

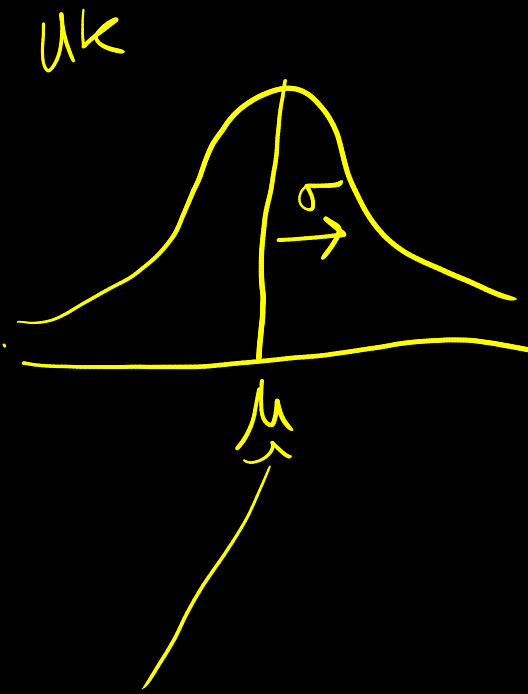
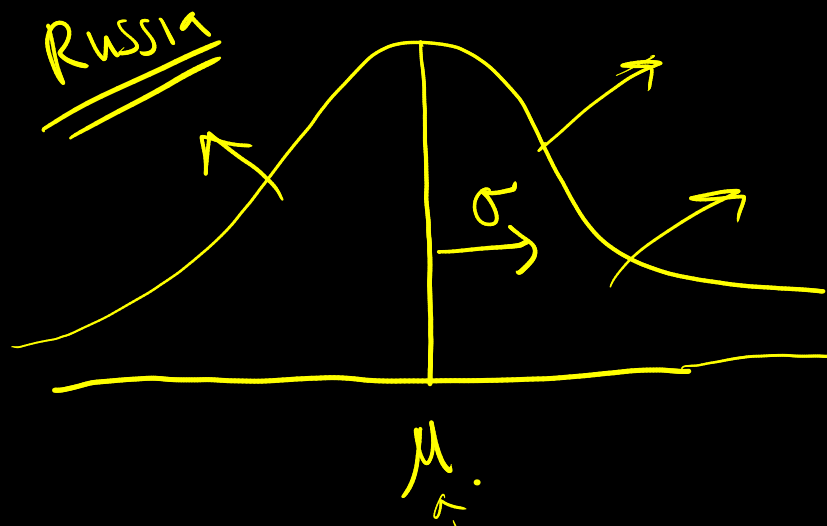
$$x \sim N(\mu, \sigma^2)$$

$$x - \mu \sim N(0, \sigma^2)$$

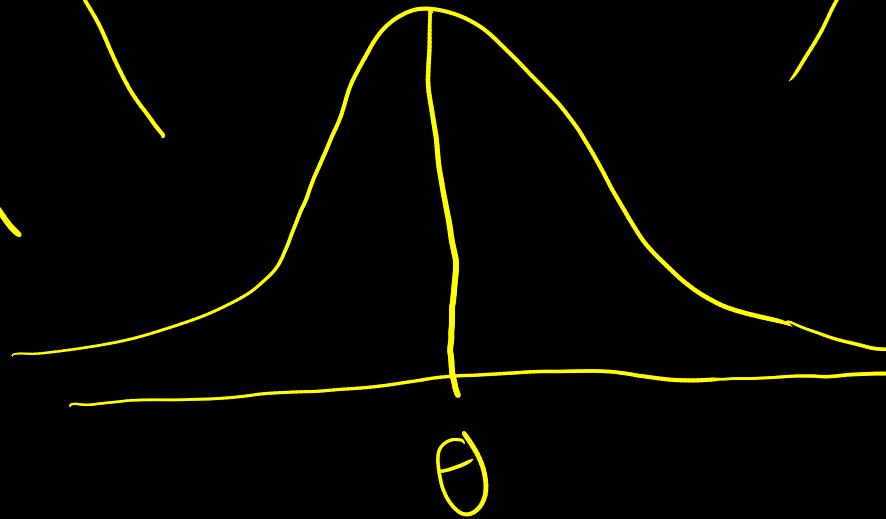
$$\beta \sim N(b, \tau^2)$$

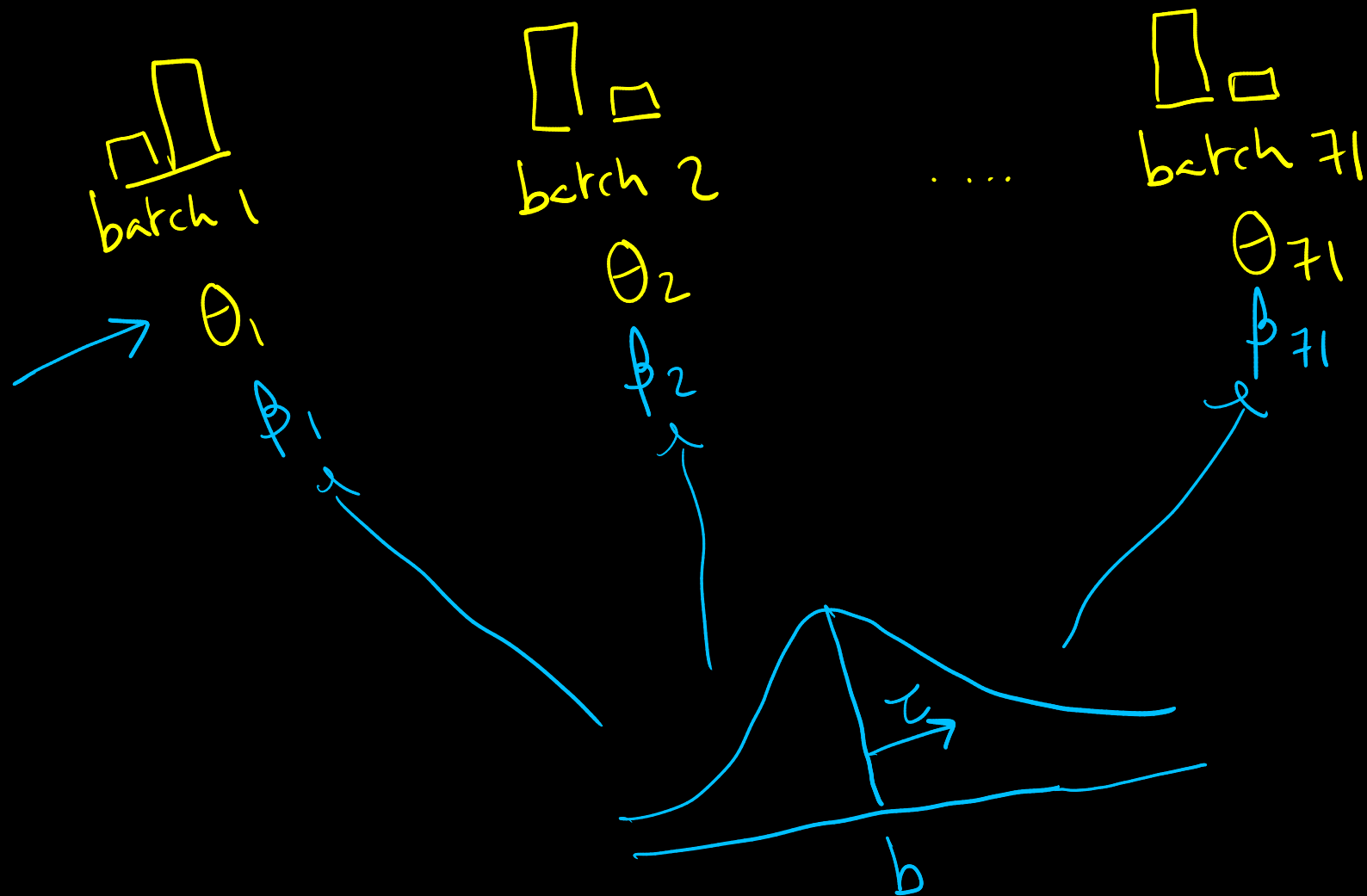
$$\beta - b \sim N(0, \tau^2)$$

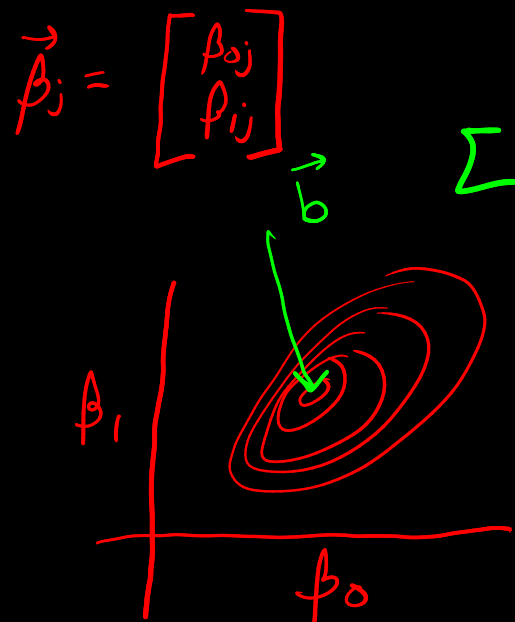
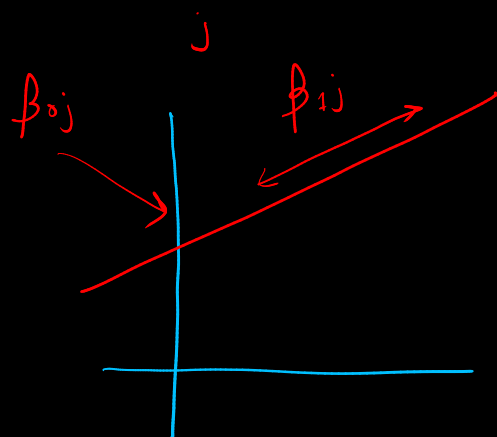
$$p(x) = \int p(x|y) p(y)$$



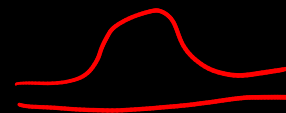
population level



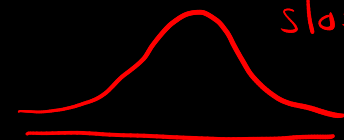




intercepts



slopes



for j in $1 \dots J$

$$\vec{\beta}_j \sim N_2(\vec{b}, \Sigma)$$

$$\beta_{j0} = b_0 + Z_{j0}$$

$$\beta_{j1} = b_1 + Z_{j1}$$

$$\begin{aligned} \mu_i &= \beta_{[s_i]0} + \beta_{[s_i]1} x_i \\ &= [b_0 + Z_{[s_i]0}] + [b_1 + Z_{[s_i]1}] x_i \end{aligned}$$

$$\begin{bmatrix} \beta_{j0} \\ \beta_{j1} \end{bmatrix} \sim \mathcal{N}(\vec{b}, \Sigma) \left[\underbrace{b_0 + b_1 x_i}_{\text{fixed}} + \underbrace{Z_{[s_i]0} + Z_{[s_i]1} x_i}_{\text{random}} \right]$$

$$\begin{bmatrix} \beta_{j0} \\ \beta_{j1} \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} + \begin{bmatrix} Z_{j0} \\ Z_{j1} \end{bmatrix}$$

$$\begin{matrix} \uparrow \\ \vec{Z}_j \end{matrix} \sim \mathcal{N}(\mathbf{0}, \Sigma)$$

$$\Sigma = \begin{bmatrix} \sigma_0^2 & \sigma_0 \sigma_1 \rho \\ \sigma_0 \sigma_1 \rho & \sigma_1^2 \end{bmatrix}$$

The image shows a handwritten covariance matrix Σ in blue ink. The matrix is a 2x2 symmetric matrix with elements σ_0^2 , $\sigma_0 \sigma_1 \rho$, $\sigma_0 \sigma_1 \rho$, and σ_1^2 . Three red arrows are drawn: one points to the top-left element σ_0^2 , another points to the top-right element $\sigma_0 \sigma_1 \rho$, and a third points to the bottom-right element σ_1^2 .

