ST308 Bayesian Inference

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Week 3: Exercises

1. A big magnetic roll tape needs tape. An experiment is being conducted in which each time 1 meter of the tape is examined randomly. The procedure is repeated 5 times and the number of defects is recorded to be 2,2,6,0 and 3 respectively. The researcher assumes a Poisson distribution for the parameter λ. From previous experience, the beliefs of the researcher about λ can be expressed by a Gamma distribution with mean and variance equal to 3. Derive the posterior distribution that will be obtained. What would be the expected mean and variance of the number of defects per tape meter after the experiment?

2. Let $x = (x_1, \ldots, x_n)$ be a random sample from a Negative Binomial Distribution(m, θ) distribution. Set the prior for θ to be a Beta(α, β) and derive its posterior distribution. Then find the Jeffreys prior and derive the corresponding posterior.

3. Let $x = (x_1, \ldots, x_n)$ be a random sample from a $\mathrm{N}(\mu, \sigma^2)$ distribution with μ known. Find the Jeffreys' prior for σ^2 and derive the corresponding posterior distribution.

4. **Optional Exercise:** Let $x = (x_1, \ldots, x_n)$ be a random sample from a $\mathrm{N}(\theta, \sigma^2)$. Assign the Jeffreys' prior $\pi(\theta, \sigma^2) \propto (\sigma^2)^{-1}$. Find the marginal posterior $\pi(\theta|x)$.

   Hint: Consider the joint posterior up to proportionality and integrate σ^2 out, to get the marginal posterior up to proportionality. Then consider the centralised verion of θ, $T = \frac{\theta - \bar{x}}{S/\sqrt{n}}$ and match its kernel to a t_{n-1} distribution.