ST308 Bayesian Inference

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Week 5: Exercises

1. Let $y = (y_1, \dots, y_n)$ be a random sample from a $\mathrm{N}(\theta, \sigma^2)$ distribution with σ^2 known. Set the prior for θ to be $\mathrm{N}(\mu, \tau^2\sigma^2)$ and derive its posterior distribution. (You can use the above result)

2. Suppose that $y_i \sim N(\mu, 1)$ for $i = 1, \dots, n$ and that the y_i's are independent.

   (a) Show that the sample mean estimator $\hat{\mu}_1 = \frac{1}{n}\sum_i^n y_i$ is obtained from minimising the least squares criterion

   $$\hat{\mu}_1 = \underset{\mu}{\operatorname{argmin}} \sum_{i=1}^{n} (y_i - \mu)^2,$$

   and that $\hat{\mu}_1$ an unbiased estimator of μ. Also find the variance of $\hat{\mu}_1$.

   (b) Consider adding a penalty term to the least squares criterion, and therefore using the estimator that minimises

   $$\hat{\mu}_2 = \underset{\mu}{\operatorname{argmin}} \sum_{i=1}^{n} (y_i - \mu)^2 + \lambda\mu^2$$

   for the mean, where λ is a non-negative tuning parameter. Derive $\hat{\mu}_2$, find it bias and show that its variance is lower than that of $\hat{\mu}_1$

   (c) Find a Bayes estimator assuming the $\mathrm{N}(0, 1/\lambda)$ as prior for μ. Compare with your answer in the previous part.