Bayes theorem: exercise

As of May 11$^{\text{th}}$, about 7% of the French population was estimated to have had COVID-19. A medical test has the following properties:

- if someone has COVID-19, its test will come out positive 71% of the time
- if someone does not have the disease, its test will come out negative 98% of the time

Given that someone got a positive result, what is his/her probability to truly have COVID-19 ?

$$\Pr(M=+) = 0.07 \qquad \Pr(T=+|M=+) = 0.71 \qquad \Pr(T=-|M=-) = 0.98$$

$$\begin{aligned}
\Pr(M=+|T=+) &= \frac{\Pr(T=+|M=+)\Pr(M=+)}{\Pr(T=+)} \\
&= \frac{\Pr(T=+|M=+)\Pr(M=+)}{\Pr(T=+|M=+)\Pr(M=+) + \Pr(T=+|M=-)\Pr(M=-)} \\
&= \frac{\Pr(T=+|M=+)\Pr(M=+)}{\Pr(T=+|M=+)\Pr(M=+) + (1-\Pr(T=-|M=-))(1-\Pr(M=+))} \\
&= (0.71\times 0.07)/(0.71\times 0.07 + (1-0.98)\times(1-0.07)) = 0.73
\end{aligned}$$

Continuous Bayes' theorem

- parametric (probabilistic) model $f(y|\theta)$
- parameters θ
- probability distribution π

Continuous Bayes' theorem:

$$p(\theta|y) = \frac{f(y|\theta)\pi(\theta)}{\int f(y|\theta)\pi(\theta)\,\mathrm{d}\theta}$$

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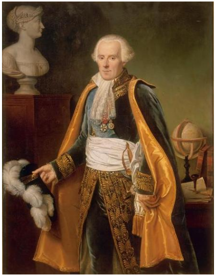


remember Pierre-Simon de Laplace !

Bayes philosophy

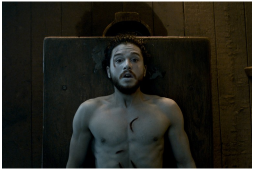
Parameters are random variables ! – *no "true" value*

⇒ induces a marginal probability distribution $\pi(\theta)$ on the parameters: the ***prior*** distribution

- 😁 allows to **formally** take into account hypotheses in the modeling
- ☹️ necessarily introduces **subjectivity** into the analysis

Bayesian vs. Frequentists: a historical note

1. **Bayes** + **Laplace** ⇒ development of statistics in the **18-19$^{\text{th}}$ centuries**
2. Galton & Pearson, then Fisher & Neymann ⇒ **frequentist** theory became dominant during the **20$^{\text{th}}$ century**
3. turn of the **21$^{\text{th}}$ century**: rise of the computer
   ⇒ **Bayes' comeback**



Bayesian vs. Frequentists: an outdated debate

Fisher firmly rejected Bayesian reasoning

$\Rightarrow$ community split in 2 in the 20^{th}

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To be, or not to be, Bayesian, that is no longer the question: it is a matter of wisely using the right tools when necessary

Gilbert Saporta