## Kruskal-Wallis Test

This tests if *k* independent samples are drawn from the same population. As the Kruskal-Wallis test ranks the values, it is more powerful than the Median test. The Kruskal-Wallis test is derived from the one-way ANOVA, but uses ranks rather than actual observations. It is also the extension of the Mann-Whitney U test.

The assumptions of the Kruskal-Wallis test are:

1. The data have been collected from a randomly selected set of observations.
2. The dependent variable is at least at the ordinal level of measurement.
3. There are more than two independent groups.
4. There is independence of observations within each group and between the groups. There are no repeated measures or multiple response categories.
5. The shapes of the distributions of the groups are similar.

If the last assumption holds then the hypotheses are:

H0: The medians in the *k* groups are equal.

HA: There is a difference in medians between the *k* groups.

If the last assumption does not hold:

H0: The *k* groups have the same shape and location.

HA: The *k* groups have a different shape and location.

The alternative hypothesis can be directional or non-directional. If a significant result is obtained then post hoc testing can be used to see where any differences lie.

If the assumptions are met the test can be used in the following way:

1. Determine the null and alternative hypothesis and α the level of significance for the test.
2. Rank the whole sample from lowest to highest.
3. Calculate the sum of the ranks for each group.
4. Calculate the average rank in each group, , and the average rank for the whole sample, .
5. Calculate the test statistic *H*,



where, *ni* = the number of observations in group *i*

*N* = the total sample size

1. Compare this value with the chi-square distribution with *k–1* degrees of freedom. If the statistic is bigger than the critical value in the chi-square table, the result is significant. If the result is significant, then pairwise post hoc test can be carried out.

### Example

The data are the reduction in weekly headache activity for three treatment groups, expressed as a percentage of the baseline data (example from Altman).

1. H0: The three samples come from populations with the same median.

HA: At least one sample comes from a population with a different median.

α = 0.05.

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| Relaxation/ response feedback | Rank | Relaxation alone | Rank | Untreated | Rank |
| 62 | 8 | 69 | 9 | 50 | 7 |
| 74 | 10.5 | 43 | 6 | –120 | 2 |
| 86 | 12 | 100 | 17 | 100 | 17 |
| 74 | 10.5 | 94 | 14 | –288 | 1 |
| 91 | 13 | 100 | 17 | 4 | 4 |
| 37 | 5 | 98 | 15 | –76 | 3 |
| Rank sum (mean) | 59  (9.83) |  | 78  (13) |  | 34  (5.67) |

4. 

5. 

1. Therefore, there is not sufficient evidence to reject the null hypothesis at the 5% level of significance, p=0.06.