Problem Set 2

August 20, 2019

1. Calculate the following limits.
   (a) $\lim_{x\to 4}(x-3)(x+5)$
   (b) $\lim_{x\to 2}\frac{x^3-8}{x-2}$
2. Calculate the following derivatives.
   (a) $\frac{d}{dx}(5x^7+7x^4+3)$
   (b) $\frac{d}{dx}(3x^2+4)(2x^3+3x+5)$
   (c) $\frac{d}{dx}xe^{-ax^2}$
   (d) $\frac{d}{dx}\log(x^3-2)$
   (e) $\frac{d^n}{dx^n}x^n$
   (f) $\frac{d}{dx}\{\log(e^x+1)\}^2$
3. Find all the extrema (local and global) of the following functions on the specified domains, and state whether each extremum is a minimum or maximum and whether each is only local or global on that domain.
   (a) $f(x)=4x^3+x^2-2x+3$ $(x\in[-1,1])$
   (b) $f(x)=6\log(x+1)-\frac{1}{2}x^2+1$ $(x\in[0,4])$ (Hint: $\log(5)\approx 1.6$)
4. (Cournot game) Suppose that only two firms, A and B, are selling product j. In this market, the price, p is determined by the function
$$p=1000-(q_A+q_B)$$
where q_A and q_B are the quantities of j produced by firm A and B, respectively. Assume that the total cost of producing q_i $(i\in\{A,B\})$ equals q_i (i.e., marginal cost of producing one product is just 1 for both firms). Let $q_i^\star$ $(i\in\{A,B\})$ denote the quantity which maximizes the profit for firm i.
   (a) What is the equation representing profit for firm A, π_A, when the firm B produces a quantity $q_B^\star$? (Hint: Profit is calculated as the total revenue minus the total cost)
   (b) Assume that firms A and B determine the quantities they produce at the same time. How many j should they produce to maximize their profits?