ST308 Bayesian Inference

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Week 2: Exercises

1. Consider the vaccination example in the lecture slides.

   (a) Assume that a person is tested positive for immunity. Which of the decision rules have the lower posterior risk?

   (b) Repeat the above for the case that the person was tested negative.

   (c) Combine the two above cases and choose the optimal decision rule. Compare with the Bayes risk outcome.

2. Consider the quadratic error, absolute error and $0-1$ loss functions. Find the Bayes estimator for θ in the case of

   (a) A random sample $x = (x_1, ..., x_n)$ from a Normal(θ, 1). Assign a N(μ, τ^2) prior to θ.

   (b) A single observation x from a Binomial(n, θ). Assign a Beta(α, β) prior to θ.

3. Show that the bayes risk $r(\delta(x), \pi(\theta))$ can be written as averaging the posterior risk over x. In other words show that

$$\int R(\delta(x), \theta)\pi(\theta)d\theta = \int \rho(\delta(x), \pi(\theta))m(x)dx,$$

   assuming certain regularity conditions under which

$$\int_\Theta \int_\mathcal{X} L(\delta(x), \theta)h(x, \theta)dxd\theta = \int_\mathcal{X} \int_\Theta L(\delta(x), \theta)\pi(\theta|x)m(x)d\theta dx.$$

4. **Optional Exercise:** Let $x = (x_1, \ldots, x_n$ be a random sample from a Poisson(λ) distribution. Assign a Gamma(α, β) prior to λ. Consider the LINEX (LINear-EXponential) loss function.

$$L(a, \lambda) = \exp(k(a - \lambda)) - k(a - \lambda) - 1$$

   where k is a known positive constant. Find the Bayes estimator λ.