ST308 Bayesian Inference

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Week 4: Exercises

1. Let $y = (y_1, \ldots, y_{10})$ be a sample of independent and identically distributed (iid) random variables from the Normal distribution with unknown mean θ and known variance 1. The sample mean $\bar{y}$ is recorded to be 0.3.

   (a) Suppose that we are interested in only two values of θ s, i.e. 0 and 1, and no prior knowledge is available. Assign a suitable prior on θ, justifying your choice, and derive the corresponding posterior. Consider the hypotheses $H_0 : \theta = 0$ and $H_1 : \theta = 1$, calculate the Bayes factor for comparing H_1 in reference to H_0 and interpret its value.

   (b) Now consider the hypotheses $H_0 : \theta \leq 0$ and $H_1 : \theta > 0$. Calculate the Bayes factor for comparing H_1 in reference to H_0 and interpret its value.

   (c) Finally, consider the hypotheses $H_0 : \theta = 0$ and $H_1 : \theta \neq 0$. Can you calculate the Bayes factor in this case? Justify your answers.

2. Find the predictive distribution based on x for a future observation y in the following cases

   (a) An observation x from a Binomial(n, θ) likelihood with a Beta(α, β) prior for θ.

   (b) An random sample $x = (x_1, \ldots, x_n)$ from a Poisson(λ) likelihood with a Gamma(α, β) prior for λ.