

# Comparative Statics Exercises

- 1) Prove that if  $f$  is differentiable, then  $f$  has increasing differences if and only if  $\frac{\partial^2 f(x, \theta)}{\partial x \partial \theta} \geq 0$ .
- 2) Consider the parameterized optimization problem

$$\max_{x \in [0,1]} (1-x)p(x) + q(1-p(x))$$

where  $p(\cdot) > 0$  is strictly increasing and concave. Assume that  $x^*$  is on the interior of  $[0, 1]$ .

i) Use the implicit function theorem to show how the optimal choice of  $x$  given  $q$ ,  $x^*(q)$  changes as  $q$  changes.

ii) How does the value function change as  $q$  changes?

- 3) Consider the parameterized optimization problem

$$\max_{x,z} f(x, z; \theta)$$

where  $x, y, \theta \in \mathbb{R}$ . Assume  $f$  is twice continuously differentiable. Let  $f_{ij}$  denote  $\frac{\partial^2 f}{\partial i \partial j}$  for  $i, j \in \{x, z, \theta\}$  i.e.  $f_{xx}$  is the second derivative of  $f$  and  $f_{xz}$  is the cross partial derivative of  $f$  with respect to  $x$  and  $z$ . Let  $(x^*(\theta), z^*(\theta))$  be a solution.

i) What conditions on  $f_{xx}$ ,  $f_{zz}$ , and  $f_{xz}$  must hold for  $(x^*(\theta), z^*(\theta))$  to be a local maximum? (Hint: what must be true of the Hessian matrix with respect to choice variables at a local maximum?)

ii) Use the implicit function theorem to characterize  $\frac{\partial}{\partial \theta} x^*(\theta)$  and  $\frac{\partial}{\partial \theta} z^*(\theta)$  in terms of  $f_{ij}$ .

iii) Let  $f_{x\theta} = 0$ ,  $f_{z\theta} < 0$ ,  $f_{xz} > 0$ . Describe the comparative statics. Now let  $f_{xz} < 0$

and describe the comparative statics. Interpret this result.

iv) Show that if  $f$  is supermodular, then  $\frac{\partial}{\partial \theta} x^*(\theta) > 0$  and  $\frac{\partial}{\partial \theta} z^*(\theta) > 0$ .