

## Smooth Functions: Solutions

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1. Prove that every continuous linear functional; is differentiable with  $Df[x] = \alpha$ .<sup>1</sup>
2. Prove that if a differentiable functional  $f$  is increasing, then  $Df[x_0](x) \geq 0$  for all  $x \in X$ .<sup>2</sup>
3. Let  $f$  be a differentiable functional. Prove that the  $\nabla f(x_0)$  is orthogonal to the hyperplane tangent to the contour through  $f(x_0)$ .
4. Let the policy production function discussed above be written

$$f(x, y) = x^\beta + y^\alpha$$

Give a sufficient condition for this function to be concave on  $\{\mathbb{R}_{++} \times \mathbb{R}_{++}\}$ .

<sup>1</sup> Carter 4.6

<sup>2</sup> Carter 4.15, recall the definition of increasingness from the lecture on monotonic functions.