

Continuous Functions: Solutions

September 12, 2018

1. Use the Delta-Epsilon method to prove the continuity of $f(x) = |x|$ under the d_1 (absolute value) metric.
2. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are continuous functions, then $g \circ f : X \rightarrow Z$ is also continuous.¹
3. Let (X, d) be a compact metric space and f be a continuous function. Prove $f(X)$ is closed and bounded.²
4. Prove: If a function $f : X \rightarrow Y$ is continuous, then every open subset S of a metric space Y , $f^{-1}(S)$ is open in metric space X .³

¹ Carter 2.72

² Hint: The limit definition of continuity and the Bolzano-Weierstrass theorem will be useful.

³ Hint: $\bigcup_{i=1}^{\infty} O_i$ with O_i open for all i is open.