

## Continuous Functions: Solutions

September 12, 2018

1. Use the Delta-Epsilon method to prove the continuity of  $f(x) = |x|$  under the  $d_1$  (absolute value) metric.
2. If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are continuous functions, then  $g \circ f : X \rightarrow Z$  is also continuous.<sup>1</sup>
3. Let  $(X, d)$  be a compact metric space and  $f$  be a continuous function. Prove  $f(X)$  is closed and bounded.<sup>2</sup>
4. Prove: If a function  $f : X \rightarrow Y$  is continuous, then every open subset  $S$  of a metric space  $Y$ ,  $f^{-1}(S)$  is open in metric space  $X$ .<sup>3</sup>

<sup>1</sup> Carter 2.72

<sup>2</sup> Hint: The limit definition of continuity and the Bolzano-Weierstrass theorem will be useful.

<sup>3</sup> Hint:  $\bigcup_{i=1}^{\infty} O_i$  with  $O_i$  open for all  $i$  is open.