Final Exam (100 points QM, 50 points BER)

- Show your work for partial credit. It is very difficult to give partial credit if the only thing on your page is "x = 3".
- If possible, take the exam during an uninterrupted period of no more than 6 hours. (It should not take anywhere near that long.) In any case, do not spend more than 6 hours on the exam.
- Other than this cheat sheet, all you are allowed to use for help are the basic functions on a calculator (or a spreadsheet program. Partial translation: no books, no notes, no websites, no talking to other people, and no advanced calculator features like programmable functions or present value formulas.
- People who have taken the exam can talk to each other all they want, and people who have not taken the exam can talk to each other all they want, but communication between the two groups about class should be limited to three phrases: "Yes", "No", and "Have you taken the exam?"
- Expected value is given by summing likelihood times value over all possible outcomes:

$$\begin{array}{lll} \text{Expected Value} & = & \displaystyle \sum_{\text{Outcomes } i} \text{Probability}(i) \cdot \text{Value}(i). \end{array}$$

- A fair bet is a bet with an expected value of zero.
- The future value of a lump sum payment of \$x\$ invested for n years at interest rate s is $FV = x(1+s)^n$. The present value of a lump sum payment of \$x\$ after n years at interest rate s is $PV = \frac{x}{(1+s)^n}$. (Note that this formula also works for values of n that are negative or zero.)
- The present value of an **annuity** paying x at the end of each year for n year at interest rate s is

$$PV = x \left[\frac{1 - \frac{1}{(1+s)^n}}{s} \right].$$

The present value of the related **perpetuity** (with annual payments forever) is

$$PV = \frac{x}{s}$$
.

• The **inflation rate**, *i*, is the rate at which prices rise. The **nominal interest rate**, *n*, is the interest rate in terms of dollars. The **real interest rate**, *r*, is the interest rate in terms of purchasing power. These are related by

$$1+r = \frac{1+n}{1+i}.$$

When the inflation rate is small, we can approximate this as

$$r \approx n - i$$
.

- A **Pareto efficient** (or **Pareto optimal**) allocation or outcome is one in which it is not possible find a different allocation or outcome in which nobody is worse off and at least one person is better off. An allocation or outcome B is a **Pareto improvement over A** if nobody is worse off with B than with A and at least one person is better off.
- A (strictly) **dominant strategy** is a strategy which yields higher payoffs than any other strategy regardless of the other players' strategies.
- In an ascending price auction, the price starts out at a low value and the bidders raise each other's bids until nobody else wants to bid. In a descending price auction, the price starts out at a high value and the auctioneer lowers it until somebody calls out, "Mine." In a first-price sealed-bid auction, the bidders submit bids in sealed envelopes; the bidder with the highest bid wins, and pays an amount equal to his or her bid (i.e., the highest bid). In a second-price sealed-bid auction, the bidders submit bids in sealed envelopes; the bidder with the highest bid wins, but pays an amount equal to the second-highest bid.
- Total revenue is price times quantity: TR = pq.
- The **price elasticity of demand at point A** measures the percentage change in quantity demanded (relative to the quantity demanded at point A) resulting from a 1% increase in the price (relative to the price at point A). The formula is

$$\varepsilon(A) = \frac{\% \text{ change in } q}{\% \text{ change in } p} = \frac{\frac{\Delta q}{q_A}}{\frac{\Delta p}{p_A}} = \frac{\Delta q}{\Delta p} \cdot \frac{p_A}{q_A} = \frac{q_B - q_A}{p_B - p_A} \cdot \frac{p_A}{q_A}.$$

- In English If, at point A, a small change in price causes the quantity demanded to increase by a lot, demand at point A is elastic; if quantity demanded only changes by a little then demand at point A is inelastic; and if quantity demanded changes by a proportional amount then demand at point A has unit elasticity.
- In math If, at point A, the price elasticity of demand is less than -1 (e.g., -2), then demand at point A is elastic; if the elasticity is greater than -1 (e.g., $-\frac{1}{2}$), then demand at point A is inelastic; if the elasticity is equal to -1 then demand at point A has unit elasticity.

- 1. Imagine that you own some land and you decide to manage it as a tree farm: you plant some trees, and then you cut them down and sell the lumber. Your objective is to make as much money as possible, i.e., to maximize the present value of the lumber. (Assume for simplicity that replanting or other land uses are not possible.)
 - (a) (5 points) Is the interest rate at the bank going to affect your decision about when to cut down the trees? Circle one (Yes No) and explain briefly.

(b) (5 points) Is the amount it cost you to plant the trees going to affect your decision about when to cut them down? Circle one (Yes No) and explain briefly.

- 2. Narrowly defined, a "Prisoners' Dilemma" situation involves the following: (1) a symmetric, simultaneous-move game featuring two players; (2) the existence of a dominant strategy for each player; and (3) a predicted outcome that is Pareto inefficient.
 - (a) (5 points) Draw a payoff matrix that describes such a situation. (It may help to remember the following conventions about payoff matrices: player 1 chooses the row, player 2 chooses the column, and an outcome of (x, y) indicates that player 1 gets x and player 2 gets y.) You do not need to write any explanation, but if you cannot draw a payoff matrix then some words might get you some partial credit.

(b) (5 points) A slightly broader definition of "Prisoners' Dilemma" would include situations featuring more than two players. Provide an example of one such situation—you can describe one we've discussed in class, or make up your own—and briefly explain what the strategies are, what the predicted outcome is, and what would be a Pareto improvement over that predicted outcome.

- 3. Consider the following game featuring 4 ounces of cake and two kids, each of whom has as his or her sole objective the desire for as much cake as possible: Player 1 splits the cake by offering Player 2 either 1, 2, 3 ounces of cake; Player 2 then either accepts the offer (in which case they split the cake accordingly) or rejects the offer (in which case each player gets 1.5 ounces of cake).
 - (a) (5 points) Draw a game tree that represents this game.

(b) (5 points) Identify (with a star on the game tree, or in words if you couldn't draw a game tree) the predicted outcome of this game. Then circle *all* of the Pareto efficient outcomes in the following list, and identify a Pareto improvement for any outcome that is not Pareto efficient: (3,1), (2,2), (1,3), (1.5,1.5).

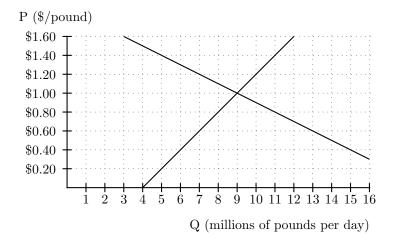
- 4. The Intergovernmental Panel on Climate Change reports that human activity (especially the burning of fossil fuels such as coal, oil, and natural gas) is warming the earth. (Note: With the exception of this fact, all of the numbers &etc in this question are completely made up.)
 - (a) (5 points) Assume that global warming will raise sea levels and increase the frequency of hurricanes, leading to damages of \$1 trillion (= $10^{12} = 1,000,000,000,000)$ at the end of each year for the next seven years. What is the present value of that damage if the relevant interest rate is 4%? [Note: If all the zeroes confuse you or your calculator, use \$1,000,000 or \$1,000 instead.] Also: is the relevant interest rate nominal or real? (Circle one.)

(b) (5 points) Next, assume that the full damages you've calculated above will only occur with probability 1/3. With probability 1/3 the damages will be only half as big, and with probability 1/3 the damages will be zero. What is the expected value of the damage caused by global warming? [Note: If you didn't answer part 4a above, just assume for this part that the total damage is \$1,000,000.]

(c) (5 points) Next, assume that the hurricanes &etc won't happen for 100 years. So: take the expected damages you calculated in part 4b and compute the present value of having that amount of damage occur 100 years in the future if the relevant interest rate is 4%. [Note: If you didn't answer part 4b, assume for this part that the total damage is \$1,000,000.]

5. (5 points) Consider a market with a demand curve of q=220-20p and a supply curve of q=60p-100. Determine the price and quantity at the market equilibrium and then show how (if at all) a 25% sales tax on the sellers will affect both the equation for the supply curve and the equation for the demand curve.

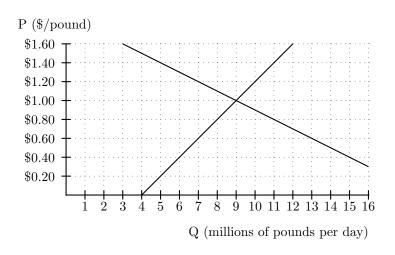
6. Below is a hypothetical market for oranges.



Suppose that the government decides to impose a per-unit tax of \$.60 per pound on the buyers of oranges.

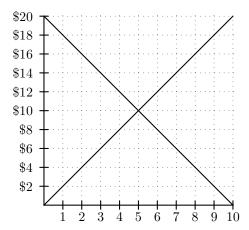
(a) (5 points) Show the impact of this tax on the supply and demand curves above **and explain** (as if to a non-economist) why the tax shifts the curves the way it does.

(b) (5 points) Imagine that the government imposes a 50% tax on the buyers instead of a \$.60 per-unit tax. Use the graph below to show how this changes the supply and demand curves. You do not need to explain.



7. (5 points) "When I buy a \$20 radio that was made in China, it clearly adds \$20 to C, the household consumption part of the GDP equation Y = C + I + G + (Ex - Im)." Is this statement true or false? (Circle one in the preceding sentence.) Regardless of what you circled, explain how (if at all) the \$20 radio purchase affects the six variables in the equation above, e.g., by writing "Y increases by \$20" or "Y is unchanged".

- 8. Consider the labor market below, with the price measuring the hourly wage in dollars per hour and the quantity measuring millions of workers.
 - (a) (5 points) Use a dot and label with an "F" the market outcome in a free market. Then use a dot and label with an "M" the market outcome in a market with a minimum wage of \$14 per hour.



(b) (5 points) Explain (as if to a non-economist) why the free market outcome is where it is.

(c) (5 points) Explain why the \$14 minimum wage creates unemployment. Also: *quantify* the amount of unemployment created by writing down the number of unemployed people resulting from the minimum wage. Please circle your answer.

9.	Imagine that the Federal Reserve decides to engage in open-market oper-
	ations by buying government bonds.

(a) (5 points) Use a graph to demonstrate the effect of this action on the money market; use an arrow to indicate the direction(s) of movement of the affected curve(s). Clearly indicate what variables are being measured on the x and y axes (e.g., does one measure the wage rate and the other measure the quantity of oranges, or ???).

(b) (5 points) Use an aggregate demand / aggregate supply curve to demonstrate the impact of the Fed's action on the price level and real GDP in the short run. Then translate your answer into English by completing this sentence: "In the short run, the Fed's action..."

(c) (5 points) Use an aggregate demand / aggregate supply curve to demonstrate the impact of the Fed's action on the price level and real GDP in the long run. Then translate your answer into English by completing this sentence: "In the long run, the Fed's action..."

10. (5 points) One curiosity in economics is that the long-run aggregate supply curve is assumed to be perfectly *inelastic* but the long-run supply curve for many individual industries is assumed to be perfectly *elastic*. In the market for dry-cleaning, for example, all firms have about the same costs and there are no barriers to entry (i.e., nothing stops new firms from entering the market), so economists often picture the long-run supply curve as a horizontal line at some price p^* (e.g., \$2 per shirt), meaning that at any price below p^* firms want to supply zero, and at any price above p^* firms want to supply an infinite amount. Explain (as if to a non-economist) why this horizontal long-run supply curve makes sense for dry-cleaning.