MCMC algorithms: general principle

Approximate an integral (or another function) from a target distribution

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$\Rightarrow$ sample a Markov chain whose stationary law is the target (such as the *posterior*) distribution, then apply the Monte Carlo method

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$$\forall X_0, X_n \xrightarrow[n \to +\infty]{\mathscr{L}} \tilde{p}$$

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$$\frac{1}{N}\sum_{i=1}^{N} f(X_{n+i}) \xrightarrow[N\to+\infty]{} \mathbb{E}[f(X)]$$

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$$\overbrace{X_0 \to X_1 \to X_2 \to \cdots \to X_n}^{\text{Markov chain convergence}} \to \overbrace{X_{n+1} \to X_{n+2} \to \cdots \to X_{n+N}}^{\text{Monte Carlo sample}}$$

General framework of MCMC algorithms

MCMC algorithms uses an acceptance-rejection framework

> ① Initialise $x^{(0)}$
>
> ② For $t = 1 \dots n + N$:
>
> >   ⓐ Propose a new candidate $y^{(t)} \sim q(y^{(t)}|x^{(t-1)})$
> >   ⓑ Accept $y^{(t)}$ with probability $\alpha(x^{(t-1)}, y^{(t)})$:
> > $x^{(t)} := y^{(t)}$
> >   if $t > n$, "save" $x^{(t)}$ (as part of the final Monte Carlo sample)
>
> where q is the instrumental distribution for proposing new samples
>   and α is the acceptance probability.

Choosing the instrumental distribution

Not absolutely optimal choice for the instrumental distribution q
proposing new samples

⇒ infinite possibilities: some better than others

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To guaranty convergence towards the target $\tilde{p}$:

- the support of q has to cover the support of $\tilde{p}$
- q must not generate periodic values

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NB: *ideally q is* **easy** and **fast** to **compute**