffs. To see why this claim holds, note that the publication rule with $\overline{s}=s^h$ and $\lambda=\lambda^h$ would be weakly improved upon by one with $\overline{s}=s^{\text{int}}$ and $\lambda=0$, supposing researcher participation. Recall that normal signals are Blackwell

[30] See Wikipedia for details: `https://en.wikipedia.org/wiki/Noncentral_chi-squared_distribution`.

ordered by their standard errors: at standard error $S = s^{\text{int}}$, the findings $X$ can be garbled into something informationally equivalent to findings from $S = s^h$. So some stochastic publication rule at $S = s^{\text{int}}$, combined with a garbling of these signals to the public, replicates the distribution of outcomes[31] that occur when a study arrives with $S = s^h$ and is published under the publication rule given by $\overline{s} = s^h$ and $\lambda = \lambda^h$. But the journal's payoffs given a study with $S = s^{\text{int}}$ are improved by removing the garbling to the public. Payoffs are further improved by publishing under the interim-optimal publication rule at $S = s^{\text{int}}$, which is exactly that given by a rule of the form in Lemma 9 with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$. Finally, by Assumption 1, the publication rule with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$ does indeed get researcher to conduct a study, since the interim outcome satisfies the researcher's participation constraint.

The final step is to show that $\lambda \geq 0$. This is because, for any publication rule of the form of Lemma 9, increasing $\lambda$ increases the publication probability at $S = \overline{s}$. Hence, it makes the researcher better off if she chooses $S = \overline{s}$ and slackens her incentive constraints. Moreover, starting from $\lambda < 0$, increasing $\lambda$ to 0 improves the journal's payoff, since again $\lambda = 0$ is interim optimal and hence optimal conditional on a study being submitted at $S = \overline{s}$. $\square$

*Proof of Lemma 9.* Take an arbitrary publication rule $\tilde{p}$. We will show that it can be replaced by a rule $p$ of the desired form that weakly increases the journal's payoff.

First suppose that $\tilde{p}$ does not induce the researcher to conduct a study. Then define some $p$ of the form in the statement of the Lemma by setting $\overline{s}$ arbitrarily and setting $\lambda = 0$. If the publication rule $p$ induces the researcher not to participate, then the journal's payoffs are unchanged from $\tilde{p}$. If the rule $p$ induces the researcher to conduct a study with standard error $S = s$, then the journal's payoffs are weakly higher than before, since under $p$ the journal never publishes studies that give negative net interim payoff.

So, for the rest of the proof, assume that $\tilde{p}$ does in fact induce the researcher to conduct a study with $S$ equal to some level $\overline{s}$. We show that there exists $\lambda$ such that we can replace $\tilde{p}$ with a publication rule $p$ satisfying the following properties and weakly improve the journal's payoff:

1. At $s > \overline{s}$, $p(x, s) = 0$:
   Let $p(x, s) = \tilde{p}(x, s)$ at $s \leq \overline{s}$ and 0 at $x > \overline{s}$. The publication rule $p$ gives the researcher the same payoff from choosing $S = \overline{s}$ and weakly reduces her payoff from choosing other values of $S$, and so under $p$ the researcher's behavior is unchanged. She continues to conduct a study with $S = \overline{s}$ and the journal's payoff given the choice of $S = \overline{s}$ is also unchanged.
2. At $s = \overline{s}$, $p(X, \overline{s}) = 1$ if $\Delta(\pi_1^{(X,\overline{s})}, a^*(\pi_0)) > c - \lambda$, and $p(X, \overline{s}) = 0$ if $\Delta(\pi_1^{(X,\overline{s})}, a^*(\pi_0)) < c - \lambda$:
   Let $p(x, s) = \tilde{p}(x, s)$ at all $s \neq \overline{s}$. Denote the probability of publication under $\tilde{p}$

[31] I.e., the probability of publication at each state, and the joint distribution over public actions and states conditional on publication.

at $S = \overline{s}$, given by $\mathbb{E}[\tilde{p}(X,S)|S = \overline{s}]$, by $y \in [0,1]$. If $y = 0$ then $\tilde{p}$ is equivalent to a publication rule $p$ of the appropriate form with $\lambda = \infty$. If $y = 1$ then $\tilde{p}$ is equivalent to a publication rule $p$ of the appropriate form with $\lambda = -\infty$.

For interior $y$, define $p(\cdot, \overline{s})$ so as to maximize the journal's payoff subject to accepting a share $y$ of papers at this standard error. To do so, first set $\lambda \in \mathbb{R}$ as the supremum over values of $l$ such that $P(\Delta(\pi_1^{(X,\overline{s})}, a^*(\pi_0)) > c - l|S = \overline{s}) \leq y$. Next, let $p(x,\overline{s}) = 0$ if $\Delta(\pi_1^{(x,\overline{s})}, a^*(\pi_0)) < c - \lambda$ and let $p(x,\overline{s}) = 1$ if $\Delta(\pi_1^{(x,\overline{s})}, a^*(\pi_0)) > c - \lambda$. Finally, if $\Delta(\pi_1^{(x,\overline{s})}, a^*(\pi_0)) = c - \lambda$, set $p(x,\overline{s})$ such that the publication probability at $S = \overline{s}$, $\mathbb{E}[p(X,S)|S = \overline{s}]$, is equal to $y$. (This last step is only relevant if $\Delta(\pi_1^{(X,\overline{s})}, a^*(\pi_0)) = c - \lambda$ with positive probability at $S = \overline{s}$.)

The publication rules $p$ and $\tilde{p}$ publish with the same probability as each other conditional on any choice $S$ by the researcher. Hence, the researcher continues to be willing to pick $S = \overline{s}$. Moreover, given the constraint of publishing with probability $y$ at $S = \overline{s}$, the journal's expected payoff given a researcher choice of $S = \overline{s}$ is maximized by $p$. Hence, the journal weakly prefers $p$ to $\tilde{p}$ if the researcher is to choose $S = \overline{s}$.

3. At $s < \overline{s}$, $p(x,s) = 1$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) \geq c$ and $p(x,s) = 0$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) < c$:

   Let $p(x,s) = \tilde{p}(x,s)$ at $s \geq \overline{s}$; at $s < \overline{s}$, let $p(x,s) = 1$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) \geq c$ and $p(x,s) = 0$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) < c$.

   Under publication rule $p$, the researcher will either continue to choose $S = \overline{s}$ or will switch to $s' < \overline{s}$. If the researcher continues to choose $S = \overline{s}$, then the journal's payoffs are as before. If the researcher now chooses $s' < \overline{s}$, we claim that the journal must be weakly better off. (This argument exactly follows an argument in the proof of Proposition 6.) To show the claim, recall that normal signals are Blackwell ordered by their standard errors: at standard error $S = s'$, the finding $X$ can be garbled into something informationally equivalent to a finding from $S = \overline{s}$. So some stochastic publication rule at $S = s'$, combined with a garbling of these signals to the public, replicates the distribution of outcomes (probability of publication at each state, and joint distribution over public actions and states conditional on publication) that occur when a study arrives with $S = \overline{s}$ and is published under the publication rule given by $p(X, \overline{s})$. But the journal's payoffs given a study that has been published with $S = s'$ are improved by removing the garbling to the public. Payoffs are further improved by publishing under the interim-optimal publication rule at $S = s'$, which is exactly that under $p$.

The only remaining item to prove is that it is without loss of generality to suppose that if the researcher chooses to conduct a study, she chooses $S = \overline{s}$; applying step 3 above could possibly have changed the researcher's choice of $S$ to something below $\overline{s}$. However, iterating step 1 (with $\overline{s}$ redefined to the new choice of $S$) recovers a publication rule of the appropriate form in which the researcher does choose $S =$

$\overline{s}$. $\square$

## C.5 Proofs for Appendix B

### C.5.1 Proofs for Appendix B.1

*Proof of Lemma 2.* Follows from arguments in the text of Appendix B.1.1. $\square$

*Proof of Lemma 3.* Follows from arguments in the text of Appendix B.1.1. $\square$

*Proof of Proposition 7.* By Lemma 3 part 1, it suffices to show that $a = a^*(\pi_1^{0,p^{I(a)}})$ is uniquely solved by $a = \mu_0$ – in other words, that $a^0 = \mu_0$ is the unique fixed point when we map default actions to interim optimal publication rules, and then map publication rules back to default actions.

Conditional on a study $(X, S)$ arriving when the default action is $a^0$, the journal will not publish if $(\mu_1^{(X,S)} - a^0)^2 < c$, i.e., if $\mu_1^{(X,S)}$ lies in the interval $(a^0 - \sqrt{c}, a^0 + \sqrt{c})$. Let $\bar{\mu}(a^0)$ indicate $\mathbb{E}[\theta|\mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})]$, the expected state conditional on a study arriving and not being published. If this expectation is undefined due to the event $\mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})$ occurring with zero probability, let $\bar{\mu}(a^0) = \mu_0$.

The mean of the default belief – and therefore the implied default action – conditional on nonpublication will be a convex combination of $\bar{\mu}(a^0)$ (with weight $q$) and $\mu_0$ (weight $1 - q$). Therefore, to show that $a^0 = \mu_0$ is the unique fixed point, it is sufficient to show the following three items: (i) for $a^0 = \mu_0$, it holds that $\bar{\mu}(a^0) = a^0$; (ii) for any $a^0 < \mu_0$, it holds that $\bar{\mu}(a^0) > a^0$; and (iii) for any $a^0 > \mu_0$, it holds that $\bar{\mu}(a^0) < a^0$. (If we had assumed $q < 1$ then it would be sufficient to show (ii) and (iii) with weak inequalities.)

Item (i) follows from the fact that $\mu_1^{(X,S)}$ is symmetric about $\mu_0$, and therefore it remains symmetric when this random variable is truncated outside of the interval $(\mu_0 - \sqrt{c}, \mu_0 + \sqrt{c})$. The proofs of items (ii) and (iii) will be identical to each other, up to the direction of inequalities, so let us focus on proving (ii). Fix $a^0 < \mu_0$. First, if there is a zero probability that $\mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})$, then $\bar{\mu}(a^0) = \mu_0 > a^0$ and we are done. Otherwise, notice that symmetry about $\mu_0$ combined with single-peakedness means that the pdf of $\mu_1^{(X,S)}$ is larger at $a^0 + k$ than at $a^0 - k$ for any $k > 0$, with the inequality being strict for any $\epsilon$ such that either pdf value is nonzero. Hence the mean of $\mu_1^{(X,S)}$ conditional on being in the interval $(a^0 - \sqrt{c}, a^0 + \sqrt{c})$ is strictly above the midpoint $a^0$. That completes the proof of item (ii). $\square$

*Proof of Proposition 8.* By Lemma 3 part 2, it suffices to show that the payoff under default action $a^0 = 0$ is higher than under default action $a^0 = 1$, i.e., that $EW\left(p^{I(0)}, 0\right) \geq EW\left(p^{I(1)}, 1\right)$. The interim optimal publication rule $p^{I(a^0)}$ is as follows: for $a^0 = 0$ publish if $\mu_1^{(X,S)} \geq c$, and for $a^0 = 1$ publish if $\mu_1^{(X,S)} \leq -c$.

Expanding out $EW(p, a^0)$ from (5) for each possible value of $a^0$,

$$EW(p^{I(0)}, 0) = q\mathbb{E}\left[\begin{cases} \mu_1^{(X,S)} - c & \text{if } \mu_1^{(X,S)} \geq c \\ 0 & \text{if } \mu_1^{(X,S)} < c \end{cases}\right]$$
$$EW(p^{I(1)}, 1) = q\mathbb{E}\left[\begin{cases} \mu_1^{(X,S)} & \text{if } \mu_1^{(X,S)} > -c \\ -c & \text{if } \mu_1^{(X,S)} \leq -c \end{cases}\right] + (1-q)\mu_0.$$

Taking the difference,

$$EW(p^{I(0)}, 0) - EW(p^{I(1)}, 1) = q\mathbb{E}\left[\begin{cases} -c & \text{if } \mu_1^{(X,S)} \geq c \\ -\mu_1^{(X,S)} & \text{if } \mu_1^{(X,S)} \in (-c, c) \\ c & \text{if } \mu_1^{(X,S)} \leq -c \end{cases}\right] - (1-q)\mu_0. \tag{29}$$

We seek to show that this difference is nonnegative. Since $\mu_0 \leq 0$ by assumption, it is sufficient to show that the expectation term is nonnegative.

To show that the expectation term is nonnegative, first define a weakly increasing function $l : \mathbb{R} \to \mathbb{R}_+$ as follows:

$$l(k) = \begin{cases} 0 & \text{if } k \leq 0 \\ k & \text{if } k \in (0, c) \\ c & \text{if } k > c \end{cases}.$$

The expectation term in (29) can be rewritten as $\mathbb{E}[l(-\mu_1^{(X,S)})] - \mathbb{E}[l(\mu_1^{(X,S)})]$, and so it is sufficient to show that this difference is nonnegative.

Next, observe that the distribution of $-\mu_1^{(X,S)}$ first order stochastically dominates that of $\mu_1^{(X,S)}$:

$$\begin{aligned} P(-\mu_1^{(X,S)} \leq k) &= 1 - P(\mu_1^{(X,S)} \leq -k) \\ &\leq 1 - P(\mu_1^{(X,S)} \geq k) \\ &= P(\mu_1^{(X,S)} \leq k), \end{aligned}$$

where the inequality comes from the assumption of $P(\mu_1^{(X,S)} \leq -k) \geq P(\mu_1^{(X,S)} \geq k)$. By FOSD, then, the expectation of $l(-\mu_1^{(X,S)})$ is weakly larger than the expectation of $l(\mu_1^{(X,S)})$, completing the proof. □

### C.5.2 Proofs for Appendix B.2

*Proof of Proposition 9.*

1. All comparative statics are immediate from the formula.

2. The only comparative static that is not immediate is that for the t-statistic cutoff, $\left(\frac{1}{S}+\frac{S}{\sigma_0^2}\right)(c-\mu_0)$, with respect to $S$. The argument for this result follows identically as the argument for the analogous result in the proof of Proposition 1 part 3.

$\square$

*Proof of Proposition 10.*

1. This result is immediate.
2. The derivative of the benefit with respect to $\sigma_0$ evaluates to

$$(1-\alpha)\frac{2s_2^4\sigma_0^7(S_1^4\sigma_0^4+2s_2^4(S_1^2+\sigma_0^2)(2S_1^2+\sigma_0^2)+s_2^2(5S_1^4\sigma_0^2+4S_1^2\sigma_0^4))}{(s_2^2+\sigma_0^2)^3(S_1^2+\sigma_0^2)^2(S_1^2\sigma_0^2+s_2^2S_1^2+s_2^2\sigma_0^2)^2}$$

which is positive. As $\sigma_0\to 0$, the numerator goes to 0 while the denominator goes to a positive limit.
3. The derivative of the benefit with respect to $S_1$ evaluates to

$$-(1-\alpha)\frac{2s_2^4S_1\sigma_0^8(2S_1^2\sigma_0^2+\sigma_0^4+2s_2^2S_1^2+2s_2^2\sigma_0^2)}{(s_2^2+\sigma_0^2)^2(S_1^2+\sigma_0^2)^2(S_1^2\sigma_0^2+s_2^2S_1^2+s_2^2\sigma_0^2)^2}$$

which is negative. As $S_1\to\infty$, the numerator is constant while the denominator goes to infinity.
4. The derivative of the benefit with respect to $s_2$ evaluates to

$$(1-\alpha)\frac{2\sigma_0^8s_2^3(-s_2^4(S_1^2+\sigma_0^2)+s_2^2\sigma_0^2(S_1^2+\sigma_0^2)+2S_1^2\sigma_0^4)}{(s_2^2+\sigma_0^2)^3(S_1^2+\sigma_0^2)(S_1^2\sigma_0^2+s_2^2S_1^2+s_2^2\sigma_0^2)^2}.$$

This has the sign of $-s_2^4(S_1^2+\sigma_0^2)+s_2^2\sigma_0^2(S_1^2+\sigma_0^2)+2S_1^2\sigma_0^4$. This expression is a concave quadratic in $s_2^2$, which is positive at $s_2^2=0$ and maximized at $s_2^2=\frac{\sigma_0^2}{2}>0$. In particular, the derivative in $s_2$ is positive and then negative. As $s_2\to 0$, the numerator goes to zero while the denominator goes to a positive limit. As $s_2\to\infty$, the numerator increases at a rate of $s_2^4$ while the denominator increases at a rate of $s_2^6$, so the ratio goes to 0. $\square$