Introduction to Mathematics for Political Science

Problem Set 8: Solving Systems of Equations

Instructions: You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. You may use a calculator for basic arithmetic, but do not search the internet for solutions. Show all of your work. Submit typed solutions using the link on the course page.

1. Solve the following systems of equations using substitution or elimination.

   - $$\begin{aligned} 2x + 3y &= 10 \\ 2y &= 4 \end{aligned}$$

   - $$\begin{aligned} 3x + 5y - 2z &= 1 \\ 3y + z &= 8 \\ 5z &= 25 \end{aligned}$$

   - $$\begin{aligned} 2x + 2y &= 8 \\ 6x + 2y &= 4 \end{aligned}$$

   - $$\begin{aligned} x - 3y - 2z &= 6 \\ 2x - 4y - 3z &= 8 \\ -3x + 6y + 8z &= -5 \end{aligned}$$

2. Provide a set of *basis* or *spanning* vectors for the following vector spaces.
   - $\mathbb{R}^2$
   - $\mathbb{R}^3$
   - A 45 degree line intersecting the origin in $\mathbb{R}^2$

3. Consider the following systems $\boldsymbol{Ax} = \boldsymbol{b}$. Find the *rank* of $\boldsymbol{A}$. If the solution ($\boldsymbol{x}$) to the system is unique, solve for it. If not, explain why not.

   - $$\underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 7 & 4 & 7 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 10 \\ 12 \\ 34 \end{bmatrix}}_{\boldsymbol{b}}$$

   - $$\underbrace{\begin{bmatrix} 3 & 1 & 4 \\ 9 & -3 & -2 \\ 1 & 1 & 1 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 8 \\ 4 \\ 3 \end{bmatrix}}_{\boldsymbol{b}}$$

4. Provide a non-zero column vector $\boldsymbol{a}_3$ such that the following matrices $\boldsymbol{A}$ are *singular*.

   - $$\boldsymbol{A} = \begin{bmatrix} 3 & 1 & \\ 4 & 2 & \boldsymbol{a}_3 \\ 1 & 1 & \end{bmatrix}$$

   - $$\boldsymbol{A} = \begin{bmatrix} 9 & 12 & \\ 7 & 8 & \boldsymbol{a}_3 \\ 3 & 4 & \end{bmatrix}$$

5. For what values of the parameter k does the following system have a) no solution, b) one solution, and c) more than one solution?[1]

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 - kx_2 &= 1 \end{aligned}$$

[1] **Source:** Moore and Siegel 13.4 #3