Midterm Examination

POL 500 - Introduction to Mathematics for Political Science

August 28, 2019

This is a closed book examination. Calculators are not permitted. Attempt to answer all questions. Each question is worth ten points and should take about ten minutes. The exam ends sharply at 11:50.

Question 1

Evaluate the following limits:

a.

$$\lim_{x \to 5} \frac{\sqrt{x^2 - 9} - 4}{x - 5}$$

b.

$$\lim_{x \to -\infty} \frac{\sqrt{4x^6 + 9}}{2x^3 + 6x + 1}$$

c.

$$\lim_{x \to 0} \frac{|x|}{x}$$

Question 2

Let $g(x) = \ln(x)$. Use the limit definition of the derivative to find $g'(x)$. **Hint:** $\lim_{k \to 0} -(1 + \frac{k}{x})^{1/k} = -e^{1/x}$

Question 3

Compute the following derivatives with respect to x:

a) $f(x) = \sqrt{x^3 - 7x}$

b) $f(x) = x^2 e^{-x}$

c) $f(x) = x^2\sqrt{1 - x^2}$

Question 4

Let $f(x) = x^2$ and $g(x) = x$. Let $A = \{(x, y) \in \mathbb{R}^2 : y \geq f(x)\}$ and $B = \{(x, y) \in \mathbb{R}^2 : y \leq g(x)\}$.

a) $A \cap B$ is a region in $\mathbb{R}^2$. Find the area of this region.

b) Now assume that a point is drawn completely at random from $A \cap B$. Find the joint probability density function that represents this process.

Question 5

Evaluate the following integrals

a) $\int_0^2 (x^2 - xb)dx$

b) $\int x \ln(x) dx$

Question 6

A family has two children. Given that one of the children is a boy and that he was born on a Tuesday, what is the probability that both children are boys?

Question 7

Prove the following statement: If A is orthogonal, then the rows of A are orthogonal to each other and each row has a norm of 1. **Hint: A matrix is orthogonal if** $A^T A = I$

Question 8

Use the Cauchy-Schwartz Inequality

$$\boldsymbol{u} \cdot \boldsymbol{v} \leq \|\boldsymbol{u}\|\|\boldsymbol{v}\|$$

to prove the Triangle Inequality

$$\|\boldsymbol{u} + \boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$$

Hint: Convince yourself that $\|\boldsymbol{u} + \boldsymbol{v}\|^2 = (\boldsymbol{u} + \boldsymbol{v}) \cdot (\boldsymbol{u} + \boldsymbol{v})$