Smooth Functions: Solutions

September 12, 2018

1. Prove that every continuous linear functional; is differentiable with $Df[x] = \alpha$.[1]

    [1] Carter 4.6

2. Prove that if a differentiable functional f is increasing, then $Df[x_0](x) \geq 0$ for all $x \in X$.[2]

    [2] Carter 4.15, recall the definition of increasingness from the lecture on monotonic functions.

3. Let f be a differentiable functional. Prove that the $\nabla f(x_0)$ is orthogonal to the hyperplane tangent to the contour through $f(x_0)$.

4. Let the policy production function discussed above be written

    $$f(x,y) = x^\beta + y^\alpha$$

    Give a sufficient condition for this function to be concave on $\{\mathbb{R}_{++} \times \mathbb{R}_{++}\}$.