An Introduction to Mathematics for Political Science

Problem Set 1

You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Submit typed solutions or scans of handwritten solutions as a PDF. Note that starting with problem set 3 all solutions must be written in LaTeX.

Set Basics

1. Let $A = (40, 60)$ and $B = [50, 70]$.

   a) Is $A \subset B$, $B \subset A$, both, or neither?

   b) What is $A \cup B$?

   c) What is $A \cap B$?

   d) Write three elements of the Cartesian product $A \times B$.

2. Identify whether the following sets are (a) open, closed, or neither; (b) bounded; (c) compact; (d) convex:

   a) $(0, 1)$

   b) $[0, 1]$

   c) $(0, 1]$

   d) $[0, \infty)$

   e) $(0, \infty)$

f) $[0,1] \cup [2,3]$

g) $[0,1] \times [0,1]$

h) $(0,3] \cap [1,4]$

i) $[0,5] \backslash \{1,2\}$

3. Express the following sentences in mathematical notation:

a) A is the set of all real numbers less than or equal to seven, excluding zero and four.

b) B is the intersection of the natural numbers and the real numbers between π and 30.5.

c) For all epsilon greater than zero, there exists a delta greater than zero.

d) The set of all even integers between 5 and 21.

Algebra

4. Simplify into one term or evaluate the following:

a) $y \cdot y \cdot y \cdot y$

b) $(-a)(-b)^3 - b^2 + a^3$

c) $(4b + 2)(a - 5)$

d) $\frac{5!}{2!}$

e) $\sum_{i=1}^{3} (\frac{1}{3})^i$

f) $\sum_{i=2}^{5} 2^i$

g) $\prod_{i=1}^{3} (\frac{1}{3})^i$

h) $\frac{48}{4} - 6 \cdot 9$

i) $(3^3 + (-5)) \cdot 3 - (-7)$

j) $[6 + (\frac{-66}{11})] \cdot (-2)^3$

k) $\frac{y-11}{5} + \frac{y+12}{3}$

5. Solve the following for x:

a) $5(-3x - 2) - (2x - 3) = -4(4x + 5) + 13$

b) $8x^2 = 15 - 14x$

c) $x^2 - 3x + 4 = 2(x - 1)$

d) $-6 > 5x + 5 + 4$

e) $-2(x + 1) + 4 < 10$

f) $2\ln(2x + 1) - 10 = 0$

g) $3^x e^{4x} = e^7$

Functions

6) Find the image/range of x^2 on the domain $[-3, 3]$.

7) Let $f(x) = x^2 - 4x + 2$ and $g(x) = 3x - 7$. Find (a) $f(x) - g(x)$; (b) $f(x) + g(x)$; (c) $f(g(x))$; and (d) $g(f(x))$. Evaluate each for $x = 2$.

8) Identify whether each of the following functions $f : \mathbb{R} \to \mathbb{R}$ is (a) surjective/onto; (b) injective/one-to-one; (c) bijective.

a) $f(x) = x^2$

b) $f(x) = x^3 - x$

c) $f(x) = e^x$

d) $f(x) = x^3$

9) Find an equation for the inverse for each of the following functions:

a) $f(x) = (5x - 1)^3$

b) $f(x) = \frac{x+4}{3x-5}$

c) $f(x) = e^{5x-1}$

10) Evaluate each of the following limits or show that they do not exist:

a) $\lim_{x\to 5} \frac{x^2-25}{x^2+x-30}$

b) $\lim_{x\to -1} \frac{x^3}{(x+1)^2}$

c) $\lim_{x\to 2} \frac{x^2+4x-12}{|x-2|}$

d) $\lim_{x\to \infty} \frac{x^2-1}{2x^2+1}$

e) $\lim_{x\to \infty} (\frac{x^3}{x^2+2} - x)$

f) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$

11) Determine whether the following functions are continuous at the specified value. Use limits where appropriate. You may find sketching the graph of each function to be helpful.

a) $f(x) = \frac{x^2+1}{x^3+1}$ at $x = -1$

b) $f(x) = \begin{cases} 3x - 5 \text{ if } x \neq 1 \\ 2 \text{ if } x = 1 \end{cases}$ at $x = 1$

c) $f(x) = \begin{cases} \frac{x-6}{x-3} \text{ if } x < 0 \\ 2 \text{ if } x = 0 \\ \sqrt{4+x^2} \ x > 0 \end{cases}$ at $x = 0$