# Linear Spaces Exercises

1) Let $S$ be a basis for $X$ so that for every $x \in X$, there exist elements $x_1, x_2, ..., x_n \in X$ and scalars $\alpha_1, \alpha_2, ..., ....\alpha_n \in \mathbb{R}$ such that

$$x = \sum_{i=1}^{n} \alpha_i x_i$$

Prove that $\alpha_i$ is unique for all $i$. (Hint: use Proposition 1).

2) Prove or disprove the following statement: any vector space $X$ has a unique basis.

3) Prove that if $X$ is an $n$-dimensional linear space, then any set $S \subset X$ of $n+1$ elements is linearly dependent. (Hint: use Propositions 1 and 5).