Introduction to Mathematics for Political Science

Problem Set 7: Scalars, Vectors, and Matrices

Due: August 2

Instructions: You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. You may use a calculator for basic arithmetic, but do not search the internet for solutions. You do not need to replicate drawings in your answers if you describe them verbally. Show all of your work. Submit typed solutions using the link on the course page.

Vectors, Dot Products, Norms

1. Calculate $\boldsymbol{u} \cdot \boldsymbol{v}$

- $\boldsymbol{u}$= $(1, 2, 3, 4, 5)$ and $\boldsymbol{v}$=$(1, 2, 3, 4, 5)$
- $\boldsymbol{u}$= $(1, 2)$ and $\boldsymbol{v}$=$(-1, 2)$
- $\boldsymbol{u}$= $(0, 0, 0, 0, 1)$ and $\boldsymbol{v}$=$(1, 2, 3, 4, 7)$
- $\boldsymbol{u}$= $(3, 4, 5)$ and $\boldsymbol{v}$=$(4, 2, 1)$

2. Calculate $\|\boldsymbol{u}\|$

- $\boldsymbol{u}$=$(1, 0, 0)$
- $\boldsymbol{u}$=$(5, 2, 4, 2)$
- $\boldsymbol{u}$=$(5, 5, 5, 5)$
- $\boldsymbol{u}$=$(3, 4)$

3. Solve for θ, the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ (degrees or radians are acceptable). **Hint: If you can visualize the vectors, you may not have to calculate lengths and dot products.**

- $$\boldsymbol{u} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- $$\boldsymbol{u} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} 9 \\ 15 \\ 6 \end{bmatrix}$$
- $$\boldsymbol{u} = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$
- $$\boldsymbol{u} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

4. Show that the Cauchy-Schwarz inequality ($|\boldsymbol{u} \cdot \boldsymbol{v}| \leq \|\boldsymbol{u}\|\|\boldsymbol{v}\|$) holds for the following two dimensional vectors.
$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

5. Solve for a

- $$a \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 9 \end{bmatrix}$$
- $$a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (1-a) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{4} \end{bmatrix}$$

6. Let $\boldsymbol{y} = \boldsymbol{x}\beta + \boldsymbol{\epsilon}$ where
$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 6 \\ 2 \\ 10 \\ 10 \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \quad \beta = 2$$

   a. Find $\boldsymbol{\epsilon}$.
   b. Calculate $(\boldsymbol{x}^T\boldsymbol{x})^{-1}\boldsymbol{x}^T\boldsymbol{y}$. Interpret your result. (Hint: Try drawing a picture.)

Matrix Properties

1. Calculate AB.

- $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 11 \\ 4 \\ 6 \end{bmatrix}$$

- $$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$
- $$A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 0 & 8 \\ 7 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
- $$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 0 & 5 \\ 2 & 1 & 2 & 5 \end{bmatrix}$$

2. Calculate the dimensions of the resulting matrix.

- $A_{3\times2}B_{2\times3}$
- $A_{3\times2}B_{2\times3}C_{3\times1}$
- $A_{2\times3}A^T_{2\times3}B_{2\times6}$
- $A_{1\times3}A_{3\times1} + B_{1\times1}$

3. What does it mean for a set of matrices to be *conformable*? Why are A^TA and AA^T always conformable?