

Ordered Sets Exercises

- 1) Prove that for the set of positive integers, the relation “ m is a multiple of n ” is an order relation.
- 2) Let $X = \{1, 2, \dots, 9\}$, ordered by the relation “ m is a multiple of n ”. Find all maximal and best elements of this ordered set and its least upper bound in \mathbb{Z} .
- 3) Show that $x \sim y$ is an equivalence relation if \succsim is rational.
- 4) Prove or disprove the following statements
 - i) Every best element is a maximal element.
 - ii) Every maximal element is a best element.
 - iii) An element is a best element if and only if it is a maximal element.
- 5) Let $X = \Delta^1$ and \succsim be defined such that for any $(a, b), (c, d) \in X$, $(a, b) \succsim (c, d)$ if and only if $\max\{a, b\} \geq \max\{c, d\}$.
 - i) Find all maximal elements and best elements if they exist.
 - ii) Find all least upper bounds of the set in \mathbb{R}^2 .
 - iii) Use the properties of binary relations to identify whether the set is partially ordered, totally ordered, and/or weakly ordered.
- 6) Prove that if X is finite, (X, \succsim) has at least one maximal element for all order relations.