Homework 4

By signing my name, I agree to abide by the Stern Code of Conduct ________________________

Question 1

a. Compute the mean $\bar{x}$ of the list of numbers 4, 3, 4, 4, 5

b. Consider the random variable X which has probability distribution function $p_X(1) = 1/10, p_X(2) = 1/10, p_X(3) = 2/10, p_X(4) = 4/10, p_X(5) = 2/10$. Draw a graph of the probability distribution function of X. Label the axes of your graph.

c. Is this graph a histogram? Why or why not?

d. Compute $E[X]$.

e. Compute Var(X). You can use `R`, or any calculator, to simplify the final answer.

f. Do you think it's plausible that the list from part (a) might have been sampled from the distribution of X?

Question 2

Suppose $Y \sim \text{Bin}(n, p)$, and recall these facts from class:

- $\mu = E[Y] = np$
- $\sigma^2 = \text{Var}(Y) = np(1-p)$
- Chebyshev's inequality: $P(|Y - \mu| \geq a) \leq \sigma^2/a^2$.
- This inequality says the probability that Y falls *outside* the range from $\mu - a$ to $\mu + a$ can't be too high, where the specific meaning of too high depends on σ^2 and a^2.

For this problem, let $p = 1/2$.

a. Simplify the formulas for μ and σ^2 using $p = 1/2$.

b. What value of a can you plug in to Chebyshev's inequality to make the right hand side less or equal to 1/2? Write out the inequality for that value of a, simplifying as much as you can, and using the formulas above for μ and σ^2.

c. Now let $n = 100$, so $np = 50$. According to Chebyshev's inequality, $P(y_L \leq Y \leq y_U) \leq 1/2$ for some specific numbers y_L and y_U. What are these two numbers? Round up y_U and round down y_L to the nearest integers.