An Introduction to Mathematics For Political Science

Problem Set 3

You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Starting this week all solutions must be written in LaTeX.

Integration

1. Evaluate the following:

a) $\int x^2 + 2x + 2\, dx$

$\frac{x^3}{3} + x^2 + 2x + C$

b) $\int x^7 + x^{-3} - \frac{1}{x}\, dx$

$\frac{x^8}{8} - \frac{1}{2x^2} - \ln(|x|) + C$

c) $\int_4^9 \frac{1}{x\sqrt{x}}\, dx$

$\int_4^9 x^{-3/2} = \frac{1}{-1/2} x^{-1/2}]_4^9 = -2(9)^{-1/2} - (-2(4)^{-1/2}) = -2/3 + 2/2 = 1/3$

d) $\int e^{-x/3}\, dx$

$\frac{1}{-1/3} e^{-x/3} + C = -3e^{-x/3} + C$

e) $\int e^{2x} - 2e^x - 2ex^2\, dx$

$\frac{e^{2x}}{2} - 2e^x - \frac{2ex^3}{3} + C$

f) $\int_{-e^2}^{-1} \frac{4}{x}\, dx$

$\int_{-e^2}^{-1} \frac{4}{x} dx = 4\ln|x|]_{-e^2}^{-1} = 4\ln|-1| - 4\ln|-e^2| = 4\ln(1) - 4\ln(e^2) = 8$

g) $\int_0^2 2x^2 + 3x + 1\, dx$

The indefinite integral is $\frac{x(4x^2+9x+6)}{6} + C$. Now evaluate $\frac{2(4(2)^2+9(2)+6)}{6} - \frac{0(4(0)^2+9(0)+6)}{6} = \frac{40}{3}$

h) $\int_{-a}^{a} x^5\, dx$

$\int_{-a}^{a} x^5 = \frac{x^6}{6}]_{-a}^{a} = \frac{a^6}{6} - \frac{(-a)^6}{6} = \frac{a^6}{6} - \frac{a^6}{6} = 0$

i) $\int e^{\sqrt{x}}\, dx$

Let $u = \sqrt{x}$. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \rightarrow dx = 2\sqrt{x}\, du = 2u\, du$. Substitute u and dx into the original integral to get $\int 2ue^u du = 2\int ue^u du$. Now integrate by parts: $\int fg' = fg - \int f'g$ where we let $f = u$ and $g' = e^u$. Note that $f' = \frac{\partial}{\partial u}u = 1$. To find g, take the antiderivative of e^u which is simply e^u. Now we have $\int ue^u\, du = ue^u - \int e^u\, du = ue^u - e^u$. Finish by plugging everything back into $\int 2ue^u du$:

$\int 2ue^u du = 2ue^u - 2e^u + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = 2(\sqrt{x} - 1)e^{\sqrt{x}} + C$

j) $\int \frac{e^{2x}}{2} dx$

Let $u = 2x$. $\frac{du}{dx} = 2 \rightarrow dx = \frac{1}{2}du$. Substitution gives us $\int \frac{e^{2x}}{2} dx = \frac{1}{4}\int e^u du = \frac{e^u}{4}$ Now finish by substituting $u = 2x$ and adding a constant: $\frac{e^{2x}}{4} + C$.

k) $6\int xe^{2x} dx$

We need to integrate by parts. Let $f = x$ and $g' = e^{2x}$ so that $f' = 1$ and $g = \frac{e^{2x}}{2}$. Now we have $fg - \int f'g = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2} dx$. In the previous problem we found that $\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + C$ so the solution is $6(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}) = 3xe^{2x} - \frac{3e^{2x}}{2} + C$.

l) $\int \frac{(6x^2+5)e^{2x}}{2} dx$

Note that $\frac{1}{2}\int (6x^2+5)e^{2x} dx$. Integrate by parts: $f = 6x^2 + 5$, $g' = e^{2x}$, $f' = 12x$, $g = \frac{e^{2x}}{2}$.

$fg - \int f'g = \frac{(6x^2+5)e^{2x}}{2} - 6\int xe^{2x} dx$

In the previous problem we found that $6\int xe^{2x} dx = 3xe^{2x} - \frac{3e^{2x}}{2} + C$ so our solution is

$\frac{1}{2}(\frac{(6x^2+5)e^{2x}}{2} - 3xe^{2x} + \frac{3e^{2x}}{2}) = \frac{(6x^2+5)e^{2x}}{4} - \frac{3xe^{2x}}{2} + \frac{3e^{2x}}{4} + C$

m) $\int (2x^3 + 5x + 1)e^{2x}\, dx$

Use integration by parts: $\int fg' = fg - \int f'g$. Let $f = 2x^3 + 5x + 1$ and $g' = e^{2x}$.

$f' = 6x^2 + 5$ and $g = \frac{e^{2x}}{2}$. Now we just follow the formula:

$\int fg' = fg - \int f'g = \frac{(2x^3+5x+1)e^{2x}}{2} - \int \frac{(6x^2+5)e^{2x}}{2} dx.$

In the previous problem we found that

$\int \frac{(6x^2+5)e^{2x}}{2} dx = \frac{(6x^2+5)e^{2x}}{4} - \frac{3xe^{2x}}{2} + \frac{3e^{2x}}{4} + C$

so we conclude that

$\int (2x^3 + 5x + 1)e^{2x}\, dx = \frac{(2x^3+5x+1)e^{2x}}{2} - \frac{(6x^2+5)e^{2x}}{4} + \frac{3xe^{2x}}{2} - \frac{3e^{2x}}{4} + C$

which simplifies to

$\frac{(2x^3-3x^2+8x-3)e^{2x}}{2} + C$