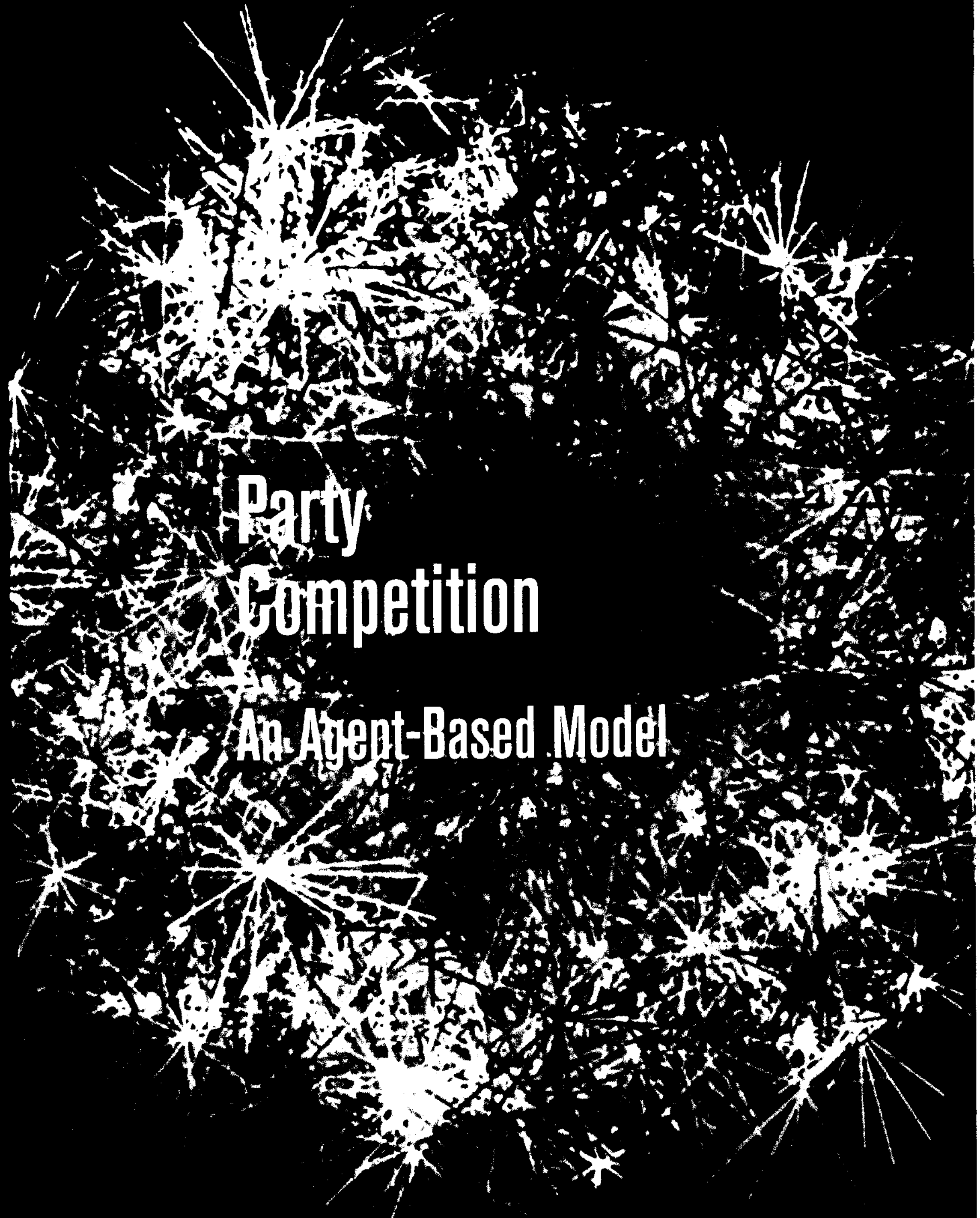
Michael Laver and Ernest Sergenti



Party Competition

An Agent-Based Model

PRINCETON STUDIES IN COMPLEXITY

CHAPTER ONE

Modeling Multiparty Competition

We hold these truths to be self-evident:

- *Politics is dynamic.* It evolves. It never stops; It is never at, nor *en route* to, some static equilibrium. Politics evolves.
- *Politics is complex.* Political outputs today feed back as input to the political process tomorrow.
- *Politicians are diverse.* In particular, different politicians attack the same problem in different ways.
- *Politics is not random.* Systematic patterns in political outcomes invite systemic predictions, making a political "science" possible.

Politics in modern democracies is largely the politics of representation. It concerns how the needs and desires, the hopes and fears of ordinary citizens affect national decision making at the highest level, doing this via public representatives who are chosen by citizens in free and fair elections. Representative politics is to a large extent about party competition: about how a small number of organized political parties offer options to a large number of voters, who choose at election time between alternative teams of public representatives. Party competition is therefore a core concern for everyone, be they professional political scientist or ordinary decent civilian, who cares about politics in democratic societies.

We believe that party competition is a complex and evolving dynamic process that can be analyzed in a rigorous scientific manner. More precisely, we analyze the dynamics of *multi*party competition, by which we mean competition for voters' support among more than two parties, opening up the possibility that no single party wins a majority of votes cast. Figure 1.1 plots some observations of multiparty competition in the Netherlands over the period 1970–2005. The left panel shows positions of the three main Dutch parties on a left-right scale of party ideology, estimated from their party manifestos.[1] The right panel shows support for these same parties in the Dutch electorate, estimated using Eurobarometer surveys.[2] While some of the plotted "variation" in party sizes and

[1] These parties are the Liberals (VVD), the Christian Democrats (CDA), and the Labour Party (PvdA).

[2] The Comparative Manifestos Project and the Eurobarometer survey series, the sources of these data, are discussed at some length in chapter 11 below. Several smaller parties represented in the Dutch legislature have been omitted in the interest of clarity.

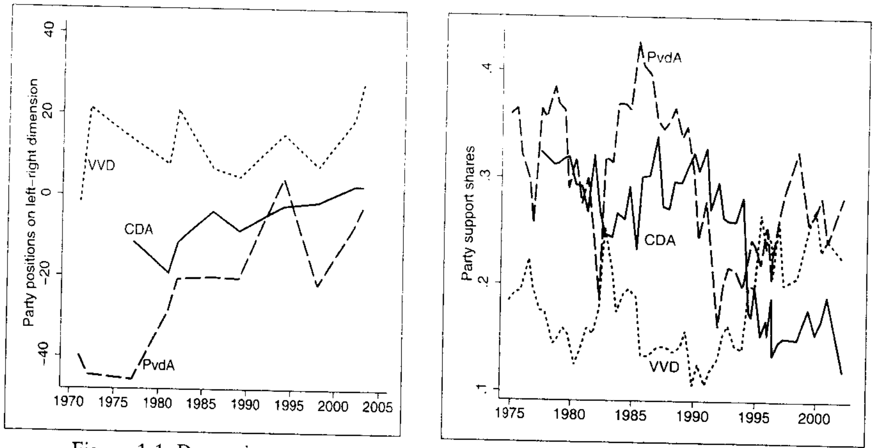


Figure 1.1. Dynamic party competition in the Netherlands, 1970–2005.

policy positions is surely the result of measurement error, by no stretch of the imagination was the Dutch party system "flatlining" in steady state during the period under observation. It was clearly a dynamic system and, as a result, there were frequent changes in the partisan composition of Dutch governments. These dynamics are clearly a central concern for all political scientists analyzing Dutch politics during this period, be they theorist or country specialist. Equivalent plots can be generated for any party system in which we might be interested.

We Need a New Approach to Modeling Party Competition

Formal models of party competition have been an abiding preoccupation of political scientists since the early 1960s. A vast body of existing work has added hugely to our understanding of party competition. Our own *substantive* interest, however, and we believe the substantive interest of most people who want to understand party competition in democratic societies, concerns crucial features of party competition that these models typically assume away as a price to be paid for analytical tractability. We ourselves are interested in party competition among many more than two parties. We are interested in "multidimensional" political environments in which politicians and voters care about more than one type of issue. We see politics as a continuously evolving dynamic process that never settles at some static equilibrium, to be perturbed only by random shocks. Pursuing these interests poses formidable theoretical challenges.

We show in chapter 2 that dynamic models of multiparty competition, especially when voters care about a diverse set of issues, are analytically intractable. They are not just "difficult" to solve, they *cannot* be solved using conventional analytical techniques.

The *analytical* intractability of the relevant theoretical models does not make us any less interested, *substantively*, in dynamic multiparty competition. Indeed, this very intractability gives us an important and liberating theoretical insight. If analysts cannot use tractable formal models to find optimal courses of action in this setting, then *neither can real people making real decisions about real party competition.* These people still need to make decisions about what to do. If no formally provable best-response strategy is available, real humans must employ informal decision rules or heuristics.[3] To preview a decision rule we investigate extensively in this book, a party leader might decide to move party policy toward the position currently advocated by some larger rival party, on the grounds there must be more voters who prefer this rival's policy position. We find that this decision rule (which we call Predator) is sometimes very, very good and sometimes perfectly horrid. It is certainly not a "best" response in any conceivable situation but, in the analytically intractable setting of dynamic multiparty competition, it is one of many potentially good rules that politicians may use in certain circumstances when they set party policy positions.

Agent-Based Modeling

Analytical intractability of the decision-making environment, and the resulting need for real politicians to rely on informal decision rules, suggests strongly that we use *agent-based modeling* to study multiparty competition in an evolving dynamic party system. Agent-based models (ABMs) are "bottom-up" models that typically assume settings with a fairly large number of autonomous decision-making agents. Each agent uses some well-specified decision rule to choose actions, and there may well be considerable diversity in the decision rules used by different agents. Given the analytical intractability of the decision-making environment, the decision rules that are specified and investigated in ABMs are typically based on adaptive learning, rather than forward-looking strategic analysis, and agents are assumed to have bounded rather than perfect rationality (Gigerenzer and Selten 2001; Rubinstein 1998; Simon 1957). ABM is a modeling technology that is ideally suited to investigate outcomes that may emerge when large numbers of boundedly rational agents, us-

[3] We use these terms interchangeably in what follows.

ing adaptive decision rules selected from a diverse portfolio of possibilities, interact with each other continuously in an evolving dynamic setting (MacGregor et al. 2006).

Putting a particular ABM to work by manipulating its parameters and observing the associated outcomes typically involves *computing* the outcomes of these interactions if the underlying model is analytically intractable—as is usually the case. Such computation, does not, of its essence, involve *electronic* computers. One of the most influential early ABMs analyzed housing segregation by scattering black and white chips and then moving them around on what amounted to a large chess board (Schelling 1978). This model was computational in the sense that an abacus is a computer, implemented by moving pieces around a chessboard. As originally published, it did not rely on using an *electronic* computer.[4] Scatter a number of black and white chips at random on a chessboard; these chips represent people of different color. Assume people have some view about the color of their neighbors; say, for example, they are unhappy if fewer than a quarter of their neighbors are the same color as them. The modeled behavior is simply that unhappy agents move to a randomly chosen close-by empty square that makes them happy. A model "run" begins with chips scattered at random. With an equal number of black and white chips, the typical person will find that 50 percent of neighbors are the same color and will be happy to stay put. There will however be some people in the random scatter who find that fewer than a quarter of their neighbors are the same color; they will move to a square that makes them happy. Everyone is given a chance to move, and to move again, using this rule until there is no unhappy agent who wants to move. The results are striking and unexpected. Even if everyone merely wants at least a quarter of their neighbors to be the same color, modeled population movement typically results in a steady state in which on average about 60 percent of a typical agent's neighbors are of the same color. If we change the key model parameter and assume people to be unhappy, and to move, when they are in a local minority (fewer than 50 percent of neighbors are the same color) then people find that on average 88 percent of neighbors are the same color in the typical steady state that emerges. The deep substantive insight from Schelling's ABM is that intense spatial segregation can arise when people do not seek this at all, but simply prefer not to be in a small minority. More generally, this model shows very nicely that simple decision heuristics can interact to generate complex and unexpected "emergent" patterns of social behavior. This is the core insight of agent-based modeling.

[4] A version of this model implemented in NetLogo for electronic computers can however be found in the NetLogo models library.

All good things come at a price. The price paid for using computational as opposed to formal analytical models, and thus for using agent-based modeling, is that computation involves calculating model outputs for particular parameter settings. An analytical result, if it is general, is a beautiful thing that is good for all valid parameter settings. Strictly speaking, computational results are good only for those parameter settings that have actually been investigated. Inferences about parameter settings that have not been investigated—and thus more general theoretical inferences we might want to draw from the model—are, in effect, interpolations. This is one reason why we never use computational methods when analytical results are available for the substantive problem that interests us.

The distinction between analytical and computational methods should not be overdrawn, however. A longstanding set of observations that compare models of computation with systems of formal logic, collectively known as the "Curry-Howard isomorphism," shows us that computer programs and formal proofs are in essence the same thing (De Groote 1995). Both take a set of explicit premises and manipulate these, using some system of formal logic, to prove theorems based on these premises. Consider, for example, the area, A, of a circle with radius r. It is well known that we can prove analytically the proposition: $A = \pi \cdot r^2$ for *any* positive real r. We can also prove $A \approx \pi \cdot r^2$ for *any given* positive real r by various computational methods. With *infinite* computing power at our disposal, we could prove $A \approx \pi \cdot r^2$ for *any* positive real r.[5] This would not be an "elegant" proof according to most standards of elegance, but now we are talking about aesthetics. With less-than-infinite computing power, we can sample a huge number of positive real values of r, compute A, and show in every single case that $A \approx \pi \cdot r^2$. We can draw the *statistical* inference, at a specified level of confidence, that $A \approx \pi \cdot r^2$ for any positive real r. If for some reason it happened that we could not prove analytically that $A = \pi \cdot r^2$, then this computational/statistical inference would be immensely valuable to us. If we wanted to increase our confidence in this inference, we could simply do more computing and sample more values of r. Of course, we could never be *perfectly* confident in this conclusion. We can show that $A \approx \pi \cdot r^2$ when $r = 2.0000001$ and 2.0000002; you could claim it is possible $A \neq \pi \cdot r^2$ when r is set between these values, at

[5] The approximation arises because π is a transcendental number that cannot be stored to any arbitrary level of precision in a digital computer. However, a number very close to π can be stored as a high-precision floating point number. For the same reason, the area of any circle calculated using the classical formula $A = \pi \cdot r^2$ can be *written down* as a real number only using an approximation that deploys some arbitrary level of precision specifying the number of decimal places we are prepared to use.

2.00000015. Strictly speaking, this would be true.[6] We could however show statistically, with access to enough computing power, that the *probability* of this exception is extraordinarily small. Furthermore, we could drive down this probability to as low a level as makes you feel happy—simply by doing more computing.

This is an issue we take very seriously indeed in this book since we do want our computational results to have effectively the same scope and precision as those derived from analogous analytical work. We address this by specifying careful procedures for systematically varying parameter settings, and rigorous methods for estimating model outputs of interest associated with these settings. If we carefully design and execute our computational work in this way, then the scope and precision of our results depend only on the volume of computation we are willing and/or able to deploy. Since we want our own results to have the same scope and precision as typical results from formal models in this field, we are both willing and able to deploy a huge amount of computing power, taking advantage of the Harvard-MIT Data Center's high-performance cluster in order to do this. An important consequence of this is that we are confident that the computational results we present in this book can be "taken to the bank," in the formal statistical sense that, if we were to do very much more computing, or if many other people were to repeat our procedures, essentially identical results would arise. Thus, while this is a book above all about the substantively fascinating topic of multiparty competition, it is also an exercise in how to use computational methods in general, and ABMs in particular, in a way that allows us to draw confident general conclusions.

To summarize, the substantively important real-world problem that interests us is the dynamics of multiparty competition. Theoretical models are no more than intellectual tools designed to help us understand substantively important real-world problems. The technology of classical formal modeling is not a good tool to help us understand the dynamics of multiparty competition, since the resulting models are analytically intractable, with consequences for analysts and more importantly for real humans making decisions in these settings. In contrast, the empowering new technology of agent-based modeling is well suited to investigating problems that are of great substantive interest to us. Impatient for results and problem focused as we are, this book is about how agent-based modeling helps us think systematically about the dynamics of multiparty competition. We start simple and build an increasingly complex model of party competition that deals with a range of substantive matters we have

[6] Although our advice to you in this case would be that you should get out more.

wanted to think about for a long time but had not really been able to think about in a systematic way before the emergence of ABM.

Plan of Campaign

Chapter 2 sets up the core problem in which we are interested. To demonstrate that this problem is analytically intractable, we use compelling results from a subfield of geometry that deals with "Voronoi tessellations" (or tilings) and has powerful applications in many disciplines. Largely unnoticed by political scientists, this work addresses a problem of "competitive spatial location" that is directly analogous to the problem of dynamic competition between a set of political parties competing with each other by offering rival policy programs. One result from this field is that the problem of competitive spatial location is intractable if the space concerned has more than one dimension (we return below to discuss the meaning of a "dimension" in models of party competition), implying that there are no formally provable best-response strategies for this. This is an important and widely recognized justification for deploying computational methods, and the study of Voronoi tessellations is a major subfield in *computational* geometry.

Chapter 3 specifies our "baseline" ABM of dynamic multiparty competition, which derives from an article published by one of us (Laver 2005). This assumes that each voter has in mind some personal ideal "package" of policy positions and supports the political party that offers the policy package closest to this. The dynamic system at the heart of our model is as follows: voters support their "closest" party in this sense; party leaders adapt the policy packages they offer in light of the revealed pattern of voter support; voters reconsider which party they support in light of the revealed pattern of party policy packages; and this process continues forever. This recursive model describes policy-based party competition as a complex system, and our baseline model specifies three decision rules that party leaders may deploy when they choose party policy positions in such a setting. These rules are Sticker (always keep the same position), Aggregator (move policy to the centroid of the ideal policy positions of your current supporters), and Hunter (if your last policy move increased your support, make another move in the same direction; or else change heading and move in a different direction). These rules model, in a simple way, an "ideologically intransigent" party leader who *never* changes party policy, no matter how unpopular this might be; a "democratic" party leader who always adapts the party position to the preferences of *current* supports; and a "vote-seeking" party leader who is always looking for *new* supporters and does not care what policies must be chosen in order to do this. These decision rules were specified in Laver (2005); the innovation in this chapter concerns our assumptions about the preferences of voters. Rather than assuming a single coherent voting population with a perfectly symmetrical multivariate normal distribution of ideal policy positions, we now assume that electorates comprise a number of distinct subgroups. Combining subgroups into an aggregate voting population, we produce an aggregate distribution of ideal points that is no longer perfectly symmetric. This more generic assumption about voter preferences makes a big difference to what our model predicts.

Chapter 4 develops our methods for designing, executing, and analyzing large suites of computer simulations that generate stable and replicable results. We start with a discussion of the different methods of experimental design, such as grid sweeping and Monte Carlo parameterization. Next, we demonstrate how to calculate mean estimates of output variables of interest. In order to do so, we must first discuss, among other things, stochastic processes, Markov Chain representations, and model burn-in. As we see below, we are especially interested in three stochastic process representations: nonergodic deterministic processes that converge on a single state, nondeterministic stochastic processes for which a time average provides a representative estimate of the output variables, and nondeterministic stochastic processes for which a time average does not provide a representative estimate of the output variables. The estimation strategy we employ depends on which stochastic process the simulation follows. Last, we present a set of diagnostic checks, used to establish an appropriate sample size for the estimation of the means. More observations obviously lead to more precise estimates. However, given a fixed computational budget, in terms of computer processing time and storage space, as well as the opportunity costs of not executing other simulations, we want to gather enough observations to allow precise estimates, but no more than is needed.

We report our benchmark results in chapter 5. Perhaps the most striking of these concerns the "representativeness" of any given configuration of party policy positions and uses a second result that comes from the Voronoi geometry of competitive spatial location. A set of n points arranged so as to generate a "centroidal Voronoi tessellation" (CVT) is an "optimal representation" of the space in which these points are located. By this we mean that the *aggregate* distance between all points in the space and their closest generating point can never be less than when the n generating points are arranged in a CVT (Du et al. 1999).[7] If we think that voters are

[7] The analogous problem in digital imaging is to find the most representative set of n points (party positions) to represent a much more detailed image comprising m points (voters). More generally, a CVT can be seen as a "best" simple representation of any spatially structured dataset.

more satisfied at election time the closer their own ideal policy is to the policy position of their closest party, then this implies that the electorate as a whole is most satisfied when party policy positions are arranged in a CVT. Since the "representativeness" of any party system is an important matter, both normatively and in terms of practical politics, the notion of an optimal representation gives us an important benchmark for assessing evolved configurations of party policy positions. A robust conjecture in computational geometry, concerning what is known as Lloyd's Algorithm (Lloyd 1982), is very relevant in this context. If all party leaders use the Aggregator rule for setting party policy positions, continuously adapting party policy to the centroid of *current* supporters' ideal points, then Lloyd's Algorithm tells us that the set of party policy positions will converge on a steady state that is a CVT. *Party positions in all-Aggregator party systems thus evolve to configurations that are optimal representations of the space.*[8] Other configurations of party policy positions will generically imply suboptimal representation, in this precise sense.

Thus far we have treated the set of competing political parties as exogenously given to us by God or Nature. We move beyond this in chapter 6 and define a model of endogenous party "birth" and "death" (Laver et al. 2011; Laver and Schilperoord 2007) that has the implication that *the set of surviving political parties is endogenous* to the system of party competition. We now also model competition between party leaders using different decision rules, extending work on this using computer "tournaments" (Fowler and Laver 2008). All of this requires us to extend our model to define a de facto survival threshold for political parties; an updating regime that specifies how voters feel about the party system today, given what happened today and how they felt about the system yesterday; and a distinction between "campaign ticks" of the model, during which party leaders make choices that do not have long-term consequences for their survival, and "election ticks" that do have a bearing on party survival. The resulting more realistic model of party competition with endogenous parties is *evolutionary*, describing a *survival-of-the-fittest* environment in which more successful parties survive and less successful parties do not.

Up to this stage in the argument, we have extended, improved, and generalized previously published work based on three simple decision rules: Sticker, Aggregator, and Hunter. We break completely new ground in chapter 7, defining new "species" of vote-seeking decision rule (Predator and Explorer) and specifying both these and existing rule species in terms of a set of parameterized rule "features," including satisficing and speed of adaptation. Predator rules, specified in a flawed form in Laver (2005)

[8] In this context it is very important to note that there is typically no unique optimal representation.

and redefined by us here, in essence attack the closest more successful party by moving their policy position toward it. Explorer rules are generalizations of "hill climbing" algorithms. Explorers randomly poll positions in some local policy neighborhood during campaign ticks—moving on an election tick to the best position they found during the campaign. The net result of these extensions is that we now consider competition between party leaders who may choose from one of 111 different decision rules—or, strictly speaking, parameterizations of rule-agent pairings. This dramatically expands the state space of our model and forces a major modification in the method we use to estimate characteristic model outputs. We find that which particular vote-seeking rule is most effective depends critically on parameters of the competitive environment. Chapter 7 reports another result we feel is particularly important, concerning what happens when *satiable* and *insatiable* vote-seeking party leaders compete with each other. We find well-defined circumstances in which satiable leaders, who do nothing until their party vote share falls below some "comfort threshold," systematically win higher vote shares than insatiable leaders, who always seek more votes no matter how many they currently have. This is a good example of the classic "exploitation-exploration trade-off" in reinforcement learning (Sutton and Barto 1998). Insatiable party leaders always explore the space in search of more votes, whereas satiable leaders exploit their good fortune whenever vote share is above their comfort threshold. This is the type of insight that can be derived only from a *dynamic* model of party competition.

In chapter 8, we extend our survival-of-the-fittest evolutionary environment to take account of the possibility that new political parties, when they first come into existence, do not pick decision rules at random but instead choose rules that have a track record of past success. We do this by adding *replicator-mutator dynamics* to our model, according to which the probability that each rule is selected by a new party is an evolving but noisy function of that rule's past performance. Estimating characteristic outputs when this type of positive feedback enters our dynamic model creates new methodological challenges. Having addressed these challenges, the simulation results we report in chapter 8 show that it is very rare for one decision rule to drive out all others over the long run. While the diversity of decision rules used by party leaders is drastically reduced with such positive feedback in the party system, and while some particular decision rule is typically prominent over a certain period of time, party systems in which party leaders use different decision rules are sustained over substantial periods. More generally, we continue to find party leaders choosing from a diverse rule set in this evolutionary setting. We find no evidence whatsoever of evolution toward the dominance of a single decision rule for setting party policy positions.

Moving beyond the assumption that voters care about only the party policy positions on offer, chapter 9 models the possibility that they also care about perceived "nonpolicy" attributes of political candidates: competence, charisma, honesty, and many other things besides. These characterize what have become known as "valence" models of party competition. Voters balance utility derived from each candidate's nonpolicy valence against utility derived from the candidate's policy position. The contribution of valence models has been to explain why all parties do not all converge on regions of the policy space with the highest densities of voter ideal points. Higher valence parties tend to go to regions of the policy space with higher voter densities, while lower valence parties are forced to steer well clear of these parties and pick policy positions in regions with lower voter densities. We replicate and extend the findings of traditional static valence models, with one important twist. Over the long run, lower valence parties tend to die and higher valence parties tend to survive, a finding that suggests a reappraisal of valence models as currently specified. These essentially static models show a snapshot of the party system at a given time; but the tendency of low-valence parties to disappear in an evolutionary setting suggests that these snapshots are not dynamic equilibriums that can be sustained over time.

Moving beyond voters who care about more than policy, we look in chapter 10 at party leaders who care about their own private policy preferences as well as about winning votes. In the spirit of our existing model of endogenous party birth, we take the preferred policy position of a party leader as the founding policy position of his or her party. In an intriguing echo of our findings on satisficing in an evolutionary setting, we find that party leaders who care somewhat about their own policy position may do somewhat better *at winning votes* in competition with party leaders who care exclusively about vote share. This may arise from the fact that, in an evolutionary setting, *the ideal points of surviving party leaders are endogenous*. Each surviving party leader was once a new entrant into the system at a policy position for which there was demonstrable voter "demand." Leaders who stay close to this founding position continue to satisfy the demand that originally caused the party birth. They thereby also, effectively though not intentionally, forestall new party births in this region of the policy space.

Having specified theoretical models of multiparty competition in the first ten chapters of the book, we investigate empirical implications of these models in chapter 11, comparing model predictions with changes in observed party policy positions and vote shares in ten real European party systems. Confronting theoretical models with empirical data is a central part of the definition of political science as a "science," highlighted by the influential Empirical Implication of Theoretical Models project. This is easy to say but hard to do well, and it is even harder for dynamic models that have many parameters whose values are not directly observable in the real world. We face serious problems of model *calibration*, of finding parameter settings for our theoretical models that plausibly correspond to those in the real party systems for which we can observe empirical observations. Calibration problems are compounded in this case by an acute shortage of high-quality time series data on party system outputs of interest. Given all of these problems, the best we can hope for is to find "plausible" model calibrations generating predictions that are close to reality. This is of course not a full-dress scientific test of our model, which is not feasible given the lack of good time series data on party policy positions, combined with the lack of reliable independent data for model calibration. We prefer, however, to be honest about the calibration and data problems that arise with any dynamic model of multiparty competition rather than, dishonestly we feel, making "assumptions" about model calibration that will give us lovely empirical results but that are, in effect, assumptions chosen to give those lovely results. What we call the "auto-calibration" of our model, searching for model calibrations consistent with accurate predictions, does allow us to conclude that the model *can* be calibrated to generate accurate predictions and that the calibration values associated with good predictions do have good face validity.

Putting all of this together, our fundamental interest in this book is in multiparty competition, seen as an evolving dynamic system. Our fundamental intellectual objective is to explore some of the puzzles about this that can be addressed using techniques of agent-based modeling. Substantively, while readers are the ultimate judges of this, we do feel that agent-based modeling empowers us to tackle interesting and important questions that cannot be addressed so fruitfully using the techniques of classical analytical modeling. Methodologically, we do feel that carefully designed and executed computational work can generate results that have a scope and precision equivalent to those generated by more traditional techniques.

Our sincere hope is that we open an intellectual window for at least some readers, who will take the ideas and suggestions in what follows and improve them beyond all recognition.

CHAPTER TWO

Spatial Dynamics of Political Competition

Spatial Models of Political Competition

Politicians compete with each other in many different ways. They trade on personal popularity; they attack the integrity and character of their opponents. They exploit, or are victims of, biased coverage in the news media. They hire advertising agencies to manipulate these same media. They practice dark arts and dirty tricks. Political competition in any given setting has many idiosyncratic features that are not at all amenable to general explanations. If we want to understand the close-in detail of any particular political system, we do best to ask people who have particular local knowledge and/or insight—"gurus" who specialize in the exceptional features and minutiae of the domestic politics of interest.

Notwithstanding this, key features of political competition look remarkably similar in different domestic settings. Politicians make promises about the policies they will implement if elected. Voters have views about the different policy promises on offer and choose public representatives, at least in part, on the basis of these promises. Naïve voters may take these promises at face value. More sophisticated voters may treat them as signals from which to draw inferences about what, given what politicians *say* they would do, they actually *will* do if given the chance. All of this characterizes party competition in many different domestic settings. This characterization of democratic politics, as competition among politicians for the support of voters on the basis of the public policies they offer, underpins the account of party competition we consider in this book.

"Spatial" descriptions of the preferences of key decision makers are widely used by political scientists modeling in a wide range of different settings, from elections and legislatures, to government formation, to international organizations, to the U.S. Supreme Court, and many other things besides. References to "left" and "right," and to changing policy "positions," are part of day-to-day political discourse and not just terms of art for political scientists. Spatial models based on this way of describing politics have provided such a fruitful basis for rigorous thinking about political competition that this approach is now seen as "the workhorse theory of modern legislative studies" (Cox 2001) and has gen-

CHAPTER FIVE

Benchmarking the Baseline Model

Having set out our methods in chapter 4, we are now at last in a position to start investigating multiparty competition using the baseline model we specified in chapter 3. As we said when setting out our plan of campaign, we start simple in this chapter and build up to a more complex and realistic model of party competition in subsequent chapters. We need to understand the basic processes of dynamic multiparty competition in a simple setting before we move on to study complications such as endogenous political parties, diverse sets of decision rules, voters who care about more than policy, and politicians who care about more than votes. Our aim in this chapter is to analyze our baseline model of multiparty competition and develop benchmarks against which to measure future results.

Specifically, we begin here by modeling party systems with an exogenously fixed number of parties, all of which use the same decision rule. While current models of party competition in the classical analytical tradition do also typically deal with exogenously fixed sets of parties in which all parties use the same decision rule,[1] our ambition in this book is to go very far beyond this. We reign in our ambitions in the current chapter, however, in the interests of an orderly program of model development. The results we report below are useful for two main reasons. First, they provide a benchmark against which to evaluate results generated by the increasingly complex models we develop in subsequent chapters. Second, they provide a point of reference that allows us to compare results generated by our model to those generated by more traditional models of party competition. By the end of the book, in contrast, the substantive reach of our model will be so far beyond models in the classical tradition that there will be nothing to compare it with.

We first benchmark our model for the baseline assumption that voters' ideal points are distributed over the policy space with a perfectly symmetric bivariate normal density. This essentially replicates and confirms already published work (Laver 2005), always an important starting point for any scientific project. We then move beyond existing computational work to exercise our model under the new assumption about ideal point

[1] As we note in the next chapter there are classical formal models of "party entry" in very spare settings, in which the set of competing parties may be endogenous.

distributions that we specified in chapter 3. This leads us to investigate multiparty competition in the more general setting with distinct subpopulations of voters and consequently *asymmetric* aggregate distributions of ideal points. Before we do any of this, we specify design and estimation procedures for our computational work that build on the methodological conclusions we reached in the previous chapter.

Experimental Design

We specify different designs for experiments with symmetric and with asymmetric distributions of voter ideal points—given the more complex parameterization of the model in the latter case.

Symmetric Populations

Our computational experiments for symmetric populations have the simple grid-sweeping design set out in Figure 4.1, since we are manipulating two model parameters, each with a small number of discrete and "natural" settings. We investigate the effects of three different rule sets (all-Sticker, all-Aggregator, all-Hunter) in party systems of eleven different sizes, ranging from two to twelve parties. This gives us a thirty-three-point parameter grid and hence a total of thirty-three model runs for the experiment.

Asymmetric Populations

The description of distributions of voter ideal points is now generalized to comprise aggregations of two subpopulations, each with normally distributed ideal points. This mandates a more general experimental design since the dimensionality of our parameter space is now considerably expanded to include two new parameters, as shown in Figure 4.2. The new parameters are the relative size of the two subpopulation, n_l / n_r, and their ideal point means, μ_r ($= -\mu_l$). Given the increased size of the parameter space, we use a design based on Monte Carlo parameterization. For each of the three rule sets under investigation, we run a suite of one thousand runs. Each run in each suite uses a Monte Carlo parameterization of our model that is specified as follows:

- the number of parties is sampled from a uniform distribution of integers on the interval [2, 12]
- the relative size of subpopulations, n_l / n_r, is sampled from a uniform distribution of real numbers on the interval [2.0, 1.0]
- the ideal point means μ_r ($= -\mu_l$), are sampled from a uniform distribution of real numbers on the interval [0.0, 1.5]

Run Design

The suites of runs for all-Sticker, all-Aggregator, and all-Hunter party systems all generate different stochastic processes, and for this reason, each requires a different run design.

All-Sticker Runs

Political parties in an all-Sticker system never change position. Since voters in our model do not change their ideal points, "nothing happens" in all-Sticker simulations once initial party positions have been generated. Immediately after the random scatter of party positions at the start of each run repetition, therefore, the repetition has reached a single deterministic steady state: no party will change position, none of the party vote shares will change, and none of the values for the aggregate measures, mean eccentricity, ENP, and representativeness will change. There is no transient state, hence no need for burn-in.

In addition, given that the steady state obtained for any particular run repetition depends entirely on the initial random scatter of parties (which in turn depends on the random seed used by the repetition), these run processes are nonergodic. Each all-Sticker run process, therefore, is a nonergodic deterministic process that converges on a single state. Our estimation procedure for the output means of interest, therefore, is to execute several repetitions, each with a single iteration, recording the value for each of the output variables at the first iteration as being representative of the repetition. We then calculate the ensemble average of the output values across the repetitions we perform. Using the diagnostics we developed in chapter 4 for this run procedure, we determined that one thousand repetitions per run were sufficient to ensure a representative estimate with the level of precision we desire.[2]

All-Aggregator Runs

There is no random component in the Aggregator rule: set party policy on each dimension at the mean preference of all current party supporters. An all-Aggregator run process is thus also deterministic. In addition, as we saw in chapter 2 and develop more fully below, we have the theoretical result from Voronoi geometry that party positions in all-Aggregator systems always converge on a centroidal Voronoi tessellation of the policy space. Each all-Aggregator run process therefore tends toward a single steady state. As with all-Sticker runs, different steady states are

[2] Section E5.1 in the electronic appendix gives details of the diagnostic tests that led to this conclusion.

possible, depending on initial conditions. As with all-Sticker runs, therefore, all-Aggregator processes are nonergodic deterministic processes that converge on a single state. Hence, we calculate the mean by taking an ensemble average of the steady-state values of the output variables from several repetitions. Unlike all-Sticker runs, however, all-Aggregator runs do not start out in steady state. Aggregator parties adapt away from their initial random starting positions, eventually settling after a series of iterations into a deterministic steady-state configuration.

Our design for all-Aggregator runs is therefore to execute several repetitions until each run repetition has reached steady state, collect final representative values for each of the output variables for the run repetition at which steady state is achieved, and calculate an ensemble average of all of the collected values as an estimate of μ for each output variable of the run. Using the diagnostics we discuss in chapter 4, we settled on a sample size of one thousand repetitions.[3] As we see throughout this book, one thousand observations are sufficient to satisfy the first four diagnostic checks with all of the theoretical simulations that we execute. Hence, with all simulations, we always collect one thousand observations to calculate mean estimates. In so doing, we achieve the same relative precision of 3.2 percent across all runs and all output variables analyzed in this book.

All-Hunter Runs

All-Hunter runs are more complex. First, the Hunter rule stipulates that, if the last policy move increased party support, the party should continue in the same direction, otherwise it should reverse heading and make a unit move in a heading chosen randomly from the 180-degree arc centered on the direction it is currently facing. This random component of the Hunter rule makes all-Hunter run processes stochastic and ensures that they are ergodic.

Given that the run processes are stochastic, determining run burn-in for all-Hunter runs is a more complicated matter. To facilitate the burn-in analysis, we create a Markov representation of the run process. One such representation is to define the state space at time t, using the set of variables that summarizes the x- and y-coordinate positions for all-Hunter parties at times t and $t-1$. Thus, the *vector of state space variables* at time t, X_t, comprises the configuration of party positions at *both* time t and time $t-1$, $X_t = \{P(t), P(t-1)\}$. For example, with an all-Hunter benchmark run with 12 parties, the vector of variables would comprise, for each party competing, the x-coordinate at time t, the y-coordinate at time t, the x-coordinate at time $t-1$, and the y-coordinate at time $t-1$. This is a total of $4 \times 12 = 48$ variables. This combination of variables satisfies the necessary conditions for a Markov process. First, combined with information about the initial distributions of voter ideal points, all other output variables at time t can be determined from these variables. The vote shares at time t are determined by the party positions at time t and the distribution of voter ideal points. The heading for each of the Hunter parties at time t can be determined by comparing its position at time t to its position at time t-1 and the change in its vote share from time $t-1$ to time t (which itself is derived from the positions of all of the parties at times t and $t-1$). Last, all of the aggregate measures at time t, mean eccentricity, ENP, and representativeness, can be calculated using the party positions at time t and the distribution of voter ideal points. Second, the vector of state space variables satisfies the Markov property. The probability that the process will be in any state of the state space at time $t+1$, Prob $(X_{t+1} = j)$, is conditional on solely the state of the process at time t, X_t. This is because (1) the probability of a particular configuration of Hunter parties at time $t+1$, $P(t+1)$, is completely determined by the positions of the parties at times t and t-1, $\text{Prob}(P(t+1)) = f(P(t), P(t-1))$ and the (2) the positions of the parties at time t are already known, $P(t) = P(t)$. Hence, the probability that the vector of state space variables will take on a particular value at time $t+1$, Prob $(X_{t+1} = j)$ is completely determined by the vector of state space variables at time t, X_t.

Given this Markov representation of the process, we use two summary variables, mean eccentricity and ENP, to diagnose when the run process is burnt in. Both of these variables ultimately depend on the positions of all parties. When these parties have reached a stochastic steady-state configuration, the output variables mean eccentricity and ENP will also be in stochastic steady state. The next step is to determine whether or not a time average provides a representative estimate of μ for all of the output variables. In fact, we already examined this case in the empirical burn-in subsection of chapter 4. As we described there, one thousand post-burn-in iterations are sufficient to map out the steady-state vector for the summary variables, and hence a time average provides a representative estimate of μ for all output variables. Also, as we see with Figure 4.4, which plots each summary variable by iteration, one hundred iterations are more than enough to reach the stochastic steady state with the most extreme run with twelve competing parties.[4] Therefore, our procedure for estimating the mean of the output variables for the all-Hunter runs is to execute a single repetition per run, collect

[3] Section E5.2 in the electronic appendix gives details of the diagnostic tests that led to this conclusion.

[4] Runs with fewer parties required fewer iterations for burn-in.

one thousand observations starting from iteration 101, and calculate a time average of the collected values.[5]

Multiparty Competition in Symmetric Voter Populations

We are now finally in a position to run computational experiments using our baseline model of dynamic multiparty competition. We analyze the results of these computational experiments in the rest of this chapter. We begin with competition among parties for the support of voters whose ideals points have a symmetric normal distribution in the policy space. We move on to consider multiparty competition in settings where the distribution of ideal points is asymmetric, as a result of the existence of at least two distinct subpopulations of voters.

Evolved Eccentricities of Party Policy Positions

Figure 5.1 shows two sample screen shots from our simulation runs. The shaded area in the background shows the bivariate normal distribution of voter idea points; lighter areas have higher ideal point densities, darker areas have lower densities. The seven gray arrowheads in the left panel show the policy positions of a deterministic steady-state configuration of seven Aggregators. These party positions evolved, as a result of iterated decisions taken by party leaders using the Aggregator rule, from a set of seven completely random policy positions to a configuration of positions that are very evenly and symmetrically distributed in the policy space. Since all party leaders are using the Aggregator rule, we know from Lloyd's Algorithm that this steady-state configuration of party positions is a centroidal Voronoi tessellation and is therefore an optimal seven-point representation of the set of voter ideal points.

The right panel shows a snapshot of the positions of seven Hunters. These policy positions, again shown by gray arrows, are never at rest but are in continual motion with headings indicated by the arrowheads. This continual motion arises because each Hunter is "insatiable," to use a term we define more precisely in chapter 7. Whatever the state of the process, a Hunter always makes some policy move in search of more votes. This snapshot of a configuration in constant motion shows Hunter party positions that are unevenly distributed and closer to the center of the space than the Aggregator positions.

These configurations of party positions are typical, and Figure 5.2 plots typical policy eccentricities in our simulations of political parties

[5] Section E5.3 in the electronic appendix gives details of the supporting diagnostic tests.

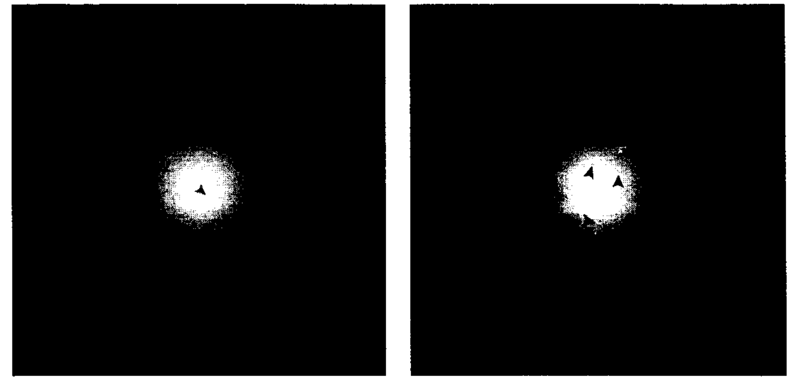


Figure 5.1. Screen shots of locations of seven Aggregators (left) and seven Hunters (right) in benchmark runs.

competing for votes in all-Sticker, all- Aggregator, and all-Hunter party systems.[6] Setting these results in context, recall from Figure 3.3 that mean policy eccentricity of *voters* in the baseline symmetric population is 1.25 and that we know analytically, given the algorithm for randomly scattering parties, that the typical policy eccentricity of random Stickers will be 1.50.

The conclusions we draw from Figure 5.2 are clear-cut and substantively important. Compared to parties in equivalent all-Aggregator party systems, vote-seeking parties in all-Hunter systems systematically choose more central policy positions. Party systems with two vote-seeking Hunters are somewhat atypical; parties tend to search for votes close to the centroid of voter ideal points (typically 0.2 standard deviation units away from this). When more parties compete for votes, confirming results reported in Laver (2005), Hunter parties typically compete for votes at locations that are fairly close to the voter centroid but very definitely not at it. The solid line in Figure 5.1 shows that parties in all-Hunter systems typically promote policy positions that are 0.75–0.95 standard deviations from the voter centroid, positions significantly closer to the center than the ideal point of the typical voter.[7] In all-Aggregator party systems with four or more parties, in contrast, party leaders tend to promote posi-

[6] Standard errors of all estimates are less than 0.002. The detailed estimates generating Figure 5.2 are reported in Table E5.4.

[7] Since our estimates are very precise, whenever we use the word "significant" in reporting results this also indicates statistical significance.

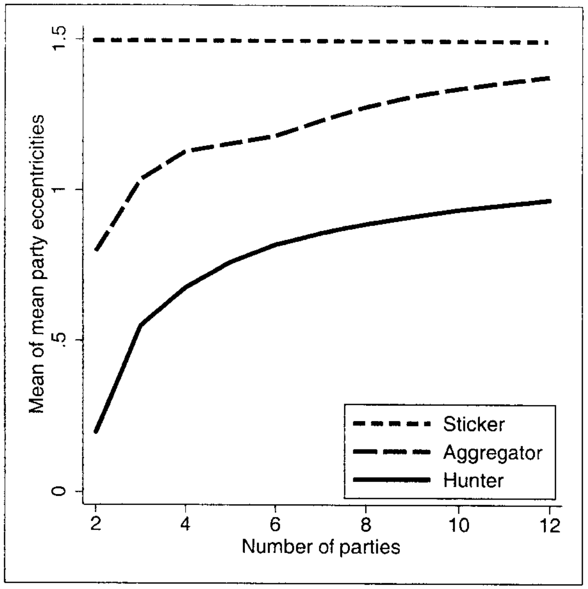


Figure 5.2. Mean party policy eccentricity, by decision rule and number of parties; benchmark runs in symmetric populations.

tions close to the mean policy eccentricity of the underlying voter population. Our simulations also show clearly that, whether in all-Aggregator or all-Hunter party systems, mean party policy eccentricity increases with the number of parties. When there are more parties, some of these tend to take more eccentric policy positions.

One of the seminal Downsian intuitions about party competition is that competitive spatial location by parties in *one-dimensional* policy spaces tends to result in centripetal position taking and a resulting convergence of parties on the center of the policy space. Given this, avoidance of the center of the policy space by vote-seeking Hunters, even though this is the location of the highest voter densities, is substantively striking; it is also a very robust result. To gain some intuition about this, we return to the geometry of dynamic Voronoi diagrams. Figure 5.3 builds on the example used in chapter 3, showing the Voronoi dynamics of a move by the Fianna Fáil party toward the voter centroid. The move is from FF_1 to FF_2, and the voter centroid is at x. As before, solid lines show the Voronoi diagram generated by the party configuration with FF_1. Dotted straight lines show changes in the Voronoi diagram resulting from the change in

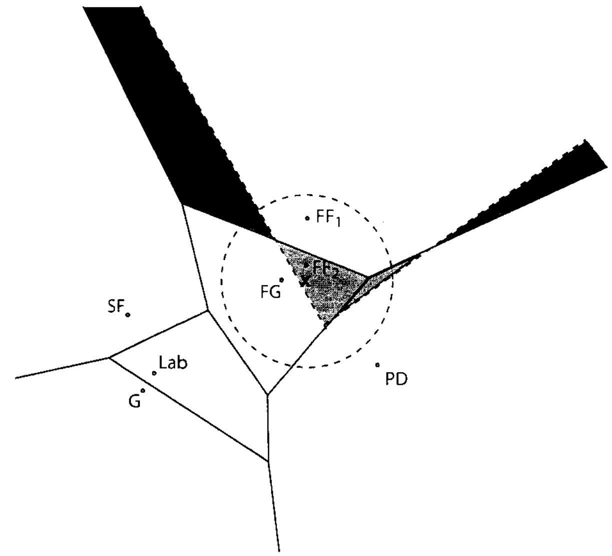


Figure 5.3. Voronoi dynamics of a move by FF toward the voter centroid at x.

Fianna Fáil policy. Note that FF_2 is closer than FF_1 to x, that is, the move is toward the voter centroid. The regions *added* to Fianna Fáil's Voronoi region as a result of this move are shaded lighter gray; the regions *lost* are shaded darker gray. Whether Fianna Fáil's move toward the center results in a net gain or loss of votes depends on the distribution of voters' ideal points

The dotted circle plots an ideal point density contour for this symmetric distribution. If ideal point densities outside this contour are very low, then Fianna Fáil's move toward the center would on balance win the party more votes. If the contour plots three standard deviations from the voter centroid, for example, FF_1 would be about two standard deviations from the voter centroid and there would be relatively few voter ideal points (less than 0.5 percent) outside the dotted circle. In this case, Fianna Fáil's move toward the center would be rewarded. Gains in densely populated regions inside the circle would greatly outweigh losses in sparsely populated regions outside this. In contrast, if the contour plots 0.5 standard deviations from the voter centroid, then FF_1 would be about 0.4 standard deviations from the voter centroid and about 62 percent of voter ideals would be outside the circle. Now, if Fianna Fáil moves from FF_1 to FF_2, gains of votes in the light gray regions would be far outweighed by losses in the dark gray regions. In this case, with Fianna Fáil relatively close to the voter centroid, *a move closer to the centroid would be punished by a net loss of vote share.*

More generally in this case, if the Fianna Fáil leader is using a Hunter rule and the party starts at an eccentric policy position, say two standard deviations from the voter centroid, and moves toward the center, then such a move will at first tend to be rewarded and the move will therefore be repeated. After a series of such moves party policy will be close enough to the centroid that a further centripetal move will be punished with a lower vote share. At this point, the Hunter rule implies reversing direction and moving away from the voter centroid.

Figure 5.4 gives a more general feel for the complex dynamics generated by vote-seeking multiparty competition in multidimensional policy spaces. It plots typical vote shares against policy eccentricities for each party, over the course of a burnt-in five-Hunter simulation run. The solid horizontal line shows the mean long-run vote share for each party. For five parties this is, axiomatically, 20 percent. The dashed lines show that each party's vote share is maximized, at about 22.5 percent, when it locates about 0.4 standard deviations from the voter centroid. Why then do these five vote-seeking parties typically locate, as we see from Figure 5.2, at 0.76 standard deviations from the voter centroid? Is this location not suboptimal for them?

The answer is no. It is arithmetically impossible for five parties to average 22.5 percent of the vote over the long run. The solid vertical line in Figure 5.4 shows that each of the five parties averages its long-run expectation of 20 percent of the vote when it is 0.76 standard deviations from the voter centroid. Since all parties use the same rule and thus expect the same average vote share over the long run, the *average* long-run positions for each of the five Hunters should be 0.76 standard deviations from the center, the location at which they all tend to get the five-party mean vote share of 20 percent. Figure 5.4 does show that a *single* Hunter can expect to do better than this if it locates closer to the center, but it also shows that this situation is not dynamically stable in the long run. If one vote-seeking party wins more than an average share of the vote, *then other parties using the same decision rule must win less*; their vote-seeking behavior will correct this situation over the course of dynamic party competition in the long run. Figure 5.4 shows us that perfect symmetry between the five Hunters means that they should, over the long run, tend to locate at 0.76 standard deviations from the voter centroid, and Figure 5.2 shows us that this is indeed the case.

We have taken some time working though this example because it gives us a clear sense of why we must think about party competition in very different ways when this is set in an intrinsically *dynamic* environment, in contrast to the static environment of the classical spatial model.

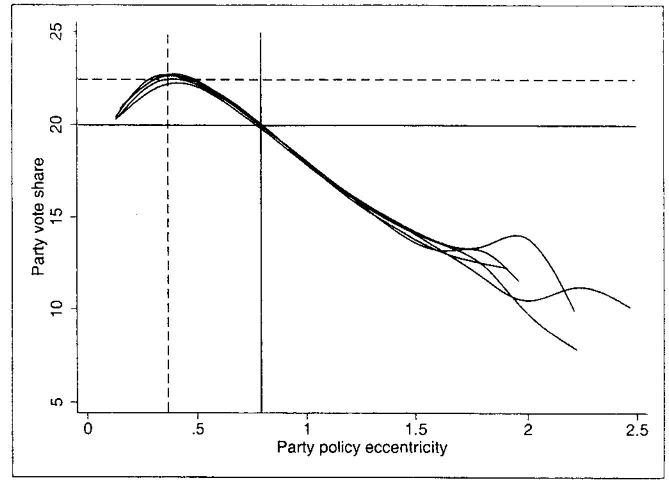


Figure 5.4. Median splines of party vote shares by policy eccentricities, burnt-in five-Hunter benchmark run. Plotted lines are twelve-band median splines and exclude cases where party policy eccentricity > 2.0.

Scholars in the classical formal modeling tradition have used many different assumptions to account for the commonplace empirical observation that vote-seeking political parties in real party systems *do not* tend to engage in "Downsian" convergence on the ideological center ground. Our ABM of dynamic multiparty competition, in contrast, shows us that observed nonconvergence of vote-seeking parties is indeed to be expected with multiparty competition in multidimensional policy spaces. No special assumption is needed to generate a model of party competition in which vote-seeking party leaders have a strong tendency to avoid the dead center of the policy space. Center avoidance by vote-seeking parties *arises directly from the Voronoi dynamics of competitive location in multidimensional policy spaces.* This substantively important intuition flows directly from our dynamic ABM of multiparty competition.

Effective Numbers of Parties

We noted when specifying our output measures that there is a distinction between the absolute number of parties engaged in party competition, which is fixed exogenously in these simulations, and the *effective* number of parties (ENP). The latter takes account of relative party sizes. For example, if five parties are in competition, then a setting in which all parties have equal vote shares implies an ENP of five, while one in which nearly

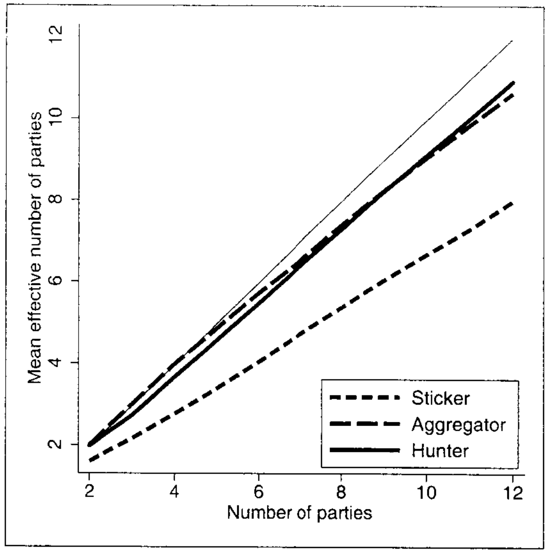


Figure 5.5. Effective and absolute number of parties, benchmark runs in symmetric populations.

all votes go to a single party implies an ENP close to unity. The more absolute and effective numbers of parties diverge, the more unequal the party vote shares. Figure 5.5 plots the relationship between the absolute number of parties and ENP.[8] The thin black 45-degree line highlights a striking pattern whereby ENP is always close to the absolute number of parties for all-Aggregator and all-Hunter party systems. These simulated party systems have a strong tendency to generate party configurations in which all parties are of roughly equal size over the long run. In all-Sticker party systems, in contrast, ENP differs significantly from the absolute number of parties because party sizes typically vary substantially

This is a striking pattern that shows another important intuition that arises from analyzing dynamic as opposed to static party competition. A dynamic party system in which all parties use the same adaptive decision rule for setting party positions and do not differ from each other in any systematic way should result, by symmetry, in a situation in which

[8] Standard errors of all estimates are less than 0.001 (Aggregators), 0.004 (Hunters), and 0.012 (Stickers). Detailed estimates generating Figure 5.5 can be found in Table E5.5 in the electronic appendix.

all parties tend *over the long run* to have the same levels of support. As we indeed find in our simulations, a *snapshot* of the dynamic process at any given point in time, such as that shown in the right panel of Figure 5.1, typically shows a situation in which parties have substantially different vote shares. But these short-run differences even out over the long run, unless there is some other, as yet unspecified, systematic difference between the parties.

This has another important substantive implication that becomes available to us only when we use dynamic models of party competition. If we observe systematic long-run differences in party sizes in some real party system, then this implies one of two things. Party leaders are not all using the same decision rule, with some rules more effective than others in finding votes in the same environment; or all things are not equal among parties using the same rule in the same environment. We analyze situations in which different party leaders use different decision rules in every subsequent chapter of this book. We return in chapter 9 to the possibility of systematic differences among parties using the same decision rule, considering effects of variations among parties nonpolicy electoral "valence."

Representativeness of the Configuration of Party Positions

Recall that we defined representativeness of a configuration of party policy positions in terms of the average closeness of voters' ideal points to the position of their closest party. We do not analyze government formation or downstream policy making; our concern here is with the representativeness of the configuration of party policy positions on offer at election time. Figure 5.6 plots representativeness of the configuration of evolved party policy positions, by number of parties and party decision rule.[9] We see two clear patterns. First, the upward slope of all lines shows that, whatever decision rule party leaders use, *representativeness increases with the number of parties in contention.* The more unique party positions there are, the closer any random ideal point is likely to be to some party position. Second, we see from the dashed line plotting representativeness in all-Aggregator party systems that, for any number of parties, *voters are significantly better represented when party leaders use the Aggregator rule than when they use Hunter.*

This computational finding comports perfectly with the twin theoretical results from Voronoi geometry, reported in chapter 3, that an n-Aggregator system reaches steady state when party policy positions are

[9] Standard errors of all estimates are less than 0.0001 (Aggregators), 0.002 (Hunters), and 0.005 (Stickers). The detailed estimates generating Figure 5.6 can be found in Table E5.6.

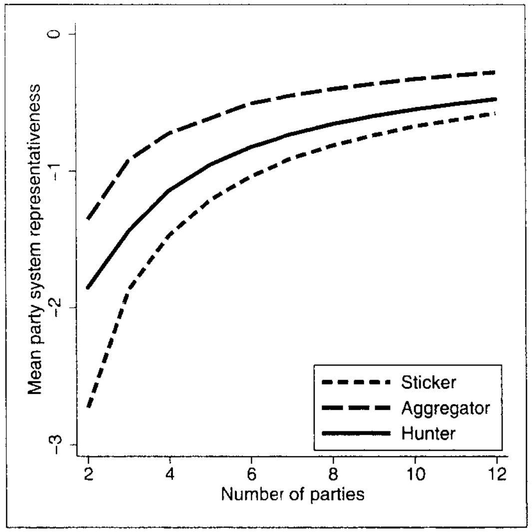


Figure 5.6. Party system representativeness, by decision rule and number of parties in symmetric populations.

at the centroids of their Voronoi regions; the resulting centroidal Voronoi tessellation (CVT) of the policy space is an optimal *n*-party representation of the set of voter ideal points. Leaders of individual Aggregator parties never seek to increase representativeness of the party system *as a whole*. Nonetheless, an all-Aggregator system in which each party leader seeks to maximize representation of her *own party supporters* evolves, given Lloyd's Algorithm, to a configuration of party policy positions that optimizes representativeness of the party system as a whole.

An equivalent number of Hunter parties will almost never be at the centroids of their Voronoi regions. Configurations of Hunter party policy positions will typically not be a CVT and representation will typically not be optimal. Strikingly, this means that the policy preferences of the voting population *as a whole* are not best represented by a set of political parties that compete for voters' support on the basis of trying to find the most popular policy positions. When parties do behave in this way, they tend to compete for support "too close" to the center of the policy space to maximize overall representativeness of the party system. On one view this might be seen as contradicting a standard normative justification for competitive elections, which is that competition among parties for votes will tend to result in representative election results.

The big picture in Figure 5.6, given Lloyd's Algorithm and the fact that CVTs are optimal representations, is that the line plotting representativeness of all-Aggregator party systems also plots *the optimal level of representativeness that can be achieved by party systems of equivalent size*. Voters, in short, are better served by a set of political parties whose prime concern is to satisfy *current* supporters rather than search restlessly for *new* supporters.

Multiparty Competition in Asymmetric Populations

Thus far we retrieved and consolidated results on dynamic multiparty competition with symmetric voter populations that were previously reported by Laver (2005). We did this using much more robust computational experiments, and we introduced a substantively important new measure of party system representativeness, with its implied notion of optimal representation. We now go well beyond this to analyze party competition in a more general setting with asymmetric and quite possibly multimodal distributions of voter ideal points. As we saw in chapter 3, these typically arise from aggregating distinct subpopulations of voters reflecting, for example, different ethnic, linguistic, or social groups. We use two subpopulations in the rest of this book. We experimented with more subpopulations and found that the important effects on model results arise from the departure from symmetric ideal point distributions that can be achieved with two distinct subpopulations. In our experience—though this could be a subject for future work—moving to three or more voter subpopulations adds many new parameters and thereby increases the computational budget by orders of magnitude, without adding substantive insight.

Experimental Design

As we saw in chapter 4 when discussing the difference between grid-sweeping designs and Monte Carlo model parameterizations, moving to two distinct subpopulations adds two new real-valued parameters that do not have "natural" discrete settings. These parameters are polarization of subpopulation ideal point centroids and the relative sizes of constituent subpopulations. As specified in chapter 3, we calibrate our computational work to the type of party system in which we are substantively interested by simulating party competition in settings where the relative size of subpopulations, n_l / n_r, is in the range [2.0, 1.0] and subpopulation ideal point means, $\mu_r = - \mu_l$ are in the range [0.0, 1.5]. Our experimental design therefore involves one thousand different Monte Carlo parameter-

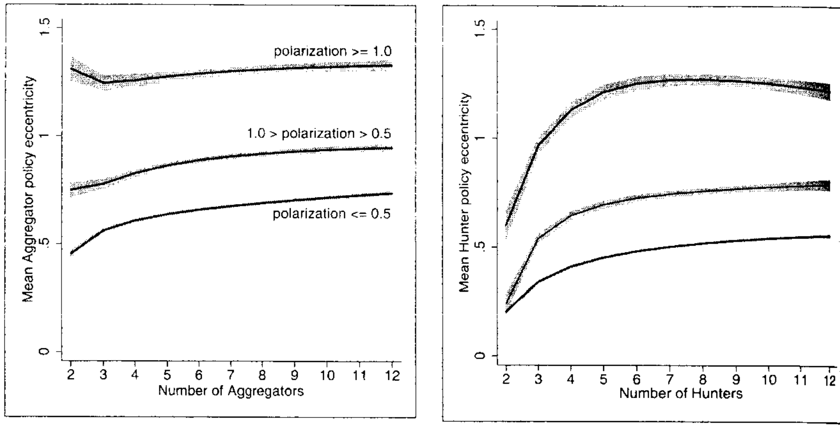


Figure 5.7. Mean party policy eccentricity, by decision rule, number of parties, and subpopulation polarization; asymmetric populations.

izations of the distribution of voter ideal points. For each of the one thousand model runs, values of n_l / n_r and μ_r are high-precision floating point numbers randomly drawn from a uniform distribution over the specified range. The method we employ to estimate the run output parameters is the same as the method used with the symmetric voter population.

Party Policy Positions

Figure 5.7 is analogous to Figure 5.2 and shows evolved party policy eccentricities, now with asymmetric distributions of voter ideals points, by party decision rule and number of parties. In contrast to Figure 5.2, which summarizes results for a single symmetric bivariate normal population, Figure 5.7 summarizes results from one thousand different Monte Carlo parameterizations of asymmetric populations, with different panels for all-Aggregator (left) and all-Hunter (right) party systems.[10] Each parameterization specifies a different level of polarization between the two subpopulations.[11] Given the highly nonlinear nature of many of the results we report in the rest of this book, we summarize these using

[10] Plots of party policy eccentricity against relative subpopulation sizes reveal no noticeable pattern and are shown in Figure E5.1 of the electronic appendix. Party positions in all-Sticker benchmarking runs contain no new information since they reflect simply the random positions that Stickers are assigned in simulations.

[11] Strictly speaking, we mean a different distance between subpopulation ideal point centroids.

fractional-polynomial prediction plots. The three bands in each panel of Figure 5.7 plot 95 percent confidence intervals around fractional polynomial fit lines summarizing model output for highly polarized (top), moderately polarized (middle), and relatively unpolarized (bottom) voter subpopulations.

Comparing the two panels of Figure 5.7, we see that Hunters continue to take significantly more central policy positions than Aggregators, as they did in symmetric voter populations. Comparing the upper bands in these two panels, however, we see that this difference is far less striking when voter subpopulations are highly polarized and the number of parties is relatively large. In polarized settings such as these, typical Hunter and Aggregator policy positions are relatively similar. The reason for this can be seen clearly in the right panel of Figure 5.7, which gives the big news from these simulations. *Party systems with four or more Hunters are very different from those with two or three.* Moving from two to three to four parties, the typical policy eccentricity of Hunter parties increases rapidly, leveling off dramatically as the number of parties increases beyond four. The reason why this happens can be seen easily by watching simulations in motion. With four or more Hunter parties in contention, we typically see two or more parties competing for support in each subpopulation. When this happens, Hunters tend to be punished for moving away from "their" subpopulation toward the center of the space, and thus tend not to do this. The right panel of Figure 5.8 is a screen shot of precisely this situation as it emerged in a four-Hunter benchmark run with two highly polarized voter populations—as before, lighter background colors show regions with higher ideal point densities. With only two or three parties, at least one voter subpopulation can "host" at most one party and this party cannot be punished for moving away from the subpopulation ideal point centroid, toward the center. We see this in the left panel of Figure 5.8, which shows a screenshot of Hunter positions in a three-party benchmark run. Once two Hunters are competing in one subpopulation, *there is nothing to deter the third Hunter from moving toward them.*

This striking pattern is documented systematically in Figure 5.9, which shows histograms of parties' x-coordinates, in both two- and four-Hunter systems, over benchmark runs for an asymmetric population with subpopulations of equal sizes and ideal point centroids at ±1.0. The left panel shows that, with only two Hunters in contention, both party leaders have a strong tendency to choose policy positions near the center of the x-dimension, *completely ignoring the two high-density subpopulation centroids of citizens' ideal points.* There is no punishment in this two-party environment for parties that choose policy positions far away from the ideal points of most voters and evolved Hunter policy positions are perversely unrepresentative. In stark contrast, with four Hunters in con-

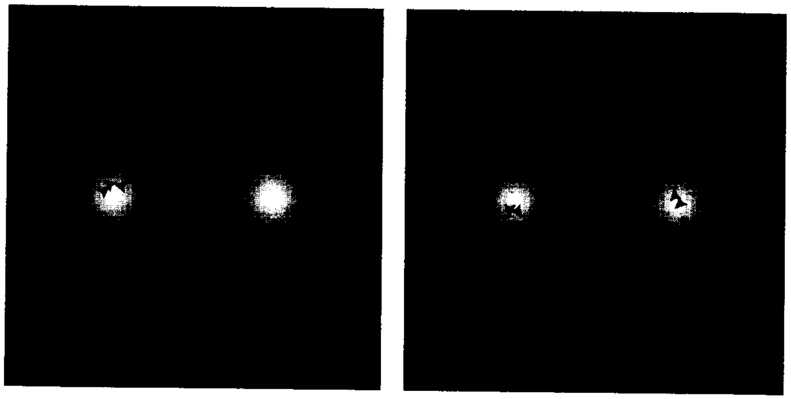


Figure 5.8. Screen shots of party positions in three-Hunter (left) and four-Hunter (right) benchmark runs with asymmetric voter populations: subpopulation centroids at −1.5 and +1.5 on the x-axis.

tention, the right panel of Figure 5.9 shows that all parties tend to stay close to one subpopulation centroid or the other, *almost never choosing policy positions at the center of the* x*-dimension.*

Party System Representativeness

It is quite remarkable that the two panels in Figure 5.9 characterize vote-seeking party competition in *precisely the same aggregate population.* In contrast to the perversely unrepresentative positions taken by two vote-seeking Hunters, when there are four or more Hunters, competition among parties *within each subpopulation* now results in party positions that are close to the bulk of voters' ideal points. Figure 5.10 plots representativeness of evolved configurations of party positions, by decision rule, number of parties, and polarization of voter ideal points. The left panel plots results for all-Aggregator party systems that we now know generate optimally representative configurations of party positions. This panel thus shows upper bounds on the representation of voter ideal points in the different environments we investigate. The right panel of Figure 5.10 is worth a thousand words. It shows that *representativeness of evolved Hunter policy positions goes through a sea change as the number of these vote-seeking parties hits four or more.* Hunter party policy positions tend to be *very* unrepresentative in asymmetric populations when there are fewer than four vote-seeking Hunters. The lower band shows that this pattern is especially strong when subpopulations are highly polarized.

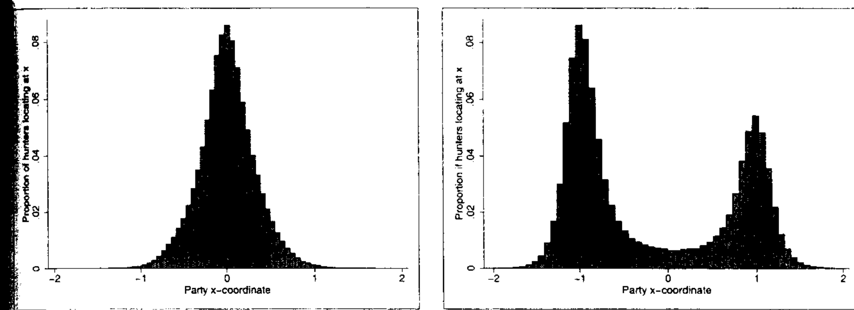


Figure 5.9. Histograms of party x-coordinates, for two- and four-Hunter benchmark runs in asymmetric population.

As we have seen, two vote-seeking Hunters tend, perversely, to compete for votes in the almost unpopulated center ground. With four or more parties, competition between pairs of Hunters keeps them close to subpopulation centroids and generates a much more representative configuration of party positions. Indeed it is striking that, as the number of parties increases in asymmetric populations, there is increasingly little difference between the representativeness of all-Aggregator and all-Hunter party systems. This is in clear contrast to the situation in symmetric populations, in which all-Aggregator party systems are systematically more representative than all-Hunter systems. As a general rule, these results suggest that highly polarized voter populations may tend to be well represented by evolved configurations of party policy positions, except when the party system comprises just two or three vote-seeking Hunter parties.[12]

Conclusions

We set out in this chapter to use methods and procedures we specified in chapter 4 to get started on the analysis of the model of party competition we develop in the rest of this book. We also set out to move beyond the

[12] The relationship between the effective and absolute number of parties, shown in Figure E5.2 of the electronic appendix, was not affected by subpopulation polarization and was essentially the same as for symmetric populations.

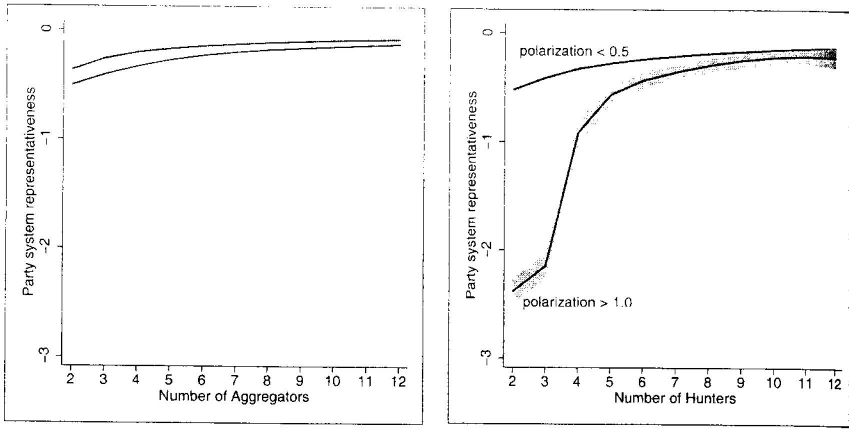


Figure 5.10. Party system representativeness, by decision rule, number of parties, and subpopulation polarization.

assumption of a symmetric bivariate normal distribution of voters' ideal points to investigate the more general ideal point distributions that arise in settings where the electorate comprises two distinct subpopulations of voters, for example representing ethnic, religious, or other social groups.

Our substantive headline results concern the *representativeness* of evolved configurations of party policy positions. In symmetric populations we find, paradoxically, that the ideal points of voters are *not* best represented by a set of (Hunter) parties who compete for their support by trying to find popular policy positions. Instead, voter preferences *are* better represented by a set of (Aggregator) parties that do not compete *with each other* on policy at all but instead seek to represent the policy preferences only of their current supporters. This happens because the dynamics of vote-seeking competition in this setting cause parties to set policy positions *closer* to the center of the policy space than would be needed for optimal representation—while at the same time avoiding the dead center of the space.

The situation is quite different in asymmetric voter populations. Vote-seeking competition between two Hunters now moves parties *away* from the centroids of subpopulation ideal points, resulting in typical configurations of party positions that are perversely unrepresentative. However, competition among four or more vote-seeking Hunters tends to result in party positions that are *close* to subpopulation centroids, making evolved configurations of party positions *much* more representative than

in two- or three-Hunter systems. The shift from two- to four-Hunter competition generates a sea change in the representativeness of party positions in the vote-seeking party competition we model. In these circumstances, we see the emergence of what are to all intents and purposes two independent systems of party competition, one for each voter subpopulation. *Parties tend to "serve" one subpopulation or the other and to compete for votes with other parties serving the same subpopulation, rather than competing directly with parties serving other subpopulations.* We find this theoretically intriguing, as well as an empirically plausible account of real party competition in settings with polarized electorates.