Comparative Statics Exercises

1) Let $f(x, y; \theta) = v(x; \theta) + w(y; \theta) - c(x, y)$ where $x, y, \theta \in \mathbb{R}$. Assume that there exists a solution to the program

$$\max_{x,y} \ f(x, y; \theta)$$

$$\text{s.t.} \quad g(x, y) = 0$$

where f and g are twice continuously differentiable. Use the implicit function theorem to identify how $x^*(\theta)$ and $y^*(\theta)$ change for a small increase in θ. Assume that $v(x; \theta)$ and $v(y; \theta)$ are increasing and concave in their first argument and that $c(x, y)$ is increasing and convex in each argument.

2) Prove that if f is differentiable, then f has increasing differences if and only if $\frac{\partial^2 f(x,\theta)}{\partial x \partial \theta} \geq 0$.