Metric Spaces Exercises

1) Show that $d_\infty(x, y) = \max_{i=1}^{n} |x_i - y_i|$ is a metric for $\mathbb{R}^n$.

2) Consider the metric space $(\mathbb{R}, d_1)$ and the sequence $(\frac{1}{n^2})$. Prove that $\lim \frac{1}{n^2} = 0$.

3) Prove that every convergent sequence in a metric space is bounded.

4) Prove that every convergent sequence in a metric space is Cauchy.

5) Prove that every Cauchy sequence is bounded.

6) The Bolzano-Weierstrass theorem states that every bounded sequence of real numbers has a convergent subsequence. Use the theorem to prove that $\mathbb{R}$ is complete.