

Mathematics for Political Science

Day 2: Algebra

Solutions

1. (a) $x = \frac{1}{3}$
(b) $x = \frac{3}{4}$
2. (a) $\alpha = \beta + 4\theta$
(b) $\alpha = \frac{4}{(x+y-x^2-y^2)}$
3. (a) $x > -18$
(b) $t < 6$
(c) $y \leq \frac{29}{22}$
4. (a) $x = 2$ or $x = -7$
(b) $x = 4$
(c) $x = 2$ or $x = -5$
5. (a) $x = \frac{1}{9}$ or $x = -1.5$
(b) $x = -\frac{2}{7}$ or $x = \frac{4}{5}$
6. (a) $a = 0, b = 2$
(b) $a = 5, b = 5$
7. (a) $c = 7, d = -2$
(b) $c = -3, d = 4$
8. $x = 4\alpha + 2, y = 2\alpha + 1$
9. $q = 1, r = -1, s = 3$
10. (a) 25
(b) 22
11. Odd powers are identical to the matrix given; even powers are the identity matrix.
12.
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a*1+b*0 & a*0+b*1 \\ c*1+d*0 & c*0+d*1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
13. (a) $\begin{bmatrix} 56 & 70 \end{bmatrix}$
(b) $\begin{bmatrix} ap + bq + cr \\ dp + eq + fr \\ gp + hq + ir \end{bmatrix}$
(c) The inner dimensions do not conform, so these matrices cannot be multiplied in this order.
14. Multiply the matrices below to show that order matters for matrix multiplication:

$$(a) \begin{bmatrix} 17 \end{bmatrix} \text{ or } \begin{bmatrix} 12 & 21 & 3 \\ 0 & 0 & 0 \\ 20 & 35 & 5 \end{bmatrix}$$

$$(b) \begin{bmatrix} 44 & 64 & 36 \\ 15 & 36 & 21 \\ 20 & 22 & 12 \end{bmatrix} \text{ or } \begin{bmatrix} 48 & 114 \\ 15 & 44 \end{bmatrix}$$

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