Continuity Exercises Solutions

1) Let $f : [0, \infty) \to \mathbb{R}$ be defined by $f(x) = \sqrt{x}$. Use the $\epsilon - \delta$ definition of continuity to show that $f(x)$ is continuous at $c > 0$.

2) Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$\frac{x + x^3 + 5x^5}{1 + x^2}$$

Prove that $f(x)$ is continuous.

3) Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} e^x & \text{for} \quad x \leq 0 \\ 0 & \text{for} \quad x > 0 \end{cases}$$

Show that $f(x)$ is discontinuous.

4) Let f and g be continuous at x_0 in $\mathbb{R}$. Prove that $\max(f, g)$ is continuous at x_0 (Hint: first show that for any $a, b \in \mathbb{R}$, $\max\{a, b\} = \frac{1}{2}(a + b) + \frac{1}{2}|a - b|$).

5) Prove that if f and g are continuous at x_0, then their product fg is continuous at x_0.