Ordered Sets Exercises

1) Prove that for the set of positive integers, the relation "m is a multiple of n" is an order relation.

2) Let $X = \{1, 2, ..., 9\}$, ordered by the relation "m is a multiple of n". Find all maximal and best elements of this ordered set and its least upper bound in $\mathbb{Z}$.

3) Show that $x \sim y$ is an equivalence relation if $\succsim$ is rational.

4) Prove or disprove the following statements

   i) Every best element is a maximal element.

   ii) Every maximal element is a best element.

   iii) An element is a best element if and only if it is a maximal element.

5) Let $X = \Delta^1$ and $\succsim$ be defined such that for any $(a, b), (c, d) \in X$, $(a, b) \succsim (c, d)$ if and only if $\max\{a, b\} \geq \max\{c, d\}$.

   i) Find all maximal elements and best elements if they exist.

   ii) Find all least upper bounds of the set in $\mathbb{R}^2$.

   iii) Use the properties of binary relations to identify whether the set is partially ordered, totally ordered, and/or weakly ordered.

6) Prove that if X is finite, $(X, \succsim)$ has at least one maximal element for all order relations.