An Introduction to Mathematics For Political Science

Problem Set 4

You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. All solutions must be written in LaTeX.

Optimization

1. Find all extrema (local and global) of the following functions on the specified domains, and state whether each extremum is a minimum or maximum and whether each is only local or global on that domain. In your answer, report both the max/min and argmax/armin.

a) $f(x) = x^2 - 4x + 2$ on $[0, 3]$

b) $f(x) = 2x^3 - x$ on $(-1, 1]$

c) $f(x) = \sqrt{x}$ on $[0, 4)$

d) $f(x) = -x^2 + 4$ on $(-2, 2)$

2. Explain (in words) the difference between a global maximum and a supremum.

3. Find the second derivative with respect to x of the following functions:

a) $2x^3 - 4x^2 + x$

b) $x^4 + e^{2x}$

c) $\ln^2(x)$

d) $-(x - a)^2$

e) xe^{-x}

4. Find all critical points and inflection points of the following functions. Identify whether each critical point is a local maximum, local minimum, or inflection point.

a) $f(x) = x^3 - 3x^2$

b) $f(x) = x^3 - 6x^2 + 9x + 15$

c) $f(x) = -(x - b)^2$

d) $f(x) = -x^3$

5. Identify the regions of $\mathbb{R}$ on which the following functions are weakly concave and/or convex:

a) $f(x) = -x^3$

b)$f(x) = \frac{1}{x}$

c) $f(x) = x^3 - 3x^2$

d) $f(x) = 4x - 5$