Problem Set 3

August 21, 2019

1. Find $x^\star$ and $y^\star$ which minimize the function $f(x, y) = x^2 + 2xy + 2y^2 - 6x + 4y + 10$

2. Calculate the following integrals.
   (a) $\int (4x^4 + 3x^3 + 2x^2 + x + 1)dx$
   (b) $\int (x^{-1} + 3x^2)dx$
   (c) $\int (\log 3x)dx$
   (d) $\int ((\log x)^2)dx$
   (e) $\int (x + 2)(x^2 + 4x + 3)^2 dx$
   (f) $\int x\sqrt{1 - x}dx$ (Hint: set $t = \sqrt{1 - x}$ and perform integration by substitution)
   (g) $\int_2^3 \frac{1}{x^2-1}dx$ (Hint: first factor $x^2 - 1$, then express $\frac{1}{x^2-1}$ as a difference of two fractions)
   (h) $\int_0^a \lambda e^{-\lambda x}dx$
   (i) $\int_0^\infty x\lambda e^{-\lambda x}dx$
   (j) $\int_0^\infty x^2\lambda e^{-\lambda x}dx$

3. The function

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt \quad (x > 0)$$

   is called the Gamma function. Show that

$$\Gamma(x + 1) = x\Gamma(x)$$