

## Smooth Functions

1. Prove that every continuous linear functional; is differentiable with  $Df[\mathbf{x}] = \boldsymbol{\alpha}$ .<sup>1</sup>
2. Prove that if a differentiable functional  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is increasing, then  $Df[\mathbf{x}_0](\mathbf{x}) \geq 0$  for all  $\mathbf{x} \in X$ , or  $\frac{\partial f}{\partial x_i} \geq 0$  for all  $i \in \{x_1, \dots, x_n\}$ .
3. Let  $f$  be a differentiable functional. Prove that the  $\nabla f(\mathbf{x}_0)$  is orthogonal to the hyperplane tangent to the contour through  $f(\mathbf{x}_0)$ .
4. Let the policy production function discussed above be written

$$f(x, y) = x^\alpha y^\beta$$

Give a sufficient condition for this function to be concave on  $\{\mathbb{R}_{++} \times \mathbb{R}_{++}\}$ .

**Hint:** A  $2 \times 2$  symmetric matrix  $A$  is negative definite if  $A_{11} < 0$  and  $A_{11}A_{22} - A_{12}A_{21} > 0$ .

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<sup>1</sup>Carter 4.6