An Introduction to Mathematics For Political Science Problem Set 6

You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. All solutions must be written in LaTeX.

Random Variables and Distributions

- 1. A popular state lottery game requires participants to select a three-digit number (leading 0s allowed). Then three balls, each with one digit, are chosen at random from well-mixed bowls. The sample space here consists of all triples (i_1, i_2, i_3) where $i_j \in \{0, ..., 9\}$ for j = 1, 2, 3. If $s = (i_1, i_2, i_3)$, define $X(s) = 100i_1 + 10i_2 + i_3$. For example, X(0, 1, 5) = 15. Find Pr(X = x) for each integer $x \in \{0, 1, ..., 999\}$.
- 2. Now consider a lottery in which participants select a real number, x, between 0 and 1. A real number, X, is then randomly selected from the interval with equal probability, i.e. X is a draw from the uniform distribution on [0,1]. Find Pr(X=x) for some real number x.
 - 3. Suppose that the p.d.f. of a certain random variable X has the following form:

$$f(x) = \begin{cases} cx & \text{for } 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant. Find c.

4. Suppose that the p.d.f. of a certain random variable X has the following form:

$$f(x) = \begin{cases} \frac{x}{8} & \text{for } 0 < x < 4\\ 0 & \text{otherwise} \end{cases}$$

Find $Pr(1 \le X \le 2)$ and Pr(X > 2).

- 5. The c.d.f. of a random variable X is given by $F(x) = 1 e^{-\lambda x}$ on \mathbb{R}_+ . Find $Pr(x \ge \lambda)$ and $Pr(x \le \lambda^2)$.
 - 6. Find the p.d.f. of the random variable X from the previous problem.
 - 7. Let the c.d.f. of a random variable X be given by

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^{\frac{2}{3}} & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x > 1 \end{cases}$$

Find the p.d.f. of X.

- 8. Suppose that a point (X, Y) is selected at random from inside the circle, S, defined by $x^2 + y^2 \le 9$. Find the joint p.d.f. of (X, Y).
- 9. An investor is considering whether or not to invest 18 dollars in a stock for one year. The value of the stock after one year in dollars will be 18+X where X is a random variable. Suppose X is distributed such that Pr(X=-2)=.1, Pr(X=0)=.4, Pr(X=1)=.3, and Pr(X=4)=.2. Alternatively, the investor can place her 18 dollars in the bank at 4 percent interest. Which use of her money is optimal if the investor is risk neutral?

- 10. Let X be a random variable with p.d.f. f(x) = 2x with support only on (0,1). Find E[X].
- 11. A product has a warranty of one year. Let X be the time at which the product fails. Suppose that X has a continuous distribution with the p.d.f.

$$f(x) = \begin{cases} 0 & \text{for } x < 1\\ \frac{2}{x^3} & \text{for } x \ge 1 \end{cases}$$

Find the expected time to failure.

- 12. A random variable Y is a linear function of random variables X_1 and X_2 and ϵ . In particular, let $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$. Let X_1 be distributed according to the p.d.f from question 10 and let X_2 be a Bernoulli random variable with parameter $p = \frac{1}{4}$. Let $\epsilon \sim N(0,1)$. The terms β_i are real-valued scalars. Find E[Y].
- 13. Suppose a politician's utility function is described by $U(x) = -(z x)^2$ where z is her preferred policy $z \in \mathbb{R}$ and x is a random policy shock with p.d.f. f(x). Let f(x) be the uniform distribution on [0, 1]. Find her expected utility.
 - 14. Find the variance for the following series of numbers:
 - a) 12, 6, 7, 3, 15, 10, 18, 5
 - b) 2, 3, 6, 8, 11
 - c) 3, 5, 2, 7, 6, 4, 9, 1
- 15. X is a random variable with equal probability of taking any one of five values, -2, 0, 1, 3, and 4. Compute the variance of X.