Introduction

Statistics:

- a **mathematical** science
- to **describe** what has happened and
- to assess what **may** happen in **the future**
- relies on the **observation** of natural phenomena in order to propose an interpretation, often through **probabilistic models**

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Frequentist statistics:

- Neyman & Pearson
- **deterministic** view of the parameters
- **Maximum Likelihood Estimation**
- statistical **test theory** & **confidence interval**

Bayes' theorem

Reverend Thomas Bayes posthumous article in 1763

$$\Pr(A|E) = \frac{\Pr(E|A)\Pr(A)}{\Pr(E|A)\Pr(A) + \Pr\left(E|\overline{A}\right)\Pr\left(\overline{A}\right)} = \frac{\Pr(E|A)\Pr(A)}{\Pr(E)}$$

(conditional probability formula: $Pr(A|E) = \frac{Pr(A \cap E)}{Pr(E)}$)

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In practice:

Last time you visited the doctor, you got **tested for a rare disease**. Unluckily, the result was positive...

Given the test result, what is the probability that I actually have this disease?

(Medical tests are, after all, not perfectly accurate.)

→ *Seeing Theory*, Brown University

Bayes theorem: exercise

As of May 11th, about 7% of the French population was estimated to have had COVID-19. A medical test has the following properties:

- if someone has COVID-19, its test will come out positive 71% of the time
- if someone does not have the disease, its test will come out negative 98% of the time

Given that someone got a positive result, what is his/her probability to truly have COVID-19 ?

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