

# Midterm Examination

POL 500 - Introduction to Mathematics for Political Science

September 10, 2019

This is a closed book examination. Calculators are not permitted. Attempt to answer all questions. Each question is worth ten points and should take about ten minutes. The exam ends sharply at 11:50.

## Question 1

Evaluate the following limits:

a.

$$\lim_{x \rightarrow 5} \frac{\sqrt{x^2 - 9} - 4}{x - 5}$$

b.

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 + 9}}{2x^3 + 6x + 1}$$

c.

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

## Question 2

Let  $g(x) = \ln(x)$ . Use the limit definition of the derivative to find  $g'(x)$ . **Hint:**  $\lim_{k \rightarrow 0} -(1 + \frac{k}{x})^{1/k} = -e^{1/x}$

### Question 3

Compute the following derivatives with respect to  $x$ :

a)  $f(x) = \sqrt{x^3 - 7x}$

b)  $f(x) = x^2 e^{-x}$

c)  $f(x) = x^2 \sqrt{1 - x^2}$

### Question 4

Let  $f(x) = x^2$  and  $g(x) = x$ . Let  $A = \{(x, y) \in \mathbb{R}^2 : y \geq f(x)\}$  and  $B = \{(x, y) \in \mathbb{R}^2 : y \leq g(x)\}$ .

a)  $A \cap B$  is a region in  $\mathbb{R}^2$ . Find the area of this region.

b) Now assume that a point is drawn completely at random from  $A \cap B$ . Find the joint probability density function that represents this process.

### Question 5

Evaluate the following integrals

a)  $\int_0^2 (x^2 - xb) dx$

b)  $\int x \ln(x) dx$

### Question 6

A family has two children. Given that one of the children is a boy and that he was born on a Tuesday, what is the probability that both children are boys?

### Question 7

Prove the following statement: If  $A$  is orthogonal, then the rows of  $A$  are orthogonal to each other and each row has a norm of 1. **Hint: A matrix is orthogonal if  $A^T A = I$**

## Question 8

Use the Cauchy-Schwartz Inequality

$$\mathbf{u} \cdot \mathbf{v} \leq \|\mathbf{u}\| \|\mathbf{v}\|$$

to prove the Triangle Inequality

$$\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$$

**Hint:** Convince yourself that  $\|\mathbf{u} + \mathbf{v}\|^2 = (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$