Monotone, Linear, and Convex Functions Exercises (Solutions)

1) Let $f_1, f_2, ..., f_n$ be convex functions and $\alpha_1, \alpha_2, ...\alpha_n \geq 0$. Prove that $f(x) = \alpha_1 f_1(x) + ... + \alpha_n f_n(x)$ is convex. Is $\alpha_1 f_1 - \alpha_2 f_2$ convex? Prove your answer.

2) Prove the Cauchy-Schwarz inequality for $\mathbb{R}^n$.

3) Prove the following: $L : \mathbb{R}^l \to \mathbb{R}$ is a continuous, linear functional if and only if there exists a $y \in \mathbb{R}^l$ such that for all $x \in \mathbb{R}^l$, $L(x) = y^T x$.