

# Inner Product Spaces, Orthogonality, Projection

## Exercises

1. Let  $X$  and  $Y$  be normed linear spaces. Let  $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  be a basis for  $X$  and  $\{\mathbf{y}_1, \dots, \mathbf{y}_m\}$  a basis for  $Y$ . Prove that if  $x_i \perp y_j$  for all  $i \in \{1, \dots, n\}$ ,  $j \in \{1, \dots, m\}$ , then  $X$  and  $Y$  are orthogonal spaces.
2. Prove: If a vector  $\mathbf{a}$  is in the null space of a set of vectors  $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ , then it is orthogonal to the space spanned by  $\{\mathbf{y}_1, \dots, \mathbf{y}_m\}$  where

$$\mathbf{y}_i = \{x_{1i}, \dots, x_{ni}\}$$

3. Donald Trump tweeted 100 times in April, 150 times in May, and 110 times in June.<sup>1</sup> Let  $\mathbf{b} = (100, 150, 110)$  represent the number of tweets in each month. Project  $\mathbf{b}$  onto the linear space spanned by  $\mathbf{a} = (1, 1, 1)$ . Interpret your result.

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<sup>1</sup>Disclaimer: these data are of suspect quality.