

Application to the historical example

① the likelihood

...

② the prior

...

③ the posterior

...

Application to the historical example

① the likelihood

$$f(\mathbf{y}|\theta) = \prod_{i=1}^n \theta^{y_i} (1-\theta)^{(1-y_i)} = \theta^S (1-\theta)^{n-S} \quad \text{where } S = \sum_{i=1}^n y_i$$

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Uniform: $\pi(\theta) = 1$

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Construction of a Bayesian model

Application to the historical example

1 the likelihood

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2 the prior

Uniform: $\pi(\theta) = 1$

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$$p(\theta|\mathbf{y}) = \frac{\theta^S(1-\theta)^{n-S}}{f(\mathbf{y})} = p(\theta|\mathbf{y}) = \binom{n}{S}(n+1)\theta^S(1-\theta)^{n-S}$$

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To answer the question of interest, we can then calculate: ...

Construction of a Bayesian model

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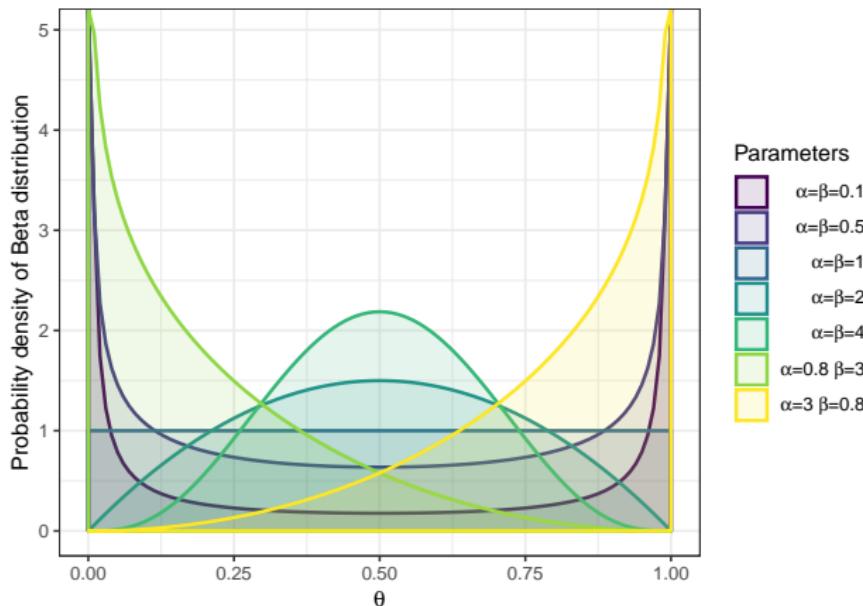
To answer the question of interest, we can then calculate:

$$P(\theta \geq 0.5 | \mathbf{y}) = \int_{0.5}^1 p(\theta | \mathbf{y}) = \binom{n}{S} (n+1) \int_{0.5}^1 \theta^S (1-\theta)^{n-S} d\theta \approx 1.15 \cdot 10^{-42}$$

Construction of a Bayesian model

The Beta distribution

$$f(\theta) = \frac{(\alpha + \beta - 1)!}{(\alpha - 1)!(\beta - 1)!} \theta^{\alpha-1} (1-\theta)^{\beta-1} \text{ for } \alpha > 0 \text{ and } \beta > 0$$



Examples of various parametrizations for the Beta distribution

Introduction to Bayesian statistics
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Bayesian modeling
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Bayesian Inference
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Conclusion
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Construction of a Bayesian model

Conjugacy of the Beta distribution

Beta prior: $\pi = \text{Beta}(\alpha, \beta)$

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This is called a **conjugated distribution** because the **posterior** and the **prior** belong to the **same parametric family**

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Impact of the *prior* choice

Interpretation of the prior	Parameters of the Beta distribution	$P(\theta \geq 0.5 \mathbf{y})$
#boys > #girls	$\alpha = 0.1, \beta = 3$	$1.08 \cdot 10^{-42}$
#boys < #girls	$\alpha = 3, \beta = 0.1$	$1.19 \cdot 10^{-42}$
#boys = #girls	$\alpha = 4, \beta = 4$	$1.15 \cdot 10^{-42}$
#boys ≠ #girls	$\alpha = 0.1, \beta = 0.1$	$1.15 \cdot 10^{-42}$
non-informative	$\alpha = 1, \beta = 1$	$1.15 \cdot 10^{-42}$

For 493,472 newborns including 241,945 girls

Construction of a Bayesian model

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Interpretation of the <i>prior</i>	Parameters of the Beta distribution	$P(\theta \geq 0.5 \mathbf{y})$
#boys > #girls	$\alpha = 0.1, \beta = 3$	0.39
#boys < #girls	$\alpha = 3, \beta = 0.1$	0.52
#boys = #girls	$\alpha = 4, \beta = 4$	0.46
#boys \neq #girls	$\alpha = 0.1, \beta = 0.1$	0.45
non-informative	$\alpha = 1, \beta = 1$	0.45

For 20 newborns including 9 girls

Impact of the *prior* choice for 20 observed births – continued