ST308 Bayesian Inference

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Week 1: Exercises

1. Let $x = (x_1, \dots, x_n)$ be a random sample from an Exponential(λ) distribution. Set the prior for λ to be a Gamma(α, β) and derive its posterior distribution.

2. Let $x = (x_1, \dots, x_n)$ be a random sample from a $\mathrm{N}(\theta, \sigma^2)$ distribution with σ^2 known.

   (a) Show that the likelihood is proportional to

   $$f(x|\theta) \propto \exp\left(-\frac{n(\bar{x}-\theta)^2 + (n-1)S^2}{2\sigma^2}\right).$$

   where $\bar{x}$ is the sample mean and S^2 is the sample variance

   $$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

   Hence the likelihood simplifies to

   $$f(x|\theta) \propto \exp\left(-\frac{(\theta-\bar{x})^2}{2\frac{\sigma^2}{n}}\right)$$

   (b) Set the prior for θ to be $\mathrm{N}(\mu, \tau^2)$ and derive its posterior distribution. (You can use the above result)

3. Let $x = (x_1, \dots, x_n)$ be a random sample from a $\mathrm{N}(0, \sigma^2)$ distribution. Set the prior for σ^2 to be IGamma(α, β) and derive its posterior distribution.