Inner Product Spaces, Orthogonality, Projection: Solutions

4 September 2018

Exercises

1. Let X and Y be normed linear spaces. Let $\{\boldsymbol{x}_1, ..., \boldsymbol{x}_n\}$ be a basis for X and $\{\boldsymbol{y}_1, ..., \boldsymbol{y}_m\}$ a basis for Y. Prove that if $x_i \perp y_j$ for all $i \in \{1, ..., n\}$, $j \in \{1, ..., m\}$, then X and Y are orthogonal spaces.

2. Prove: If a vector $\boldsymbol{\alpha}$ is in the null space of a set of vectors $\{\boldsymbol{x}_1, ..., \boldsymbol{x}_n\}$, then it is orthogonal to the space spanned by $\{\boldsymbol{y}_1, ..., \boldsymbol{y}_m\}$ where

$$\boldsymbol{y}_i = \{x_{1i}, ..., x_{ni}\}$$

3. Donald Trump tweeted 100 times in April, 150 times in May, and 110 times in June.[1] Let $\boldsymbol{b} = (100, 150, 110)$ represent the number of tweets in each month. Project $\boldsymbol{b}$ onto the space spanned by $\boldsymbol{a} = (1, 1, 1)$. Interpret your result.

[1] Disclaimer: these data are of suspect quality.