The informational content of prices when policy makers react to financial markets*

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Job Market Paper

December 5, 2015

Abstract

I analyze settings where a policy maker needs information that financial market traders have in order to implement her optimal policy, and market prices may reveal this information. Policy decisions, in turn, affect asset values, possibly punishing the informed for trading on information. Applications include central bank policy reactions to asset prices or regulator reactions to bond prices. In the first part without noise, I derive a necessary and sufficient condition for the possibility of fully revealing equilibria, which identifies all situations where learning from prices for policy purposes works, and where it does not. Full revelation may be impossible because the pricing problem is a self-defeating prophecy, impairing incentives to trade on information. I demonstrate how the condition can be used for asset design that supports information revelation by markets. In the second part, I develop a noisy model of trader-policy maker interaction. Noise can solve the problem of self-defeating prophecies in specific situations, but in general it may still occur. Using a novel solution approach, I derive a necessary and sufficient condition for the existence of revealing equilibria in a general class of noisy equilibria that allows for nonlinear price and policy functions.

Keywords: Asymmetric Information, Financial Markets, Policy, Policy Risk, Price Informativeness, Rational Expectations Equilibrium, Self-Defeating Prophecy

*I am grateful to Klaus Adam, Marco Bassetto, Johannes Bubeck, Pierre Boyer, Antonio Cabrales, Hans Peter Grüner, Felix Jarman, Xavier Lambin, Justin Leduc, Edward S. Prescott, Andreas Rapp, Philipp Zahn, and seminar/conference participants at University of Oxford (Nuffield), Tilburg University, University of Mannheim, University College London, ENTER Jamboree 2015, VfS 2015 Münster, and SAET 2015 Cambridge for very helpful comments. This research was supported by the German Research Foundation (DFG) via SFB 884.

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1 Introduction

Economists have long recognized that markets can aggregate and reveal diverse information among market participants via market prices (Hayek, 1945; Fama, 1970). The recent advance of prediction markets has extended the set of forecasting problems that allow for market-based forecasts to virtually all areas involving uncertain outcomes, such as election outcomes or armed conflicts. Thus, financial markets can potentially be a valuable tool helping policy makers to make decisions by providing information or forecasts.

For example, a central bank may use asset prices to infer information about inflation expectations or future demand shocks, and adapt policy in response (Bernanke and Woodford, 1997). A regulator may learn about the financial health of a bank from bond prices, and use this information for regulatory purposes (e.g., contingent capital with market trigger, Sundaresan and Wang, 2015). Or a company could use internal prediction markets—where asset values depend on the launch date of a new product—to predict whether deadlines can be met, and react if forecasts indicate major delays (e.g., Cowgill and Zitzewitz, 2015).

In all of these examples, one or several agents—which I shall call policy makers—react to the information contained in asset prices, and the reaction in turn affects asset values. This simultaneous feedback from prices to asset values and asset values to prices presents problems both in theoretical and practical terms. In practical terms, the (asset value) forecasts implicit in prices may be falsified by the policy reaction, which can diminish incentives to reveal information if traders are forward looking, thus making market prices less informative and reducing their usefulness for policy makers. A theoretical problem is that such policy maker-trader interactions may not have an equilibrium. The main goal of this paper is to identify if and when it is possible for markets to inform policy makers if traders correctly anticipate the policy maker reaction.[1]

In the first part, I develop a general model of trader and policy maker interaction in competitive financial markets without noise, and derive a sufficient and necessary condition where traders reveal their information by trading, and policy makers use this information for policy purposes in equilibrium. Thus, the condition identifies all situations where we can expect financial markets to work as policy maker tools without compromising the informational content of prices, and all situations where we cannot. Informally, the condition specifies whether the pricing problem is a self-defeating prophecy. With an uninformed policy maker, the condition is very simple and requires invertibility of the expected asset value given optimal policy reaction in a sufficient statistic of trader information. If the condition is not fulfilled, then traders anticipate that informative market prices will trigger a policy maker reaction that leads to trader losses, making market prices less informative. The model unifies several applications from the literature such as Bernanke and Woodford (1997) and

[1]Clearly, an unanticipated policy maker intervention does not diminish the incentives to trade on information. This paper instead focuses on the possibility of policy makers *systematically* using the information revealed by market prices.

Bond et al. (2010), generalizes their results, and shows that there is a common cause to the problem of uninformative prices: self-defeating prophecies. The theoretical results can be used to identify asset payoff functions and policy maker objectives that are more supportive of information revelation. For example, I show that 'deadline securities' in corporate prediction markets, which forecast whether a project will be completed on time, may provide improper incentives to share information about the project if management reacts to these forecasts, and I suggest alternatives with proper incentives.

In the second part of the paper, I develop a model with policy maker-trader interaction where prices are affected by noise. Adapting a new solution approach for noisy rational expectations equilibria (Breon-Drish, 2015), I can solve for equilibria with uninformed policy maker in closed-form even if the functional form of the equilibrium price function is unknown at first. The standard guess-and-verify approach, on the other hand, requires knowledge of the functional form, which depends on the policy reaction to prices. Similar to the no-noise model, I derive a sufficient and necessary condition for the possibility of revealing equilibria in a fairly general class of equilibria that includes and generalizes the usual linear equilibria. I show that noise can solve the problem of self-defeating prophecies in some settings such as the one considered by Bernanke and Woodford (1997): Because noise prevents full revelation of trader information, the informed retain incentives to trade on information, making prices informative while they would not be without noise.

If the policy maker is informed, i.e., receives private signals about the state, then policy maker preferences not only determine whether informative equilibria exist, but also how informative they are. Because asset values are affected by policy decisions, a policy maker with independent information introduces additional 'policy risk' in asset returns beyond the usual risk over asset fundamentals, which influences how aggressively the informed trade on information. Policy risk can induce strategic complementarity leading to multiple equilibria. Interestingly, a higher quality of the policy maker signal can decrease the overall information available to the policy maker, because the informed react by trading less due to policy risk.

I derive measures for price informativeness and policy maker welfare to determine the impact of information contained in financial market prices on real decisions. Comparative statics show that more information in the financial market, less risk aversion among informed traders, and less noise in the market all increase price informativeness and the policy maker welfare gain due to financial market information. The quality of independent policy maker information and prior information decrease price informativeness and the welfare gain. Less extreme policy maker intervention preferences make prices more informative, but also make information less important to the policy maker, so the effect on welfare is ambiguous.

These results can be useful to identify situations where policy makers can use financial market information, and in designing institutions/assets that allow for better information revelation. Moreover, the problem raised here is a more fundamental challenge for the possibility of informationally efficient markets. Financial market prices may not reflect trader information even if all traders have perfect information, are perfectly rational, and obtain their perfect information for free. Similar to the Grossman and Stiglitz (1980)-paradox and its resolution, prices may not be informative in noiseless markets due to self-defeating prophecies, and even if prices are informative in noisy markets, they cannot be fully revealing.

The paper is organized as follows. The next subsections describe two applications in more detail, give a simple example illustrating the problem of self-defeating prophecies, and review the related literature. The first main part analyzes a general model without noise. The second main part analyzes a model with noise. The last section concludes.

1.1 Applications

This section briefly discusses two applications that fit the model, which are by no means the only ones. The first is corporate prediction markets (e.g., Wolfers and Zitzewitz, 2004; Cowgill and Zitzewitz, 2015). Corporate prediction markets are designed to elicit information dispersed among employees about business-relevant future outcomes such as whether project deadlines can be met, what next quarter's demand for a product will be, or whether a competitor will enter a particular market segment. Prediction markets typically trade simple assets whose value depends on these outcomes. For example, a project deadline prediction market asset may pay \$1 if and only if the deadline is missed, otherwise it pays \$0. The price of this asset can be interpreted as a market forecast of the probability that the deadline is missed: If many traders think the deadline is not feasible, then they buy the asset, driving the price up.

The information revealed by these markets is most useful if it is used to improve corporate decisions: If a deadline is likely not going to be met, then the company can assign additional resources. If a competitor is about to enter a market segment, then a price reduction might deter him. However, this "policy maker" reaction to market prices is exactly what can create self-defeating prophecies, because it also affects asset values. If traders buy the asset to signal that the deadline cannot be met, and the company reacts by assigning additional resources, then the asset becomes worthless, diminishing the incentives to trade on information in the first place! The results in this paper help to design these markets properly, so that traders have incentives to use their information if they anticipate that the market information is used for real decisions.

One example where these prediction markets affected company policy is mentioned in Cowgill and Zitzewitz (2015): Ford decided against introducing several new products after prediction market forecasts revealed that these would not be popular among consumers. Indeed, improving decisions was the main reason for using these markets: "Ford Motor Company [turned to prediction markets] to improve their ability to make decisions that would be in line with customer interests" (HPC Wire, 2011).

One caveat is that the model uses a competitive equilibrium concept, where traders act as price takers. In very small corporate prediction markets, this may not be realistic: A single trader can affect prices, which possibly introduces incentives for price manipulation. A lot of these corporate markets at Google and Ford are quite large (Cowgill and Zitzewitz, 2015)—at Google, all employees were eligible to participate—so the price taker assumption is plausible in many cases.

A second major application is central bank reaction to market prices. It is well known that central banks monitor asset prices, which can reveal information about inflation expectations or future inflation shocks (e.g., Bernanke and Woodford, 1997). Moreover, a large literature on Taylor rules finds that central banks react to asset prices, housing prices, or oil prices (e.g., Rigobon and Sack, 2003; Castro and Sousa, 2012; L'œillet and Licheron, 2012; Finocchiaro and Heideken, 2013). While these empirical results do not always explain why the central bank reacts to these asset prices, they do establish that prices affect policy decisions. The example in the next subsection illustrates how a self-defeating prophecy can arise in the central bank-trader interaction.

1.2 Self-defeating prophecies

How do self-defeating prophecies arise in a financial market context? Consider a very simple example in the spirit of Bernanke and Woodford (1997) for illustration.

Example 1. Suppose the future interest rate π is a function of (random) inflation pressures θ and interest rate i set by the central bank (CB), with $\pi = \theta - i$. Suppose $\theta \in \{0, 1\}$ with full support and $i \in \{0, 1\}$. The CB is inflation targeting and wants $\pi(\theta, i) = 0 \; \forall\theta$. Suppose the CB does not have any information on the realization of θ, while traders know θ. The financial market trades one asset, which is worth 1 if $\pi = 0$ and 0 otherwise. Consider the situation where the realization is $\theta = 1$ and the current policy is $i = 0$, which will remain unless the CB receives new information about θ. Can traders profit from their information about θ, and how would they invest to do so?

Suppose traders buy the asset up to price 1, leading to a market price of 1. Then the CB infers from asset prices that the target rate is reached ($\pi = 0$), and does not change policy.[2] Without the policy change, $\pi = \theta - i = 1$, hence the target rate is missed, the asset is worthless, and the traders lose everything they spent buying the asset. Clearly, this is not a behavior that forward looking traders would engage in. Now, instead suppose traders sell the asset at any positive price, leading to a market price of 0. Then the CB infers from asset prices that there are strong inflation pressures $\theta = 1$, hence the CB changes policy to $i = 1$. The target rate is therefore reached, $\pi = 0$, assets have value 1, and since traders sold the asset below value, they again lose money.

[2]Clearly, the CB expectations about how θ maps into prices is endogenous in equilibrium. In this example, the equilibrium candidate is that traders buy if $\theta = 1$, and sell otherwise, which generates the expectations described.

It is easy to see that there exists no price in the example that equals the eventual asset value, because the CB reaction leads to value 0 if the price is 1 and to value 1 if the price is 0; the pricing problem is a self-defeating prophecy. This problem does not occur in standard models where the asset value is exogenously fixed. □

1.3 Related literature

The paper probably closest to the first part is Bond et al. (2010). The authors consider the problem where a board of directors has no or only imperfect information about the quality of their agent, the company CEO, whereas traders have perfect information. A low quality CEO reduces the firm value, hence should be replaced to increase the firm value, whereas medium and high quality CEOs should not be replaced, since the intervention is costly. In this setting, there is a difficulty in inferring CEO quality from the company stock price, which is a function of the company value, if traders know that the board might react to it: If traders observe a low CEO quality and trade only at low prices, then the board infers low CEO quality from low prices, fires the CEO, increases the stock value, and effectively punishes traders for revealing the information. But then they have no incentive to reveal the information in the first place. The model without noise in this paper generalizes their setting to more flexible information structures and arbitrary policy maker preferences and policy variable spaces. The main addition provided here is the proof that derives a necessary and sufficient condition for the possibility of revealing equilibria, which also provides the link to other applications describing similar problems. Moreover, the model with noise introduced here shows that similar problems can occur even if prices are not fully revealing.

Bernanke and Woodford (1997), in an extension of the Woodford (1994) model, consider a central bank (CB) that attempts to infer a state variable θ from private forecasts or forecasts implicit in asset prices to reach a constant inflation target. Forecasters directly observe θ, the CB does not. In their static model, there is no rational expectations equilibrium that fully reveals θ to the CB. This follows from the impossibility that a forecast simultaneously reveals the state and correctly forecasts inflation, which will not depend on θ if the state is revealed to the CB. They show, moreover, that no equilibrium exists in their setting. Again, their application is an example of a self-defeating prophecy. Their static model is a special case of the model without noise in this paper, and the model with noise in this paper shows that their conclusions do not carry over to a noisy setting: With noise, an inflation targeting central bank can learn from prices in equilibrium.

Birchler and Facchinetti (2007) address a similar problem in banking supervision, and give a nice description of self-defeating prophecies and the "double endogeneity" problem of asset values affecting prices and prices affecting asset values via policy. They model a kind of prediction market that predicts bank failure, and the banking supervisor can react to information contained in these asset prices. As in the noiseless models above, full revelation may fail to occur because forward looking traders take into account that the supervisor will react to prices.

The setting considered here is strongly related to the recent literature on contingent capital with market triggers. The idea of contingent capital with market triggers is that information revealed via prices (typically financial health of a bank) is used for real decisions (convert debt into equity, helping struggling banks raise equity), but this in turn affects asset values (returns to equity are diluted). The argument for market triggers is that they provide more current information than accounting measures, which tend to have a considerable lag. The contingent capital models can also suffer from equilibrium non-existence due to self-defeating prophecies (Prescott, 2012; Sundaresan and Wang, 2015). The main difference is that real decisions in these models are not made by a utility maximizing policy maker, but by a mechanical rule that reacts to market prices. Still, many of the problems encountered in the policy maker settings carry over to the contingent capital setting; this applies in particular to the problem of self-defeating prophecies. The main technical difference besides the mechanical decision rule rather than a policy maker in Sundaresan and Wang (2015) is that they consider a continuous time pricing problem. Both papers model a market without noise.

The paper probably closest to the noisy financial market model in the second part is Bond and Goldstein (2015). They also consider a CARA-normal noisy REE model where traders have information about a state θ that the government would like to have, and the government action affects asset values. However, in their model, the state does not directly affect asset values, only indirectly via the government action. Consequently, given their linear policy rule, even if there were no noise, there is no possibility of self-defeating prophecies in their model. Unlike here, Bond and Goldstein (2015) do not analyze the case of an uninformed government. Thus, they only consider linear policy rules whereas this paper also allows for nonlinear policy rules if the policy maker is uninformed. An interesting difference is one of their policy conclusions on transparency: In their model, the government should not disclose its information about θ, as it takes away all incentives to trade on information; in my model, since θ also affects the asset values directly, disclosure can help the government to get more information from the market. Bond and Goldstein (2015) also analyze the question whether the government should commit itself to using financial market information more or less, which is not the focus of this paper.

This paper contributes to the growing literature of the real effects of financial markets via an informational channel, which mostly consists of studies without self-defeating prophecies. In most of this literature, the 'real effect' is the financial market information impact on corporate decisions, as in Dow and Gorton (1997); Subrahmanyam and Titman (1999); Goldstein and Guembel (2008); Foucault and Gehrig (2008); Goldstein et al. (2013); Dow et al. (2015); Edmans et al. (2015).

2 The model without noise

2.1 Set-up

Consider a financial market with a single riskless asset with rate normalized to 1, and a single risky asset. The optimal policy and the risky asset value depend on a state θ, which is the realization of a random variable distributed according to a common prior distribution on support Θ, where Θ contains at least two elements. The policy maker sets policy $i \in I$. The value of the risky asset is a function $a : \Theta \times I \to \mathbb{R}$, determined by state θ and policy i. Consequently, asset values are directly affected only by policy maker actions (for outcome manipulation by traders, see Ottaviani and Sørensen, 2007). Throughout I assume $\Theta, I \subseteq \mathbb{R}$.

The financial market consists of a continuum of risk neutral traders with a common prior. Every trader j receives an informative i.i.d. signal s_j on the realization of the state variable θ, distributed according to density $f(s_j|\theta) \neq f(s_j|\theta')\ \forall \theta \neq \theta' \in \Theta$. Since different trader signal profiles can contain the same information, denote the summary statistic of the signal profile by $\boldsymbol{s}$, and the set of all possible unique realizations of $\boldsymbol{s}$ by $\boldsymbol{S}$, so that $\forall \boldsymbol{s} \neq \boldsymbol{s}' \in \boldsymbol{S} : h(\theta|\boldsymbol{s}) \neq h(\theta|\boldsymbol{s}')$, where h is the conditional probability density function of θ. $\boldsymbol{s}$ is a sufficient statistic for signal profile $\{s_j\}_j$ if and only if $h(\theta|\{s_j\}_j, \boldsymbol{s}) = h(\theta|\boldsymbol{s}) = h(\theta|\{s_j\}_j)$. The following are three examples of commonly used information structures with corresponding summary statistic that are consistent with this setup.

Example 2.

1. Traders receive perfect signals, i.e., $s_j = \theta$ for all j, as for example in Bernanke and Woodford (1997) or Bond et al. (2010). The summary statistic is $\boldsymbol{s} = \theta$.

2. State $\theta \in \{0, 1\}$ is binomially distributed, $s_j \in \{0, 1\}$, and traders receive imperfect signals, i.e., $1 > \Pr(s_j = 1|\theta = 1) = \Pr(s_j = 0|\theta = 0) > 1/2$. The summary statistic is $\boldsymbol{s} = \int s_j dj$.

3. The state space is the entire real line, $\theta \in \mathbb{R}$, $s_j \sim \mathcal{N}(\theta, \sigma^2)$, and traders receive imperfect signals, i.e., $\sigma^2 > 0$. The summary statistic is $\boldsymbol{s} = \int s_j dj$.

Let $p(\boldsymbol{s}) : \boldsymbol{S} \to A$ be the price function mapping trader information $\boldsymbol{s}$ into an asset price. For example, $a(\theta, i)$ may represent the value of a company, while $p(\boldsymbol{s})$ is the price of the publicly traded company stock. The timing of decisions is illustrated in Figure 1: first, trading among all j leads to a market price $p(\boldsymbol{s})$, then, observing the price, the policy maker sets i. The results would be the same for simultaneous trading and policy-making, since policy can condition on the price but traders (prices) cannot condition on policy. In order to keep the analysis focused, I only consider the market aggregate (market price function) and not individual trader strategies for now.[3]

[3] Section 3, Appendix B, and Appendix C provide explicit microfoundations in terms of trader endowments, strategy spaces etc.

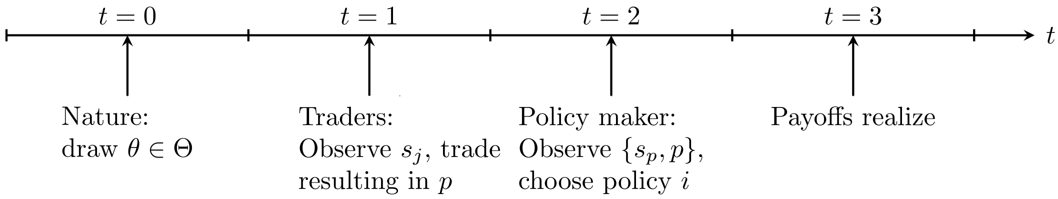


Figure 1: Timeline with traders and policy maker.

The policy maker receives an imperfect signal $s_p \in S_p$ on the realization of θ. In a special case the policy maker receives a completely uninformative signal, i.e., $|S_p| = 1$, which is equivalent to no signal. The utility function u represents the rational preference ordering of the policy maker over the tuple (θ, i). Thus, if trader information $\boldsymbol{s}$ were known to the policy maker, she would choose policy

$$i(\boldsymbol{s}) \in \arg\max_i \mathbb{E}[u(\theta, i)|\boldsymbol{s}, s_p].$$

To simplify the exposition, I will assume throughout this paper that i exists and is single-valued. Results can be adapted for multiple solutions and mixed policy strategies in a straightforward manner.

In a backward-induction-like step, define $v(\boldsymbol{s}) : \boldsymbol{S} \to A$, the expected asset value if $\boldsymbol{s}$ were known to the policy maker, who then implements her optimal policy $i(\boldsymbol{s})$,

$$v(\boldsymbol{s}) := \mathbb{E}[a(\theta, i(\boldsymbol{s}))|\boldsymbol{s}].$$

The objects $i(\boldsymbol{s})$ and $v(\boldsymbol{s})$ are defined *assuming* $\boldsymbol{s}$ is known to the policy maker, even though it is not. The reason is that once prices are fully revealing (see definition 2 below), then $\boldsymbol{s}$ is known to the policy maker. Hence, the policy maker will implement policy $i(\boldsymbol{s})$ leading to expected asset value $v(\boldsymbol{s})$. These are reactions that forward looking traders are going to anticipate if prices are fully revealing.

Given this information structure, policy i cannot be conditioned on $\boldsymbol{s}$ directly, only on $p(\boldsymbol{s})$, so the resulting asset value is $a(\theta, i(p(\boldsymbol{s}), s_p))$. Equilibrium (to be formally defined below) will require policy given beliefs to be optimal. Policy maker behavior, in particular u, is common knowledge. For non-triviality, I assume the optimal policy depends on $\boldsymbol{s}$, so the policy maker is interested in additional information which traders have.

The next definitions introduce two properties of price functions.

Definition 1. *A price function $p(\boldsymbol{s})$ is accurate if and only if*

$$p(\boldsymbol{s}) = \mathbb{E}[a(\theta, i(p, s_p))|p(\boldsymbol{s}) = p, s_j] \text{ for all } j.$$

In words, an accurate price function requires that asset prices equal asset values from the perspective of all traders, where the information set of trader j is both the information contained in prices and his private information s_j. The condition can be interpreted as requiring no systematic mispricing, which becomes clearer in Lemma 1 below.

In the present setting, $\boldsymbol{s}$ can only be indirectly revealed to the policy maker via price $p = p(\boldsymbol{s})$ in combination with the policy maker signal s_p. If the policy maker knows the price function (knows trader behavior), as she does for example in the perfect Bayesian Nash or rational expectations equilibrium concept, then $\boldsymbol{s}$ can always be inferred from the tuple $\{p = p(\boldsymbol{s}), s_p\}$ if the Bayesian posterior probability is positive for at most one $\boldsymbol{s} \in \boldsymbol{S}$.

Definition 2. *A price function $p(\boldsymbol{s})$ and policy maker signals are jointly fully revealing if and only if*

$$|\{t \in \boldsymbol{S} : \Pr(\boldsymbol{s} = t | p(\boldsymbol{s}) = p, s_p) > 0\}| \leq 1 \ \forall s_p, \forall p.$$

According to this definition, *prices need not be fully revealing to traders or outsiders*, only to the policy maker who can combine $p = p(\boldsymbol{s})$ and s_p. If policy maker signals are uninformative, i.e., $|S_p| = 1$, then full revelation reduces to invertibility of the price function in $\boldsymbol{s}$, because prices alone have to reveal $\boldsymbol{s}$. In this case, the information is fully revealed to anybody who observes the price and knows the price function.

The next lemma establishes that if a price function is fully revealing, then accuracy implies $p(\boldsymbol{s}) = v(\boldsymbol{s})$ and vice versa. Hence, if prices are fully revealing, then accurate prices must fully reveal the expected asset value given all trader information $v(\boldsymbol{s})$.

Lemma 1. *A fully revealing price function $p(\boldsymbol{s})$ is accurate if and only if $p(\boldsymbol{s}) = v(\boldsymbol{s})$.*

Proof. Necessity: Accuracy implies $p(\boldsymbol{s}) = v(\boldsymbol{s})$. First, $\mathbb{E}[a(\theta, i(p, s_p)) | p(\boldsymbol{s}) = p, s_j] = \mathbb{E}[a(\theta, i(\boldsymbol{s})) | p(\boldsymbol{s}) = p, s_j]$ since prices are fully revealing by assumption, and $\mathbb{E}[a(\theta, i(\boldsymbol{s})) | p(\boldsymbol{s}) = p, s_j] = \mathbb{E}[v(\boldsymbol{s}) | p(\boldsymbol{s}) = p, s_j]$ by the law of iterated expectations. Plugging into the definition of accurate prices, $p(\boldsymbol{s}) = \mathbb{E}[v(\boldsymbol{s}) | p(\boldsymbol{s}) = p, s_j]$ for all j (and all $\boldsymbol{s} \in \boldsymbol{S}$), and taking the conditional expectation on both sides, $\mathbb{E}[p(\boldsymbol{s}) | \boldsymbol{s}] = \mathbb{E}[\mathbb{E}[v(\boldsymbol{s}) | p(\boldsymbol{s}) = p, s_j] | \boldsymbol{s}]$. Again using iterated expectations yields $p(\boldsymbol{s}) = v(\boldsymbol{s})$.

Sufficiency: $p(\boldsymbol{s}) = v(\boldsymbol{s})$ implies accurate prices. This immediately follows:

$$\mathbb{E}[a(\theta, i(p, s_p)) | p(\boldsymbol{s}) = p, s_j] = \mathbb{E}[a(\theta, i(p, s_p)) | v(\boldsymbol{s}) = p, s_j] = v(\boldsymbol{s}) = p(\boldsymbol{s}) \text{ for all } j. \qquad \square$$

Finally, we are going to need a definition to characterize the policy maker signal structures that allow for full revelation. First, define the full inverse of $v(\boldsymbol{s})$, i.e., the set of $\boldsymbol{s} \in \boldsymbol{S}$ for which the expected asset value equals p if $\boldsymbol{s}$ is known to the policy maker as

$$v^{-1}(p) := \{\boldsymbol{s} \in \boldsymbol{S} : v(\boldsymbol{s}) = p\},$$

and the set of p for which $v^{-1}(p)$ contains more than one distinct element as

$$X := \{p \in \text{Image}(v(\boldsymbol{s})) : |v^{-1}(p)| > 1\}.$$

Thus, set X contains all prices $p = v(\boldsymbol{s})$ (assuming accurate prices) that are consistent with more than one distinct state $\boldsymbol{s} \in \boldsymbol{S}$. If X is empty, then $v(\boldsymbol{s})$ is invertible, i.e., $v^{-1}(p)$ is single-valued for all p. If prices alone do not always reveal $\boldsymbol{s}$, then X is non-empty. To achieve full revelation, the policy maker signal s_p has to discriminate between the possible states $v^{-1}(p)$ that are consistent with the observed price $p \in X$.

The following condition states that if $v(\boldsymbol{s})$ is not invertible, i.e., set $v^{-1}(p)$ contains more than one element for some price p, then the probability of receiving any signal s_p must be zero in all $\boldsymbol{s} \in v^{-1}(p)$ except for at most one.

Condition 1 (**Excluding signal structure in case of non-invertibility of $v(\boldsymbol{s})$**)**.**

$$|X| > 0 \implies \Pr(s_p|\boldsymbol{s} = t) \cdot \Pr(s_p|\boldsymbol{s} = t') = 0 \ \forall s_p \in S_p, \ \forall t \neq t' \in v^{-1}(p), \ \forall p \in X.$$

The condition has similarity to what Cabrales et al. (2014) call 'excluding signal structure,' but it is not identical, because in their meaning it is sufficient for signals s_p to exclude one state, whereas here policy maker signals might have to exclude several.

The condition is fulfilled if $v(\boldsymbol{s})$ is invertible, so that $|X| = 0$. The following example illustrates how full revelation may still occur even if the price function $p(\boldsymbol{s}) = v(\boldsymbol{s})$ is not invertible, because the policy maker can combine the information revealed by prices with her private information to rule out all states but one.

Example 3. Suppose $\theta \in \{1, 2, 3\}$ and traders observe the state, i.e., $s_j = \theta \ \forall j$. The policy maker receives the imperfect signal $s_p = 1$ if $\theta = 1$ and $s_p = 0$ if $\theta \in \{2, 3\}$.

Now suppose that asset values (if the policy maker knew θ) are $v(\theta) = 1$ if $\theta \in \{1, 2\}$ and $v(\theta = 3) = 0$. Consequently, $v(\theta)$ is not invertible and $|X| > 0$, since $v^{-1}(1) = \{1, 2\}$. Thus, imposing accurate prices, observing merely $p = p(\theta) = v(\theta)$ does not reveal θ, since $p(\theta = 1) = p(\theta = 2) = 1$. However, condition 1 still holds, because if the policy maker observes $p = p(\theta) = v(\theta) = 1$, then she can discriminate between the two states consistent with these prices, $\theta \in \{1, 2\}$, using her private information, since $s_p = 1$ if $\theta = 1$ and $s_p = 0$ if $\theta \in \{2, 3\}$. Both prices and signal s_p are necessary for full revelation; neither one on its own is sufficient to reveal θ. □

Bond et al. (2010) provide another example that fulfills condition 1, where $\theta \in \mathbb{R}$, $v(\theta)$ is non-invertible (see Figure 2 below), and the policy maker signal rules out all states but one consistent with market prices, since s_p is distributed on a bounded support around θ.

2.2 The possibility of information revelation via prices

This section asks if a price function $p(\boldsymbol{s})$ exists which allows for both full revelation and accurate prices. There is no microfoundation for this price function yet, i.e., the section does not explain how the price function arises in some specified trading game or equilibrium concept. This foundation will be provided in subsequent sections. The analysis is separated in this way to highlight that the impossibility of fully revealing and accurate prices does not depend on this microfoundation. Instead, under some conditions it is mathematically impossible to find a price function that is both fully revealing and accurate.

When is it possible for a price function to reveal $\boldsymbol{s}$ to the policy maker (definition 2) *and* price accurately (definition 1) at the same time? Without full revelation, the policy maker has inferior information and may implement suboptimal policies, and without accurate prices, traders might lose money, hence might be better off not trading. Given correct policy maker beliefs about the price function $p(\boldsymbol{s})$, Theorem 2 shows that this is possible if and only if condition 1 is satisfied.

Theorem 2 (**Possibility of full revelation and accurate prices**). *Suppose the policy maker knows function $p(\boldsymbol{s})$ and maximizes expected utility. Then a fully revealing and accurate price function exists if and only if condition 1 holds.*

Proof. See Appendix A. □

Theorem 2 characterizes the existence of fully revealing and accurate price functions in terms of policy maker preferences u, policy maker information structures $\Pr(s_p|\theta)$, asset payoff functions $a(\theta, i)$, and trader information structures $f(s_j|\theta)$. If there exists no price function that is both accurate and fully revealing, then such a price function cannot arise in equilibrium no matter the equilibrium concept.

Corollary 3. *Suppose the policy maker knows function $p(\boldsymbol{s})$ and maximizes expected utility. Then a fully revealing and accurate price function exists if $v(\boldsymbol{s})$ is invertible.*

Full revelation and accurate prices are possible either if $v(\boldsymbol{s})$ is invertible (for any policy maker signal structure), or if $v(\boldsymbol{s})$ is not invertible but the policy maker signal structure is excluding in the sense of condition 1. The requirement on the signal structure is rather strong, as it requires that any $\boldsymbol{s}$ which is not ruled out by the "price signal" is ruled out by the private signal of the policy maker s_p. For example, if traders have perfect information ($s_j = \theta \ \forall j$), and if $\theta \in \mathbb{R}$ and $s_p \sim \mathcal{N}(\mu, \sigma^2)$ with $\sigma^2 > 0$ as in many finance models, then $\Pr(s_p|\theta = t) = \phi\left(\frac{s_p - t}{\sigma}\right) > 0$ for all $t \in \mathbb{R}$ and $s_p \in S_p$, i.e., condition 1 is not fulfilled. Hence, full revelation with normally distributed policy maker signals is possible if and only if $v(\theta)$ is invertible, because policy maker signals never rule out any state.

Moreover, in the special case where the policy maker does not receive an informative signal, invertibility of $v(\boldsymbol{s})$ is necessary and sufficient for full revelation and accurate prices, again because policy maker signals never rule out any state.

Corollary 4. *Suppose the policy maker knows function $p(\boldsymbol{s})$, does not receive informative signals ($|S_p| = 1$), and maximizes expected utility. Then a fully revealing and accurate price function exists if and only if $v(\boldsymbol{s})$ is invertible.*

This simple condition—non-invertibility of the 'expected asset value given optimal policy' function $v(\boldsymbol{s})$—explains the general difficulty of finding fully revealing equilibria for most policy maker signal structures. Prices cannot reveal $\boldsymbol{s}$ and at the same time be accurate. Thus, traders have incentives to make the forecasts implicit in their trades less revealing (rather than wrong) or not to trade at all. Mathematically, the result obtains because a price function cannot at the same time be invertible (as required by full revelation) and non-invertible (as required by accuracy, which implies $p(\boldsymbol{s}) = v(\boldsymbol{s})$, if $v(\boldsymbol{s})$ is non-invertible).

Theorem 2 implies that accurate prices and full revelation are possible if condition 1 holds. The price function equal to the expected asset value function given optimal policy is an accurate forecast of the asset value, and at the same time reveals all information to the policy maker that is need to implement this asset value. Hence, condition 1 is also the condition that determines whether the forecasting problem is self-defeating or self-fulfilling.

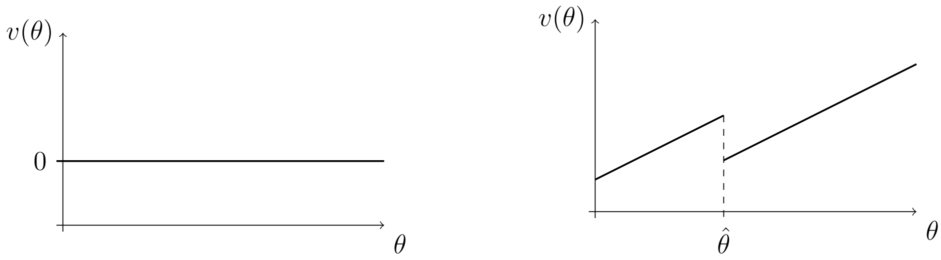
2.3 Examples

In several papers with policy maker-trader interaction from the literature section, traders have perfect information about the state variable θ. It can easily be verified that invertibility of $v(\theta)$ is not fulfilled in these papers, see Figure 2. For example (adapting their notation), in Bernanke and Woodford (1997)'s static model, the central bank wants to cancel out all variance due to inflation pressures θ, so that the asset value given the optimal policy using trader information is $v(\theta) = \theta + i = c$ for some constant c. In Bond et al. (2010) (similar in Prescott, 2012), the policy variable is binary, and an intervention is value increasing: $a(\theta, i = 1) > a(\theta, i = 0)$. The optimal policy calls for $i = 1$ if and only if $\theta \leq \hat{\theta}$ for some threshold $\hat{\theta}$, hence $v(\theta)$ has a discontinuous downward jump at $\hat{\theta}$, which makes it non-invertible (see Figure 2).

Hence, the underlying problem—non-invertibility of $v(\theta)$ or more generally failure of condition 1—is the same in these papers. Despite the same problem, preferences of the policy maker differ, which shows the problem of full revelation with self-defeating prophecies is not due to specific policy maker goals. In Bernanke and Woodford (1997), a central bank wants to minimize the variance of inflation, and in Bond et al. (2010) a board of directors wants to maximize firm value minus intervention cost. In Prescott (2012), the policy is determined by a capital conversion rule.

Despite non-invertibility of $v(\theta)$ in Bond et al. (2010), they show that full revelation may be possible under some conditions, exactly because the informative signal of the policy maker—which is uniformly distributed on a bounded support—excludes all states that are not ruled out by the price (condition 1).

Example 4. One of the most well known examples of an impressive "market forecast" was when the Challenger space shuttle exploded during take-off in 1986. While the investigators took about 4 months to officially announce the defective part and the responsible manufac-



(a) Bernanke and Woodford (1997): $i(\theta) = \arg\min_j \mathrm{Var}_\theta(\theta + i)$

(b) Bond et al. (2010), Prescott (2012): intervention below threshold $\hat{\theta}$ increases asset value

Figure 2: Examples for non-invertible asset values at the optimal policy $v(\theta)$ under full information

turing company, the stock price of the responsible company decreased about 12% on the day of the disaster (see Maloney and Mulherin, 2003 for more details). The stock prices of other companies that supplied parts to the space shuttle dropped only 2-3%. Thus, one can argue that the financial market revealed the responsible party almost instantaneously.

To formalize the situation in a very simple model, suppose the stock value of a supplier company is $a(\theta, i) = 1 - i(\theta)$, where $i = 1$ if and only if $\theta = 1$ and $i = 0$ if and only if $\theta = 0$, i.e., $i(\theta) = \mathbf{1}\{\theta = 1\}$. Variable i is the punishment a responsible company will incur, for example due to lawsuits, lost business etc., and $\theta \in \{0, 1\}$ indicates whether the company is responsible for the crash (i.e., its parts malfunctioned).

There are insiders in the market that know θ, i.e., whether the company is responsible. If θ was revealed via prices, then the asset value would be $v(\theta) = a(\theta, i(\theta)) = 1 - i(\theta) = 0$ if the company is responsible ($\theta = 1$), and $v(\theta) = 1$ if the company is not responsible. Thus, $v(\theta)$ is invertible, condition 1 holds, and the forecasting problem of finding the responsible party was not a self-defeating prophecy. Consequently, the anticipated reaction by NASA and authorities did not hamper incentives for traders to reveal their information, which may contribute to explain why the market forecast worked well in this situation.[4] □

2.4 Non-existence of fully revealing rational expectations equilibria

This section investigates whether accurate prices and full revelation can occur in a rational expectations equilibrium. That is, under which conditions can the financial market aggregate and reveal private information to the policy maker in equilibrium?

A rational expectations equilibrium in this setting is defined as follows (for a similar

[4] While the financial market information is certainly not evidence that holds up in court, the information may steer investigators in the right direction to find admissible evidence, thus affecting real decisions.

definition in the perfect information case, see Bond et al., 2010), where both the $p(\boldsymbol{s})$ and $i(p, s_p)$ function are known in equilibrium.

Definition 3. *A rational expectations equilibrium (REE) consists of*

i. A price function $p(\boldsymbol{s}) = \mathbb{E}_{\theta,s_p}[a(\theta, i(p, s_p))|p(\boldsymbol{s}) = p, s_j]$ *for almost all* j*, and*

ii. an optimal policy function $i(p, s_p)$ *given knowledge of* $p(\boldsymbol{s})$*, i.e.,*

$$i(p, s_p) = \arg\max_i \mathbb{E}_\theta[u(\theta, i)|p(\boldsymbol{s}) = p, s_p].$$

Condition (*i.*) of definition 3 requires that the equilibrium price equals the expected value of the asset given trader information for all traders. This price function is the market clearing outcome of an unmodeled noiseless competitive market with risk neutral traders and rational expectations. See Appendix B for an alternative definition explicitly modeling traders and endowments, yielding the same non-existence result, and DeMarzo and Skiadas (1998, 1999) for more details on risk neutral REE. Condition (*ii.*) requires that the policy maker acts optimally given her information s_p and the information contained in prices.

It turns out that the same condition which determines the existence of a fully revealing and accurate price function is also necessary and sufficient for the existence of a fully revealing REE. This is because Theorem 2 assumed that the policy maker knows $p(\boldsymbol{s})$ and acts optimally given her information, which is now an equilibrium requirement, and full revelation and accurate prices are also required in a fully revealing REE by definition.

Corollary 5. *A fully revealing (definition 2) rational expectations equilibrium exists if and only if condition 1 holds.*

Proof. Condition 1 of definition 3 implies that the REE price function has to be accurate (cf. definition 1). Thus, the result follows from Theorem 2. □

Thus, according to the REE concept, financial markets can both aggregate and reveal all trader information $\boldsymbol{s}$, but only if condition 1 holds. Hence, there are situations where markets cannot be strong-form informationally efficient, i.e., prices do not reflect all trader information. If prices fully revealed trader information, then in at least one state there would be mispricing, which introduces incentives to exploit the mispricing, and consequently traders do not support a fully revealing price function in equilibrium. Even in the most extreme case, where all traders perfectly know the state of the world ($s_j = \theta\ \forall j$) and perfectly know policy maker behavior, prices cannot reflect trader information if condition 1 fails to hold. The problem is not an informational one—traders know everything. Instead, accurate prices and full revelation are mutually exclusive, because the policy maker de facto “prefers to falsify trader forecasts.” This result is in strong contrast to the standard models without policy maker, where asset values are exogeneous and existence of fully revealing REE is generic (see for example Radner, 1979 or Allen, 1981).

Appendix C shows that the same condition is necessary and sufficient for full revelation using Perfect Bayesian Nash equilibrium in a continuum economy if traders receive perfectly correlated signals, so the results are relevant beyond the REE concept.

While Corollary 5 explains the non-existence of *fully revealing* equilibria in various applications, the conditions in Corollary 5 and the following Proposition 6 jointly explain the REE non-existence results (fully revealing or otherwise) for example in Bernanke and Woodford (1997) or Prescott (2012).

Proposition 6. *Suppose the policy maker does not receive any signals* ($|S| = 1$) *and traders are perfectly informed* ($s_j = \theta\ \forall j$)*. If* $a(t,i) \neq a(t',i)\ \forall t \neq t' \in \Theta,\ \forall i \in I$*, then there exists no 'not fully revealing' rational expectations equilibrium.*

Proof. Suppose there exists a not fully revealing REE. This implies there exist $t \neq t' \in \Theta$, such that $p(t) = p(t')$. Since the policy maker cannot distinguish the two states, she will take the same action i for $\theta = t$ and $\theta = t'$. But then for at least one of these two states the price must be inaccurate, since $p(t) = p(t')$ and $a(t,i) \neq a(t',i)$ implies $p(\theta) \neq \mathbb{E}[a(\theta,i)|s_j = \theta] = a(\theta,i)$ for at least one $\theta \in \{t,t'\}$, which contradicts this being an REE (condition 1 of definition 3). $\square$

Intuitively, if the asset value changes monotonically in the state θ, and prices are not fully revealing, then at least two states θ and θ' have to be 'bunched together' by the price function, $p(\theta) = p(\theta')$, so the policy maker cannot distinguish θ and θ'. But if traders know θ, then they know that the asset is mispriced for at least one of these two states, hence a not-fully revealing price function cannot be part of an equilibrium.

The last two results imply that the fully revealing REE is the unique REE if condition 1 holds, the policy maker is uninformed, and $a(\theta,i)$ invertible in θ.

2.5 Asset design and selection

The results of the previous sections state that accurate and fully revealing price functions and fully revealing equilibria exist with an invertible $v(\boldsymbol{s})$ function. From an asset design or asset selection point of view, we can ask what kind of asset supports full revelation. Formally, differences in assets are captured by the asset payoff function $a(.,.)$ that maps (θ,i) into an asset value.

In this section, I shall consider a class of assets whose value depends on some outcome o, described by a function $o : \Theta \times I \to \mathbb{R}$ that maps the realization of state θ and policy i into a real number. Consequently, the asset value indirectly depends on state θ and policy i to the degree that it influences the outcome $o(\theta,i)$. Thus, I consider the class of assets that can be written as $a(\theta,i) = A(o(\theta,i))$ for some function $A : \mathbb{R} \to \mathbb{R}$.

The A-function may be invertible, or non-invertible such as an Arrow security:

$$A(o(\theta, i)) = \begin{cases} 1 & \text{if } o(\theta, i) \geq T, \\ 0 & \text{if } o(\theta, i) < T, \end{cases}$$

for some threshold value $T \in \mathbb{R}$. Such asset payoff functions can be observed for example with credit default swaps, which have positive value if and only if the debtor solvency $o(\theta, i)$ is below a certain threshold that does not allow him to repay his debt. Bonds arguably also have a similar structure; they have positive value if and only if the issuing entity can repay. Moreover, prediction markets (e.g., Wolfers and Zitzewitz, 2004, Siemroth, 2014) often use this asset payoff function with so called 'winner take all' securities.

The following proposition shows that invertible asset payoff functions A are never worse, but may be better suited to promote fully revealing prices than non-invertible ones. Thus, we may say that invertible asset payoff functions are 'weakly preferable' in terms of information revelation. This is because a non-invertible asset payoff function may 'bunch' or 'pool' several states in a single asset value, thereby making it impossible to infer the state from the asset value, i.e., precluding invertibility of $v(\boldsymbol{s})$. Denote the set of all invertible functions $A : \mathbb{R} \to \mathbb{R}$ by $\mathcal{A}$ and the set of non-invertible functions by $\mathcal{A}'$.

Proposition 7 **(Asset design and full revelation).** *Consider the class of assets that can be written as $a(\theta, i) = A(o(\theta, i))$ for any $o : \Theta \times I \to \mathbb{R}$. If a non-invertible function $A' \in \mathcal{A}'$ allows full revelation and accurate prices, then so does any invertible $A \in \mathcal{A}$, but the converse does not hold.*

Proof. See Appendix A. □

Consequently, in this setting, we would expect assets with invertible asset payoff function $A \in \mathcal{A}$ to be more informative, all else equal, compared to assets with non-invertible asset payoff function. Proposition 7 thus provides an empirical implication of the theory, and a recommendation for anyone designing assets/markets/policies for information revelation.

Besides changing the asset value function, the proper choice of the underlying of the asset can help support information revelation. The following example investigates the project deadline assets that are used in corporate prediction markets (Cowgill and Zitzewitz, 2015).

Example 5. A project in a company may miss the deadline ($\theta = 1$) or it may meet the deadline ($\theta = 0$) with the currently available resources. Insiders—e.g., the employees that work on the project—know the state ($s_j = \theta$), while the manager (policy maker) does not. The deadline asset in the corporate prediction market pays \$1 if and only if the deadline is missed, and \$0 otherwise. The manager can react to information about whether the deadline will be met: $i = 0$ means the project does not receive additional resources (more manpower, funds, etc.), and $i = 1$ means the project receives additional resources that definitely ensure completion on time. The manager does not want to commit additional resources unless it is necessary. Consequently, the asset value is $a(\theta, i) = \mathbf{1}\{i = 0 \wedge \theta = 1\}$, i.e., the project misses the deadline if and only if the manager does not commit additional resources ($i = 0$) and the project misses the deadline without additional resources ($\theta = 1$).

To determine whether full revelation is possible in equilibrium, calculate $v(\theta)$, i.e., the asset value if θ (the trader information) were revealed to the manager. Clearly, $v(\theta = 1) = 0$, since the optimal policy is $i(\theta = 1) = 1$. Moreover, $v(\theta = 0) = 0$ with $i(\theta = 0) = 0$. Thus, the $v(\theta)$-function is not invertible in θ, and a self-defeating prophecy prevents revelation of trader information: Traders anticipate that revelation of $\theta = 1$ triggers a policy reaction that prevents the deadline being missed, hence they would lose money by trading on their information.

How can corporations solve this problem? Since they are free to design other assets in their markets, a simple adjustment fixes the problem. Consider another asset with value $a(\theta, i) = i$, i.e., the asset pays \$1 if and only if the company commits additional resources. The $v(\theta)$-function is $v(\theta = 1) = i(\theta = 1) = 1$ and $v(\theta = 0) = i(\theta = 0) = 0$, i.e., it is invertible. Thus, instead of designing an asset that predicts the outcome (deadline missed), which the company might seek to manipulate depending on state and information, another asset simply predicts the intervention decision. In equilibrium, traders have incentives to forecast the optimal policy for the policy maker, and this forecast is a self-*fulfilling* prophecy, since the policy maker wants to follow the "recommendation." In conclusion, if one is free to design assets, it will typically be possible to find outcomes and asset payoff functions such that the $v(\boldsymbol{s})$-function is invertible, which guarantees existence of a revealing equilibrium (Corollary 5), i.e., proper incentives to reveal the information. □

A natural question is whether it is always possible to find an asset payoff function $a : \Theta \times I \to \mathbb{R}$ that allows for fully revealing equilibria. This question is equivalent to asking whether an invertible function $a(\theta, i(\theta))$ always exists, but it is hard to answer at this level of generality. In theory, $a(\theta, i(\theta)) = \theta$ is always invertible in θ, yet θ may not be verifiable or contractible in all applications. For example, if θ is the quality of a company CEO, then the possibility of θ-revelation depends on whether observable measures of θ exist on which asset values might condition. If $a(\theta, i(\theta)) = \theta$ is not possible, then $a(\theta, i(\theta)) = i(\theta)$ works if $i(\theta)$ is invertible and verifiable. Consequently, this question has to be answered for specific applications.

2.6 Extension: Multiple policy makers

The setting so far assumes there is exactly one policy maker who can set policy and affect outcomes and asset values. However, in many applications, it is not just one non-trader who influences asset values. For example, it may not just be the central bank chairman that sets policy, but the board that determines policy with a voting procedure. Indeed, the players that affect policy do not even have to be part of the same organization or have the same goals. In this section, I briefly outline how the model can be easily extended for arbitrary sets of policy makers, i.e., players who can potentially affect policy, and arbitrary interaction between them. In short, instead of one policy maker setting policy, many potential policy makers take part in a game whose outcome is a policy, taking as given the financial market outcome $p = p(\boldsymbol{s})$ and the information contained therein.

To model this extension, I replace the policy maker decision (so far a subgame with one node) from the main section with a new subgame with arbitrarily many nodes and players. For example, the new extended policy maker subgame may be a voting game between board members. The outcome of the interaction between potential policy makers is some policy $i \in I$. The reaction function $i(p)$—previously the action that maximized the single policy maker's utility—is now the outcome in a (perfect) Bayesian Nash equilibrium of the policy maker subgame, taking financial market price $p = p(\boldsymbol{s})$ as given. Clearly, there may not be a unique equilibrium to the policy maker subgame, nor are the equilibria restricted to pure strategies. All we require is that the policy maker subgame has an equilibrium for all information that can be potentially revealed via prices.

Let $m \in M$ index all potential policy makers. The policy maker subgame consists of

- strategy sets I_m for each $m \in M$,
- signal structures s_m distributed according to $f_m(s_m|\theta)$,
- utilities over outcomes $u_m(i_1, i_2, \ldots, \theta)$ that may be affected by actions of others, and
- rules of the game g, specifying the timing of decisions within the subgame and how strategies map into policy i, i.e., $g : I_1 \times \ldots \times I_M \to I$.

The outcome function of the game $g(.)$ may be stochastic, e.g., because ties are broken. A (perfect) Bayesian Nash equilibrium of the policy maker subgame, where all agents $m \in M$ take financial market clearing prices $p = p(\boldsymbol{s})$ as given and understand the mapping from states to prices, requires that all $m \in M$ choose $i_m \in \arg\max_j \mathbb{E}_{\theta,g,i_{-m}}[u_m(i_m, i_{-m}, \theta)|p = p(\boldsymbol{s}), s_m]$. The equilibrium policy is $i(p) = g(i_1(p, s_1), i_2(p, s_2), \ldots)$.

The only additional difficulty compared to the main section is to solve for the equilibria of the policy maker subgame for each $p = p(\boldsymbol{s})$ in order to determine $i(p)$. The following example illustrates a simple application.

Example 6. Suppose there are two periods, the present and the future. The oil spot price P in the future is determined by equating oil supply i and future aggregate oil demand $\theta - \beta \cdot P$ with $\beta > 0$ (common knowledge) and $\theta \in \mathbb{R}^+$ (realization unknown to oil producers). θ can be interpreted as a future oil demand shock. Three oil producing countries $m \in \{1, 2, 3\}$ determine $i \in \mathbb{R}^+$ jointly in a majority vote. Each of the three countries has single peaked preferences over oil prices P, with peaks at P_m (these are common knowledge).

Consequently, country m would prefer $i(P_m) = \theta - \beta P_m$. Clearly, the optimal decision over oil supply i requires knowledge of future demand including θ. For simplicity, suppose traders know the realization of θ perfectly, and the value of oil futures $a(\theta, i)$ equals future oil prices (i.e., $a(\theta, i) = P = (\theta - i)/\beta$). Can the oil producing countries learn the realization of θ from prices of oil futures on a financial market and use it to determine supply?

To answer this, we determine the policy assuming the oil future prices perfectly reveal the realization of θ in a backward induction step. By the median voter theorem, the Nash equilibrium outcome of the policy maker subgame is $i = \theta - \beta \cdot \text{Median}(P_1, P_2, P_3)$ and $P = \text{Median}(P_1, P_2, P_3)$. In the notation of the main section, $v(\theta) = a(\theta, i) = P = \text{Median}(P_1, P_2, P_3)$. Thus, $v(\theta)$ is independent of θ and therefore non-invertible. Consequently, Corollary 5 implies that there exists no fully revealing REE. The intuition is quite clear: The policy makers want an oil price that is independent of θ, so oil futures prices that reveal θ would necessarily have to be mispriced in some states θ.

The proposition also helps to quickly determine how the assumptions of the example affect the results. For example, it might be more realistic to assume that the bliss points P_m are strictly increasing functions of θ, e.g., because larger oil demand θ would require more exploration at higher marginal costs to keep prices stable. Then $v(\theta) = \text{Median}(P_1(\theta), P_2(\theta), P_3(\theta))$, which is a strictly increasing function of θ, hence a fully revealing REE exists (Corollary 5). $\square$

3 The model with noise affecting market prices

3.1 Setup

This section presents a financial market model with noise where a policy maker reacts to information contained in prices and thereby changes asset values. Besides making the model more realistic, noise solves some undesirable features of noiseless models such as no trade, and allows us to investigate whether the previous results are an artifact of noiseless markets. I extend a standard constant absolute risk aversion (CARA)-normal model by introducing a policy maker. The equilibrium market clearing price p is affected by the realization of a random noise variable u, which is independent of the state θ. In the common interpretation, u is the aggregate net demand of noise traders, whose trading activity (due to exogenous reasons such as liquidity shocks) is independent of the price/state.

For rational traders and the policy maker, the noise shocks introduce a difficulty in extracting information from the price: A high asset price may indicate favorable information about the fundamental θ, or it may indicate a lot of noise trader purchases u which are unrelated to fundamentals. Consequently, traders and the policy maker will only be able to make stochastic inferences about the realization of θ from the market price, and cannot perfectly infer θ from the market price as in the previous section without noise.

More specifically, I solve for a noisy REE, where[5]

- a riskless asset (return normalized to zero) and a risky asset is traded in the financial market,
- the risky asset value is the sum of fundamental and policy,

$$a(\theta, i) = \theta + i,^{6}$$

- fundamental $\theta \sim \mathcal{N}(\bar{\theta}, 1/\tau_\theta)$, private trader signals $s_j = \theta + \varepsilon_j$, $\varepsilon_j \sim \mathcal{N}(0, 1/\tau_\varepsilon)$, private policy maker signal $s_p = \theta + \varepsilon_p$, $\varepsilon_p \sim \mathcal{N}(0, 1/\tau_\varepsilon)$, and aggregate noise trader net demand $u \sim \mathcal{N}(0, 1/\tau_u)$ are normally distributed, with $0 < \tau_\theta, \tau_\varepsilon, \tau_u < \infty$, and $\varepsilon_p, \varepsilon_j, u$ are independent of θ,
- there is a continuum of traders $j \in [0, 1]$, and all j have a CARA utility function defined over investment returns π_j, $U_j(\pi_j) = -\exp(-\rho_j \pi_j)$, with $\pi_j = (\theta + i - p)x_j$ and net demand x_j for the risky asset,
- a share $\mu \in (0, 1]$ of traders are informed, receive i.i.d. signals s_j about the realization of θ, have a coefficient of absolute risk aversion $\rho_I > 0$, and use demand strategies $X_I(p, s_j) : \mathbb{R}^2 \to \mathbb{R}$ to be specified later,
- a share $(1 - \mu) \in [0, 1)$ of traders are uninformed, have a coefficient of absolute risk aversion $\rho_U > 0$, and use demand strategies $X_U(p) : \mathbb{R} \to \mathbb{R}$ to be specified later,
- the policy maker has a utility function defined over state θ and policy i, $U(\theta, i)$, for which $i(z) = \arg\max_i \mathbb{E}[U(\theta, i)|z]$ exists for any normally distributed signal z,
- the environment (i.e., all of the above except for the realization of random variables $(\{s_j\}_{j=0}^{\mu}, s_p, u, \theta)$) is common knowledge.

CARA-utility functions exhibit no wealth effects, hence I normalize wealth to zero without loss of generality. As is standard, there are no budget constraints in this model; demands are bounded by the degree of risk aversion.

The timing is still as depicted in Figure 1: First, nature draws θ, then traders trade leading to a market clearing price p, the policy maker sets i, and finally asset values and payoffs realize. Since the policy maker can condition on the price, but traders cannot condition on the policy, the model yields the same equilibria if we assume simultaneous decisions of traders and policy maker.

[5] I heavily borrow notation from Vives (2010), who provides a detailed derivation of the standard linear noisy REE without policy maker.

[6] In the case with an uninformed policy maker, the results can be generalized any asset payoff function with an additively separable form $a(\theta, i) = \theta + g(i)$ with arbitrary $g : I \to \mathbb{R}$. In the case of an informed policy maker, however, this linear asset payoff function is critical. See also the discussion on quasi-linear equilibria below.

This extended model nests the standard CARA-normal REE, which is the special case $i = 0$, i.e., where the policy maker does not affect asset values. I choose the CARA-normal parametrization to facilitate comparison with the existing literature, which heavily relies on this framework since Grossman and Stiglitz (1980) and Hellwig (1980). Indeed, Vives (2010) calls it the workhorse model in the study of financial markets with asymmetric information.

If the policy maker is informed, it is crucial that demand and policy reaction functions are linear in price p and state θ to characterize the equilibrium at least implicitly. In these cases, I will use the following utility function,

$$U(\theta, i) = \psi_1 + \psi_2 i - \psi_3 i^2/2 + \psi_4 i \cdot \theta + \psi_5 \theta, \tag{1}$$

with $\psi_3 > 0$ to ensure a unique policy reaction and $\psi_4 \neq 0$ for non-triviality, which leads to a linear policy function $i(p, s_p) = \beta_1 + \beta_2 p + \beta_3 s_p$. This policy maker utility function is one flexible parameterization that yields a linear reaction function, although it is not the only one. One advantage of this particular parametrization is that it yields a simple and intuitive measure of the welfare impact of the additional information provided by the financial market, as is shown below. The reaction function of an informed policy maker is

$$\frac{\partial \mathbb{E}[U(\theta, i)|p, s_p]}{\partial i} = \psi_2 - \psi_3 i + \psi_4 \mathbb{E}[\theta|p, s_p] \stackrel{!}{=} 0 \iff i(p, s_p) = \frac{\psi_2 + \psi_4 \mathbb{E}[\theta|p, s_p]}{\psi_3}.$$

A noisy rational expectations equilibrium with an informed policy maker is defined as follows (the definition for an uninformed policy maker is easily adapted).

Definition 4. *A noisy rational expectations equilibrium is a set of trading strategies contingent on the available information, $X_I(p, s_j)$ for all $j \in [0, \mu]$ and $X_U(p)$ for all $j \in (\mu, 1]$, an optimal policy function $i(p, s_p)$, and a measurable price functional $P(\theta, u)$ such that*

1. *the market for the risky asset clears:*

$$\int_0^\mu X_I(p = P(\theta, u), s_j) dj + \int_\mu^1 X_U(p = P(\theta, u)) dj + u = 0 \text{ a.s.},$$

2. *all traders j use optimal demand strategies given the available information,*

$$X_I(p, s_j) \in \arg\max_x \ \mathbb{E}_{\theta, s_p}[U_j((\theta + i(p, s_p) - p)x)|p = P(\theta, u), s_j] \ \forall j \in [0, \mu],$$
$$X_U(p) \in \arg\max_x \ \mathbb{E}_{\theta, s_p}[U_j((\theta + i(p, s_p) - p)x)|p = P(\theta, u)] \ \forall j \in (\mu, 1],$$

3. *the policy maker sets an optimal policy given the available information,*

$$i(p, s_p) \in \arg\max_i \ \mathbb{E}_\theta[U(\theta, i)|p = P(\theta, u), s_p].$$

Any set of information structures and policy maker preferences that fulfills the parametric assumptions in this noisy REE setup can be translated to the model without noise, but not vice versa, since the model without noise allows for more general signal structures and policy maker preferences.

3.2 Measures of price informativeness and policy maker welfare

3.2.1 A new measure of price informativeness

Because the realization of the noise variable u introduces price movements unrelated to fundamentals or information, the price cannot perfectly reveal trader information or the state. Thus, a different concept than full revelation (definition 2) as in the model without noise is required to assess the informational content of market prices. The new measure is how much the additional information revealed by the market price reduces the policy maker θ-forecast error on average.

To derive this measure, first note that price informativeness is affected by how aggressively informed traders use their information to trade, which is captured by the coefficient a in the informed traders' net demand strategies,

$$X_I(p, s_j) = as_j - g_I(p).$$

Given $a > 0$, and holding noise variance $1/\tau_u > 0$ constant, a larger coefficient a means informed traders increase their net demand for the asset the more favorable their information s_j. Thus, the relative weight of information compared to noise increases, and prices become more informative. To see this more clearly, consider the market clearing condition if net demand strategies take the forms $X_I(p, s_j) = as_j - g_I(p)$ and $X_U(p) = -g_U(p)$ with equilibrium price function $P(\theta, u)$, which requires aggregate excess demand to be zero,

$$\begin{aligned}
&\int_0^\mu (as_j - g_I(P(\theta, u)))dj - (1-\mu)g_U(P(\theta, u)) + u = 0 \\
\iff\ &\frac{1}{\mu a}\left[\mu g_I(P(\theta, u)) + (1-\mu)g_U(P(\theta, u))\right] = \theta + u/(\mu a)
\end{aligned}$$

where I use a law of large numbers to evaluate the integral over i.i.d. trader signals s_j (Sun, 2006). Since the left hand side depends on (θ, u) only via $P(\theta, u)$, any price function that clears the market must reveal the term on the right hand side, $z := \theta + u/(\mu a)$. Hence, the 'price signal' is an additive statistic of the fundamental θ and a normally distributed noise term $u/(\mu a)$. This information contained in the price is independent of the endogenous demand strategy parts unrelated to private information (g_I and g_U), and only depends on how aggressively the informed trade on information (captured by strategy coefficient a).

Now, $\mathrm{Var}(u) = 1/\tau_u$, $\mathrm{Var}(u/(\mu a)) = 1/((\mu a)^2\tau_u)$, thus its precision is $\mathrm{Var}(u/(\mu a))^{-1} = (\mu a)^2\tau_u$. Using the standard Bayesian updating rule for normal distributions, the information z revealed by the price leads to the estimate

$$\mathbb{E}[\theta|p] = \mathbb{E}[\theta|z] = \frac{\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u z}{\tau_\theta + (\mu a)^2 \tau_u}, \tag{2}$$

which is the precision weighted sum of prior mean and price signal. To define a measure of price informativeness, consider the mean squared error of the policy maker "estimator" of θ given p,

$$\mathrm{MSE}(\mathbb{E}[\theta|p]) := \mathbb{E}\left[(\theta - \mathbb{E}[\theta|p])^2 | p\right],$$

which is a well known measure of the deviation or forecast error of an estimator from the variable to be estimated. To measure price informativeness, take the difference of the mean squared error of the policy maker estimate *without* the information contained in the market price and the mean squared error of the estimate *with* the information contained in the market price. Thus, the measure directly captures the differences in "forecast errors" of θ due to access to the price and the information contained therein: The larger the measure, the more the price information helps to estimate θ.[7] For an uninformed policy maker, the price informativeness measure is

$$\begin{aligned} \mathrm{PI}_{\text{uninformed}} :=& \mathrm{MSE}(\mathbb{E}[\theta]) - \mathrm{MSE}(\mathbb{E}[\theta|p]) = \mathbb{E}\left[(\theta - \mathbb{E}[\theta])^2\right] - \mathbb{E}\left[(\theta - \mathbb{E}[\theta|p])^2 | p\right] \\ =& \mathrm{Var}(\theta) - \mathrm{Var}(\theta|p) = \frac{1}{\tau_\theta} - \frac{1}{\tau_\theta + (\mu a)^2 \tau_u} = \frac{1}{\tau_\theta^2/((\mu a)^2 \tau_u) + \tau_\theta}, \end{aligned} \tag{3}$$

which is decreasing in prior precision τ_θ and increasing in $\mu, |a|, \tau_u$. Intuitively, if there is less uncertainty in the θ-realization (τ_θ increases), then more information (contained in p) does not help as much in predicting θ, thus the price informativeness measure decreases. $\mathrm{PI}_{\text{uninformed}}$ is not just price informativeness for the (uninformed) policy maker, but all uninformed traders or outsiders.

Similarly, we can define price informativeness if the policy maker is informed, which is the difference in MSE given her private information s_p and the MSE given her private information and the market price,

$$\begin{aligned} \mathrm{PI}_{\text{informed}} :=& \mathrm{MSE}(\mathbb{E}[\theta|s_p]) - \mathrm{MSE}(\mathbb{E}[\theta|p, s_p]) = \mathrm{Var}(\theta|s_p) - \mathrm{Var}(\theta|p, s_p) \\ =& \frac{1}{(\tau_\theta + \tau_\varepsilon)^2/((\mu a)^2 \tau_u) + \tau_\theta + \tau_\varepsilon}, \end{aligned} \tag{4}$$

[7] An alternative measure of price informativeness is $|\mu a|$, i.e., how aggressively the informed trade on information weighted by the share of informed traders (e.g., Bond and Goldstein, 2015). While this measure is appealing, it does not exactly capture how informative prices are for the policy maker. For example, consider the limit case of perfect information for the policy maker, i.e., the variance of the error term in the policy maker signal approaches zero. Then prices do not reveal any new information to the policy maker, but traders will still gain by trading on information ($a \neq 0$). Similarly, $|\mu a|$ does not capture changes in the prior distribution, while the MSE measure does.

which is decreasing in τ_θ and τ_ε, and increasing in $\mu, |a|, \tau_u$.

3.2.2 A measure of welfare gains

A central question of this paper is to what degree information from financial markets can help policy makers to improve real decisions. To make this assessment, I need to impose more structure on the policy maker utility function, and I will use the quadratic policy maker utility function (1) for this purpose. The welfare measure of interest is how much the financial market information improves the utility of the policy maker by improving decisions. We can think of the policy maker utility function as a welfare measure for all the (unmodeled) individuals that are affected by the policy maker decision.[8] For example, if the financial market can reveal information about future inflation shocks to the central bank, then the central bank will be better able to hit the target inflation rate or maintain price stability, thus improving welfare.

Since the optimal policy depends on the realization of θ, a utility loss can result if the policy maker sets a wrong policy due to imperfect information about θ. Define the expected utility loss of a policy maker with information set $\mathcal{I}$ due to imperfect information, i.e., the expected utility difference of a perfectly informed policy maker observing the realization θ and a policy maker observing only $\mathcal{I}$, as

$$\begin{aligned}
L(\mathcal{I}) &:= \mathbb{E}[U(\theta, i(\theta)) - U(\theta, i(\mathcal{I}))|\mathcal{I}] \\
&= \mathbb{E}[\psi_2(i(\theta) - i(\mathcal{I})) - (i(\theta)^2 - i(\mathcal{I})^2)\psi_3/2 + \psi_4\theta(i(\theta) - i(\mathcal{I}))|\mathcal{I}] \\
&= \frac{\psi_2\psi_4(\mathbb{E}[\theta|\mathcal{I}] - \mathbb{E}[\mathbb{E}[\theta|\mathcal{I}]|\mathcal{I}])}{\psi_3} - \frac{\mathbb{E}[(\psi_2 + \psi_4\theta)^2 - (\psi_2 + \psi_4\mathbb{E}[\theta|\mathcal{I}])^2|\mathcal{I}]}{2\psi_3} \\
&\quad + \frac{\psi_4^2(\mathbb{E}[\theta^2|\mathcal{I}] - \mathbb{E}[\theta\mathbb{E}[\theta|\mathcal{I}]|\mathcal{I}])}{\psi_3} \\
&= -\frac{\psi_4^2(\mathbb{E}[\theta^2|\mathcal{I}] - \mathbb{E}[\theta|\mathcal{I}]^2)}{2\psi_3} + \frac{\psi_4^2(\mathbb{E}[\theta^2|\mathcal{I}] - \mathbb{E}[\theta|\mathcal{I}]^2)}{\psi_3} \\
&= \frac{\psi_4^2}{2\psi_3}(\mathbb{E}[\theta^2|\mathcal{I}] - \mathbb{E}[\theta|\mathcal{I}]^2) = \frac{\psi_4^2}{2\psi_3}\mathrm{Var}(\theta|\mathcal{I}).
\end{aligned}$$

Thus, the utility loss due to imperfect policy maker information is proportional to the variance of θ given policy maker information, multiplied by a factor depending on how strongly the bliss point of the utility function reacts to a change in θ, i.e., how important θ is in making policy. Now we can define a welfare measure that directly captures the welfare impact of financial market information on policy maker utility. The measure is the difference in utility loss if the policy maker does not have access to the information provided by the financial market and if she does. For an uninformed policy maker, this leads to a welfare

[8] Clearly, using policy maker utility as welfare measure assumes a benevolent policy maker.

improvement due to financial market information

$$\begin{aligned}\Delta W_{\text{uninformed}} :=& L(\varnothing) - L(p) = \frac{\psi_4^2}{2\psi_3}(\mathrm{Var}(\theta) - \mathrm{Var}(\theta|p)) \\ =& \frac{\psi_4^2}{2\psi_3}\mathrm{PI}_{\text{uninformed}} = \frac{\psi_4^2}{2\psi_3}\frac{1}{\tau_\theta^2/((\mu a)^2\tau_u) + \tau_\theta},\end{aligned} \tag{5}$$

which is linear and increasing in price informativeness. Intuitively, the more informative the prices, the more they improve decisions and therefore welfare. The similarity of the expressions for price informativeness and welfare impact is a consequence of the quadratic utility function of the policy maker. The welfare impact of financial market information on an informed policy maker is

$$\begin{aligned}\Delta W_{\text{informed}} :=& L(s_p) - L(p, s_p) = \frac{\psi_4^2}{2\psi_3}(\mathrm{Var}(\theta|s_p) - \mathrm{Var}(\theta|p, s_p)) \\ =& \frac{\psi_4^2}{2\psi_3}\mathrm{PI}_{\text{informed}} = \frac{\psi_4^2}{2\psi_3}\frac{1}{(\tau_\theta + \tau_\varepsilon)^2/((\mu a)^2\tau_u) + \tau_\theta + \tau_\varepsilon}.\end{aligned} \tag{6}$$

3.3 Results with an uninformed policy maker

I begin the analysis with a policy maker who does not receive an independent signal on the fundamental θ, hence the only information available to her (apart from the prior) is contained in market prices. Usually, the CARA-normal setup is used for its tractability and simple closed-form solutions. However, the typical guess-and-verify approach guessing a linear equilibrium price function can only analyze cases with linear policy reaction functions. For nonlinear reaction functions, the approach requires knowledge of the functional form of the equilibrium price function. Thus, the approach is unsuitable to derive a condition for equilibrium existence depending on policy maker preferences. Breon-Drish (2015) recently demonstrated a novel solution approach in a model without policy maker where the typical CARA-normal assumptions can be relaxed. Adapting this new approach in an extended model with policy maker, I can solve for equilibria with nonlinear policy reaction functions if the policy maker is uninformed, and can derive a condition for equilibrium existence depending on policy maker preferences. Since the new approach does not require knowledge of the form of the equilibrium price function, it is simpler and more useful to analyze the model with arbitrary policy maker preferences. The only other noisy model of policy maker-trader interaction that I am aware of (Bond and Goldstein, 2015) only considers linear policy functions. Since the equilibria can be solved in closed-form, the model is useful for many applications. The prime application is monetary policy: There is considerable evidence that central bank policy reacts nonlinearly or asymmetrically to changes in price levels and stock prices (e.g., Weise, 1999; Kim et al., 2005; Surico, 2007; Ravn, 2012).

3.3.1 Equilibrium

With an uninformed policy maker, I will consider the class of equilibria where demand functions of the informed traders are possibly nonlinear in the price but additively separable from signal s_j,

$$X_I(s_j, p) = as_j - g_I(p),\ X_U(p) = -g_U(p),$$

and where the equilibrium price function $P(\theta, u)$ is continuous.[9] This class includes and generalizes the linear equilibria solved for in the standard guess-and-verify approach. Equilibrium existence or non-existence is understood within this equilibrium class.

Definition 5 (**Quasi-linear equilibrium**). *The class of equilibria where the net demand function of the informed traders takes the form $X_I(s_j, p) = as_j - g_I(p)$ for constant a and function $g_I : \mathbb{R} \to \mathbb{R}$, with continuous price function $p = P(\theta, u)$, is called quasi-linear.*

Recall that the equilibrium price function $P(\theta, u)$ maps all possible realizations (θ, u) into a price $p \in \mathbb{R}$. The market clearing condition given the demand functions is

$$\begin{aligned} &\int_0^{\mu} as_j - g_I(P(\theta, u))dj - (1-\mu)g_U(P(\theta, u)) + u = 0 \\ \iff\ &\frac{1}{\mu a}(\mu g_I(P(\theta, u)) + (1-\mu)g_U(P(\theta, u))) = \theta + u/(\mu a). \end{aligned} \tag{7}$$

Only the right hand side directly depends on (θ, u). Since the market clearing condition has to hold for all realizations (θ, u), it implies that the left hand side has to react to any change in $\theta + u/(\mu a)$. Since the left hand side depends on θ and u only via the equilibrium price function $P(\theta, u)$, it further implies that any equilibrium price function must reveal the linear statistic $\theta + u/(\mu a)$, i.e., a noisy signal of the state θ. Consequently, the price function must be invertible in $\theta + u/(\mu a)$, and defining $z := \theta + u/(\mu a)$, I shall write $P(z)$ instead of $P(\theta, u)$ in the following. Thus, if there is an equilibrium, the policy maker can infer at least the realization z from the price. Continuity of the price function ensures that prices reveal z and no more, as later shown in the proof. The contribution of Breon-Drish (2015) is to recognize that this approach allows us to pin down the information set of the uninformed (which in this setting includes the policy maker) without knowing the functional form of the equilibrium price function $P(\theta, u)$. The standard guess-and-verify approach, on the other hand, heavily relies on guessing the correct functional form of the equilibrium price function.

The utility function $U(\theta, i)$ of the uninformed policy maker is an arbitrary function of θ and i so that $i(p) \in \arg\max_i \mathbb{E}_\theta[U(\theta, i)|p = P(z)]$ always exists for normally distributed signals p and is continuous (to ensure that the price function is continuous). Moreover, for

[9] While equilibria in the class of continuous equilibria are unique (see Breon-Drish, 2015 and Proposition 8 below), Pálvölgyi and Venter (2015) show for the classical Grossman and Stiglitz (1980)-model that a continuum of discontinuous equilibria with varying price informativeness exist besides the standard linear equilibrium. Thus, we may view the focus on continuous price functions as a reasonable equilibrium refinement.

equilibrium uniqueness I require $i(p)$ to be unique. $i(p)$ is the policy maker reaction function to prices. The asset value $\theta + i(p)$ conditional on the price is normally distributed, even if the policy reaction function $i(p)$ is nonlinear, since it only depends on p. For a normally distributed asset value, the optimal demand of the informed traders with CARA utility is

$$
\begin{aligned}
X_I(s_j, p) &= \frac{\mathbb{E}[\theta + i(p) - p | s_j, p]}{\rho_I \text{Var}(\theta + i(p) - p | s_j, p)} = \frac{\mathbb{E}[\theta | s_j, p] + i(p) - p}{\rho_I \text{Var}(\theta | s_j, p)} \\
&= \frac{\tau_\theta + \tau_\varepsilon + (\mu a)^2 \tau_u}{\rho_I} \left(\frac{\tau_\theta \bar{\theta} + \tau_\varepsilon s_j + (\mu a)^2 \tau_u P^{-1}(p)}{\tau_\theta + \tau_\varepsilon + (\mu a)^2 \tau_u} + i(p) - p \right) \\
&= \left[\tau_\theta \bar{\theta} + \tau_\varepsilon s_j + (\mu a)^2 \tau_u P^{-1}(p) + (i(p) - p)(\tau_\theta + \tau_\varepsilon + (\mu a)^2 \tau_u) \right] / \rho_I,
\end{aligned} \tag{8}
$$

which is of the form $X_I(s_j, p) = a s_j - g_I(p)$ as assumed, with

$$
g_I(p) = -\left[\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u P^{-1}(p) + (i(p) - p)(\tau_\theta + \tau_\varepsilon + (\mu a)^2 \tau_u) \right] / \rho_I, \tag{9}
$$

and, matching the coefficient, $a = \tau_\varepsilon / \rho_I$. The demand of the uninformed traders is similarly

$$
X_U(p) = \frac{\mathbb{E}[\theta | p] + i(p) - p}{\rho_U \text{Var}(\theta | p)} = \left[\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u P^{-1}(p) + (i(p) - p)(\tau_\theta + (\mu a)^2 \tau_u) \right] / \rho_U, \tag{10}
$$

with

$$
g_U(p) = -\left[\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u P^{-1}(p) + (i(p) - p)(\tau_\theta + (\mu a)^2 \tau_u) \right] / \rho_U. \tag{11}
$$

These demand functions are uniquely determined given that prices reveal z. This fact together with the requirement that $P(z)$ be invertible implies equilibrium uniqueness in the class of quasi-linear equilibria. Substituting $g_I(p), g_U(p)$ into the market clearing condition and setting $p = P(z)$,

$$
\begin{aligned}
-\frac{1}{\mu a} \big(\mu \left[\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u P^{-1}(P(z)) + (i(P(z)) - P(z))(\tau_\theta + \tau_\varepsilon + (\mu a)^2 \tau_u) \right] / \rho_I \\
+ (1 - \mu) \left[\tau_\theta \bar{\theta} + (\mu a)^2 \tau_u P^{-1}(P(z)) + (i(P(z)) - P(z))(\tau_\theta + (\mu a)^2 \tau_u) \right] / \rho_U \big) = z.
\end{aligned} \tag{12}
$$

As established before, market clearing requires the equilibrium price function $P(z)$ to be invertible. Thus, we can rewrite $P^{-1}(P(z)) = z$ and, abusing notation, $i(P(z)) = i(z)$ with $i(z) = \arg\max_i \mathbb{E}[U(\theta, i) | z]$. These are the price terms that only depend on the information contained in prices and do not change if we change the price function (assuming it remains invertible and thus reveals the same information z). The market clearing condition is therefore

$$
\begin{aligned}
-\frac{1}{\mu a}\big(\mu\left[\tau_\theta\bar{\theta}+(\mu a)^2\tau_u z+(i(z)-P(z))(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)\right]/\rho_I \\
+(1-\mu)\left[\tau_\theta\bar{\theta}+(\mu a)^2\tau_u z+(i(z)-P(z))(\tau_\theta+(\mu a)^2\tau_u)\right]/\rho_U\big)=z \\
\iff -\frac{1}{\mu a}\big(\mu\left[\tau_\theta\bar{\theta}+(i(z)-P(z))(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)\right]/\rho_I \\
+(1-\mu)\left[\tau_\theta\bar{\theta}+(i(z)-P(z))(\tau_\theta+(\mu a)^2\tau_u)\right]/\rho_U\big)=z(1+\mu^2 a\tau_u/\rho_I+(1-\mu)\mu a\tau_u/\rho_U) \\
\iff P(z)\left[\mu(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)/(\rho_I\mu a)+(1-\mu)(\tau_\theta+(\mu a)^2\tau_u)/(\rho_U\mu a)\right] \\
=z(1+\mu^2 a\tau_u/\rho_I+(1-\mu)\mu a\tau_u/\rho_U)+\mu[\tau_\theta\bar{\theta}+i(z)(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)]/(\rho_I\mu a) \\
+(1-\mu)[\tau_\theta\bar{\theta}+i(z)(\tau_\theta+(\mu a)^2\tau_u)]/(\rho_U\mu a) \\
\iff P(z)=\Big\{z(1+\mu^2 a\tau_u/\rho_I+(1-\mu)\mu a\tau_u/\rho_U)+\mu[\tau_\theta\bar{\theta}+i(z)(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)]/(\rho_I\mu a) \\
+(1-\mu)[\tau_\theta\bar{\theta}+i(z)(\tau_\theta+(\mu a)^2\tau_u)]/(\rho_U\mu a)\Big\} \\
/\Big\{\mu(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)/(\rho_I\mu a)+(1-\mu)(\tau_\theta+(\mu a)^2\tau_u)/(\rho_U\mu a)\Big\},
\end{aligned}
\tag{13}
$$

which is an explicit expression for the price function given optimal demands if the price function reveals z to all traders and policy maker, and if traders anticipate the policy reaction $i(z)$. For equilibrium, we still have to confirm that the price function is in fact invertible, as required for market clearing. Only the numerator depends on z, so to determine invertibility we can drop the denominator, which is a positive constant. We can also drop additive terms from the numerator. Thus, $P(z)$ is invertible if and only if

$$
\begin{aligned}
z(1+\mu^2 a\tau_u/\rho_I+(1-\mu)\mu a\tau_u/\rho_U)+i(z)[\mu(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)/(\rho_I\mu a) \\
+(1-\mu)(\tau_\theta+(\mu a)^2\tau_u)/(\rho_U\mu a)]
\end{aligned}
\tag{14}
$$

is invertible in z. These terms capture how aggregate net demand changes if z changes. But (14) might not be invertible if $i(z)$ is at least locally decreasing, since the terms in brackets are positive. Thus, the one and only possible cause of non-invertibility and equilibrium non-existence is the policy reaction function $i(z)$ to information z: In the standard case without policy maker, $i(z)=0$ for all z, so the equilibrium price function is linearly increasing and an equilibrium always exists. Notice the similarity to the invertibility condition in the model without noise, which requires that the expected asset value given optimal policy and trader information $v(\boldsymbol{s})$ is invertible. If we let the noise variance go to zero here ($\tau_u\to\infty$), then (14) reduces to invertibility of $z+i(z)$, and since $z=\theta+u/(\mu a)=\theta$ a.s. for $\tau_u\to\infty$, $\theta+i(\theta)$ is the asset value given optimal policy and trader information.

The approach taken in this section is similar to the approach taken in the section without noise: I first assume the price function is invertible in $z=\theta+u/(\mu a)$, a noisy signal of θ, and therefore fully reveals the noisy signal. Second, I obtain the reaction function $i(z)$ of the policy maker to that information, which determines how the expected asset value is affected by policy. Finally, I check whether the price function given the optimal demands of the traders is in fact invertible. If it is not, then traders are not willing to clear the market for any price function that fully reveals z and triggers policy reaction $i(z)$, so that no equilibrium in the class of quasi-linear equilibria exists. As in the model without noise, it is a self-defeating prophecy that precludes revealing financial market prices: If market prices were revealing, they would lead to policy reactions for which traders would not want to clear the market.

The following proposition gives the formal result.

Proposition 8. *An equilibrium in the class of quasi-linear equilibria exists if and only if there exists a continuous reaction function $i(z) \in \arg\max_i \mathbb{E}[U(\theta, i)|z]$ such that, for $a = \tau_\varepsilon/\rho_I$,*

$$z(1 + \mu^2 a\tau_u/\rho_I + (1-\mu)\mu a\tau_u/\rho_U) + i(z)[\mu(\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u)/(\rho_I\mu a) \\ +(1-\mu)(\tau_\theta + (\mu a)^2\tau_u)/(\rho_U\mu a)]$$

is invertible in z. If it exists, then the equilibrium is unique in the class of quasi-linear equilibria and market prices fully reveal the noisy signal $z = \theta + u/(\mu a)$.

The trader equilibrium strategies are $X_I(s_j, p) = as_j - g_I(p)$ and $X_U(p) = -g_U(p)$ with $a = \tau_\varepsilon/\rho_I$, $g_I(p)$ given in (9)*, and $g_U(p)$ given in* (11)*. The equilibrium policy reaction is $i(P^{-1}(p)) = i(z)$. The equilibrium price function $P(\theta, u) = P(z)$ is given in* (13)*.*

Proof. See Appendix A. □

Interestingly, market prices are informative, but the trading aggressiveness a of the informed traders is constant and unaffected by policy maker preferences. The explanation is given below in example 7.

The next result makes the requirement on the reaction function $i(z)$ more salient: Assuming differentiability, its derivative has to be either large enough or small enough for all z, but it may not have both positive and large negative slopes on $z \in \mathbb{R}$. Consequently, quadratic or quartic reaction functions will produce self-defeating prophecies, whereas almost all linear functions $i(z)$ and all weakly increasing functions $i(z)$ allow for revealing equilibria. Moreover, "weak" non-invertibilities in $i(z)$ such that $\sup_z i'(z) > 0$ and $0 > \inf_z i'(z) > \xi$, where $\xi < 0$ is defined in (15), can be supported in equilibrium.

Corollary 9. *If the policy reaction function $i(z)$ is continuous and differentiable, then there exists an equilibrium in the class of quasi-linear equilibria if and only if $i'(z) > \xi$ almost everywhere or $i'(z) < \xi$ almost everywhere, with $a = \tau_\varepsilon/\rho_I$ and*

$$\xi := -\frac{1 + \mu^2 a\tau_u/\rho_I + (1-\mu)\mu a\tau_u/\rho_U}{\mu(\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u)/(\rho_I\mu a) + (1-\mu)(\tau_\theta + (\mu a)^2\tau_u)/(\rho_U\mu a)} < 0. \tag{15}$$

Proof. A continuous function on a convex set is invertible if and only if it is strictly monotone. Thus, taking the derivative of (14) with respect to z and requiring it to be strictly greater or smaller than zero for almost all z (allowing for saddle points on a measure zero set) yields the result. $\square$

In the noiseless limit as $\mathrm{Var}(u) = 1/\tau_u \to 0$, $\xi \to -1$, i.e., the policy reaction $i(z)$ may not exactly offset any increase in $z = \theta + u/(\mu a) = \theta$ (a.s.) for an informative equilibrium to exist. An example of $i'(z) = -1$ in the noiseless case is Bernanke and Woodford (1997), as discussed below.

To summarize the results so far, the new approach of solving for noisy rational expectations equilibria allows for nonlinear policy maker reaction functions, which covers a larger set of policy maker preferences and settings with self-defeating prophecies from the no-noise model. This section is therefore a large step towards modeling and analyzing the policy maker-trader interactions in the presence of noise, providing the tools for many applications beyond linear policy rules. While the results here focused on continuous reaction functions $i(z)$, there are no functional form restrictions beyond that. Proposition 8 shows that noise cannot generally solve the problem of self-defeating prophecies, thus the problem of self-defeating prophecies also occurs in more realistic models with noise.

Still, situations with policy maker objectives that implied a self-defeating prophecy without noise may not be a self-defeating prophecy with noise, hence noise may support rather than hamper information revelation via prices in some instances. To understand why, consider an equilibrium where the policy maker has the quadratic utility function (1), which yields a linear reaction function, with $\psi_4/\psi_3 = -1$. $\psi_4/\psi_3 = -1$ means that the policy maker sets $i(p)$ such that $\mathbb{E}[\theta + i(p)|p] = c$ for some $c \in \mathbb{R}$, and consequently the asset value $\theta + i(p)$ would be constant for every realization of θ if prices were fully revealing (i.e., condition 1 does not hold, so there is no revealing equilibrium without noise). Bernanke and Woodford (1997) is one example from the literature which investigates exactly this case. Consider an example in the noisy REE setup, where an equilibrium with the following strategies exists.

Example 7. The parameters are $\psi_4/\psi_3 = -1$, $\psi_2 = 0$, $\bar{\theta} = 3$, $\tau_\varepsilon = 2$, $\tau_\theta = 1$, $\tau_u = 1$, $\mu = 0.5$, $\rho_I = 2$, $\rho_U = 2$. The resulting noisy REE strategies are

$$X_I(p, s_j) = s_j - 2.1875p - 3, \; X_U(p) = -0.625p,$$
$$i(p) = -3 - 0.5625p.$$

Figure 3 plots the equilibrium price function at the mean/median value of the noise variable ($u = 0$), which is also the average/median price function due to linearity in u. The price function is increasing in θ, even though the policy maker wants to implement a constant asset value. The figure further displays the expected value of θ given the information contained in price p, which is also increasing, but at a relatively flat slope. This line is

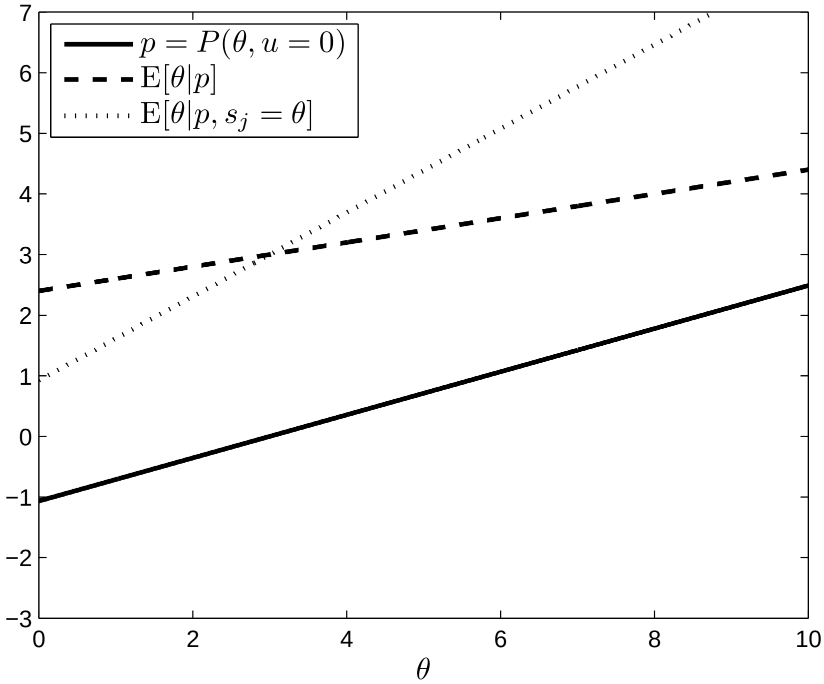


Figure 3: Plot of the average equilibrium price function $P(\theta, u = 0)$ (solid line), the expected value of the state variable θ for the uninformed (dashed), and the expected value of the state variable for the average informed trader (dotted).

the inference that the policy maker and uninformed traders draw from the price. If prices were fully revealing, then the expected value of θ given the price would be the 45° line, but the noise prevents full revelation of the state. The line of the expected value for the average informed trader is steeper than the line of the uninformed, reflecting their superior information about θ, but still smaller than one.

Figure 4 explains why the equilibrium price is informative, but not perfectly informative. It plots the average equilibrium price function, as well as the expected asset value given p (dashed)—this is the expected asset value for uninformed traders and the policy maker—and the expected asset value given p and signal $s_j = \theta$ (dotted), which is the expected asset value for the mean/median informed trader. First consider the expected asset value for the uninformed. It is constant, which just reflects the policy maker preferences $\psi_4/\psi_3 = -1$, where she sets $i(p)$ such that $\mathbb{E}[\theta + i(p)|p]$ is constant. Uninformed traders sell the asset for any price above 0—where the expected profit from buying is negative—and buy for any price below 0.[10]

The important line is the expected asset value for the average informed trader. His asset valuation and expected profit from buying increases the larger θ, since the expected asset value function is steeper than the price function. The reason is that the equilibrium

[10]This example may seem to suggest that uninformed traders lose money, but this is just because the graphs consider the realization $u = 0$, which implies that noise traders do not trade. For any other realization of u (measure 1) they do trade and lose money in expectation to the uninformed (and informed) traders.

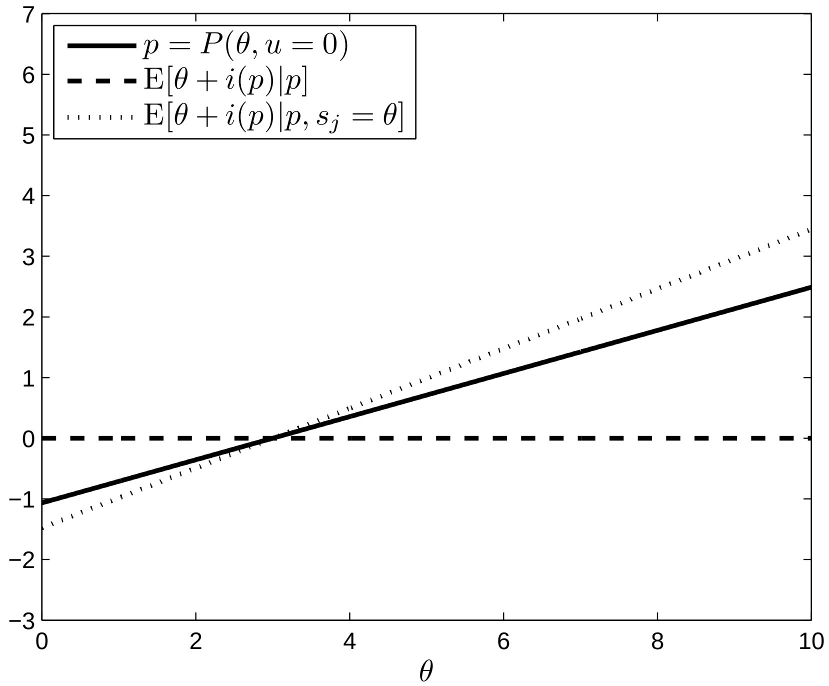


Figure 4: Plot of the average equilibrium price function $P(\theta, u = 0)$ (solid line), the expected value of the asset for the uninformed (dashed), and the expected value of the asset for the average informed trader (dotted).

price function only imperfectly reveals information about θ, which is why the policy maker and uninformed traders tend to underestimate θ for θ-realizations above the prior mean $\bar{\theta} = 3$, and tend to overestimate for θ-realizations below the prior mean (see Figure 3). The informed traders have a better estimate of θ due to their signals. The expected asset value for most informed traders exceeds the price for $\theta > \bar{\theta}$, because their signals tend to be closer to the actual θ than what the price implies, so they buy more of the asset the larger their realization of s_j, which increases their estimate of θ. This is why $a > 0$ and why the equilibrium price function is informative: Informed traders have incentives to trade on their information, because noise prevents full revelation of their information, which otherwise would lead to a self-defeating prophecy. □

Thus, similar to the solution of the Grossman and Stiglitz (1980) paradox, prices in this instance are informative *because* of noise, rather than despite of noise. The explanation that the informed traders profit from information applies beyond this example, since the estimate of θ for the uninformed tends to be more tilted towards the prior mean $\bar{\theta}$, so it always pays for the informed to increase net demand for larger s_j in equilibrium.

Finally, it is also interesting to consider the limit as the variance of the noise variable u approaches zero, or equivalently, if the precision diverges to infinity. The next result shows that, in the limit, market prices are fully revealing if an equilibrium exists. Thus, as the noise variance becomes smaller and smaller, market prices become arbitrarily close to fully revealing.

Corollary 10. *If the policy maker is uninformed, then equilibrium market prices become fully revealing in the limit as the noise variance* $1/\tau_u$ *approaches zero, i.e.,*

$$\mathbb{E}[\theta|p] \xrightarrow{a.s.} \theta \text{ for } \tau_u \to \infty.$$

Proof. From Proposition 8, $a = \tau_\varepsilon/\rho_I > 0$ in equilibrium, which is independent of u or τ_u. Thus,

$$\lim_{\tau_u\to\infty} \mathbb{E}[\theta|p] = \lim_{\tau_u\to\infty} \frac{\tau_\theta\bar{\theta} + (\mu a)^2\tau_u(\theta + u/(\mu a))}{\tau_\theta + (\mu a)^2\tau_u} = \lim_{\tau_u\to\infty} \theta + u/(\mu a)) = \theta \text{ (a.s.)},$$

since $\mathbb{E}[u] = 0$ and $\mathrm{Var}(u) = 1/\tau_u \to 0$. □

3.3.2 Comparative statics

The last results for an uninformed policy maker determine the factors that influence price informativeness and policy maker welfare if an equilibrium exists.

Proposition 11 (**Comparative statics**). *In the quasi-linear noisy REE with uninformed policy maker,*

- *larger trader signal precision* τ_ε*, share of informed traders* μ*, or noise precision* τ_u*, and*
- *smaller prior distribution precision* τ_θ*, or risk aversion* ρ_I*,*

increase price informativeness (3)*, all else equal.*

Proof. From (3), price informativeness does not depend on demand strategies except for coefficient a. From Proposition 8, the equilibrium strategy coefficient is $a = \tau_\varepsilon/\rho_I$. Thus, price informativeness is increasing in $|a|, \mu$ and τ_u, and $|a|$ in turn is increasing in τ_ε and decreasing in ρ_I. Moreover, price informativeness (3) decreases in τ_θ. □

To assess the welfare impact of additional information from financial market prices, we need to be more explicit about policy maker preferences. The following corollary derives the comparative statics if the policy maker has quadratic utility. The results are identical to those of price informativeness, since more information about θ helps the policy maker to implement better policies, except for the effect of $|\psi_4/\psi_3|$. This coefficient in the utility function of the policy maker (1) captures how strongly the optimal policy depends on the realization of θ. If it is larger, then better information about θ improves welfare more.

Corollary 12 (**Comparative statics welfare**). *In the linear noisy REE with quadratic policy maker utility function* (1)*,*

- *larger $|\psi_4/\psi_3|$, trader signal precision τ_ε, share of informed traders μ, or noise precision τ_u, and*

- *smaller prior distribution precision τ_θ, or risk aversion ρ_I,*

increase the policy maker utility gain due to financial market information (5)*, all else equal.*

Proof. From (5), the welfare gain is an increasing linear function of price informativeness, hence the same comparative statics as in Proposition 11 hold. In addition, the factor ψ_4^2/ψ_3 is increasing in $|\psi_4/\psi_3|$ ($\psi_3 > 0$ by construction). □

3.4 Results with an informed policy maker

3.4.1 Equilibrium

If the policy maker is informed, i.e., receives independent information s_p, then demand functions of the informed are not of the quasi-linear form as in definition 5 unless the policy reaction function is linear. Consequently, throughout this section, I assume the policy maker has the quadratic utility function (1), which implies a linear reaction function of the form $i(p, s_p) = \beta_1 + \beta_2 p + \beta_3 s_p$. The ratio of two parameters from the utility function, ψ_4/ψ_3, will be of particular interest in this section, which determine how strongly and in which direction the optimal policy reacts to the realization of θ. Traders have linear demand functions of the form $X_I(p, s_j) = as_j - c_I p + b_I$ and $X_U(p) = -c_U p + b_U$.

Despite these stronger assumptions compared to the previous sections, the equilibrium cannot be explicitly solved, since the equilibrium condition (28) determining a is a fifth degree polynomial, which does not generally admit an analytical solution. Without solving explicitly, I first establish existence of equilibrium for almost all parameter profiles, but there need not be a unique equilibrium. Then I give conditions for equilibrium uniqueness and derive comparative statics in these unique equilibria. Finally, I explain how "policy risk" affects equilibrium trading behavior and price informativeness, and how policy risk can create a form of strategic complementarity and induce multiple equilibria.

Proposition 13 (**Equilibrium existence**)**.** *If the policy maker is informed, then a linear noisy REE exists for almost all parameter values.*

Proof. See Appendix A. □

There may be different equilibrium values of a, but all other strategy coefficients $c_I, c_U, b_I, b_U, \beta_1, \beta_2, \beta_3$ are unique given a. Thus, a—how aggressively the informed trade on information—is the only source of equilibrium multiplicity. An equilibrium may fail to exist for a negligible set of parameter values because of a self-defeating prophecy, where—if prices were informative—the policy reaction to prices $i(p, s_p)$ would exactly cancel out any favorable information about θ, so traders have no reason to change their positions for different prices, which means prices cannot clear the market. The intuition is very much as in the case of an uninformed policy maker or as in the noiseless case. Due to the restriction to linear reaction functions, self-defeating prophecies are only a knife-edge case in $\psi_4/\psi_3 \in \mathbb{R}$ space.

Without solving a explicitly, it is obvious from the equilibrium condition that $a = 0$, i.e., a completely uninformative equilibrium, can never be a solution. Thus, noisy REE are always informative if they exist.

Corollary 14. *In any equilibrium, $a \neq 0$, i.e., market prices are informative.*

Proof. Setting $a = 0$ in equilibrium condition (28) yields a contradiction for any $\tau_\varepsilon > 0$. $\square$

We can again consider the limit as the noise variance approaches zero. As with an uninformed policy maker, prices come arbitrarily close to fully revealing in any equilibrium.

Corollary 15. *If the policy maker is informed, then equilibrium market prices become fully revealing in the limit as the noise variance $1/\tau_u$ approaches zero, i.e.,*

$$\mathbb{E}[\theta|p] \xrightarrow{a.s.} \theta \text{ for } \tau_u \to \infty.$$

Proof. Corollary 14 proved that $a \neq 0$ in any equilibrium. Thus,

$$\lim_{\tau_u\to\infty} \mathbb{E}[\theta|p] = \lim_{\tau_u\to\infty} \frac{\tau_\theta\bar{\theta} + (\mu a)^2\tau_u(\theta + u/(\mu a))}{\tau_\theta + (\mu a)^2\tau_u} = \lim_{\tau_u\to\infty} \theta + u/(\mu a)) = \theta \text{ (a.s.)},$$

since $\mathbb{E}[u] = 0$ and $\text{Var}(u) = 1/\tau_u \to 0$. $\square$

Although existence of an equilibrium is generic (Proposition 13), uniqueness (within the class of linear equilibria) is not guaranteed. While determining a necessary and sufficient condition for uniqueness may be possible, e.g., using Sturm's theorem on the fifth degree polynomial (28), the resulting condition will be too complex to be of value here. Instead, I shall give simple sufficient conditions on the parameters for uniqueness. Not only are the predictions of the model sharper with a unique equilibrium, it also makes comparative statics unambiguous while they are typically not if multiple equilibria exist (see below).

The next proposition shows that sufficiently strong policy maker preferences for intervention in either direction ($\psi_4/\psi_3 >> 0$ or $\psi_4/\psi_3 << 0$) or strong risk aversion among informed traders guarantee a unique equilibrium in the class of unique equilibria. Technically, these conditions guarantee that the right hand side of equilibrium condition (28) plotted in Figure 5 is very flat, so that it intersects the 45° line (left hand side of the equilibrium condition) only once and the equilibrium is unique. Intuitively, these conditions guarantee that the policy maker reaction to a change in how aggressively informed traders use their information (a) changes trader payoffs only slightly, thus the complementarity between a and policy maker reaction is weak.

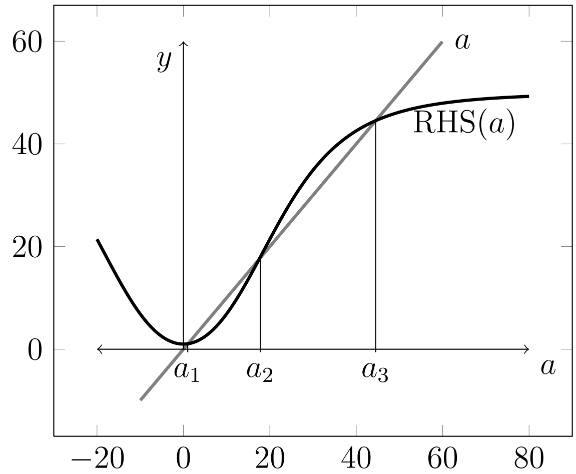


Figure 5: Equilibrium multiplicity. The graph plots the left hand side ($y = a$) and right hand side ($y = \text{RHS}(a)$) of equilibrium condition (28); the three intersections determine three different equilibrium values of a.[11]

Strong enough strategic complementarity can induce multiple equilibria as follows. If informed traders increase $|a|$, i.e., trade more aggressively on information, then equilibrium prices become more informative. Consequently, the policy maker relies more on the price (rather than her private information) when inferring the state and making policy. This can be directly seen in the weight the policy maker places on her private information when making inferences about θ using Bayes' rule and the resulting reaction to private information in (27). From the perspective of the traders, a smaller weight on private information by the policy maker in determining policy (and thus asset values) typically reduces the asset variance conditional on the price (22), because the risk introduced by the private information of the policy maker is reduced. Thus, larger $|a|$ induces a policy maker reaction that reduces the asset variance, which makes it more attractive to trade on information, i.e., increase $|a|$. Consequently, multiple equilibria with small or large $|a|$ may exist.

Proposition 16 (**Equilibrium uniqueness**).

i. There exists $r^* > 0$ such that, for all $\psi_4/\psi_3 > r^*$, the linear equilibrium is unique with $a > 0$.

ii. There exists $r^* < 0$ such that, for all $\psi_4/\psi_3 < r^*$, the linear equilibrium is unique with $a < 0$.

iii. There exists $\rho^* > 0$ such that, for all $\rho_I > \rho^*$, the equilibrium is unique.

Proof. See Appendix A. □

[11]The parameter profile used for the graph is $\psi_4/\psi_3 = -1$, $\tau_\varepsilon = 5$, $\tau_\theta = 0.1$, $\tau_u = 0.6$, $\mu = 0.1$, $\rho_I = 0.1$.

3.4.2 Comparative statics

The next proposition derives the comparative statics for the price informativeness for the cases where ψ_4/ψ_4 is either sufficiently positive or sufficiently negative, i.e., where the policy maker has strong preferences for intervention, which guarantees a unique equilibrium (Proposition 16). These are the most interesting cases, since ψ_4/ψ_3 close to zero implies that the optimal policy barely depends on θ, so the proposition considers the cases where additional information really matters for policy.

It is possible to derive comparative statics under other conditions that guarantee uniqueness, but then the comparative statics may differ for $\psi_4/\psi_3 < 0$ close to zero and $\psi_4/\psi_3 << 0$. It is also possible to derive comparative statics if there are multiple equilibria, but then comparative statics will be equilibrium specific and there will typically exist at least one other equilibrium where the statics are reversed. If there are multiple equilibria as plotted in Figure 5 and if $\psi_4/\psi_3 > 0$, then the comparative statics results of (*i.*) carry over to all equilibria where the slope of the RHS is smaller than 1, i.e., the RHS crosses from above (a_1 and a_3 in Figure 5), and are reversed for those where the RHS crosses from below (a_2 in Figure 5). Due to this possible ambiguity of the comparative statics, the proposition focuses on cases where a unique equilibrium and unambiguous comparative statics are guaranteed.

For the purpose of comparative statics, I allow the quality of information for informed traders and policy maker to vary separately. τ_ε is the precision of the trader signals and τ_ε^P is the precision of the policy maker signal.

Proposition 17 (**Comparative statics**).

i. *For $\psi_4/\psi_3 > 0$ sufficiently large,*

- *decreasing ψ_4/ψ_3, policy maker information precision τ_ε^P, prior distribution precision τ_θ, or informed trader risk aversion ρ_I, and*
- *increasing the share of informed traders μ, noise precision τ_u, or trader signal precision τ_ε*

increases price informativeness (4) *for the policy maker in the unique linear equilibrium. Moreover, all of these except for ψ_4/ψ_3 unambiguously increase the welfare gain due to financial market information* (6).

ii. *For $\psi_4/\psi_3 < 0$ sufficiently negative,*

- *decreasing policy maker information precision τ_ε^P, prior distribution precision τ_θ, or informed trader risk aversion ρ_I, and*
- *increasing ψ_4/ψ_3, the share of informed traders μ, noise precision τ_u, or trader signal precision τ_ε*

increases price informativeness in the unique linear equilibrium. Moreover, all of these except for ψ_4/ψ_3 unambiguously increase the welfare gain due to financial market information.

Proof. See Appendix A. □

The intuition for these results is as follows. If $\psi_4/\psi_3 > 0$, then the policy maker has a preference for implementing policies that increase the asset value more for larger θ. Thus, the policy maker intervention amplifies the conditional asset value variance $\mathrm{Var}(\theta + i(p, s_p)|p, s_j)$ from the perspective of the traders beyond the usual uncertainty over the fundamental value, because of the "policy reaction risk." The variance amplification occurs because the policy maker has private information, which influences policy and asset value, and traders do not know the policy maker reaction from observing the market clearing price p (unlike the case of an uninformed policy maker, where p completely determines policy). This additional risk in the policy reaction leads the informed—who are risk averse—to trade less aggressively on their private information, i.e., $a > 0$ decreases and price informativeness suffers. If $\psi_4/\psi_3 << 0$, then reducing it further similarly amplifies the return variance and traders react by increasing $a < 0$, i.e., also trade less aggressively and make prices less informative. Bond and Goldstein (2015) also document policy risk (what they call endogenous risk) in their model.

If the share of informed traders μ increases, then the informed trade more aggressively on information. This is because the price becomes more informative with more informed traders, and the policy maker reacts by placing a larger weight on the price rather than her private information when inferring the state θ and making her policy decision. This reduces the return variance from the perspective of the traders, because observing p they can better infer the policy maker reaction. Similarly, less noise affecting the price (larger τ_u) and better information for traders (larger τ_ε) leads to the informed trading more aggressively, again because the policy maker uses her private information less in making policy, which reduces the variance from the perspective of the traders, i.e., reduces "policy risk." Finally, a smaller risk aversion ρ_I leads to more aggressive trading and more informative prices, which is a standard result and not directly related to the policy maker reaction.

The effect of an increased precision of the policy maker information τ_ε^P on price informativeness is potentially ambiguous. On the one hand, more precise policy maker information means the additional information from the financial market is less important, in the sense that it does not reduce the posterior variance over θ for the policy maker as much (direct effect). On the other hand, it can potentially increase $|a|$, i.e., how aggressively the informed trade on information, which in turn increases price informativeness (indirect effect). The proof shows that the direct effect dominates for large enough $|\psi_4/\psi_3|$. The same intuition applies to the prior distribution precision τ_θ.

Interestingly, considering the variance of the policy maker estimate $\mathrm{Var}(\theta|p, s_p)$ rather

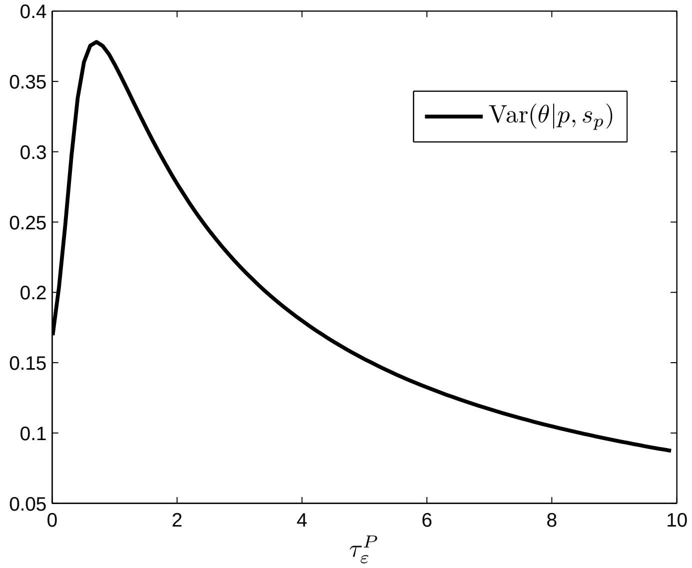


Figure 6: Plot of the variance of the policy maker estimate of the state θ, depending on policy maker information precision τ_ε^P and taking into account equilibrium effects on price informativeness. The parameter values of the example are $\mu = 1$, $\rho_I = 1$, $\tau_\theta = 1$, $\tau_\varepsilon = 1$, $\tau_u = 5$, $\psi_4/\psi_3 = 2$.

than price informativeness, more policy maker information is not uniformly better. Taking equilibrium effects into account, i.e., how informed traders change $|a|$ and hence the informational content of market prices, better information for the policy maker (as measured by a lower policy maker signal variance $1/\tau_\varepsilon^P$) can increase the variance of the policy maker estimate. Figure 6 plots one example, where the conditional variance is increasing in τ_ε^P for small values of τ_ε^P, because the informed trade less aggressively on information ($|a|$ decreases) due to increased policy risk. However, the figure also shows that τ_ε^P decreases the conditional variance for large enough τ_ε^P, since the gain in private information precision outweighs the loss in financial market information precision eventually.

3.4.3 Policy maker transparency

The comparative statics results showed that price informativeness can be strongly affected by policy risk, i.e., the risk due to the policy maker reaction, since the informed traders are risk averse. To reduce the policy risk, which exists solely because traders do not know the independent information of the policy maker s_p, the policy maker could make her information s_p public. In this case, the expected asset value for informed traders is $\mathbb{E}[\theta + i(p, s_p)|p, s_j, s_p] = \mathbb{E}[\theta|p, s_j, s_p] + i(p, s_p)$, where private information s_j is still useful in predicting part of the asset value. Moreover, the conditional asset variance becomes $\mathrm{Var}(\theta + i(p, s_p)|p, s_j, s_p) = \mathrm{Var}(\theta|p, s_j, s_p) < \mathrm{Var}(\theta + i(p, s_p)|p, s_j)$, so there is no more policy risk, since traders know all factors that determine policy (p, s_p). Therefore, if s_p is publicly disclosed, then trading aggressiveness is $a = \tau_\varepsilon/\rho_I$, as in the standard model without policy maker or with an uninformed policy maker (see above). And exactly the same reasons as in the section with uninformed policy maker explain why the informed have higher net demand for the asset with larger signal s_j.

If the policy maker does not disclose her information as assumed in all other sections so far, then a is implicitly defined as (28):

$$a = \frac{\tau_\varepsilon}{\rho_I \left[1 + \frac{\psi_4}{\psi_3} \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_\theta + (\mu a)^2 \tau_u} + \frac{\left(\frac{\psi_4}{\psi_3}\right)^2 \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_\theta + (\mu a)^2 \tau_u}}{1 + \frac{\psi_4}{\psi_3} \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_\theta + (\mu a)^2 \tau_u}} \right]}.$$

Clearly, for large $|\psi_4/\psi_3|$, the equilibrium a becomes very small (see also the proof of Proposition 16), because policy risk severely diminishes the incentives to trade on information for the risk averse traders. Indeed, $a \to 0$ as $|\psi_4/\psi_3| \to \infty$. Hence, a policy maker with preferences for large interventions will be better off by disclosing her information, since it removes policy risk for traders and thereby makes prices more informative.

Bond and Goldstein (2015) find the exact opposite. In their model, the policy maker should never disclose her information s_p about θ, for then prices become completely uninformative. This is because the asset value in their model depends on θ only indirectly via policy, so once the policy is known, traders have no reason to use their information about θ to trade.

4 Conclusion

This paper analyzes a general problem where traders have information about a state variable θ that a policy maker needs in order to implement her optimal policy i. A financial market trades an asset whose value depends both the state θ and the policy i. Under which conditions can the equilibrium prices of these assets be revealing, in the sense that trader information (or a noisy signal thereof) is revealed to the policy maker, if traders correctly anticipate the policy maker reaction?

In a model without noise, I show that there are situations where it is impossible to find a price function that both fulfills a no-mispricing condition and reveals trader information to the policy maker. Consequently, since competitive markets require no mispricing, there is no fully revealing equilibrium in these situations. The main result of this section shows that a condition on asset values, policy maker preferences, and trader/policy maker information structures is necessary and sufficient for the existence of fully revealing equilibria. Hence, the main result identifies all situations where we can expect financial markets and prediction markets to work as tools guiding policy makers in making real decisions, and where we cannot. If the condition is not fulfilled, then market prices can be fully revealing only as long as the policy maker does not react to the information contained therein. The main result can be used to design policies or assets that are more supportive of information revelation by traders.

To investigate how these results depend on the assumption of a noiseless financial market, I develop a model where prices are affected by noise shocks. Adapting a new solution approach, I can explicitly solve for equilibria with uninformed policy maker even if reaction and price functions are nonlinear. In a class of equilibria with continuous price functions, I derive a necessary and sufficient condition for the existence of revealing equilibria. Thus, the section provides useful tools for applications that allow for a broad set of policy maker preferences. In specific cases, noise can solve the problem of self-defeating prophecies, because prices do not fully reveal trader information. Hence, traders retain incentives to trade on information, making market prices at least partially revealing. Similar to Grossman and Stiglitz (1980), market prices in these cases can be informative *because* of noise, rather than despite of noise, only the reason why strong-form informational efficiency fails is self-defeating prophecies and not costly information.

A question for future research is how the necessary and sufficient conditions change if there are large strategic traders who can move prices, which may introduce price manipulation motives to trigger different policy reactions and achieve larger trading profits. Moreover, there are many important welfare aspects left to explore. For example, if the ex post optimal policy reaction function leads to a self-defeating prophecy, when can commitment to another reaction function be superior, because it supports information revelation? This question is difficult to answer in cases where a self-defeating prophecy implies equilibrium non-existence, where welfare cannot be evaluated.

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A Proofs

Theorem 2 (**Possibility of full revelation and accurate prices**). *Suppose the policy maker knows function $p(\boldsymbol{s})$ and maximizes expected utility. Then a fully revealing and accurate price function exists if and only if condition 1 holds.*

Proof. Necessity: Existence of a fully revealing and accurate price function implies that condition 1 holds. First note that full revelation implies that the expected asset value given trader information equals $v(\boldsymbol{s}) = \mathbb{E}[a(\theta, i(\boldsymbol{s}))|\boldsymbol{s}]\ \forall \boldsymbol{s} \in \boldsymbol{S}$. As shown in Lemma 1, if prices are fully revealing, then accuracy implies $p(\boldsymbol{s}) = v(\boldsymbol{s})$.

Full revelation with accurate prices by definition implies

$$|\{t \in \boldsymbol{S} : \Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p) > 0\}| \leq 1\ \forall s_p, \forall p. \tag{16}$$

By Bayes' rule and the law of iterated expectations,

$$\begin{aligned}
\Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p) &= \frac{\Pr(\boldsymbol{s} = t) \cdot \Pr(p(\boldsymbol{s}) = p \cap s_p|\boldsymbol{s} = t)}{\Pr(p(\boldsymbol{s}) = p \cap s_p)} \\
&= \frac{\Pr(\boldsymbol{s} = t) \cdot \mathbb{E}[\Pr(p(\boldsymbol{s}) = p \cap s_p)|\boldsymbol{s} = t)]}{\Pr(p(\boldsymbol{s}) = p \cap s_p)} \\
&= \frac{\Pr(\boldsymbol{s} = t) \cdot \mathbb{E}[\Pr(s_p|p(\boldsymbol{s}) = p) \cdot \Pr(p(\boldsymbol{s}) = p)|\boldsymbol{s} = t)]}{\Pr(p(\boldsymbol{s}) = p \cap s_p)} \\
&= \frac{\Pr(\boldsymbol{s} = t) \cdot \Pr(p(t) = p) \cdot \Pr(s_p|\boldsymbol{s} = t)}{\Pr(p(\boldsymbol{s}) = p \cap s_p)}
\end{aligned} \tag{17}$$

First, if $\Pr(p(\boldsymbol{s}) = p|\boldsymbol{s} = t) = \Pr(p(t) = p) > 0$ for exactly one $t \in \boldsymbol{S}$ for all $p \in \text{Image}(v(\boldsymbol{s}))$, then $v(\boldsymbol{s})$ is invertible and the price alone fully reveals $\boldsymbol{s}$. Hence $\nexists p \in \text{Image}(v(\boldsymbol{s})) : |v^{-1}(p)| > 1$, i.e., the antecedent of condition 1 is false and the condition holds.

Second, if there exists $p \in \text{Image}(v(\boldsymbol{s}))$ such that $\Pr(p(\boldsymbol{s}) = p|\boldsymbol{s} = t) > 0$ for more than one $t \in \boldsymbol{S}$, then $|X| > 0$, i.e., the antecedent of condition 1 is true. Thus, for condition 1 to hold, the consequent must be true as well. Since there is full revelation, i.e., (16) holds, we must have $\Pr(s_p|\boldsymbol{s} = t) \cdot \Pr(s_p|\boldsymbol{s} = t') = 0\ \forall s_p \in S_p, \forall t \neq t' \in v^{-1}(p), \forall p \in X$, so that $\Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p) > 0$ for at most one $t \in v^{-1}(p)$ for each $p \in \text{Image}(v(\boldsymbol{s}))$. Thus, the consequent is true and condition 1 holds.

Sufficiency: Condition 1 implies the existence of a fully revealing and accurate price function. Condition 1 is true either if the antecedent is false, or if both antecedent and consequent are true. First, if the antecedent of condition 1 is false, then $v(\boldsymbol{s})$ is invertible. In this case, the price function $p(\boldsymbol{s}) = v(\boldsymbol{s})$ is both fully revealing and accurate.

Second, consider the case where the antecedent and consequent of condition 1 is true. The consequent is $\Pr(s_p|\boldsymbol{s} = t) \cdot \Pr(s_p|\boldsymbol{s} = t') = 0\ \forall s_p \in S_p,\ \forall t \neq t' \in v^{-1}(p),\ \forall p \in X$, which implies that $\Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p)$ in (17) is positive for exactly one $t \in \boldsymbol{S}$ for every $p \in \text{Image}(v(\boldsymbol{s}))$, i.e., the price function $p(\boldsymbol{s}) = v(\boldsymbol{s})$ is both fully revealing and accurate. □

Proposition 7 (Asset design and full revelation). *Consider the class of assets that can be written as $a(\theta, i) = A(o(\theta, i))$ for any $o : \Theta \times I \to \mathbb{R}$. If a non-invertible function $A' \in \mathcal{A}'$ allows full revelation and accurate prices, then so does any invertible $A \in \mathcal{A}$, but the converse does not hold.*

Proof. Define $v(\boldsymbol{s}, A) := \mathbb{E}_\theta[A(o(\theta, i(\boldsymbol{s}, s_p)))|\boldsymbol{s}]$. We need to show that there is no pair $\boldsymbol{s} \neq \boldsymbol{s}'$ such that $v(\boldsymbol{s}, A') \neq v(\boldsymbol{s}', A')$ but $v(\boldsymbol{s}, A) = v(\boldsymbol{s}', A)$, i.e., whenever a pair of states is revealed solely via prices under $A' \in \mathcal{A}'$, then it must also be revealed solely by prices under all $A \in \mathcal{A}$. Formally,

$$v(\boldsymbol{s}, A') \neq v(\boldsymbol{s}', A') \implies v(\boldsymbol{s}, A) \neq v(\boldsymbol{s}', A)\ \forall A \in \mathcal{A},\ \forall A' \in \mathcal{A}'. \tag{18}$$

If this condition holds, then if condition 1 holds under A', it will also hold under any A, since a price function $p(\boldsymbol{s}) = v(\boldsymbol{s}, A)$ of asset A must always rule out at least as many states $\boldsymbol{s} \in \boldsymbol{S}$ as a price function $p'(\boldsymbol{s}) = v(\boldsymbol{s}, A')$ of asset A'.

To show (18) holds, note that for any $A' \in \mathcal{A}'$,

$$\begin{aligned} &\mathbb{E}_\theta[A'(o(\theta, i(\boldsymbol{s}, s_p)))|\boldsymbol{s}] \neq \mathbb{E}_\theta[A'(o(\theta, i(\boldsymbol{s}', s_p)))|\boldsymbol{s}'] \\ &\implies \mathbb{E}_\theta[o(\theta, i(\boldsymbol{s}, s_p))|\boldsymbol{s}] \neq \mathbb{E}_\theta[o(\theta, i(\boldsymbol{s}', s_p))|\boldsymbol{s}']. \end{aligned} \tag{19}$$

Moreover, because $\mathcal{A}$ is the set of invertible functions,

$$\begin{aligned}&\mathbb{E}_\theta[o(\theta, i(\boldsymbol{s}, s_p))|\boldsymbol{s}] \neq \mathbb{E}_\theta[o(\theta, i(\boldsymbol{s}', s_p))|\boldsymbol{s}'] \\ &\quad \Longrightarrow \mathbb{E}_\theta[A(o(\theta, i(\boldsymbol{s}, s_p)))|\boldsymbol{s}] \neq \mathbb{E}_\theta[A(o(\theta, i(\boldsymbol{s}', s_p)))|\boldsymbol{s}'] \; \forall A \in \mathcal{A}.\end{aligned} \tag{20}$$

Combining (19) and (20) results in (18), which as argued above implies that condition 1 holds for any $A \in \mathcal{A}$ whenever it holds for some $A' \in \mathcal{A}'$.

We still need to show that $A \in \mathcal{A}$ can allow for full revelation and accurate prices but $A' \in \mathcal{A}'$ might not. This immediately follows from the fact that invertibility of A is necessary (but not sufficient) for the invertibility of $v(\boldsymbol{s}) = \mathbb{E}_\theta[A(o(\theta, i(\boldsymbol{s}, s_p)))|\boldsymbol{s}]$, which guarantees that condition 1 holds. And according to Theorem 2, full revelation and accurate prices are possible if and only if condition 1 holds. □

Proposition 8. *An equilibrium in the class of quasi-linear equilibria exists if and only if there exists a continuous reaction function $i(z) \in \arg\max_i \mathbb{E}[U(\theta, i)|z]$ such that, for $a = \tau_\varepsilon/\rho_I$,*

$$\begin{aligned} z(1 + \mu^2 a\tau_u/\rho_I + (1-\mu)\mu a\tau_u/\rho_U) + i(z)[\mu(\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u)/(\rho_I \mu a) \\ +(1-\mu)(\tau_\theta + (\mu a)^2\tau_u)/(\rho_U \mu a)] \end{aligned}$$

is invertible in z. If it exists, then the equilibrium is unique in the class of quasi-linear equilibria and market prices fully reveal the noisy signal $z = \theta + u/(\mu a)$.

The trader equilibrium strategies are $X_I(s_j, p) = as_j - g_I(p)$ and $X_U(p) = -g_U(p)$ with $a = \tau_\varepsilon/\rho_I$, $g_I(p)$ given in (9)*, and $g_U(p)$ given in* (11)*. The equilibrium policy reaction is $i(P^{-1}(p)) = i(z)$. The equilibrium price function $P(\theta, u) = P(z)$ is given in* (13)*.*

Proof. Conjecturing demand functions of the quasi-linear form in definition 5, the market clearing condition is (7). The market clearing condition has to hold for any realization of (θ, u), which appears directly only on the right hand side of (7). Since (θ, u) appears on the left hand side only indirectly via the equilibrium price function $P(\theta, u)$, this price function must at least reveal the statistic $z = \theta + u/(\mu a)$, so that I now write $P(z)$. That is, all (θ, u) realizations with $p = P(\theta, u)$ must be on the line $z = \theta + u/(\mu a)$. Hence, any equilibrium price function reveals at least the realization of z to the uninformed, which includes the policy maker. Below, I shall also confirm that the price function reveals at most z. Invertibility of $P(z)$ also implies that there is a unique market clearing price for every realization of z.

Using the information set for the uninformed $\{z\}$ and the information set for the informed $\{s_j, z\}$, the expected asset values are normally distributed conditional on this information. Consequently, the optimal demand of the informed is uniquely determined and given by (8), which is of the conjectured quasi-linear form. Matching coefficient a, $a = \tau_\varepsilon/\rho_I$. Matching g_I yields (9). Similarly, the optimal demand for the uninformed is uniquely given by (10), and matching g_U yields (11). Consequently, given that z is revealed by prices and that $i(z)$ is unique, the equilibrium must be unique in the class of quasi-linear equilibria if it exists, since demand functions and hence the price function (via market clearing condition) is unique. Substituting $g_I(p)$ and $g_U(p)$ into the market clearing condition (7) yields (12). Given the assumption that $P(z)$ is invertible, rewrite $P^{-1}(P(z)) = z$ and, abusing notation, $i(P(z)) = i(z)$, so that the market clearing condition can be rearranged for an explicit expression of $P(z)$ given in (13). Dropping constant factors and terms from the right hand side of (13), it is immediate that $P(z)$ is invertible as required if and only if (14) is invertible in z. If (14) is non-invertible, it contradicts market clearing and no equilibrium exists.

It remains to be shown that the continuous equilibrium price function $P(\theta, u)$ does not reveal more than the realization of z, i.e., $P(\theta, u)$ depends on (θ, u) only via $\theta + u/(\mu a)$. Lemma 2 in Pálvölgyi and Venter (2015) shows this for the Grossman and Stiglitz (1980)-model, and the proof can be applied directly to the problem here. For completeness, I will translate their proof into my notation.

I already established that all (θ, u) realizations for which $p = P(\theta, u)$ are on the line $\theta + u/(\mu a)$ (if not, then $P(\theta, u)$ would not be invertible in z as required for market clearing). Now, by contradiction, suppose $P(\theta, u)$ depends on (θ, u) not only via $z = \theta + u/(\mu a)$. This implies there exist two pairs $(\theta_1, u_1) \neq (\theta_2, u_2)$ such that $z = \theta_1 + u_1/(\mu a) = \theta_2 + u_2/(\mu a)$ with $P(\theta_1, u_1) = p_1 \neq P(\theta_2, u_2) = p_2$. Now, given continuity of $P(\theta, u)$ and using the intermediate value theorem, we can find a pair (θ^*, u^*) with $\theta^* + u^*/(\mu a) = z$ and $P(\theta^*, u^*) = (p_1 + p_2)/2$ (e.g., increase θ and decrease u). Similarly, we can find a pair (θ', u') with $\theta' + u'/(\mu a) \neq z$ and $P(\theta', u') = (p_1 + p_2)/2$ (e.g., starting at $\min\{p_1, p_2\}$, by increasing θ and keeping u constant). That is, we have two points with $P(\theta^*, u^*) = P(\theta', u') = (p_1 + p_2)/2$ and $\theta^* + u^*/(\mu a) \neq \theta' + u'/(\mu a)$. Yet this contradicts the previously established fact that all (θ, u) realizations for which $p = P(\theta, u)$ are on the same line $\theta + u/(\mu a)$. Consequently, a continuous price function $P(\theta, u)$ reveals exactly $z = \theta + u/(\mu a)$ in equilibrium. $\square$

Proposition 13 (**Equilibrium existence**). *If the policy maker is informed, then a linear noisy REE exists for almost all parameter values.*

Proof. The solution approach for the noisy REE is the typical "guess and verify" approach: First, a conjecture about the shape of demand functions is made. In this case, we conjecture the demand functions to be linear in signal s_j and price p, which—after imposing market clearing—gives a linear price function $P(\theta, u)$ with undetermined coefficients. Second, according to this price conjecture, the price function $P(\theta, u)$ gives the relationship between state θ and price, which is used to update traders' beliefs about θ via Bayes' rule. Third, demand functions given the information sets are computed. Fourth, the undetermined coefficients are identified, which gives the actual relationship between θ and prices.

I am going to derive a symmetric linear noisy rational expectations equilibrium, where the conjecture is that traders use strategies

$$X_I(p, s_j) = as_j - c_I p + b_I \text{ and } X_U(p) = -c_U p + b_U,$$

which yields the market clearing condition

$$\begin{aligned} &\int_0^\mu (as_j - c_I p + b_I)dj + \int_\mu^1 (-c_U p + b_U)dj + u = 0 \\ &\iff p = \frac{\mu a\theta + \mu b_I + (1-\mu)b_U + u}{\mu c_I + (1-\mu)c_U}, \end{aligned} \tag{21}$$

because an appropriate law of large numbers for i.i.d. random variables (Sun, 2006) yields $\int_0^\mu s_j dj = \mu\theta$. Define $\lambda \coloneqq (\mu c_I + (1-\mu)c_U)^{-1}$ and $\tilde{b} \coloneqq \mu b_I + (1-\mu)b_U$ to simplify notation, so that the market clearing condition is

$$p = \lambda(\mu a\theta + \tilde{b} + u).$$

Rearranging, the following linear statistic of the market clearing price p, which is informationally equivalent to p, is the state θ plus a normally distributed noise term $u/(\mu a)$,

$$\frac{p - \lambda\tilde{b}}{\lambda\mu a} = \theta + u/(\mu a).$$

The variance of the net asset value for informed traders conditional on their information is

$$\begin{aligned} \mathrm{Var}(\theta + i(p, s_p) - p|p, s_j) &= \mathrm{Var}(\theta + i(p, s_p)|p, s_j) = \mathrm{Var}(\theta + \beta_1 + \beta_2 p + \beta_3 s_p|p, s_j) \\ &= \mathrm{Var}(\theta(1+\beta_3) + \beta_3\varepsilon|p, s_j) = \frac{(1+\beta_3)^2}{\tau_\varepsilon + \tau_\theta + (\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}, \end{aligned} \tag{22}$$

since $\mathrm{Var}(\theta|p, s_j) = 1/(\tau_\varepsilon + \tau_\theta + (\mu a)^2\tau_u)$, as in the standard model without policy maker, and θ and ε are independent. Similarly, for the uninformed, the variance of the asset value given the price is

$$\mathrm{Var}(\theta + i(p, s_p) - p|p) = \frac{(1+\beta_3)^2}{\tau_\theta + (\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}.$$

The Bayesian updating rule for the mean of normal distributions is a precision weighted sum of prior mean and signals, where the precision of the price signal is the inverse of its conditional variance, hence the conditional expectation of the asset value is

$$\begin{aligned} \mathbb{E}[\theta + i(p, s_p) - p)|p, s_j] &= \mathbb{E}[\theta(1+\beta_3) + \beta_3\varepsilon)|p, s_j] + \beta_1 + \beta_2 p - p \\ &= (1+\beta_3)\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon s_j + (\mu a)^2\tau_u\left(\frac{p-\lambda\tilde{b}}{\lambda\mu a}\right)}{\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u} + \beta_1 + \beta_2 p - p. \end{aligned}$$

Similarly, the expectation of the asset value for uninformed traders is

$$\mathbb{E}[\theta + i(p, s_p) - p)|p] = (1 + \beta_3)\frac{\tau_\theta\bar{\theta} + (\mu a)^2\tau_u\left(\frac{p-\lambda\tilde{b}}{\lambda\mu a}\right)}{\tau_\theta + (\mu a)^2\tau_u} + \beta_1 + \beta_2 p - p.$$

The well-known CARA demand functions derived from the first order conditions are given by

$$\begin{aligned} X_I(p, s_j) &= \frac{\mathbb{E}[\theta + i(p, s_p) - p)|p, s_j]}{\rho_I \mathrm{Var}(\theta + i(p, s_p) - p|p, s_j)}, \\ X_U(p) &= \frac{\mathbb{E}[\theta + i(p, s_p) - p)|p]}{\rho_U \mathrm{Var}(\theta + i(p, s_p) - p|p)}. \end{aligned}$$

Note that the trader objective is concave even if $i(p)$ is highly convex, because a single trader does not affect p, hence $i(p)$ is a constant in the trader maximization problem. Plugging in for conditional expectations and variances,

$$X_U(p) = \frac{(1+\beta_3)\frac{\tau_\theta\bar{\theta}+(\mu a)^2\tau_u\left(\frac{p-\lambda\tilde{b}}{\lambda\mu a}\right)}{\tau_\theta+(\mu a)^2\tau_u} + \beta_1 + \beta_2 p - p}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]},$$

which is linear in p as conjectured, hence identifying coefficients using $X_U(p) = -c_U p + b_U$ we obtain

$$c_U = \frac{1 - \beta_2 - \frac{(1+\beta_3)\mu a\tau_u/\lambda}{\tau_\theta+(\mu a)^2\tau_u}}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]}, \; b_U = \frac{\beta_1 + \frac{(1+\beta_3)(\tau_\theta\bar{\theta}-\mu a\tau_u\tilde{b})}{\tau_\theta+(\mu a)^2\tau_u}}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]}. \tag{23}$$

For informed traders, plugging in for conditional expectations and variances,

$$X_I(p, s_j) = \frac{(1+\beta_3)\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon s_j+(\mu a)^2\tau_u\left(\frac{p-\lambda\tilde{b}}{\lambda\mu a}\right)}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u} + \beta_1 + \beta_2 p - p}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]},$$

which is linear in signal s_j and price p as conjectured. Matching coefficients of $X_I(p, s_j) = as_j - c_I p + b_I$,

$$\begin{aligned} a &= \frac{(1+\beta_3)\tau_\varepsilon}{\rho_I(\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u)\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]}, \\ c_I &= \frac{1 - \beta_2 - \frac{(1+\beta_3)\mu a\tau_u/\lambda}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\rho_I\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]}, \; b_U = \frac{\beta_1 + \frac{(1+\beta_3)(\tau_\theta\bar{\theta}-\mu a\tau_u\tilde{b})}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\rho_I\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{\beta_3^2}{\tau_\varepsilon}\right]}. \end{aligned} \tag{24}$$

Now, both c_I and c_U depend on λ and vice versa. Substitute c_I and c_U into $\lambda = 1/(\mu c_I +$

$(1-\mu)c_U)$ and solve for λ to get

$$\lambda = \frac{1 + \frac{(1+\beta_3)\mu^2 a\tau_u}{\rho_I(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]} + \frac{(1+\beta_3)\mu(1-\mu)\tau_u}{\rho_U(\tau_\theta+(\mu a)^2\tau_u)\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]}}{\frac{\mu(1-\beta_2)}{\rho_I\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]} + \frac{(1-\mu)(1-\beta_2)}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]}}. \tag{25}$$

Furthermore, both b_I and b_U depend on $\tilde{b}$ and vice versa. Substitute both into $\tilde{b} = \mu b_I + (1-\mu)b_U$ and solve for $\tilde{b}$ to get

$$\tilde{b} = \frac{\mu\left[\frac{\frac{(1+\beta_3)\tau_\theta\bar{\theta}}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}+\beta_1}{\rho_I\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]}\right] + (1-\mu)\left[\frac{\frac{(1+\beta_3)\tau_\theta\bar{\theta}}{\tau_\theta+(\mu a)^2\tau_u}+\beta_1}{\rho_U\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]}\right]}{1 + \frac{\mu^2 a\tau_u(1+\beta_3)}{\rho_I(\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u)\left[\frac{(1+\beta_3)^2}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]} + \frac{(1-\mu)\mu a\tau_u(1+\beta_3)}{\rho_U(\tau_\theta+(\mu a)^2\tau_u)\left[\frac{(1+\beta_3)^2}{\tau_\theta+(\mu a)^2\tau_u}+\frac{\beta_3^2}{\tau_\varepsilon}\right]}}. \tag{26}$$

Turning to the policy maker, her utility function is strictly concave by construction, so the first order condition determines the unique reaction function

$$i(p, s_p) = \frac{\psi_2 + \psi_4\mathbb{E}[\theta|p, s_p]}{\psi_3} = \frac{\psi_2 + \psi_4\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon s_p+(\mu a)^2\tau_u\left(\frac{p-\lambda\tilde{b}}{\mu a\lambda}\right)}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\psi_3}.$$

Matching coefficients for $i(p, s_p) = \beta_1 + \beta_2 p + \beta_3 s_p$, we obtain

$$\beta_1 = \frac{\psi_2 + \psi_4\frac{\tau_\theta\bar{\theta}-\mu a\tau_u\tilde{b}}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\psi_3},\ \ \beta_2 = \frac{\psi_4\frac{\mu a\tau_u/\lambda}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\psi_3},\ \ \beta_3 = \frac{\psi_4\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+(\mu a)^2\tau_u}}{\psi_3}. \tag{27}$$

Substitute the term for β_3 in (27) into coefficient a (24), so that the equilibrium condition for a is

$$a = \frac{\tau_\varepsilon}{\rho_I\left[1 + \frac{\psi_4}{\psi_3}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{\left(\frac{\psi_4}{\psi_3}\right)^2\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}}{1+\frac{\psi_4}{\psi_3}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}}\right]}, \tag{28}$$

which depends only on one endogenous strategy variable (a). The condition can be rearranged as a fifth degree polynomial in a. Fifth degree polynomials are guaranteed to have at least one solution and at most five. Given a, β_3 in (27) is uniquely determined. Given β_3 and a, substituting β_2 into λ in (25) yields a linear condition with a unique solution for λ, which in turn determines β_2 in (27) and c_U, c_I in (23), (24) uniquely. Substituting β_1 into $\tilde{b}$ in (26) yields a linear condition with a unique solution, which in turn determines β_1 uniquely. Finally, market clearing requires $\mu c_I + (1-\mu)c_U \neq 0$ for the coefficients c_I, c_U just obtained. Thus, an equilibrium exists for almost all parameter profiles, and a is the only source of equilibrium multiplicity (i.e., all other endogenous variables are uniquely determined given a). □

Proposition 16 **(Equilibrium uniqueness).**

i. There exists $r^* > 0$ such that, for all $\psi_4/\psi_3 > r^*$, the linear equilibrium is unique with $a > 0$.

ii. There exists $r^* < 0$ such that, for all $\psi_4/\psi_3 < r^*$, the linear equilibrium is unique with $a < 0$.

iii. There exists $\rho^* > 0$ such that, for all $\rho_I > \rho^*$, the equilibrium is unique.

Proof. *i.* Using the shorthand $r := \psi_4/\psi_3$, recall equilibrium condition (28):

$$a = \frac{\tau_\varepsilon}{\rho_I \left[1 + r\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} + \frac{r^2\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}}{1+r\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u}}\right]}. \tag{28}$$

Every a that fulfills condition (28) forms an equilibrium; uniqueness requires a single intersection of a and the right hand side (RHS) of the condition. Note that the RHS of (28) is symmetric about $a = 0$, since a enters the condition only quadratically. Note also that the RHS of (28) is positive for all $a \geq 0$ at all $r > 0$. Using the shorthand $\tau := \tau_\varepsilon + \tau_\theta + (\mu a)^2\tau_u$, the slope of the RHS of (28) in a is

$$\frac{\partial \text{RHS}(a)}{\partial a} = \tau_\varepsilon\rho_I^{-1}[2r\tau_\varepsilon\tau_u\mu^2 a\tau^{-2} + 2r^2\tau_\varepsilon\tau_u\mu^2 a(\tau + r\tau_\varepsilon)^{-2}]/\left[1 + r\frac{\tau_\varepsilon}{\tau} + \frac{r^2\tau_\varepsilon}{\tau + r\tau_\varepsilon}\right]^2. \tag{29}$$

This slope is positive for $r > 0$ and approaches zero as $r \to \infty$ at all $a > 0$, since the denominator grows at a quadratic rate in r, while the numerator only grows at a linear rate. Now, at $a = 0$, $\text{RHS}(a) > a = 0$. Since a large r makes the slope of $\text{RHS}(a)$ arbitrarily small, there exists some $r^* > 0$ such that the slope of $\text{RHS}(a)$ is less than 1 for all $r \geq r^*$, which guarantees that $\text{RHS}(a)$ crosses a exactly once in $a \geq 0$, i.e., (28) has only one non-negative solution. It remains to be shown that there is no further solution $a = \text{RHS}(a)$ in $a < 0$. This immediately follows from the fact that $\text{RHS}(a) > 0$ for all $a \in \mathbb{R}$ if $r > 0$.

ii. We want to show that there exists $r^* < 0$ such that $r < r^*$ guarantees a unique solution to (28), and that this solution is $a < 0$. The RHS of the equilibrium condition (28) is negative for $a < 0$ close to zero if and only if $1 + r\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\theta+(\mu a)^2\tau_u} < 0$, which holds with $r < -\frac{\tau_\varepsilon+\tau_\theta}{\tau_\varepsilon}$. Thus, for negative enough $r < 0$, $\text{RHS}(a) < 0$ close to $a = 0$.

The slope of the RHS (29) approaches zero as $r \to -\infty$ at all $a \in \mathbb{R}$, since the denominator grows at a quadratic rate whereas the numerator changes only at a linear rate in r. Thus, there exists some $r^* < 0$ such that the slope of $\text{RHS}(a)$ is smaller than 1 and $\text{RHS}(a = 0) < 0$ for all $r \leq r^*$, which guarantees that the RHS intersects a exactly once for $a \leq 0$ and does not intersect for $a > 0$.

iii. The slope of the RHS in a given in (29) approaches zero as $\rho_I \to \infty$. The same arguments as in (*i.*) and (*ii.*) imply that we can find a threshold $\rho^* > 0$ such that, for all $\rho_I > \rho^*$, the slope of the RHS is less than 1 everywhere, which guarantees a unique equilibrium. $\square$

Proposition 17 (Comparative statics).

i. For $\psi_4/\psi_3 > 0$ sufficiently large,

- *decreasing ψ_4/ψ_3, policy maker information precision τ_ε^P, prior distribution precision τ_θ, or informed trader risk aversion ρ_I, and*
- *increasing the share of informed traders μ, noise precision τ_u, or trader signal precision τ_ε*

increases price informativeness (4) *for the policy maker in the unique linear equilibrium. Moreover, all of these except for ψ_4/ψ_3 unambiguously increase the welfare gain due to financial market information* (6)*.*

ii. For $\psi_4/\psi_3 < 0$ sufficiently negative,

- *decreasing policy maker information precision τ_ε^P, prior distribution precision τ_θ, or informed trader risk aversion ρ_I, and*
- *increasing ψ_4/ψ_3, the share of informed traders μ, noise precision τ_u, or trader signal precision τ_ε*

increases price informativeness in the unique linear equilibrium. Moreover, all of these except for ψ_4/ψ_3 unambiguously increase the welfare gain due to financial market information.

Proof. We can rewrite the equilibrium condition (28), which implicitly defines a, as $\mathrm{RHS}(a,t) - a = 0$, where $\mathrm{RHS}(a,t)$ is the right hand side of (28) as a function of a and parameter t. By the implicit function theorem, we can determine how a changes in equilibrium in response to a small change in parameter t:

$$\frac{\partial a}{\partial t} = -\frac{\mathrm{RHS}_t}{\mathrm{RHS}_a - 1},$$

where RHS_t is the partial derivative with respect to t. The uniqueness proofs of Proposition 16 show that RHS_a is less than 1 for $|\psi_4/\psi_3|$ large enough, hence it follows that a increases if and only if $\mathrm{RHS}_t > 0$ and decreases if and only if $\mathrm{RHS}_t < 0$.

i. Using the shorthands $r := \psi_4/\psi_3$ and $\tau := \tau_\varepsilon + \tau_\theta + (\mu a)^2 \tau_u$,

$$\mathrm{RHS}_r = \underbrace{-\tau_\varepsilon \rho_I^{-1}}_{<0} \underbrace{\left[1 + r\tau_\varepsilon/\tau + \frac{r^2\tau_\varepsilon/\tau}{1 + r\tau_\varepsilon/\tau}\right]^{-2}}_{>0} \left[\tau_\varepsilon/\tau + \frac{2r\tau_\varepsilon/\tau + r^2(\tau_\varepsilon/\tau)^2}{(1 + r\tau_\varepsilon/\tau)^2}\right], \qquad (30)$$

which is negative if and only if the last bracket is positive. Requiring the term to be positive is equivalent to the condition

$$r^2(\tau_\varepsilon/\tau + (\tau_\varepsilon/\tau)^2) + 2r(1 + \tau_\varepsilon/\tau) + 1 > 0, \tag{31}$$

which holds for all $r \geq 0$. Thus, $\mathrm{RHS}_r < 0$ and a decreases with larger r.

Next, RHS_μ, which is positive for $r > 0$:

$$\mathrm{RHS}_\mu = \underbrace{-\tau_\varepsilon \rho_I^{-1}}_{<0} \underbrace{\left[1 + r\tau_\varepsilon/\tau + \frac{r^2\tau_\varepsilon/\tau}{1 + r\tau_\varepsilon/\tau}\right]^{-2}}_{>0} \underbrace{\left[-2r\tau_\varepsilon a^2 \mu\tau_u/\tau^2 - 2r^2\tau_\varepsilon a^2\mu\tau_u/(\tau + r\tau_\varepsilon)^2\right]}_{<0 \text{ if } r>0}. \tag{32}$$

Moreover, $\mathrm{RHS}_{\tau_\theta} > 0$:

$$\mathrm{RHS}_{\tau_\theta} = \underbrace{-\tau_\varepsilon \rho_I^{-1}}_{<0} \underbrace{\left[1 + r\tau_\varepsilon/\tau + \frac{r^2\tau_\varepsilon/\tau}{1 + r\tau_\varepsilon/\tau}\right]^{-2}}_{>0} \underbrace{\left[-r\tau_\varepsilon/\tau^2 - r^2\tau_\varepsilon/(\tau + r\tau_\varepsilon)^2\right]}_{<0 \text{ if } r>0}. \tag{33}$$

It can similarly be verified that $\mathrm{RHS}_{\rho_I} < 0$ and $\mathrm{RHS}_{\tau_u} > 0$.

To investigate the effects of τ_ε and τ_ε^P separately, we have to derive the equilibrium condition for a again, which is more complicated in the more general case:

$$a = \frac{\left(1 + r\frac{\tau_\varepsilon^P}{\tau_\theta + \tau_\varepsilon^P + (\mu a)^2\tau_u}\right)\tau_\varepsilon}{\rho_I\left[\left(1 + r\frac{\tau_\varepsilon^P}{\tau_\theta + \tau_\varepsilon^P + (\mu a)^2\tau_u}\right)^2 + (\tau_\theta + \tau_\varepsilon + (\mu a)^2\tau_u)\left(r\frac{\tau_\varepsilon^P}{\tau_\theta + \tau_\varepsilon^P + (\mu a)^2\tau_u}\right)^2 / \tau_\varepsilon^P\right]}. \tag{34}$$

This is also a fifth degree polynomial in a, guaranteeing a solution. A tedious but straightforward computation using the quotient rule shows that the slope of the RHS in a gets arbitrarily close to zero for $|r| \to \infty$, since the denominator of RHS_a grows at a quartic rate while numerator grows only quadratically, thus we also have uniqueness for $|r|$ large enough in this more general case. It remains to determine the sign of the slope of the RHS in τ_ε and τ_ε^P for the comparative statics. Abbreviating $\beta_3 = r\frac{\tau_\varepsilon^P}{\tau_\theta + \tau_\varepsilon^P + (\mu a)^2\tau_u}$,

$$\begin{aligned}\frac{\partial \mathrm{RHS}(34)}{\partial \tau_\varepsilon} &= \frac{(1+\beta_3)\rho_I[(1+\beta_3)^2 + \tau\beta_3^2/\tau_\varepsilon^P)] - (1+\beta_3)\tau_\varepsilon\rho_I\beta_3^2/\tau_\varepsilon^P}{[\rho_I((1+\beta_3)^2 + \tau\beta_3^2/\tau_\varepsilon^P)]^2} \\ &= \frac{(1+\beta_3)\rho_I[(1+\beta_3)^2 + (\tau_\theta + (\mu a)^2\tau_u)\beta_3^2/\tau_\varepsilon^P)]}{[\rho_I((1+\beta_3)^2 + \tau\beta_3^2/\tau_\varepsilon^P)]^2} > 0,\end{aligned} \tag{35}$$

since $r > 0$ implies $\beta_3 > 0$, thus $a > 0$ in equilibrium increases with an increase of τ_ε.

Finally, denoting $\beta_3' = \partial\beta_3/\partial\tau_\varepsilon^P = r\frac{\tau_\theta+(\mu a)^2\tau_u}{(\tau_\varepsilon^P+\tau_\theta+(\mu a)^2\tau_u)^2} > 0$,

$$
\begin{aligned}
\frac{\partial \mathrm{RHS}(34)}{\partial \tau_\varepsilon^P} > 0 \iff & \beta_3'\tau_\varepsilon\rho_I[(1+\beta_3)^2 + \tau\beta_3^2/\tau_\varepsilon^P] \\
& -(1+\beta_3)\tau_\varepsilon\rho_I[2(1+\beta_3)\beta_3' + \tau(2\beta_3\beta_3'\tau_\varepsilon^P - \beta_3^2)/\tau_\varepsilon^{P2}] \\
= -(\tau_\varepsilon\rho_I(1+\beta_3)^2\beta_3' + \tau_\varepsilon\rho_I\tau 2\beta_3\beta_3'/\tau_\varepsilon^P + \tau_\varepsilon\rho_I\tau\beta_3^2\beta_3'/\tau_\varepsilon^P) & + \tau\rho_I\tau_\varepsilon(1+\beta_3)\beta_3^2/\tau_\varepsilon^{P2} > 0 \\
\iff -(1+\beta_3)^2\beta_3' - \tau 2\beta_3\beta_3'/\tau_\varepsilon^P - \tau\beta_3^2\beta_3'/\tau_\varepsilon^P & + \tau(1+\beta_3)\beta_3^2/\tau_\varepsilon^{P2} > 0 \\
\iff \frac{r(-(1+r)(\tau_\theta+(\mu a)^2\tau_u)(\tau_\varepsilon^P+\tau_\theta+(\mu a)^2\tau_u)^2 + r\tau_\varepsilon(\tau_\varepsilon^{P2}(1+r) - (t+(\mu a)^2\tau_u)^2))}{(\tau_\varepsilon^P+\tau_\theta+(\mu a)^2\tau_u)^4} & > 0,
\end{aligned}
$$

which holds for $r \to \infty$, since the positive term grows at a cubic rate while the negative terms grow at most at a quadratic rate. Thus, for $r > 0$ large enough, $\mathrm{RHS}_{\tau_\varepsilon^P} > 0$, hence $\partial a/\partial\tau_\varepsilon^P > 0$.

Price informativeness (adapting (4) for $\tau_\varepsilon^P \neq \tau_\varepsilon$)

$$
\mathrm{PI}_{\text{informed}} = \frac{1}{(\tau_\theta+\tau_\varepsilon^P)^2/((\mu a)^2\tau_u) + \tau_\theta + \tau_\varepsilon^P}
$$

increases in $|a|, \mu, \tau_u$, decreases in $\tau_\theta, \tau_\varepsilon^P$, and does not change with $\psi_4/\psi_3, \tau_\varepsilon, \rho_I$. The total effect of parameter t on price informativeness is

$$
\frac{d\mathrm{PI}}{dt} = \frac{\partial \mathrm{PI}}{\partial t} + \frac{\partial \mathrm{PI}}{\partial |a|}\cdot\frac{\partial |a|}{\partial t}.
$$

Thus, all of the above comparative statics also apply to price informativeness except for τ_θ and τ_ε^P, which decrease price informativeness directly ($\partial\mathrm{PI}/\partial\tau_\theta < 0$) but increase $|a|$ ($\partial\mathrm{PI}/\partial|a|\cdot\partial|a|/\partial\tau_\theta > 0$), making the effect potentially ambiguous. The effects of $\psi_4/\psi_3, \tau_\varepsilon, \rho_I$ are indirect via a ($\partial\mathrm{PI}/\partial t = 0$).

The indirect effect is very small for $|r| \to \infty$, which leads to $\mathrm{RHS}_{\tau_\theta} \to 0$ and $\mathrm{RHS}_a \to 0$, hence $\partial|a|/\partial\tau_\theta = \mathrm{RHS}_{\tau_\theta}/(\mathrm{RHS}_a - 1) \to 0$. The same applies to τ_ε^P. Consequently, for τ_θ and τ_ε^P, the direct effect on price informativeness dominates for large $|r|$, and price informativeness decreases with larger τ_θ and τ_ε^P.

Since the welfare gain in (6) is price informativeness times $\psi_4^2/(2\psi_3)$, all of the above comparative statics except for ψ_4/ψ_3 carry over to the welfare measure.

ii. $\mathrm{RHS}_r < 0$ for $r < 0$ sufficiently negative. This follows from the fact that $\mathrm{RHS}_r < 0$ (see (30)) if and only if (31) holds, which it does for $r << 0$ sufficiently negative. Thus, a decreases in response to a less extreme $\psi_4/\psi_3 < 0$.

$\mathrm{RHS}_\mu < 0$, since the last term in (32) is positive for $r \to -\infty$. $\mathrm{RHS}_{\rho_I} > 0$, since $\mathrm{RHS}(a) < 0$ in equilibrium (Proposition 16). $\mathrm{RHS}_{\tau_\theta} < 0$, since the last term in (33) is positive for $r \to -\infty$. $\mathrm{RHS}_{\tau_\varepsilon} < 0$, since $1+\beta_3 < 0$ for $r << 0$, which makes the slope in (35) negative. It can similarly be verified that $\mathrm{RHS}_{\tau_u} < 0$ and $\mathrm{RHS}_{\tau_\varepsilon^P} < 0$ if $r << 0$.

Since $r << 0$ implies $a < 0$ in equilibrium (Proposition 16), a decrease in a increases $|a|$. Consequently, the same arguments as in (i.) show that the same comparative statics results apply to price informativeness. As in (i.) all of these results carry over to the welfare measure except for ψ_4/ψ_3. □

B Alternative definition of rational expectations equilibrium without noise

This section gives an alternative definition of rational expectations equilibrium requiring market clearing and optimal demands of traders given the available information, instead of imposing conditions on the equilibrium price function directly. It furthermore shows that the main result can be proven using this definition of rational expectations equilibrium as well.

Before defining the equilibrium concept, we need to define endowments and budget constraints of traders, since risk neutrality would otherwise imply unbounded positions. Each trader $j \in [0, 1]$ is initially endowed with one unit of the risky asset and $0 < \omega < \infty$ units of cash, where ω is enough to match the asset value in every state of nature. Traders cannot sell more than one unit of the asset and may not spend more than ω of their cash. The net demand function of trader j is denoted by $x_j(p, s_j)$, which may depend on the private information s_j and the price (and information contained therein) p. A rational expectations equilibrium in this setting is defined as follows, where both the $p(\boldsymbol{s})$ and $i(p, s_p)$ functions are known in equilibrium.

Definition 6. *A rational expectations equilibrium (REE) requires*

i. a measurable price function $p(\boldsymbol{s})$ that clears the market:

$$\int_0^1 x_j(p = p(\boldsymbol{s}), s_j)dj = 0 \text{ a.s.},$$

ii. all traders j use optimal demand strategies given the available information,

$$x_j(p, s_j) \in \arg\max_x \; \mathbb{E}_\theta[a(\theta, i(p, s_p))x|p = p(\boldsymbol{s}), s_j] \text{ s.t. } xp \leq \omega, \; -1 \leq x, \; \forall j \in [0, 1],$$

iii. the policy maker sets an optimal policy given available information,

$$i(p, s_p) \in \arg\max_i \; \mathbb{E}_\theta[u(\theta, i)|p = p(\boldsymbol{s}), s_p].$$

With this definition, we get the same result as before.

Corollary 5. *A fully revealing (definition 2) rational expectations equilibrium exists if and only if condition 1 holds.*

Proof. Necessity: Existence of a fully revealing rational expectations equilibrium implies that condition 1 holds. If the equilibrium is fully revealing, then by definition 2,

$$|\{t \in \boldsymbol{S} : \Pr(\boldsymbol{s} = t | p(\boldsymbol{s}) = p, s_p) > 0\}| \leq 1 \ \forall s_p, \forall p.$$

Taking similar steps as in the proof of Theorem 2, if $\Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p)$ in (17) is positive for exactly one $t \in \boldsymbol{S}$ for all p in the image of $p(\boldsymbol{s})$, then the antecedent of condition 1 is false and the condition holds.

If $\Pr(\boldsymbol{s} = t|p(\boldsymbol{s}) = p, s_p) > 0$ for more than one $t \in \boldsymbol{S}$, then (17) and full revelation imply that whenever $p(t) = p(t')$ for $t \neq t' \in \boldsymbol{S}$, so that $\Pr(p(t) = p) \cdot \Pr(p(t') = p) > 0$, then $\Pr(s_p|\boldsymbol{s} = t) \cdot \Pr(s_p|\boldsymbol{s} = t') = 0$. Thus, the consequent of condition 1 is true and the condition holds.

Sufficiency: Condition 1 implies that a fully revealing rational expectations equilibrium exists. We prove this by construction. Suppose traders conjecture the (accurate) price function $p(\boldsymbol{s}) = v(\boldsymbol{s})$. Theorem 2 shows that condition 1 implies that a fully revealing and accurate price function exists. It is in fact unique, since we assumed that the optimal policy is unique, hence $p(\boldsymbol{s}) = v(\boldsymbol{s})$ is the price function that fully reveals $\boldsymbol{s}$ to the policy maker. Clearly, this price function, which reveals the expected asset value given the combined trader information $v(\boldsymbol{s})$ to traders, also clears the market, since traders are indifferent between buying and selling ($\mathbb{E}_\theta[a(\theta, i(p, s_p))|p = p(\boldsymbol{s}), s_j] = \mathbb{E}_\theta[a(\theta, i(p, s_p))|p = p(\boldsymbol{s})] = p(\boldsymbol{s}) = v(\boldsymbol{s})$), thus for example $x_j(p = p(\boldsymbol{s}), s_j) = 0$ is optimal. Finally, the policy maker sets her optimal policy given $\boldsymbol{s}$, which leads to asset value $v(\boldsymbol{s})$ and makes the conjecture self-fulfilling. □

C Implementation of full revelation as Perfect Bayesian Nash equilibrium

This section shows that condition 1, which is necessary and sufficient for fully revealing rational expectations equilibria (Corollary 5), is also necessary and sufficient for full revelation using the Perfect Bayesian Nash equilibrium concept in a continuum economy with perfectly correlated information among traders.

To derive the Perfect Bayesian Nash equilibrium (PBNE), we need to specify the trading environment. The financial market consists of a continuum (of mass 1) of risk neutral traders,[12] who have an endowment in cash $0 < \omega < \infty$ and one unit of the risky asset each, where ω is enough to match the asset value in every state of nature. In this section, I shall only consider the cases where trader signals s_j are perfectly correlated, $s_j = c \in \mathbb{R} \ \forall j$, $\boldsymbol{s} = c$, drawn from density $f(s_j|\theta)$, which includes the perfect information case ($s_j = \theta \ \forall j$).

[12]The continuum assumption implies that individual traders do not affect the price. Results are not generally the same with finitely many traders, because in such a strategic setting any single trader can change the message sent to the policy maker (price) by changing his own investment strategy, making this a market signaling game, which is beyond the scope of this paper.

I confine attention to the perfectly correlated information structure, since fully revealing REE in differential information economies are not generally implementable with a market mechanism as Bayesian Nash equilibrium (see, e.g., Vives, 2010). Thus, the role of the market in this section is not aggregation of trader information as in REE, since every trader has the same piece of information, but revelation of the information.

Denote the set of all possible excess demand functions for traders by $\mathcal{X}$. An excess demand function x_j specifies for each p and s_j by how many units trader j would like to change his initial holding of the risky asset. Unlike REE, we have to specify how prices are set in this trading environment, as a Walrasian auctioneer is not available. To be as general as possible, I merely require that prices are a function of the demand function profile. Any price setting algorithm that conditions on demand functions (e.g., market orders, limit orders, etc.) in determining the price fulfills the definition. Examples include double auctions, Dutch auctions, or automated market makers that react to orders.

Definition 7. *A generic price finding rule $\Gamma : \mathcal{X}^N \to \mathbb{R}$ maps all profiles of excess demand functions $\boldsymbol{x}(\boldsymbol{s}) := (x_1(p, s_1 = \boldsymbol{s}), x_2(p, s_2 = \boldsymbol{s}), \dots)$ into a price p^*.*

Price $p^* = \Gamma(\boldsymbol{x}(\boldsymbol{s}))$ need not be a market clearing price, since a market clearing price need not exist for all profiles of excess demand functions. However, the following definition requires market clearing in equilibrium.

Definition 8. *A Perfect Bayesian Nash equilibrium (PBNE) requires*

i. an expected utility maximizing excess demand function $x_j(p, s_j)$ using information set $\{p(\boldsymbol{s}) = p, s_j\}$ given $\boldsymbol{x}_{-j}$ s.t. $-1 \leq x_j, x_j p \leq \omega$ for all traders j at $t = 1$,

ii. market clearing in the financial market at $t = 1$, i.e., a price function $p(\boldsymbol{s}) = \Gamma(\boldsymbol{x}(\boldsymbol{s}))$ generated by a generic price finding rule, such that

$$\int x_j(p^* = p(\boldsymbol{s}), s_j = \boldsymbol{s})dj = 0 \text{ (a.s.)},$$

iii. policy maker beliefs $\mu(t|p^*) = \Pr(\boldsymbol{s} = t|p^* = p(\boldsymbol{s}), s_p)$, $\sum_{t \in \boldsymbol{S}} \mu(t|p^* = p(\boldsymbol{s}), s_p) = 1$, about financial market behavior, derived by Bayes' rule whenever possible,

iv. and an expected utility maximizing policy $i(s_p, p^*, \mu) \in I$ for the policy maker at $t = 2$.

Beliefs μ refer to the financial market as a whole, represented by the market clearing price p^*, rather than to actions of individual traders. This is a realistic way to model the interaction, as a policy maker typically cannot track investments by specific traders in anonymous markets, especially not in large markets as considered here. The price function $p(\boldsymbol{s})$ is known in equilibrium via Bayes' rule. There is no need to specify off equilibrium path beliefs, because no trader can influence the price alone, i.e., the price with a single deviation can never be off the equilibrium path.

The following proposition shows that condition 1 is necessary and sufficient for the possibility of a fully revealing PBNE.

Proposition 18. *If there is a continuum of traders and trader signals are perfectly correlated, then a fully revealing PBNE exists if and only if condition 1 is fulfilled.*

Proof. Necessity: If a fully revealing PBNE exists, then condition 1 holds. First, a fully revealing equilibrium implies that $p(\boldsymbol{s}) = v(\boldsymbol{s})$. Suppose not ($p(\boldsymbol{s}) \neq v(\boldsymbol{s})$ for some $\boldsymbol{s} \in \boldsymbol{S}$), then market clearing implies there exists at least one trader who can profit by deviating from the candidate strategy x_j to $\hat{x}_j > x_j$ if $v(\boldsymbol{s}) > p(\boldsymbol{s})$ or to $\hat{x}_j < x_j$ if $v(\boldsymbol{s}) < p(\boldsymbol{s})$, since $v(\boldsymbol{s})$ is the expected value of the asset given full revelation, and the deviation in a continuum does not change prices. Thus, full revelation implies either that $v(\boldsymbol{s})$ is invertible, so that $p(\boldsymbol{s})$ perfectly reveals the state, or that $v(\boldsymbol{s})$ is not invertible but $p(\boldsymbol{s}) = v(\boldsymbol{s})$ together with policy maker signal s_p always reveals $\boldsymbol{s}$, i.e., condition 1 holds.

Sufficiency: If condition 1 holds, then a fully revealing PBNE exists. By construction. Define $\overline{x}_j(p, \boldsymbol{s}) = \omega/p > 0$ as j's best response to $v(\boldsymbol{s}) > p$ and $\underline{x}_j(p, \boldsymbol{s}) = -1$ as j's best response to $v(\boldsymbol{s}) < p$. If condition 1 holds, then the following strategy for all j forms a fully revealing PBNE:

$$x_j(p, \boldsymbol{s}) = \begin{cases} \overline{x}_j(p, \boldsymbol{s}) > 0 & \text{if } v(\boldsymbol{s}) > p, \\ \underline{x}_j(p, \boldsymbol{s}) < 0 & \text{if } v(\boldsymbol{s}) < p, \\ 0 & \text{if } v(\boldsymbol{s}) = p. \end{cases}$$

Clearly, market clearing occurs if and only if $p = v(\boldsymbol{s})$ for all $\boldsymbol{s} \in \boldsymbol{S}$. Due to condition 1, the equilibrium candidate price function $p(\boldsymbol{s}) = v(\boldsymbol{s})$ is fully revealing. There is no profitable deviation, since x_j best responds to mispricing by going long if $p < v(\boldsymbol{s})$ and short if $p > v(\boldsymbol{s})$. At $p = v(\boldsymbol{s})$, no deviation changes the payoff, since individual traders cannot influence the price and the asset is priced at its expected value. The policy maker obtains her first best given all available information, so she has no incentive to deviate either. □

Thus, if traders hold the same information, then condition 1 determines the possibility of full revelation in both major equilibrium concepts used in financial economics, REE and PBNE, in large markets. However, if traders have differential information, then the fully revealing REE equilibrium is not implementable as PBNE using a generic price finding rule in a market mechanism, as is well known in financial economics (e.g., Vives, 2010).