Introduction to Mathematics for Political Science

Problem Set 8: Solving Systems of Equations

Instructions: You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. You may use a calculator for basic arithmetic, but do not search the internet for solutions. Show all of your work. Submit typed solutions using the link on the course page.

1. Solve the following systems of equations using substitution.

   -
     $$2x + 3y = 10$$
     $$2y = 4$$

   -
     $$3x + 5y - 2z = 1$$
     $$3y + z = 8$$
     $$5z = 25$$

   -
     $$10x - 3y - 2z = 6$$
     $$5y - 4z = 9$$
     $$3z = 12$$

2. Solve the following systems of equations using elimination.

   -
     $$2x + 2y = 8$$
     $$6x + 2y = 4$$

- $$4x + y + z = 7$$
$$x + 7y - z = 23$$
$$10x + 4y + 3z = 20$$

- $$x - 3y - 2z = 6$$
$$2x - 4y - 3z = 8$$
$$-3x + 6y + 8z = -5$$

3. If possible, solve the following systems. If not, state whether each is over- or under-determined. If the system is over-determined, find a contradiction. If the system is underdetermined, solve it up to its free variables.

- $$3x + 2y + z = 10$$
$$2x + y + 3z = 12$$
$$7x + 4y + 7z = 34$$

- $$3x + y + 4z = 8$$
$$9x - 3y - 2z = 4$$
$$x + y + z = 3$$

- $$3x + y + 4z = 8$$
$$9x - 3y - 2z = 4$$
$$x + y + z = 4$$

4. For what values of the parameter k does the following system have a) no solution, b) one solution, and c) more than one solution?[1]

$$x_1 + x_2 = 1$$
$$x_1 - kx_2 = 1$$

---

[1] **Source:** Moore and Siegel 13.4 #3