Mathematics for Political Science

Day 1: Introduction, Foundations, Pre-Calculus
Solutions

1. (a)
$$x = \frac{1}{3}$$

(b)
$$x = \frac{3}{4}$$

2. (a)
$$\alpha = \beta + 4\theta$$

(b)
$$\alpha = \frac{4}{(x+y-x^2-y^2)}$$

3. (a)
$$x > -18$$

(b)
$$t < 6$$

(c)
$$y \le \frac{29}{22}$$

4. (a)
$$x = 2$$
 or $x = -7$

(b)
$$x = 4$$

(c)
$$x = 2$$
 or $x = -5$

5. (a)
$$x = \frac{1}{9}$$
 or $x = -1.5$

(b)
$$x = -\frac{2}{7}$$
 or $x = \frac{4}{5}$

6. (a)
$$a = 0, b = 2$$

(b)
$$a = 5, b = 5$$

7. (a)
$$c = 7$$
, $d = -2$

(b)
$$c = -3, d = 4$$

8.
$$x = 4\alpha + 2$$
, $y = 2\alpha + 1$

9.
$$q = 1, r = -1, s = 3$$

11. Odd powers are identical to the matrix given; even powers are the identity matrix.

12.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a*1+b*0 & a*0+b*1 \\ c*1+d*0 & c*0+d*1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(b)
$$\begin{bmatrix} ap + bq + cr \\ dp + eq + fr \\ gp + hq + ir \end{bmatrix}$$

(c) The inner dimensions do not conform, so these matrices cannot be multiplied in this order.

14. Multiply the matrices below to show that order matters for matrix multiplication:

(a)
$$\begin{bmatrix} 17 \end{bmatrix}$$
 or $\begin{bmatrix} 12 & 21 & 3 \\ 0 & 0 & 0 \\ 20 & 35 & 5 \end{bmatrix}$

(a)
$$\begin{bmatrix} 17 \end{bmatrix}$$
 or $\begin{bmatrix} 12 & 21 & 3 \\ 0 & 0 & 0 \\ 20 & 35 & 5 \end{bmatrix}$
(b) $\begin{bmatrix} 44 & 64 & 36 \\ 15 & 36 & 21 \\ 20 & 22 & 12 \end{bmatrix}$ or $\begin{bmatrix} 48 & 114 \\ 15 & 44 \end{bmatrix}$