ST451 Bayesian Machine Learning
Kostas Kalogeropoulos

Exercises

1. A company produces big magnetic roll tapes and wants to examine the number of defect spots per meter of tape. An experiment was conducted in which one meter of the tape was selected randomly and the number of defect spots was recorded. The procedure was repeated 5 times independently to provide the data $y = (2, 2, 6, 0, 3)$. Assume that these observations are independent and distributed according to the Poisson(λ) distribution.

   (a) Find the Jeffrey's prior for this model and derive the corresponding posterior distribution. Provide the mean, variance and a 95% credible interval for the number of defect spots per tape meter taking into account the data from the experiment.

   (b) From previous experience, the beliefs of the researcher about λ can be expressed by a Gamma distribution with mean and variance equal to 3. Repeat the previous part with this prior distribution and compare the results.

2. Let $y = (y_1, \ldots, y_n)$ be a random sample from the Exponential(λ) distribution

$$f(y_i|\lambda) = \lambda \exp(-\lambda y_i), \quad y_i > 0. \;\; \lambda > 0.$$

   Set the prior for λ to be a Gamma(α, β).

   (a) Derive the posterior distribution and find a Bayes estimator for λ. Also derive the predictive distribution for a new observation y_n (assuming that it also follows the same Exponential(λ) model).

   (b) [**Computer Exercise**] The data y are given below

   $11.8, 6.7, 26.8, 5.0, 5.4, 9.7, 1.6, 8.6, 15.8, 10.8, \; 32.8, 12.9, 0.2, 0.1, 6.4, 0.3, 15.2, 3.6, 18.4, 2.1.$

   In a similar manner to activity 3 of the computer class assess the performance of the predictive distribution via the steps below

      i. Split the data into a training set (first 10 observations) and a test set (remaining observations).

      ii. Specify the predictive distribution via the training set and use it to obtain point forecasts for the test set.

      iii. Repeat the previous step but with 95% prediction intervals rather than point forecasts.

   Note that the Exponential(λ) is the same as the Gamma($1, \lambda$).