

# Monotone, Linear, and Convex Functions Exercises

## (Solutions)

- 1) Let  $f_1, f_2, \dots, f_n$  be convex functions and  $\alpha_1, \alpha_2, \dots, \alpha_n \geq 0$ . Prove that  $f(x) = \alpha_1 f_1(x) + \dots + \alpha_n f_n(x)$  is convex. Is  $\alpha_1 f_1 - \alpha_2 f_2$  convex? Prove your answer.
- 2) Prove the Cauchy-Schwarz inequality for  $\mathbb{R}^n$ .
- 3) Prove the following:  $L : \mathbb{R}^l \rightarrow \mathbb{R}$  is a continuous, linear functional if and only if there exists a  $y \in \mathbb{R}^l$  such that for all  $x \in \mathbb{R}^l$ ,  $L(x) = y^T x$ .