Mathematics for Political Science

Lesson 3: Calculus

Exercises

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1. (Gill 5.1 [adapted]) Find the following finite limits:
   (a) $\lim_{x \to 4}[x^2 - 6x + 4]$
   (b) $\lim_{x \to 0}[\frac{x - 25}{x + 5}]$
   (c) $\lim_{x \to 4}[\frac{x^2}{3x - 2}]$
   (d) $\lim_{x \to 1}[\frac{x^2 - 1}{x - 1}]$
2. (Gill 5.3 [adapted]) Find the following infinite limits and graph:
   (a) $\lim_{x \to \infty}[\frac{9x^2}{x^2 + 3}]$
   (b) $\lim_{x \to \infty}[\frac{3x - 4}{x + 3}]$
   (c) $\lim_{x \to \infty}[\frac{2^x - 3}{2^x + 1}]$
3. (Gill 5.5 [adapted]) Calculate the following derivatives:
   (a) $\frac{d}{dx} 3x^{\frac{1}{3}}$
   (b) $\frac{d}{dt}(14t - 7)$
   (c) $\frac{d}{dy}(y^3 + 3y^2 - 12)$
   (d) $\frac{d}{dx}(x^2 + 1)(x^3 - 1)$
   (e) $\frac{d}{dy}(y^3 - 7)(1 + \frac{1}{y^2})$
   (f) $\frac{d}{dy}(y - y^{-1})(y - y^{-2})$
   (g) $\frac{d}{dx} \frac{4x - 12x^2}{x^3 - 4x^2}$
   (h) $\frac{d}{dy} e^{y^2 - 3y + 2}$
   (i) $\frac{d}{dx} \ln(2\pi x^2)$
4. Consider the function $k(x) = 2(8(x^4 + 2) - 1)^2$. Find the derivative by:
   (a) Expanding the polynomial and calculating the derivative using the power rule.
   (b) Expressing $k(x)$ as the result of three nested functions $f(g(h(x)))$ and applying the chain rule.

   Show that these approaches yield the same answer.
5. For each of the functions:

$$f(x) = 3x^2 - 7x + 2 \qquad g(x) = 8x^3 - 46x^2 + 73x - 35$$

   (a) Sketch a plot the function on the interval $[0, 5]$ (calculate f(x) for integer values of x to get a general idea of the shape of the function).

(b) Identify the values of x that generate local maxima or minima (ignoring endpoints).

(c) Show mathematically whether these are maxima or minima.

6. Find the value of x that maximizes the function $\ell(x) = 2\ln(x) - x - \ln(2x+1)$ using the following approach.

   (a) Take the derivative of $\ell(x)$ and set it equal to 0.

   (b) Manipulate the expression to remove fractions and express it as a quadratic.

   (c) Solve for x.

7. Find the partial derivatives of the function $(eR(\frac{f}{f+g}))^h$ with respect to e and f.

8. (Gill 5.13 [adapted]) Calculate the following indefinite integrals:

   (a) $\int 4y^3 dy$

   (b) $\int (x^2 - x^{-\frac{1}{2}}) dx$

   (c) $\int 360t^6 dt$

9. (Gill 5.10 [adapted]) Solve the following definite integrals using the antiderivative method:

   (a) $\int_6^8 x^3 dx$

   (b) $\int_1^9 2y^5 dy$

   (c) $\int_{-1}^0 (3x^2 - 1) dx$

   (d) $\int_{-1}^1 (14 + x^2) dx$

   (e) $\int_2^4 e^y dy$

   (f) $\int_2^4 \sqrt{t} dt$

10. (Gill 5.11) Calculate the area of the following function that lies above the x-axis and over the domain $[-10, 10]$:

$$f(x) = 4x^2 + 12x - 18$$