Microeconomics III: Problem Set 9[a]

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[a] Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C	D
A	0, 2	2, 3
B	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

[Hints on the next slide. Try to independently write down the approach/criteria for a mixed-strategy NE and a pure-strategy BNE respectively.]

PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

Hint: To find a mixed-strategy NE (MSNE):
Find the probabilities q for which Player 1 is indifferent, i.e. $u_1(A, q) = u_1(B, q)$ and the probabilities p for which Player 2 is indifferent, i.e. $u_2(C, p) = u_2(D, p)$.

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

Hint: Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type, t_2). For each possible strategy $s_2(t_2)$:

1. Given the different types $t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$, write up the best response of player 1: $s_1^*(t_{1,k}) \equiv BR_1\left(s_2(t_2)|t_{1,k}\right)$.
2. If it also holds that $s_2(t_2) = BR_2\left(s_1^*(t_{1,k})|t_2\right) \equiv s_2^*(t_2)$ then $\left(s_1^*(t_{1,k}), s_2^*(t_2)\right)$ is a BNE.

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

First, find all pure-strategy NE given k.

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

k =1:
k =2:
$k \geq$3:

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

k =1: One PSNE: $\{(A, D)\}$
k =2: Two PSNE: $\{(A, D); (B, D)\}$
k ≥3: One PSNE: $\{(B, D)\}$

Then find all mixed-strategy NE given k.

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

$k = 1$: One PSNE: $\{(A, D)\}$
$k = 2$: Two PSNE: $\{(A, D); (B, D)\}$
$k \geq 3$: One PSNE: $\{(B, D)\}$

For P1 to mix, she has to be indifferent between A and B, thus we only need to look at:

$k = 2$:

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

$k = 1$: One PSNE: $\{(A, D)\}$
$k = 2$: Two PSNE: $\{(A, D); (B, D)\}$
$k \geq 3$: One PSNE: $\{(B, D)\}$

For P1 to mix, she has to be indifferent between A and B, thus we only need to look at:

$k = 2$: One MSNE: $\{(p \in (0, 1), q = 0)\}$

Optional: Write up and plot the best-response functions for k=2.

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

$k = 1$: One PSNE: $\{(A, D)\}$
$k = 2$: Two PSNE: $\{(A, D); (B, D)\}$
$k \geq 3$: One PSNE: $\{(B, D)\}$

For P1 to mix, she has to be indifferent between A and B, thus we only need to look at:

$k = 2$: One MSNE: $\{(p \in (0, 1), q = 0)\}$

Optional: For $k = 2$, BR functions are:

$$BR_1(q) = \begin{cases} p \in [0,1] & \text{if} \quad q = 0 \\ p = 0 & \text{if} \quad q > 0 \end{cases}$$

$$BR_2(p) = \{ \; q = 0 \quad \text{if} \quad p \in [0,1]$$

PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where $k \in \mathbb{N}$:

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of k, find all Nash Equilibria (pure and mixed).

P2: C is strictly dominated by D, thus D is played in any NE, pure or mixed.

P1: For P2 playing D consider:

$k = 1$: One PSNE: $\{(A, D)\}$
$k = 2$: Two PSNE: $\{(A, D); (B, D)\}$
$k \geq 3$: One PSNE: $\{(B, D)\}$

For P1 to mix, she has to be indifferent between A and B, thus we only need to look at:

$k = 2$: One MSNE: $\{(p \in (0, 1), q = 0)\}$

Optional: For $k = 2$, BR functions are:

$$BR_1(q) = \begin{cases} p \in [0, 1] & \text{if} \quad q = 0 \\ p = 0 & \text{if} \quad q > 0 \end{cases}$$

$$BR_2(p) = \{ \; q = 0 \quad \text{if} \quad p \in [0, 1]$$

FIGURE MISSING

PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C	D
A	0, 2	2, 3
B	3, 1	k, 8

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

[Hint for BNE on next slide.]

PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C	D
A	0, 2	2, 3
B	3, 1	k, 8

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

Hint: Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type, t_2). For each possible strategy $s_2(t_2)$:

Step 1: Given the different types $t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$, write up the best response of player 1: $s_1^*(t_{1,k}) \equiv BR_1\left(s_2(t_2)|t_{1,k}\right)$.

Step 2: If it also holds that $s_2(t_2) = BR_2\left(s_1^*(t_{1,k})|t_2\right) \equiv s_2^*(t_2)$ then $\left(s_1^*(t_{1,k}), s_2^*(t_2)\right)$ is a BNE.

PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C	D
A	0, 2	2, 3
B	3, 1	k, 8

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

Hint: Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type, t_2). For each possible strategy $s_2(t_2)$:

Step 1: Given the different types $t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$, write up the best response of player 1: $s_1^*(t_{1,k}) \equiv BR_1\left(s_2(t_2)|t_{1,k}\right)$.

Step 2: If it also holds that $s_2(t_2) = BR_2\left(s_1^*(t_{1,k})|t_2\right) \equiv s_2^*(t_2)$ then $\left(s_1^*(t_{1,k}), s_2^*(t_2)\right)$ is a BNE.

As C is strictly dominated, player 2 only has the viable strategy $s_2(t_2) = D$:

1. Best response of player 1, $s_1^*(t_{1,k})$:

$$BR_1\left(D|t_{1,k}\right) = (s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}) = (A, B)$$

PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where $k \in \mathbb{N}$:

	C	D
A	0, 2	2, 3
B	3, 1	k, 8

(b) Now assume that player 1 knows k, but player 2 only knows that $k = 1$ with probability $\frac{1}{2}$ and $k = 3$ with probability $\frac{1}{2}$. Find the Bayesian Nash Equilibrium of this game.

Hint: Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type, t_2). For each possible strategy $s_2(t_2)$:

Step 1: Given the different types $t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$, write up the best response of player 1: $s_1^*(t_{1,k}) \equiv BR_1\left(s_2(t_2)|t_{1,k}\right)$.

Step 2: If it also holds that $s_2(t_2) = BR_2\left(s_1^*(t_{1,k})|t_2\right) \equiv s_2^*(t_2)$ then $\left(s_1^*(t_{1,k}), s_2^*(t_2)\right)$ is a BNE.

As C is strictly dominated, player 2 only has the viable strategy $s_2(t_2) = D$:

1. Best response of player 1, $s_1^*(t_{1,k})$:

$$BR_1\left(D|t_{1,k}\right) = (s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}) = (A, B)$$

2. As $D = BR_2\left((A, B)|t_2\right) \equiv s_2^*(t_2)$ we have a unique BNE:

$$\left((s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}), s_2^*(t_2)\right) = \{((A, B), D)\}$$

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Consider the same set-up as exercise 3.4 in Gibbons, but now with the following bi-matrices for Game 1 and Game 2 respectively:

G1:

	L	R
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L	R
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Exercise 3.4 in Gibbons (p. 169). Find all the pure-strategy Bayesian Nash equilibria in the following static Bayesian game:

a. *Nature determines whether the payoffs are as in Game 1 or as in Game 2, each game being equally likely.*
b. ***Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not.***
c. *Player 1 chooses either U or D; player 2 simultaneously chooses either L or R.*
d. *Payoffs are given by the game drawn by nature.*

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L	R
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L	R
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: **Find the player 1 strategies s_1' for which player 2 will want to mix.**

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L	R
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L	R
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: **Find the player 1 strategies s_1' for which player 2 will want to mix.**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U, D) or (D, D).

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L	R
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L	R
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: **Find the player 1 strategies s_1' for which player 2 will want to mix.**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U,U),(U,D),(D,D),(D,U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U,D) or (D,D).

$BR_2((D,D)) = R$ but for (U,D) P2 is indifferent between L and R.

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L (q)	R (1-q)
U	1, 1	0, 0
D	0, 0	2, 0

G1:

	L (q)	R (1-q)
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: Find the player 1 strategies s_1' for which player 2 will want to mix.

Step 2: **Find the values of q (the probability that player 2 plays L) such that player 1 will actually play s_1'.**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U, D) or (D, D).

$BR_2((D, D)) = R$ but P2 is indifferent between L and R for $s_1' = (U, D)$.

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L (q)	R (1-q)
U	1, 1	0, 0
D	0, 0	2, 0

G1:

	L (q)	R (1-q)
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: Find the player 1 strategies s_1' for which player 2 will want to mix.

Step 2: **Find the values of q (the probability that player 2 plays L) such that player 1 will actually play s_1'.**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U,U),(U,D),(D,D),(D,U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U,D) or (D,D).

$BR_2((D,D)) = R$ but P2 is indifferent between L and R for $s_1' = (U,D)$.

2. P1 wants to play U in G1 if:

$$E[u_1|U] \geq E[u_1|D] \Leftrightarrow$$

$$q \geq 2(1-q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L (q)	R (1-q)
U	1, 1	0, 0
D	0, 0	2, 0

G1:

	L (q)	R (1-q)
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: Find the player 1 strategies s_1' for which player 2 will want to mix.

Step 2: Find the values of q (the probability that player 2 plays L) such that player 1 will actually play s_1'.

Step 3: **Write up the mixed-strategy BNE where P1 plays a pure strategy and P2 randomizes between L and R.**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U,U),(U,D),(D,D),(D,U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U,D) or (D,D).

$BR_2((D,D)) = R$ but P2 is indifferent between L and R for $s_1' = (U,D)$.

2. P1 wants to play U in G1 if:

$$E[u_1|U] \geq E[u_1|D] \Leftrightarrow$$

$$q \geq 2(1-q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

PS8, Ex. 2 (A): Mixed-Strategy Bayesian Nash Equilibria

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L (q)	R (1-q)
U	1, 1	0, 0
D	0, 0	2, 0

G1:

	L (q)	R (1-q)
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between L and R.

Step 1: Find the player 1 strategies s_1' for which player 2 will want to mix.

Step 2: Find the values of q (the probability that player 2 plays L) such that player 1 will actually play s_1'.

Step 3: **Write up the mixed-strategy BNE where P1 plays a pure strategy and**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U,U),(U,D),(D,D),(D,U)\}$$

G2: U is weekly dominated by D, thus, P1 will play D in G2 as long as P2 plays R with positive probability.

I.e. P1 either plays (U,D) or (D,D).

$BR_2((D,D)) = R$ but P2 is indifferent between L and R for $s_1' = (U,D)$.

2. P1 wants to play U in G1 if:

$$E[u_1|U] \geq E[u_1|D] \Leftrightarrow$$

$$q \geq 2(1-q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

3. For q being the probability that P2 plays L, the mixed-strategy BNE is:

$$BNE' = \left\{ \left((U,D), q \geq \frac{2}{3} \right) \right\}$$