Chapter 14: Introduction to Boosting

Modern Clinical Data Science
Chapter Guides
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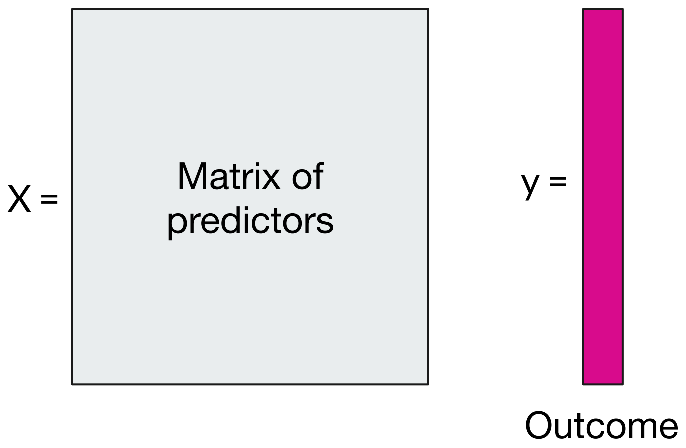
How to Use this Guide

- Read the corresponding notes chapter first
- Try to answer the discussion questions on your own
- Listen to the chapter guide (should be 30 min, max) while following along in the notes

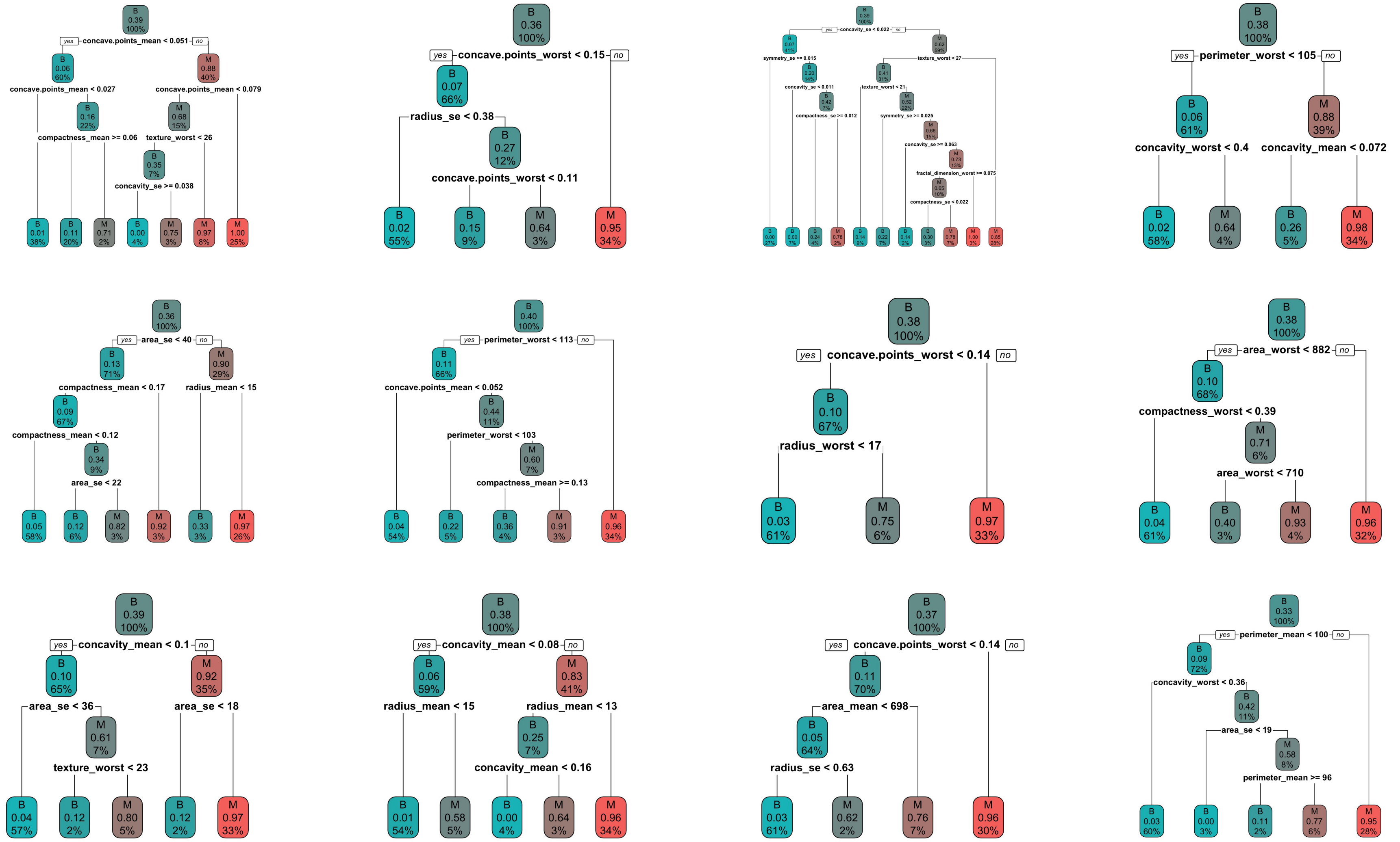
So far…

- Regression
- K-Nearest Neighbors (KNN)
- Decision trees
- Random forests

What are their stories?



Ensemble learning:
Combining multiple models that are only slightly better than random (weak learners) can produce a good model (strong learner).


B 0.39 100%
yes concave.points_mean < 0.051 no
B 0.06 60%
M 0.88 40%
concave.points_mean < 0.027
concave.points_mean < 0.079
B 0.16 22%
M 0.68 15%
compactness_mean >= 0.06
texture_worst < 26
B 0.35 7%
concavity_se >= 0.038
B 0.01 38%
B 0.11 20%
M 0.71 2%
B 0.00 4%
M 0.75 3%
M 0.97 8%
M 1.00 25%
B 0.36 100%
yes concave.points_worst < 0.15 no
B 0.07 66%
radius_se < 0.38
B 0.27 12%
concave.points_worst < 0.11
B 0.02 55%
B 0.15 9%
M 0.64 3%
M 0.95 34%
concavity_se < 0.022
symmetry_se >= 0.015
texture_worst < 27
concavity_se < 0.011
texture_worst < 21
compactness_se >= 0.012
symmetry_se >= 0.025
concavity_se >= 0.063
fractal_dimension_worst >= 0.075
compactness_se < 0.022
B 0.38 100%
yes perimeter_worst < 105 no
B 0.06 61%
M 0.88 39%
concavity_worst < 0.4
concavity_mean < 0.072
B 0.02 58%
M 0.64 4%
B 0.26 5%
M 0.98 34%
B 0.36 100%
yes area_se < 40 no
B 0.13 71%
M 0.90 29%
compactness_mean < 0.17
radius_mean < 15
B 0.09 67%
compactness_mean < 0.12
B 0.34 9%
area_se < 22
B 0.05 58%
B 0.12 6%
M 0.82 3%
M 0.92 3%
B 0.33 3%
M 0.97 26%
B 0.40 100%
yes perimeter_worst < 113 no
B 0.11 66%
concave.points_mean < 0.052
B 0.44 11%
perimeter_worst < 103
M 0.60 7%
compactness_mean >= 0.13
B 0.04 54%
B 0.22 5%
B 0.36 4%
M 0.91 3%
M 0.96 34%
B 0.38 100%
yes concave.points_worst < 0.14 no
B 0.10 67%
radius_worst < 17
B 0.03 61%
M 0.75 6%
M 0.97 33%
B 0.38 100%
yes area_worst < 882 no
B 0.10 68%
compactness_worst < 0.39
M 0.71 6%
area_worst < 710
B 0.04 61%
B 0.40 3%
M 0.93 4%
M 0.96 32%
B 0.39 100%
yes concavity_mean < 0.1 no
B 0.10 65%
M 0.92 35%
area_se < 36
area_se < 18
M 0.61 7%
texture_worst < 23
B 0.04 57%
B 0.12 2%
M 0.80 5%
B 0.12 2%
M 0.97 33%
B 0.38 100%
yes concavity_mean < 0.08 no
B 0.06 59%
M 0.83 41%
radius_mean < 15
radius_mean < 13
B 0.25 7%
concavity_mean < 0.16
B 0.01 54%
M 0.58 5%
B 0.00 4%
M 0.64 3%
M 0.96 34%
B 0.37 100%
yes concave.points_worst < 0.14 no
B 0.11 70%
area_mean < 698
B 0.05 64%
radius_se < 0.63
B 0.03 61%
M 0.62 2%
M 0.76 7%
M 0.96 30%
B 0.33 100%
yes perimeter_mean < 100 no
B 0.09 72%
concavity_worst < 0.36
B 0.42 11%
area_se < 19
M 0.58 8%
perimeter_mean >= 96
B 0.03 60%
B 0.00 3%
B 0.11 2%
M 0.77 6%
M 0.95 28%

Boosting:
A principled way of creating weak learners whose combined predictions get better over time.

AdaBoost

First practical boosting algorithm. For a long time, no one knew why it worked.

1. Initialize the observation weights to $w_i^{(1)} = \frac{1}{N}$ for $i = 1, \ldots, N$.
2. For $m = 1, \ldots, M$:
   (a) Select a classifier, $G_m(x)$, that minimizes the weighted training error according to the current set of weights, $w_i^{(m)}$. Depending on the algorithm, it may be possible to train a single classifier on the weighted training set; in other cases, one may need to select the best-performing classifier from among a predefined set.
   (b) Compute
   $$\mathrm{err}_m = \frac{\sum_{i=1}^N w_i^{(m)} \cdot \mathcal{I}(y^{(i)} \neq G_m(x^{(i)}))}{\sum_{i=1}^N w_i^{(m)}}$$
   (c) Compute voting weight for classifier m:
   $$\alpha_m = \log\left(\frac{1 - \mathrm{err}_m}{\mathrm{err}_m}\right)$$
   (d) Set
   $$w_i^{(m+1)} := w_i^{(m)} \cdot \exp\left[\alpha_m \cdot \mathcal{I}(y^{(i)} \neq G_m(x^{(i)}))\right]$$
   for $i = 1, \ldots, N$.
3. Output
$$G(x) = \mathrm{sign}\left[\sum_{m=1}^{M} \alpha_m G_m(x)\right]$$

Subject ID	friends (X_1)	money (X_2)	free time (X_3)	pet (X_4)	happy (Y)
1	1	1	0	0	-1
2	1	1	1	0	-1
3	0	1	1	0	-1
4	0	0	0	0	-1
5	1	0	0	0	-1
6	0	0	0	0	-1
7	1	2	1	0	1
8	1	0	1	0	1
9	0	0	1	1	1
10	1	0	0	1	1

$$X_1 = \begin{cases} 0 & \text{no friends} \\ 1 & \text{friends} \end{cases} \qquad X_2 = \begin{cases} 0 & \text{poor} \\ 1 & \text{enough money} \\ 2 & \text{rich} \end{cases}$$

$$X_3 = \begin{cases} 0 & \text{no free time} \\ 1 & \text{some free time} \end{cases} \qquad X_4 = \begin{cases} 0 & \text{no pet} \\ 1 & \text{has a pet} \end{cases}$$

Now, let's go through the process of applying AdaBoost to this dataset, step by step.

(a) Initialize the observation weights for the training data. (Make them uniform.)

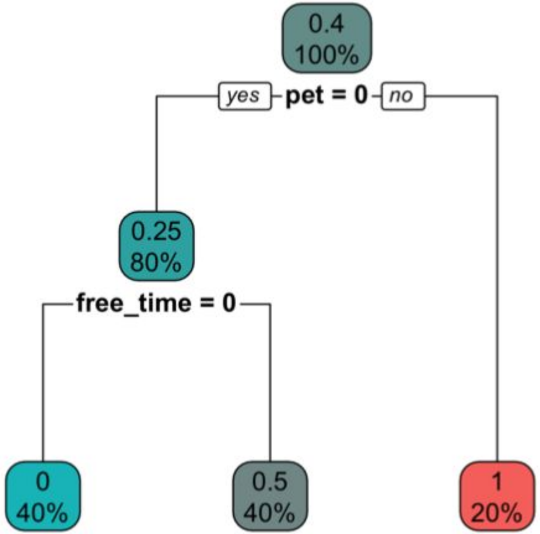
$$\begin{aligned} w_1^{(1)} &= \\ w_2^{(1)} &= \\ w_3^{(1)} &= \\ w_4^{(1)} &= \\ w_5^{(1)} &= \end{aligned} \qquad \begin{aligned} w_6^{(1)} &= \\ w_7^{(1)} &= \\ w_8^{(1)} &= \\ w_9^{(1)} &= \\ w_{10}^{(1)} &= \end{aligned}$$

Now, let's go through the process of applying AdaBoost to this dataset, step by step.

(a) Initialize the observation weights for the training data. (Make them uniform.)

$$
\begin{aligned}
w_1^{(1)} &= 0.1 & \qquad w_6^{(1)} &= 0.1 \\
w_2^{(1)} &= 0.1 & \qquad w_7^{(1)} &= 0.1 \\
w_3^{(1)} &= 0.1 & \qquad w_8^{(1)} &= 0.1 \\
w_4^{(1)} &= 0.1 & \qquad w_9^{(1)} &= 0.1 \\
w_5^{(1)} &= 0.1 & \qquad w_{10}^{(1)} &= 0.1
\end{aligned}
$$

(b) We will now grow a decision tree on this dataset, $G_1(x)$, using these weights. We'll use the `rpart` package in R, just as in Section 7.3. Here is the first tree.



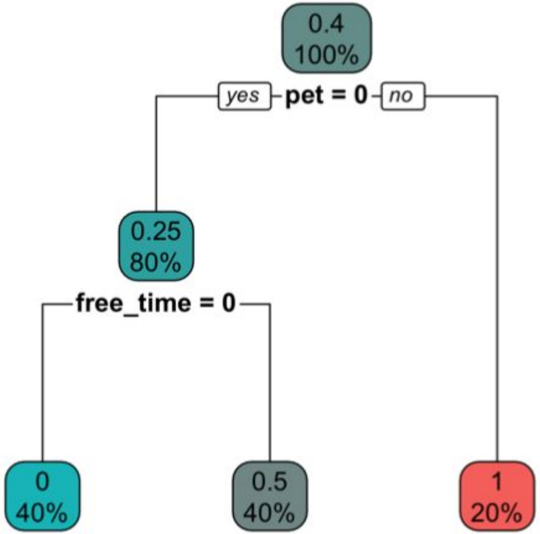
Datapoint ID	$w_i^{(1)}$	happy (Y)	$G_1(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	-1
6	0.1	-1	-1
7	0.1	1	$\boxed{-1}$
8	0.1	1	$\boxed{-1}$
9	0.1	1	1
10	0.1	1	1

Compute the misclassification error of this tree, err_1. Compare this tree to the one we constructed by hand in Chapter 7.

$$\text{err}_1 =$$

(b) We will now grow a decision tree on this dataset, $G_1(x)$, using these weights. We'll use the `rpart` package in R, just as in Section 7.3. Here is the first tree.

Datapoint ID	$w_i^{(1)}$	happy (Y)	$G_1(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	-1
6	0.1	-1	-1
7	0.1	1	-1
8	0.1	1	-1
9	0.1	1	1
10	0.1	1	1



Compute the misclassification error of this tree, err_1. Compare this tree to the one we constructed by hand in Chapter 7.

$$\text{err}_1 = \frac{0.1(2)}{0.1(10)} = 0.2$$

(c) Based on how $G_1(x)$ performs, calculate α_1, its voting weight.

$$\alpha_1 =$$

(d) Re-weight the observation weights for the training examples.

$$w_1^{(2)} = \qquad\qquad w_6^{(2)} =$$
$$w_2^{(2)} = \qquad\qquad w_7^{(2)} =$$
$$w_3^{(2)} = \qquad\qquad w_8^{(2)} =$$
$$w_4^{(2)} = \qquad\qquad w_9^{(2)} =$$
$$w_5^{(2)} = \qquad\qquad w_{10}^{(2)} =$$

(c) Based on how $G_1(x)$ performs, calculate α_1, its voting weight.

$$\alpha_1 = \log\left(\frac{1 - \text{err}_1}{\text{err}_1}\right) = 1.386$$

(d) Re-weight the observation weights for the training examples.

$w_1^{(2)} =$ 0.1

$w_2^{(2)} =$ 0.1

$w_3^{(2)} =$ 0.1

$w_4^{(2)} =$ 0.1

$w_5^{(2)} =$ 0.1

$w_6^{(2)} =$ 0.1

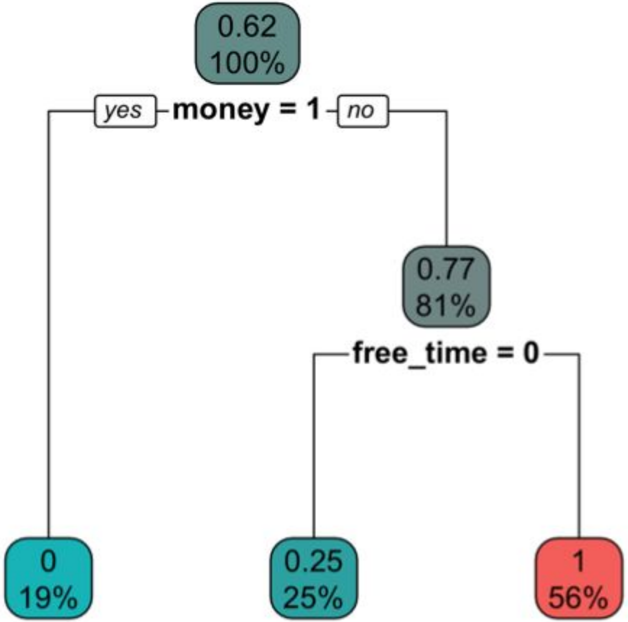
$w_7^{(2)} =$ 0.1 * exp(1.386) = 0.4

$w_8^{(2)} =$ 0.1 * exp(1.386) = 0.4

$w_9^{(2)} =$ 0.1

$w_{10}^{(2)} =$ 0.1

(e) Now we grow another decision tree, $G_2(x)$, using these new weights as inputs to the `rpart` package.

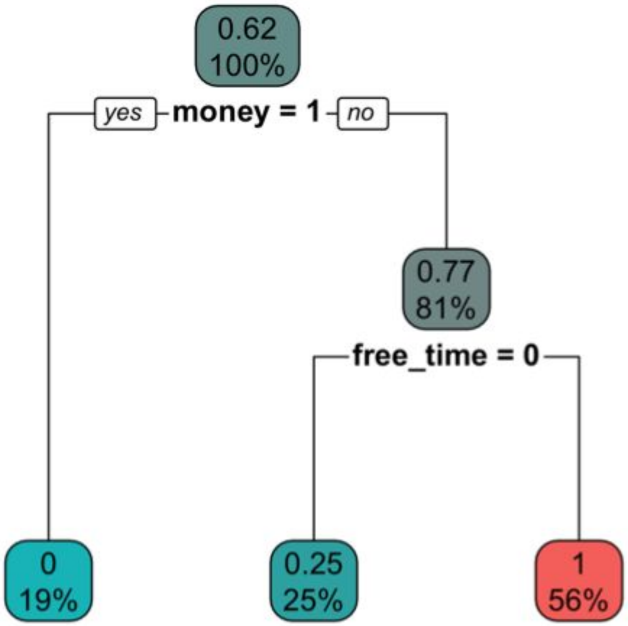


Datapoint ID	$w_i^{(2)}$	happy (Y)	$G_2(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	-1
6	0.1	-1	-1
7	0.4	1	1
8	0.4	1	1
9	0.1	1	1
10	0.1	1	$\boxed{-1}$

Compute the misclassification error of this tree, err_2.

$$\text{err}_2 =$$

(e) Now we grow another decision tree, $G_2(x)$, using these new weights as inputs to the `rpart` package.



Datapoint ID	$w_i^{(2)}$	happy (Y)	$G_2(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	-1
6	0.1	-1	-1
7	0.4	1	1
8	0.4	1	1
9	0.1	1	1
10	0.1	1	-1

Compute the misclassification error of this tree, err_2.

$$\text{err}_2 = \frac{0.1}{0.1(8) + 0.4(2)} = 0.0625$$

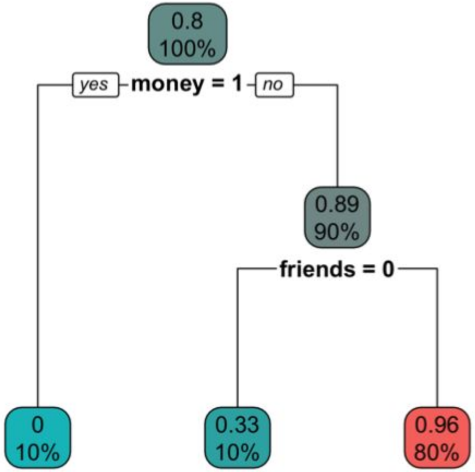
(f) Based on how $G_2(x)$ performs, calculate α_2, its voting weight.

$$\alpha_2 = \log\left(\frac{1 - \text{err}_2}{\text{err}_2}\right) = 2.708$$

(g) Re-weight the observation weights for the training examples.

$$w_1^{(3)} = 0.1$$
$$w_2^{(3)} = 0.1$$
$$w_3^{(3)} = 0.1$$
$$w_4^{(3)} = 0.1$$
$$w_5^{(3)} = 0.1$$
$$w_6^{(3)} = 0.1$$
$$w_7^{(3)} = 0.4$$
$$w_8^{(3)} = 0.4$$
$$w_9^{(3)} = 0.1$$
$$w_{10}^{(3)} = 0.1*\exp(2.708) = 1.5$$

(h) Now we grow another decision tree, $G_3(x)$, using these new weights as inputs to the `rpart` package.



Datapoint ID	$w_i^{(3)}$	happy (Y)	$G_3(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	$\boxed{1}$
6	0.1	-1	-1
7	0.4	1	1
8	0.4	1	1
9	0.1	1	$\boxed{-1}$
10	1.5	1	1

(i) Compute the misclassification error of this tree, err_3.

$$\text{err}_3 =$$

$$\frac{0.2}{0.1(7) + 0.4(2) + 1.5} = 0.067$$

(j) Based on how $G_3(x)$ performs, calculate α_3, its voting weight.

$$\alpha_3 = \log\left(\frac{1-\text{err}_3}{\text{err}_3}\right) = 2.639$$

(k) Re-weight the observation weights for the training examples.

$w_1^{(4)} =$ 0.1

$w_2^{(4)} =$ 0.1

$w_3^{(4)} =$ 0.1

$w_4^{(4)} =$ 0.1

$w_5^{(4)} =$ 0.1*exp(2.639) = 1.4

$w_6^{(4)} =$ 0.1

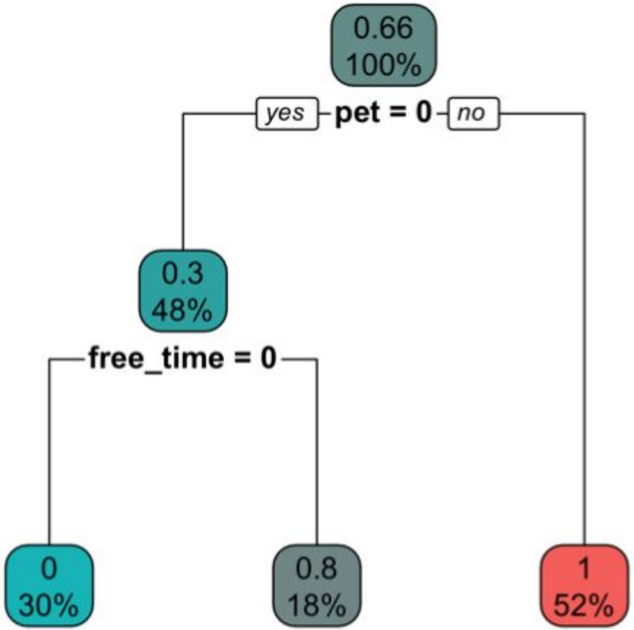
$w_7^{(4)} =$ 0.4

$w_8^{(4)} =$ 0.4

$w_9^{(4)} =$ 0.1*exp(2.639) = 1.4

$w_{10}^{(4)} =$ 1.5

(l) Now we grow another decision tree, $G_4(x)$, using these new weights as inputs to the `rpart` package.



Datapoint ID	$w_i^{(4)}$	happy (Y)	$G_4(x)$
1	0.1	-1	-1
2	0.1	-1	1
3	0.1	-1	1
4	0.1	-1	-1
5	1.4	-1	-1
6	0.1	-1	-1
7	0.4	1	1
8	0.4	1	1
9	1.4	1	1
10	1.5	1	1

(m) Compute the misclassification error of this tree, err_4.

$$\mathrm{err}_4 =$$

$$\frac{0.2}{0.1(5) + 0.4(2) + 1.4(2) + 1.5} = 0.036$$

(n) Based on how $G_4(x)$ performs, calculate α_4, its voting weight.

$$\alpha_4 = \log\left(\frac{1 - \text{err}_4}{\text{err}_4}\right) = 3.296$$

(o) Output the weighted average of the four classifiers' votes for each training example:

$$G(x) = \text{sign}\left[\alpha_1 G_1(x) + \alpha_2 G_2(x) + \alpha_3 G_3(x) + \alpha_4 G_4(x)\right]$$

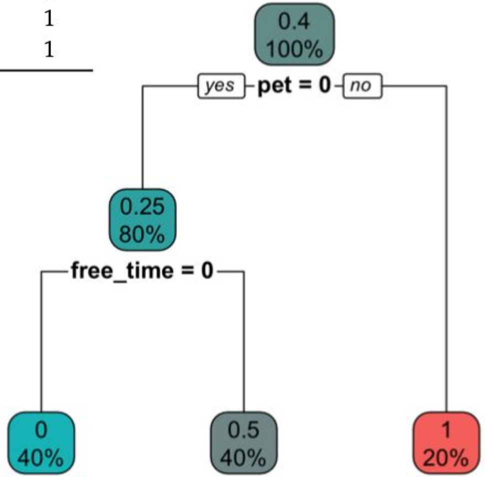
What is the final training error?

		1.386	2.708	2.639	3.296	
Datapoint ID	happy (Y)	$G_1(x)$	$G_2(x)$	$G_3(x)$	$G_4(x)$	$G(x)$
1	-1	-1	-1	-1	-1	
2	-1	-1	-1	-1	1	
3	-1	-1	-1	-1	1	
4	-1	-1	-1	-1	-1	
5	-1	-1	-1	1	-1	
6	-1	-1	-1	-1	-1	
7	1	-1	1	1	1	
8	1	-1	1	1	1	
9	1	1	1	-1	1	
10	1	1	-1	1	1	

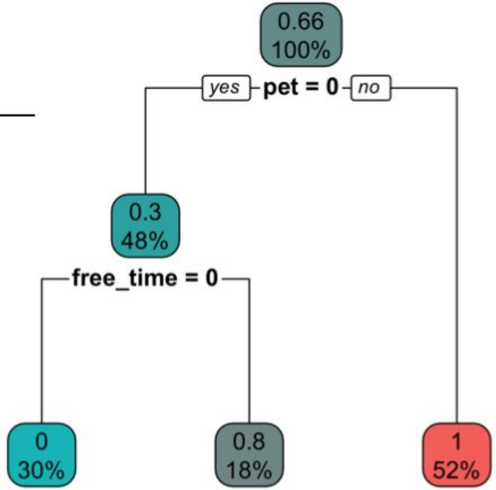
Question 14.1

The most difficult thing to understand in all of this is how the updated weights play into the construction of subsequent trees. A clue comes from the dataset percentages shown in the nodes of each tree. For example, trees 1 and 3 actually

Datapoint ID	$w_i^{(1)}$	happy (Y)	$G_1(x)$
1	0.1	-1	-1
2	0.1	-1	-1
3	0.1	-1	-1
4	0.1	-1	-1
5	0.1	-1	-1
6	0.1	-1	-1
7	0.1	1	-1
8	0.1	1	-1
9	0.1	1	1
10	0.1	1	1



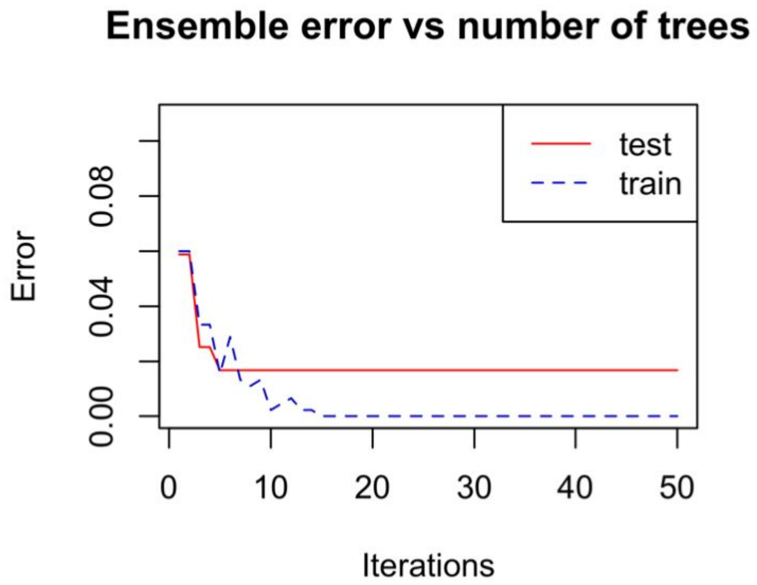
Datapoint ID	$w_i^{(4)}$	happy (Y)	$G_4(x)$
1	0.1	-1	-1
2	0.1	-1	1
3	0.1	-1	1
4	0.1	-1	-1
5	1.4	-1	-1
6	0.1	-1	-1
7	0.4	1	1
8	0.4	1	1
9	1.4	1	1
10	1.5	1	1

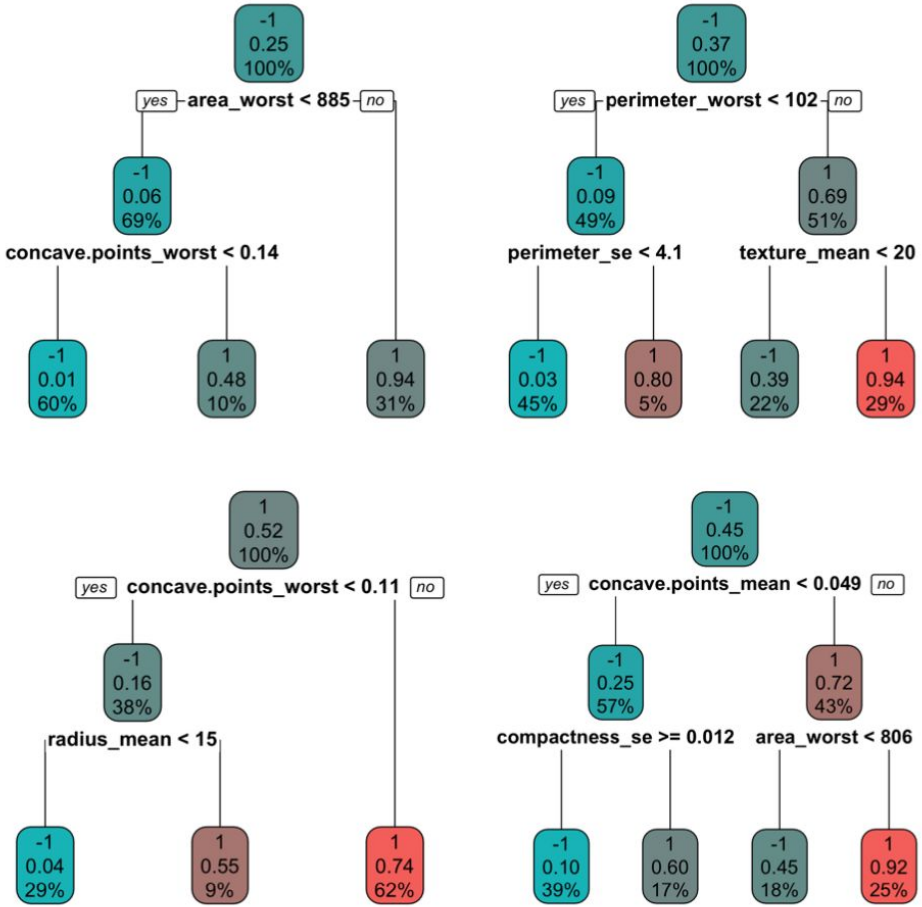


Gradient boosting:
A more general conception of boosting in which later models are fit to the errors (pseudo-residuals) of earlier models.

How does this guarantee diversity?

Wisconsin Breast Cancer dataset revisited





Question 14.2

Compare and contrast boosting and random forests on the basis of:

- Whether each tree uses all or part of the dataset
- Whether they consider a subset of the predictors at each split or all the predictors
- Whether you can parallelize the construction of different trees (i.e., build them at the same time on different processors)
- Whether the votes of different classifiers are independent
- Ease of use and interpretability