Optimization

Exercises

1. Prove Rolle's Theorem.
2. Characterize the stationary point(s) of[1]

$$f(x_1, x_2) = x_1^2 + x_2^2$$

   Are these points maxima, minima, or saddle points?
3. Characterize local optima and solve[2]

$$\max_{x_1, x_2} f(x_1, x_2) = 3x_1 x_2 - x_1^3 - x_2^3$$

4. Prove that the least squares objective function is convex, implying that the first order conditions are sufficient to characterize the $\boldsymbol{\beta}$ that solves the least squares estimator.
5. Let $g : \mathbb{R} \to \mathbb{R}$ be a monotonic transformation of $f : X \to \mathbb{R}$. Show that $g \circ f$ has the same local maxima as f.
6. Consider the problem

$$\begin{aligned} \max_{x_1, x_2} \quad & x_1 x_2 \\ \text{subject to} \quad & x_1 + x_2 = 1 \end{aligned} \tag{1}$$

   Think about the geometry of the problem. What is the constraint set? Then solve it using the method of Legrange.[3]

[1] Carter 5.10
[2] Carter Example 5.8
[3] Carter Example 5.14