Chapter 6: Introduction to Hypothesis Testing

Modern Clinical Data Science
Chapter Guides
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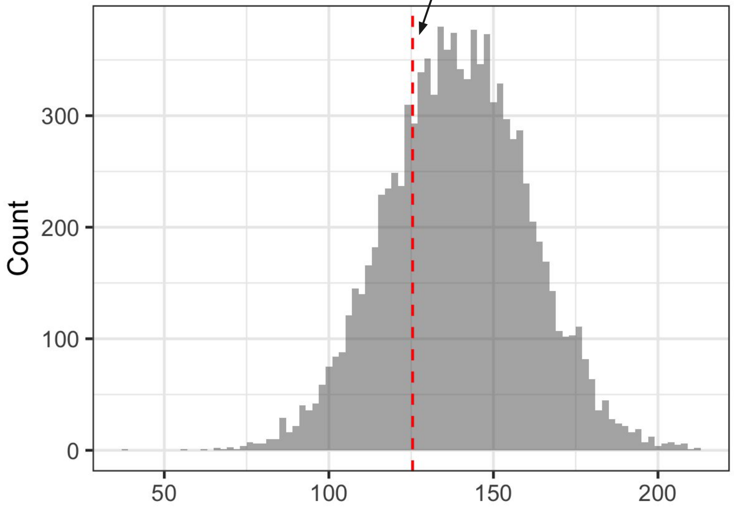
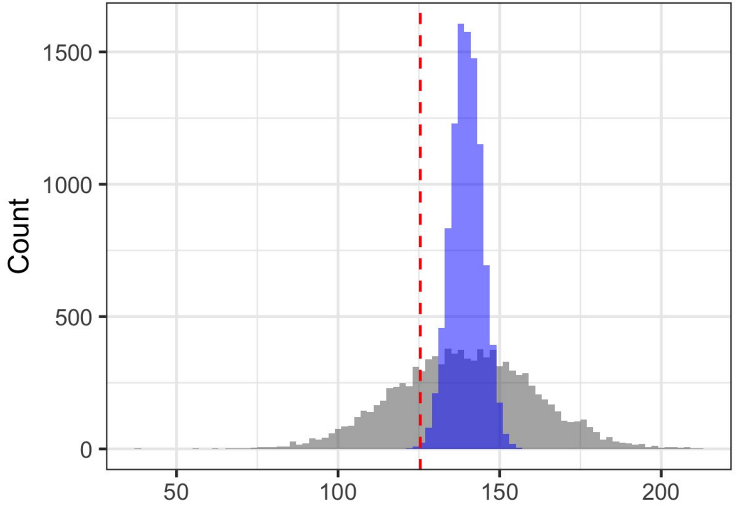
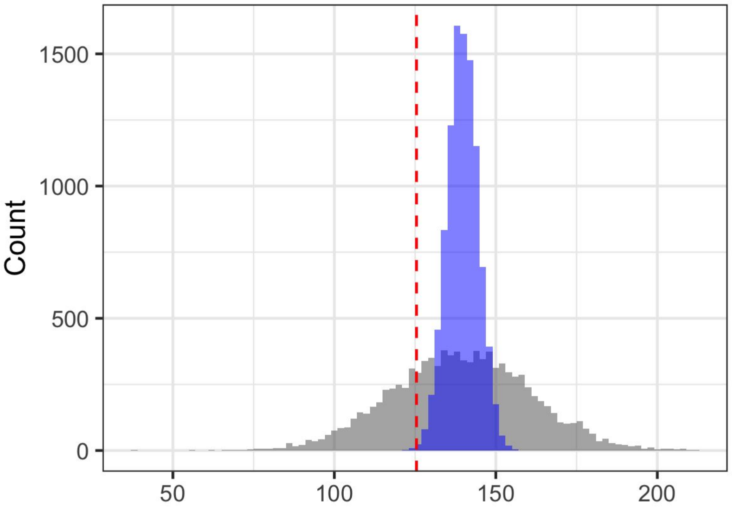
How to Use this Guide

- Read the corresponding notes chapter first
- Try to answer the discussion questions on your own
- Listen to the chapter guide (should be 15 min, max) while following along in the notes

Mean SBP for 20 men sampled
from a small town in Appalachia







$$x \sim \mathcal{N}(\mu, \sigma)$$

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x^{(i)}$$

$$\sim \mathcal{N}(\mu, \sigma/\sqrt{n})$$

Question 6.1

If $n = 1$, what is the standard deviation of the sample mean? If $n = \infty$, what is the standard deviation of the sample mean?

Question 6.2

The sample mean for our 20 sampled Appalachian men is shown as a vertical red dashed line in the figure above. Now that you know what the distribution of the sample mean looks like, do you think the observation from your Appalachian town is "weird"?

Z Test

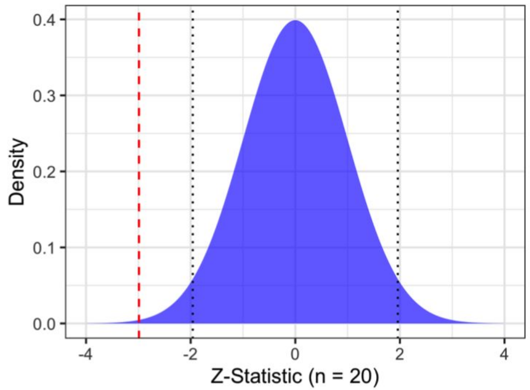
Null hypothesis:

$$H_0 : \mu_c = \mu_0$$
$$H_a : \mu_c \neq \mu_0$$

Test statistic:

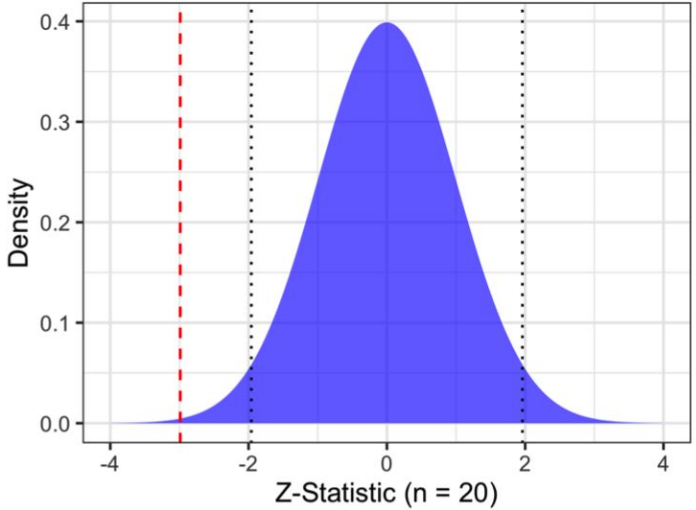
$$Z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} \qquad \text{where} \qquad \overline{x} = \frac{1}{n} \sum_{i=1}^{n} x^{(i)}$$

Null distribution w/critical values:



Question 6.3

As α gets smaller, are you more or less likely to reject the null for the same value of the test statistic? Hint: What does making α smaller do to the positions of the two black dotted lines in the figure, above?



	AA	Aa	aa	
X	52	43	5	100
Control	67	27	6	100
	119	70	11	200

Under scenario of independence (E):

	AA	Aa	aa	
X	59.5	35.0	5.5	100
Control	59.5	35.0	5.5	100
	119	70	11	200

Chi-Squared Test

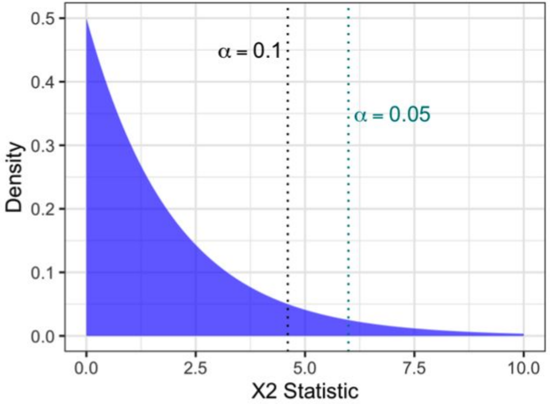
Null hypothesis:

$$H_0 : G \perp\!\!\!\perp D$$
$$H_a : G \not\perp\!\!\!\perp D$$

Test statistic:

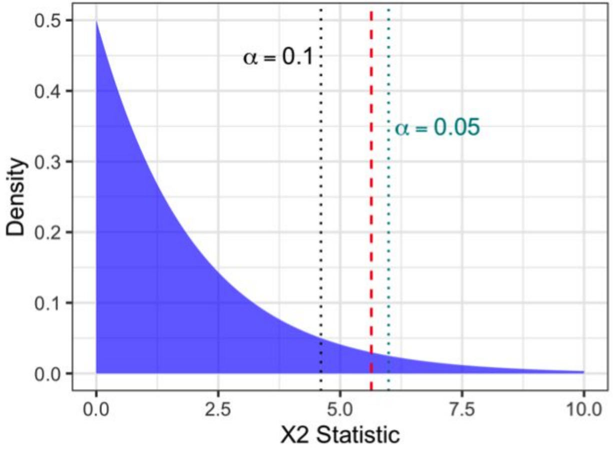
$$X^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Null distribution w/critical values:



Question 6.4

Using the formula in step 4, above, compute the actual value of the chi-squared test statistic for this example. Hint: You should end up with a value that corresponds to the position of the red dashed line in the figure below.



$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

	AA	Aa	aa	
X	52	43	5	100
Control	67	27	6	100
	119	70	11	200

Under scenario of independence (E):

	AA	Aa	aa	
X	59.5	35.0	5.5	100
Control	59.5	35.0	5.5	100
	119	70	11	200

One Sample T Test

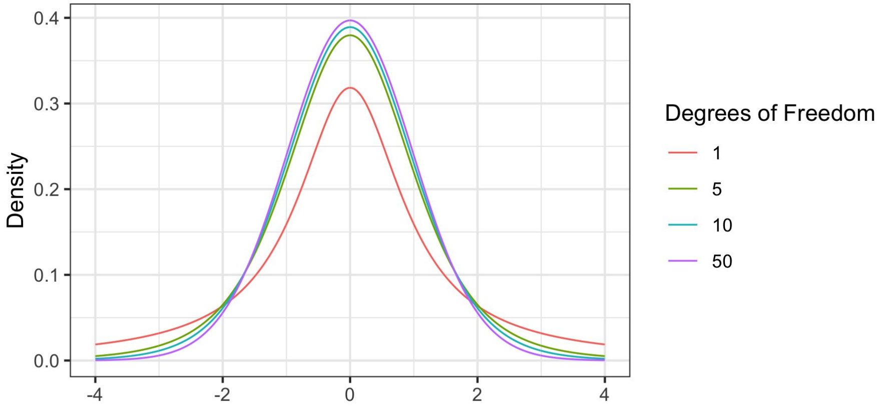
Null hypothesis:

$$H_0 : \mu_c = \mu_0$$
$$H_a : \mu_c \neq \mu_0$$

Test statistic:

$$T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x^{(i)} - \overline{x})^2}$$

Null distribution (n-1 degrees of freedom):



Question 6.5

Compare the formula for the sample standard deviation to the maximum likelihood estimate of the parameter, σ, of a normal distribution (Section 5.3.3). What is the same/different? Note in particular the use of $n-1$ in the denominator, rather than n. This arises because the MLE for σ, $\hat{\sigma}$, is a **biased** estimate of the population standard deviation (more on this later). For large n, however, the two are nearly identical.

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x^{(i)} - \hat{\mu}\right)^2} \qquad\qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x^{(i)} - \overline{x})^2}$$

Pearson's Two Sample T Test

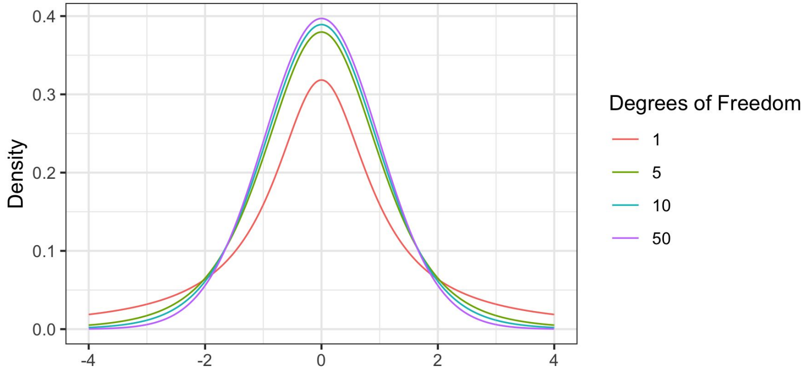
Null hypothesis:

$$H_0 : \mu_x = \mu_y$$
$$H_z : \mu_x \neq \mu_y$$

Test statistic:

$$T = \frac{\overline{x} - \overline{y}}{s_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \qquad s_p^2 = \frac{(n-1)s_x^2 + (m-1)s_y^2}{m+n-2}$$

Null distribution (m+n-2 degrees of freedom):



Welch's Two Sample T Test

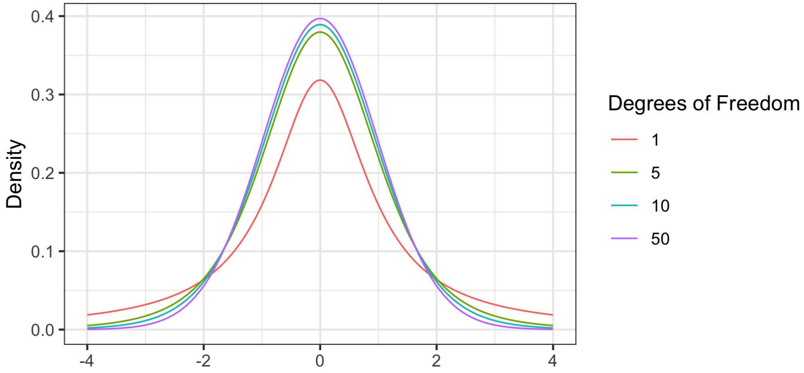
Null hypothesis:

$$H_0 : \mu_x = \mu_y$$
$$H_z : \mu_x \neq \mu_y$$

Test statistic:

$$T = \frac{\overline{x} - \overline{y}}{s_{xy}} \qquad s_{xy} = \sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}$$

Null distribution (degrees of freedom given by Welch-Sattherwaite):



T-Test for Matched Pairs

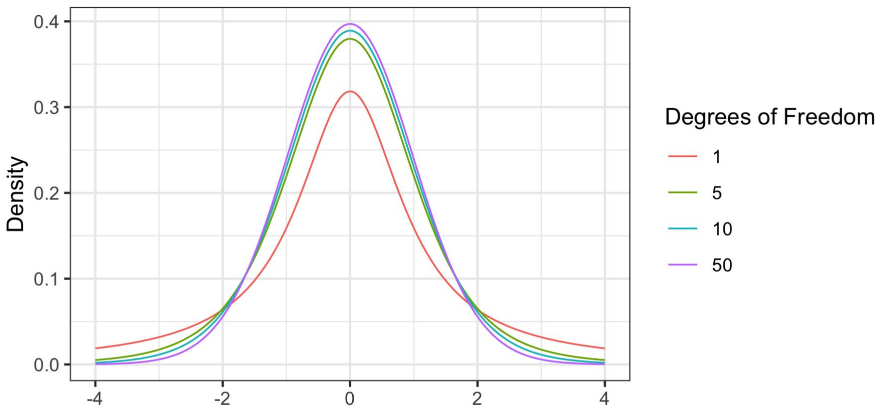
Null hypothesis (one-sample T-test of differences against μ_0=0):

$$H_0 : \mu_c = \mu_0$$
$$H_a : \mu_c \neq \mu_0$$

Test statistic:

$$T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x^{(i)} - \overline{x})^2}$$

Null distribution (n-1 degrees of freedom):



Question 6.6

Here are some sample data. They come from a study that looked at the effect of ozone, a component of smog, on the weight gain of rats. (Original source: Biometrika 63: 421-434, 1976, reproduced in Rice's *Mathematical Statistics and Data Analysis*, p. 465.) A group of 22 seventy-day-old rats were kept in an environment containing ozone for 7 days, and their weight gains were recorded. Another group of 23 rats of a similar age were kept in an ozone-free environment for a similar time and their weight gains were also recorded. Here are the data for the control group:

	group	original_weight	weight_gain
1	control	340.8	41.0
2	control	389.1	25.9
3	control	355.2	13.1
4	control	421.8	-16.9
5	control	377.1	15.4
6	control	404.3	22.4
7	control	321.2	29.4
8	control	447.5	26.0
9	control	305.9	38.4
10	control	335.9	21.9
11	control	386.3	27.3
12	control	377.0	17.4
13	control	357.2	27.4
14	control	441.7	17.7
15	control	383.7	21.4
16	control	373.7	26.6
17	control	336.0	24.9
18	control	419.4	18.3
19	control	287.1	28.5
20	control	602.8	21.8
21	control	325.4	19.2
22	control	452.4	26.0
23	control	398.9	22.7
Mean	control	384.4	22.4
St.Dev.	control	65.5	10.8

	group	original_weight	weight_gain
1	ozone	437.4	10.1
2	ozone	275.9	7.3
3	ozone	296.3	-9.9
4	ozone	295.9	17.9
5	ozone	379.7	6.6
6	ozone	274.1	39.9
7	ozone	360.0	-14.7
8	ozone	331.9	-9.0
9	ozone	531.8	6.1
10	ozone	350.5	14.3
11	ozone	345.7	6.8
12	ozone	268.1	-12.9
13	ozone	339.9	12.1
14	ozone	352.4	-15.9
15	ozone	435.8	44.1
16	ozone	476.9	20.4
17	ozone	462.5	15.5
18	ozone	368.0	28.2
19	ozone	504.3	14.0
20	ozone	188.0	15.7
21	ozone	466.9	54.6
22	ozone	288.8	-9.0
Mean	ozone	365.0	11.0
St.Dev.	ozone	88.6	19.0

(a) Imagine that the population weight distribution of rats is known to be normal with $\mu = 350$ (grams) and unknown σ. How would you test the hypothesis that the mean of the control group is equal to the population mean? How would you test the hypothesis that the mean of the ozone group is equal to the population mean?

(b) How would you test the hypothesis that the mean original weights of the ozone and control groups are equal? Do not assume equal variance.

(c) How would you test the hypothesis that the mean weight gain in the ozone group is equal to the mean weight gain in the control group? Do not assume equal variance.

(d) How would your approach in part (c) change if you assumed the weight gains in the two groups had equal variance?

one-sample T-test of control group original weight mean

one-sample T-test of ozone group original weight mean

two-sample T-test of ozone vs. control original weight means (Welch's version)

two-sample T-test of mean weight gain in ozone vs. control (Welch's version)

two-sample T-test of mean weight gain in ozone vs. control (Pearson's version)