Mathematics for Political Science

Day 1: Introduction, Foundations, Pre-Calculus

Solutions

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1. (a) all qualitative except for *coffee quality*
   (b) Nominal, Nominal, Ratio, Ordinal (interval if you can argue it)Nominal

2. (a)

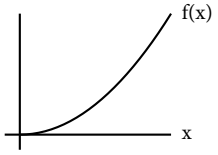
x	$f(x) = (3-x)^2$	$g(x) = 2x^3 - 4$	$f(g(x))$	$g(f(x))$
2	1	12	81	-2
4	1	124	14641	-2
5	4	246	59049	124
1	4	-2	25	124
0	9	-4	49	1454
1	4	-2	25	124

   (b)
   - $f(g(x)) = 49 - 28x^3 + 4x^6$
   - $g(f(x)) = 2(3-x)^6 - 4$

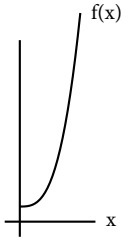
3. (a) $g(x) = 8x - 2, f(x) = 4x^3$
   (b) $g(x) = 3x - 2, f(x) = 1/x$

4. The open interval does not include the endpoint. Since the interval is limited by (but does not include) 1, it would be possible to get infinitely close to (but never reach) that point, and any number less than 1 has another point greater than that but less than 1.

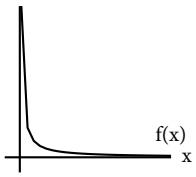
5. (a)



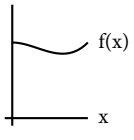
(b)



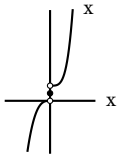
(c)



(d)



(e)



6. Approximately linear growth: $p \approx 282,200,000 + 2,800,000 * y$ (if we assume that $y = 0$ is the year 2,000)

7. ConCAVE looks like the entrance to a cave, conVex looks like a V.

8. (a) monotonic
   (b) non-monotonic
   (c) non-monotonic

9. (a) $39/2$
   (b) 288

10. (a) $-x^8y^4$
    (b) 9
    (c) $8a^6$
    (d) x
    (e) $y^3 + y^4 + y^5$
    (f) $\dfrac{10a}{77b}$
    (g) $4 - \ln(3)$
    (h) 0

11. (a) a^2
    (b) $3pq^2 + 6p^2q + 3p^3 - pq + x(4q^2 + 16pq + 16p^2)$

12. (a) B wins 29,000 to 28,000
    (b) \$265,625 more, for a total of \$1,265,625

13. $56,000$

14. (a) $x = \frac{1}{3}$
    (b) $x = \frac{3}{4}$

15. (a) $\alpha = \beta + 4\theta$
    (b) $\alpha = \frac{4}{(x+y-x^2-y^2)}$

16. (a) $x > -18$
    (b) $t < 6$
    (c) $y \leq \frac{29}{22}$

17. (a) $x = 2$ or $x = -7$
    (b) $x = 4$
    (c) $x = 2$ or $x = -5$

18. (a) $x = \frac{1}{9}$ or $x = -1.5$
    (b) $x = -\frac{2}{7}$ or $x = \frac{4}{5}$

19. (a) $a = 0, b = 2$
    (b) $a = 5, b = 5$

20. (a) $c = 7, d = -2$

(b) $c = -3, d = 4$

21. $x = 4\alpha + 2, y = 2\alpha + 1$

22. $q = 1, r = -1, s = 3$

23. (a) 25
    (b) 22

24. Odd powers are identical to the matrix given; even powers are the identity matrix.

25.
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a*1+b*0 & a*0+b*1 \\ c*1+d*0 & c*0+d*1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

26. (a) $\begin{bmatrix} 56 & 70 \end{bmatrix}$
    (b) $\begin{bmatrix} ap+bq+cr \\ dp+eq+fr \\ gp+hq+ir \end{bmatrix}$
    (c) The inner dimensions do not conform, so these matrices cannot be multiplied in this order.

27. Multiply the matrices below to show that order matters for matrix multiplication:

    (a) $\begin{bmatrix} 17 \end{bmatrix}$ or $\begin{bmatrix} 12 & 21 & 3 \\ 0 & 0 & 0 \\ 20 & 35 & 5 \end{bmatrix}$

    (b) $\begin{bmatrix} 44 & 64 & 36 \\ 15 & 36 & 21 \\ 20 & 22 & 12 \end{bmatrix}$ or $\begin{bmatrix} 48 & 114 \\ 15 & 44 \end{bmatrix}$

Thanks to Dave Ohls, Brad Jones, and Sarah Bouchat for past years' materials