Introduction to an Introduction to Mathematics for Political Science

4 September 2018

Introductions

Course Logistics, Advice

- **Lectures:** Dan and I will alternate teaching the sessions. We'll post lecture notes on the syllabus at the conclusion of each session.
- **Problem Sets:** We include exercises at the end of each set of lecture notes. Some of these will be assigned as practice problems, starting tomorrow. We'll list which problems are assigned on the syllabus Continue to type and submit problem sets using links on syllabus. For those who are new, syllabus can be found here: `brendancooley.com/imps2018` Per the syllabus, problem sets are due before the start of class the day after they are assigned.
  - Work in groups! Problems will be extremely challenging, you'll learn better if you work together. Submit your own typed work.
  - This is a good opportunity to get to know your classmates
- **Office Hours:** At least one of us will be available from 4:00pm-6:00pm each day in Fisher basement. We'll also be available electronically through Piazza, as we have been all summer.
- **Final Exam:** Comprehensive. Time TBA.
- **FQ Sequence in Politics:**
  - **POL 502:** Real analysis, optimization, social choice theory. Builds upon material covered here. You'll get comfortable reading and expressing social scientific ideas in mathematical notation. Prerequisite for game theory courses in the department.
  - **POL 571:** Probability theory – underlies statistical theory that applied researchers employ to explore their data.
  - Further opportunities in Economics and ORFE – talk to us if you're interested.
- **Questions?**

Why Are We Here?

What are DW-NOMINATE scores? How did they come about? The problem: we want to learn about a feature of legislators, in particular,

their ideology. Yet we only observe how they vote. Can we combine theory and data to learn about ideological leanings?

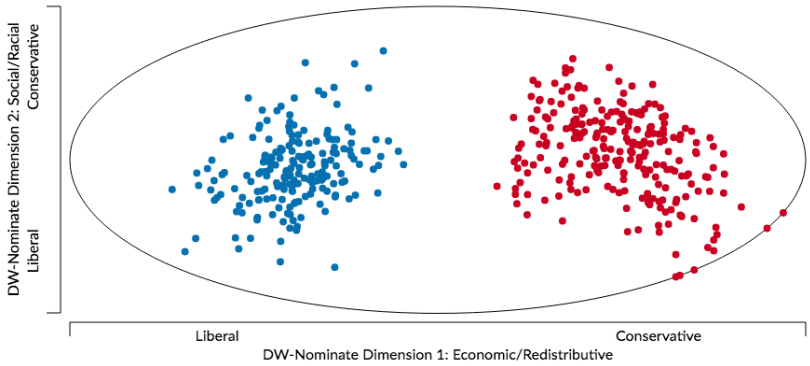

Figure 1: DW-NOMINATE Scores, House of Representatives, 115th Congress, 2017-2019. See `https://voteview.com/congress/house`

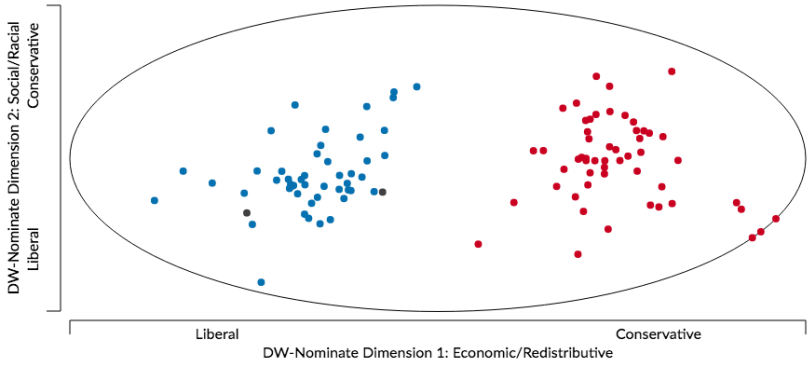

Figure 2: DW-NOMINATE Scores, Senate, 115th Congress, 2017-2019. See `https://voteview.com/congress/senate`

We can represent a given roll call as a vector... where each entry $v_{ij} \in \{0, 1\}$ represents how legislator $i \in \{1, ..., N\}$ voted on bill $j \in \{1, ..., M\}$

$$\boldsymbol{v}_j = \begin{bmatrix} v_{1j} \\ \vdots \\ v_{Nj} \end{bmatrix}$$

Combining these roll calls into a matrix of dimension $M \times N$ gives us the entire voting record for a given legislature

$$V = \begin{bmatrix} v_{11} & \cdots & v_{1M} \\ \vdots & \ddots & \vdots \\ v_{N1} & \cdots & v_{NM} \end{bmatrix}$$

where each row is a given legislator's voting record across all bills.

How can we infer a legislator's ideology from studying this matrix alone? We need theory to tell us what ideology is and to structure

how legislators behave when they vote. We'll define each legislator's ideal point as a number, $x_i \in \mathbb{R}$. We'll also define each bill by two numbers, $y_j, n_j \in \mathbb{R}$ which represent the outcomes. And we'll assume each legislator receives utility[1]

$$U(x_i, y_j)$$

$$U(x_i, n_j)$$

for voting yes and no respectively. The legislator will vote yes if

$$U(x_i, y_j) \geq U(x_i, n_j)$$

What does this model imply about voting behavior? *Perfect sorting* – legislators to the to the right of a yes-voter always vote yes and vice versa. We know legislators in fact don't vote with such precision, so we need to introduce some noise.

$$U(x_i, y_j) + \epsilon_j^y$$

$$U(x_i, n_j) + \epsilon_j^n$$

Now our problem is to find vectors

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} \qquad \boldsymbol{n} = \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix}$$

that maximize the likelihood of observing the roll call V. Then we interpret $\boldsymbol{x}$ as the legislators' *ideal points*, or ideology.

- What are some problems with this model? What would a "better" model look like? What problems would this alternative present?
- Can you think of similar problems in international relations, comparative politics, or political theory?

[1] What should these functions look like in the context of the spatial model?

Course Structure

This course will serve as an introduction to thinking mathematically and expressing social scientific ideas in the language of mathematics. Political scientists often use math to communicate ideas because doing so encourages transparency and replicability. A proof of a formal result or an empirical model can be replicated by another researcher given knowledge of the underlying mathematics and/or the source data. Per the syllabus,

> The course will provide students with basic skills necessary to express theoretical ideas in the language of mathematics, formally derive comparative static propositions about those ideas, and marry those propositions to data.

To this end, the core component course will procede as follows:

1. **Logic:** How can we formally link premises to conclusions? This is at the core of all social research. Knowledge accumulation proceeds through a combination of (sometimes untestable) assumptions and observation. We need to know how to link these assumptions and observations to conclusions.
2. **Sets and Spaces:** The building blocks of formal and empirical models of politics can be thought of as elements of sets. Each legislator in the DW-NOMINATE model is represented by a pair of numbers, part of the broader two-dimensional space of all possible ideal points. A legislator's voting record can be represented as a vector of ones and zeros, representing how the legislator voted on every bill. These vectors occupy a broader space of all possible voting records.
3. **Ordered Sets:** We usually want to make comparisons between elements of sets, making statements such as "Elizabeth Warren is more economically liberal than Joe Manchin."
4. **Metric Spaces:** With DW-NOMINATE, we can go a bit further, we can measure the ideological *distance* between two legislators. This requires attaching a *metric* to our spaces, which tells us how far apart any two elements are.
5. **Functions:** Functions connect different spaces to one another. In the spatial model underlying DW-NOMINATE, a utility function maps bills and ideal points to a number that represents the "goodness" of a bill from the perspective of a given legislator. The function is the connective tissue between the bills and ideal points and their "goodness."
6. **Optimization:** How do deliberative agents choose among competing means to achieve their political ends? Functions map choices to outcomes. Optimization is the study of how to deliver the best outcome, given some constraints. It also serves to help us choose models to fit the data best.
7. **Comparative Statics:** How do choices change in response to changes in the environment? These are usually the questions we're ultimately interested in asking as political scientists. We'll wrap up the course with the study of comparative statics, which asks how optima move as a function of parameters.

We motivated the course with an example from American politics, but the same principles apply to problems in international relations and comparative politics. Political theorists ask slightly different questions, but can use the same principles to link premises to normative conclusions. We'll cover a lot of ground in these two weeks and don't expect you to retain everything. Hopefully this will make it easier to absorb the material the next time you see it, if it doesn't sink

in immediately.

We're very open to feedback, comments, and questions. Don't hesitate to stop by our offices, send an email, or post on Piazza and let us know whow things are going. We'll check in with you as the course proceeds and adjust accordingly. We want to make sure everybody's getting something out of this and that the pace is appropriate.