An Introduction to Mathematics For Political Science

Problem Set 2

You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Typed solutions or scans of handwritten solutions in PDF form are acceptable. Note that starting next week all solutions must be written in LaTeX.

Differentiation

1) Compute derivatives for the following functions using the limit definition of derivative.

a) $f(x) = \frac{1}{1+x}$

b) $f(x) = \sqrt{2x + 5}$

c) $f(x) = 2x^3 + 5x^2 - 6$

d) $f(x) = 2x^4 - 3x^2$

Solutions:

a) $f(x + h) = \frac{1}{(x+h)+1}$

$f(x + h) - f(x) = \frac{h}{(x+h+1)(x+1)}$

$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}[\frac{1}{x+h+1} - \frac{1}{x+1}] = \frac{-1}{(x+h+1)(x+1)}$

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}$

b) $f(x + h) = \sqrt{2x + 2h + 5}$

$f(x + h) - f(x) = \sqrt{2x + 2h + 5} - \sqrt{2x + 5} = \frac{2h}{\sqrt{2x+2h+5}+\sqrt{2x+5}}$

$\frac{f(x+h)-f(x)}{h} = \frac{2}{\sqrt{2x+2h+5}+\sqrt{2x+5}}$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2}{\sqrt{2x+2h+5}+\sqrt{2x+5}} = \frac{1}{\sqrt{2x+5}}$

c) $f(x + h) = 2[x^3 + 3x^2h + 3xh^2 + h^3] + 5[x^2 + 2xh + h^2] - 6$

$f(x + h) - f(x) = 6x^2h + 6xh^2 + 2h^3 + 10xh + 5h^2$

$\frac{f(x+h)-f(x)}{h} = 6x^2 + 6xh + 2h^2 + 10x + 5h$

$\lim_{h\to 0} 6x^2 + 6xh + 2h^2 + 10x + 5h = 6x^2 + 10x.$

d) $f(x + h) = 2[x^4 + 4x^3h + 6x^2h_4^2xh^3 + h^4] - 3[x^2 + 2xh + h^2]$

$f(x + h) - f(x) = 2[4x^3h + 6x^2h^2 + 4xh^3 + h^4] - 3[2xh + h^2]$

$\frac{f(x+h)-f(x)}{h} = 8x^3 + 12x^2h + 8xh^2 + 2h^3 - 6x + 3h$

$\lim_{h\to 0} 8x^3 + 12x^2h + 8xh^2 + 2h^3 - 6x + 3h = 8x^3 - 6x$

2) Differentiate the following with respect to x:

   a) $x^3 - 2x^2 + 5$

   b) $4(x - 2)^2 - (x + 1)^3$

   c) $7^5 - x^7 + 5x + 2x^3$

   d) $(2x^3)(3x^2)$

   e) $(3x + 5)(2x^2 - 3)$

   f) $(3x^2 - 1)(x^2 + 5x + 2)$

   g) $\frac{x-1}{x+2}$

   h) $\frac{x^2+6}{2x-7}$

   i) $(x^2 - 3x + 8)^{50}$

   j) $(5x^4 + 1)^2$

   k) $(4x^5 - 1)(x + 1)^{1/3}$

   l) $(3x - 1)(-3x^2 - 4)^{-3}$

   m) e^{2x+1}

n) $\ln(x^2 + 2x)$

o) $(1 + 5x)^{1/x}$

p) $(2 - x)^{\sqrt{x}}$

q) $\frac{\ln x}{2x^2}$

r) $\frac{e^x}{1+x}$

s) $\sqrt{1 + ae^x}$ where a is an arbitrary constant

Solutions:

a) $3x^2 - 4x$

b) $-3x^2 + 2x - 19$

c) $-7x^6 + 5 + 6x^2$

d) $30x^4$

e) $18x^2 + 20x - 9$

f) $12x^3 + 45x^2 + 10x - 5$

g) $\frac{3}{(x+2)^2}$

h) $\frac{2(x^2 - 7x - 6)}{(2x - 7)^2}$

i) $50(x^2 - 3x + 8)^{49}(2x - 3)$

j) $40x^3(5x^4 + 1)$

k) $\frac{64x^5 + 60x^4 - 1}{3(x+1)^{2/3}}$

l) $\frac{3(15x^2 - 6x - 4)}{(-3x^2 - 4)^4}$

m) $2e^{2x+1}$

n) $\frac{2x+2}{x^2+2x}$

o) $\left(\frac{-\ln(1+5x)}{x^2} + \frac{5}{x(1+5x)}\right)(1 + 5x)^{1/x}$

p) $\left(\frac{\ln(2-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{2-x}\right)(2 - x)^{\sqrt{x}}$

q) $\frac{1 - 2\ln x}{2x^3}$

r) $\frac{xe^x}{(1+x)^2}$

s) $\frac{ae^x}{2\sqrt{1+ae^x}}$