

Inner Product Spaces, Orthogonality, Projection

Exercises

1. Let X and Y be normed linear spaces. Let $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ be a basis for X and $\{\mathbf{y}_1, \dots, \mathbf{y}_m\}$ a basis for Y . Prove that if $x_i \perp y_j$ for all $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m\}$, then X and Y are orthogonal spaces.
2. Prove: If a vector α is in the null space of a set of vectors $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, then it is orthogonal to the space spanned by $\{\mathbf{y}_1, \dots, \mathbf{y}_m\}$ where

$$\mathbf{y}_i = \{x_{1i}, \dots, x_{ni}\}$$

3. Donald Trump tweeted 100 times in April, 150 times in May, and 110 times in June.¹ Let $\mathbf{b} = (100, 150, 110)$ represent the number of tweets in each month. Project \mathbf{b} onto the linear space spanned by $\mathbf{a} = (1, 1, 1)$. Interpret your result.

¹Disclaimer: these data are of suspect quality.