

ST308 Bayesian Inference

Week 6

Exercises

1. Consider the case of Linear Discriminant Analysis with a scalar x with data $(y_i, x_i)_{i=1}^n$, where y_i 's are binary random variables and x_i 's continuous. Assume that $x_i \sim N(\mu_0, \sigma^2)$ in category c_0 and that $x_i \sim N(\mu_1, \sigma^2)$ in category c_1 and that they are independent. Further assume that each y_i is a Bernoulli random variable with probability of success $p(y \in c_1|x)$ and that the y_i 's are independent. Finally, the prior probability $\pi(y \in c_1) = \pi$. Write down the likelihood function and provide the maximum likelihood estimators for π, μ_0, μ_1 and σ^2 .
2. Let $y = (y_1, \dots, y_n)$ be a r.s. from a $\text{Poisson}(\lambda)$ and assign the an improper prior to λ such that $\pi(\lambda) \propto \lambda^{-1/2}$. Find the Laplace approximation to the posterior based on the mode and the Hessian matrix of $\pi^*(\lambda|y) = f(y|\lambda)\pi(\lambda)$.