

# Problem Set 3

*ECON 480 - Fall 2019*

*Due by Thursday, October 10, 2019*

## Instructions

For this problem set, you may submit handwritten answers on a plain sheet of paper, or download and type/handwrite on the PDF.

Alternatively, you may download the `.Rmd` file, do the homework in markdown, and email to me a single knitted `html` or `pdf` file (and be sure that it shows all of your code).

You may work together (and I highly encourage that) but you must turn in your own answers. I grade homeworks 70% for completion, and for the remaining 30%, pick one question to grade for accuracy - so it is best that you try every problem, even if you are unsure how to complete it accurately.

## Theory and Concepts

1. In your own words, describe what exogeneity and endogeneity mean, and how they are related to bias. What can we learn about the bias if we know  $X$  is endogenous?

2. In your own words, describe what  $R^2$  means. How do we calculate it, what does it tell us, and how do we interpret it?

3. In your own words, describe what the standard error of the regression ( $SER$ ) means. How do we calculate it, what does it tell us, and how do we interpret it?
4. In your own words, describe what homoskedasticity and heteroskedasticity mean: both in ordinary English, and in terms of the graph of the OLS regression line.

5. In your own words, describe what the variation in  $\hat{\beta}_1$  (either variance or standard error) means, or is measuring. What three things determine the variation, and in what way?

6. In your own words, describe what a  $p$ -value means, and how it is used to establish statistical significance.

7. A researcher is interested in examining the impact of illegal music downloads on commercial music sales. The author collects data on commercial sales of the top 500 singles from 2017 ( $Y$ ) and the number of downloads from a web site that allows ‘file sharing’ ( $X$ ). The author estimates the following model

$$\text{music sales}_i = \beta_0 + \beta_1 \text{illegal downloads}_i + u_i$$

The author finds a large, positive, and statistically significant estimate of  $\hat{\beta}_1$ . The author concludes these results demonstrate that illegal downloads actually *boost* music sales. Is this an unbiased estimate of the impact of illegal music on sales? Why or why not? Do you expect the estimate to overstate or understate the true relationship between illegal downloads and sales?

8. A pharmaceutical company is interested in estimating the impact of a new drug on cholesterol levels. They enroll 200 people in a clinical trial. People are randomly assigned the treatment group or into the control group. Half of the people are given the new drug and half the people are given a sugar pill with no active ingredient. To examine the impact of dosage on reductions in cholesterol levels, the authors of the study regress the following model:

$$\text{cholesterol level}_i = \beta_0 + \beta_1 \text{dosage level}_i + u_i$$

For people in the control group, dosage level <sub>$i$</sub>  = 0 and for people in the treatment group, dosage level <sub>$i$</sub>  measures milligrams of the active ingredient. In this case, the authors find a large, negative, statistically significant estimate of  $\hat{\beta}_1$ . Is this an unbiased estimate of the impact of dosage on change in cholesterol level? Why or why not? Do you expect the estimate to overstate or understate the true relationship between dosage and cholesterol level?

## Theory Problems

For the following questions, please *show all work* and explain answers as necessary. You may lose points if you only write the correct answer. You may use R to *verify* your answers, but you are expected to reach the answers in this section “manually.”

9. A researcher wants to estimate the relationship between average weekly earnings ( $AWE$ , measured in dollars) and  $Age$  (measured in years) using a simple OLS model. Using a random sample of college-educated full-time workers aged 25-65 yields the following:

$$\widehat{AWE} = 696.70 + 9.60 \text{ Age}$$

- a. Interpret what  $\hat{\beta}_0$  means in this context.
- b. Interpret what  $\hat{\beta}_1$  means in this context.
- c. The  $R^2 = 0.023$  for this regression. What are the units of the  $R^2$ , and what does this mean?
- d. The  $SER$ ,  $\hat{\sigma}_u = 624.1$  for this regression. What are the units of the  $SER$  in this context, and what does it mean? Is the  $SER$  large in the context of this regression?

- e. Suppose Maria is 20 years old. What is her predicted  $\widehat{AWE}$ ?
- f. Suppose the data shows her *actual AWE* is \$430. What is her residual? Is this a relatively good or a bad prediction?<sup>1</sup>
- g. What does the error term,  $\hat{u}_i$  represent in this case? What might individuals have different values of  $u_i$ ?
- h. Do you think that *Age* is exogenous? Why or why not? Would we expect  $\hat{\beta}_1$  to be too *large* or too *small*?

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<sup>1</sup>Hint: compare your answer here to your answer in Part D.

10. Suppose a researcher is interested in estimating a simple linear regression model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

In a sample of 48 observations, she generates the following descriptive statistics:

- $\bar{X} = 30$
- $\bar{Y} = 63$
- $\sum_{i=1}^n (X_i - \bar{X})^2 = 6900$
- $\sum_{i=1}^n (Y_i - \bar{Y})^2 = 29000$
- $\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = 13800$
- $\sum_{i=1}^n \hat{u}^2 = 1656$

a. What is the OLS estimate of  $\hat{\beta}_1$ ?

b. What is the OLS estimate of  $\hat{\beta}_0$ ?

c. Suppose the OLS estimate of  $\hat{\beta}_1$  has a standard error of 0.072. Could we probably reject a null hypothesis of  $H_0 : \beta_1 = 0$  at the 95% level?

d. Calculate the  $R^2$  for this model. How much variation in  $Y$  is explained by our model?

e. How large is the average residual?



## R Questions

Answer the following questions using R. When necessary, please write answers in the same document (knitted Rmd to html or pdf, typed .doc(x), or handwritten) as your answers to the above questions. Be sure to include (email or print an .R file, or show in your knitted markdown) your code and the outputs of your code with the rest of your answers.

11. Download the `MLBattend` dataset. This data contains data on attendance at major league baseball games for all 32 MLB teams from the 1970s-2000. We want to answer the following question:  
“How big is home-field advantage in baseball? Does a team with higher attendance at home games over their season have score more runs over their season?”
  - a. Clean up the data a bit by making a new variable to measure home attendance in millions. This will make it easier to interpret your regression later on.
  - b. Plot a scatterplot of Runs Scored (y) on Home Attendance (x). Add a regression line.
  - c. Run a regression of Runs Scored on Home Attendance. What are  $\beta_0$  and  $\beta_1$ ? Interpret them in the context of our question.
  - d. Write out the estimated regression equation.
  - e. Make a regression table of the output.
  - f. Now let's start running some diagnostics of the regression. Make a histogram of the residuals. Do they look roughly normal?
  - g. Make a residual plot.<sup>h</sup> Test the regression for heteroskedasticity. Are the errors homoskedastic or heteroskedastic? Generate robust standard errors. Make a regression output table, with one column with regular standard errors and another with robust standard errors.
  - h. Test the data for outliers. If there are any, identify which team(s) and season(s) are outliers.
  - i. What is the marginal effect of home attendance on runs scored? Is this statistically significant? Why or why not?
  - j. Now we'll try out the `infer` package to understand the  $p$ -value and confidence interval for our observed slope in our regression model. Save the (value of) our sample  $\hat{\beta}_1$  from your regression in Part C as an object. Then, install and load the `infer` package, and then calculate the slope<sup>2</sup> under the null hypothesis that there is no connection between attendance and runs.<sup>3</sup> for 1000 additional simulated samples<sup>4</sup>, and save this as an object (it's a `tibble`). Then, use this to `get_p_value()`<sup>5</sup>. Compare to the  $p$ -value given by `lm()` and `tidy()` above.
  - k. Make a histogram of the simulated slopes, and plot our sample slope on that histogram, shading the  $p$ -value.<sup>6</sup>
  - l. Get the 95% confidence interval for your slope estimate,<sup>7</sup> and then make a histogram of the simulated slopes (like part L), but instead, add `+shade_confidence_interval()`.<sup>8</sup> Compare this to what you get with `tidy()` above.\*\*

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<sup>2</sup>`calculate(stat = "slope")`

<sup>3</sup>`hypothesize(null = "independence")`

<sup>4</sup>`generate(reps = 1000, type = "permute")`

<sup>5</sup>Set `obs_stat` equal to your observed slope, and set `direction = "two_sided"`

<sup>6</sup>You can use `ggplot2` to plot a histogram in the normal way and add a `geom_vline()`, setting `xintercept` equal to your saved object with the sample  $\hat{\beta}_1$  value. *Alternatively*, you can use `infer` to pipe your `tibble` of simulations into `visualize()`, and inside `visualize()` set `obs_stat` equal to your saved  $\hat{\beta}_1$  object. *Regardless* of which method you use, add `+shade_p_value()`. Inside this, set `obs_stat` equal to your saved slope, and add `direction = "two_sided"`.

<sup>7</sup>`tidy()` your original regression, with `conf.int = TRUE` inside the command, then `select(conf.low, conf.high)` and `filter` by your `x` variable. Save this as an object.

<sup>8</sup>Inside of this, set `endpoints` equal to the object you just made with the low and high confidence interval values.