Problem Set 1

August 19, 2019

1. Simplify the following expressions as much as possible.
   (a) $\frac{2x^2+20x+50}{2x^2-50} =$
   (b) $\frac{5+17x+4x+7}{42x} =$
   (c) $((qr)^\gamma)^\delta =$
   (d) $\sqrt{x} \times \sqrt[5]{x} =$
   (e) $\log(3x) - 2\log(x+2) =$

2. Ikuma bought a total of 30 bottles of water and coke. A water bottle costs \$2, and a bottle of coke costs \$3. He spent \$77 in total. How many water did he buy?

3. Solve the following equations/inequalities for x
   (a) $x^2 + 2x - 35 = 0$
   (b) $x^2 + 14 = -14x$
   (c) $x^3 - 4x = 0$
   (d) $\frac{1}{3}x^2 + \frac{2}{3}x - 16 = 0$
   (e) $x^2 - x - 6 \geq 0$

4. Graph the following expressions.
   (a) $y = -x^2 + 2x + 5$
   (b) $y = 3|x| - 4$
   (c) $y \geq -2x + 7$
   (d) $y \leq x^2 + 3x - 10$

5. Are the expressions (i) and (ii) equal?
   (a) (i) $\sum_{i=1}^n x_i \sum_{i=1}^n y_i$ (ii) $\sum_{i=1}^n x_i y_i$
   (b) (i) $\prod_{i=1}^n \exp(x_i)$ (ii) $\exp(\sum_{i=1}^n x_i)$

6. Prove that 3 times the sum of 3 squares is also the sum of 4 squares.[1]

[1] Charles L. Dodgson (a.k.a Lewis Carroll). *Pillow Problems*. No. 14.