Microeconomics III: Problem Set 11[a]

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[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS11, Ex. 1 (A): Effect of the GED education as a signal

PS11, Ex. 2 (A): Asymmetric information (PBE)

PS11, Ex. 3: Signaling game (pooling and separating PBE)

PS11, Ex. 4: Signaling games (pooling and separating PBE)

PS11, Ex. 5: Signaling games (pooling PBE)

PS11, Ex. 6: Spence's education signaling model (PBE)

PS11, Ex. 1 (A): Effect of the GED education as a signal

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Does signaling work? Read the article by Tyler, Murnane and Willett and think about their results. What is their hypothesis for why they do not find an effect for minority groups? Come up with an example of an education program that has mostly signaling value in your country.

(This is a reflection question, no answer will be provided).

PS11, Ex. 2 (A): Asymmetric information (PBE)

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Exercise 4.11 in Gibbons (p. 250). Difficult. A buyer and a seller have valuations v_b and v_s. It is common knowledge that there are gains from trade (i.e., that $v_b > v_s$), but the size of the gains is private information, as follows: the seller's valuation is uniformly distributed on [0,1]; the buyer's valuation $v_b = kv_s$, where $k > 1$ is common knowledge; the seller knows v_s (and hence v_b) but the buyer does not know v_b (and hence v_s). Suppose the buyer makes a single offer, p, which the seller either accepts or rejects. What is the perfect Bayesian equilibrium when $k < 2$? When $k > 2$? (See Samuelson 1984.)

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Mean: $\mu = \frac{a+b}{2} \Rightarrow \mathbb{E}(x < c) = \frac{a+c}{2}$ (‡)

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Step 2: **The buyer offers a price p. Write up the seller's strategy (best response).**

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$$\max_p \mathbb{P}[v_s < p]\mathbb{E}[v_b - p | v_s < p]$$

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$$= \max_p \frac{p-0}{1-0}\mathbb{E}[kv_s - p | v_s < p] \qquad \text{using } (\dagger)$$

$$= \max_p p\left(k\mathbb{E}[v_s < p] - p\right)$$

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PS11, Ex. 3: Signaling game (pooling and separating PBE)

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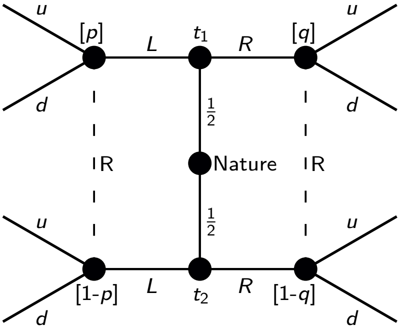
Consider the signaling game in Figure 1.

(a) Suppose there is a pooling PBE where the Sender sends message L regardless of his type. What are the beliefs in this equilibrium?

(b) Consider a possible separating PBE where t_1 sends message R, t_2 sends message L, and where the receiver chooses u if and only if he receives message L. Can you write down payoffs for this game such that nobody has an incentive to deviate?



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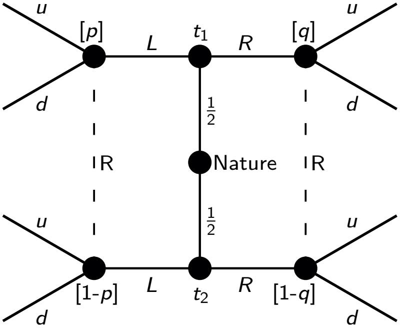
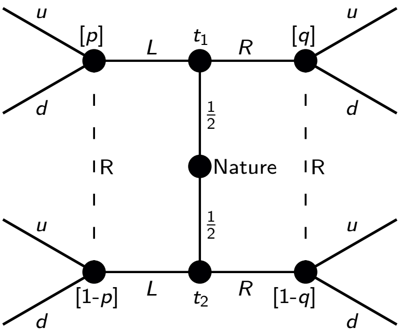
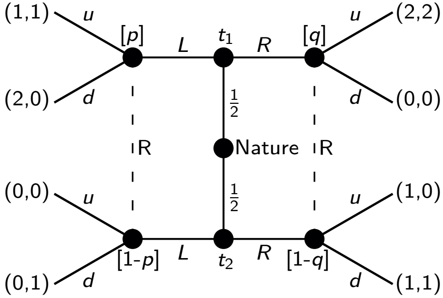


(b) Consider a possible separating PBE where t_1 sends message R, t_2 sends message L, and where the receiver chooses u if and only if he receives message L. Can you write down payoffs for this game such that nobody has an incentive to deviate?

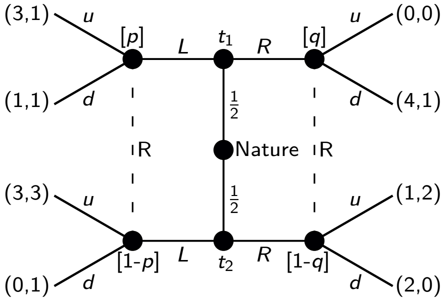


PS11, Ex. 4: Signaling games (pooling and separating PBE)

Exercise 4.4.a in Gibbons (p. 248). Describe all the pure-strategy pooling and separating perfect Bayesian equilibria in the following signaling game.

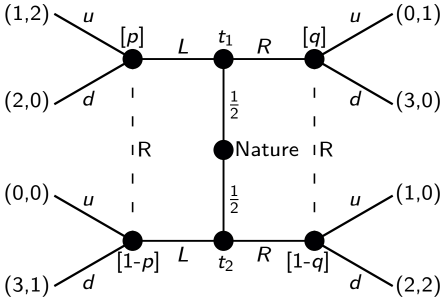


Exercise 4.4.b in Gibbons (p. 248). Describe all the pure-strategy pooling and separating perfect Bayesian equilibria in the following signaling game.

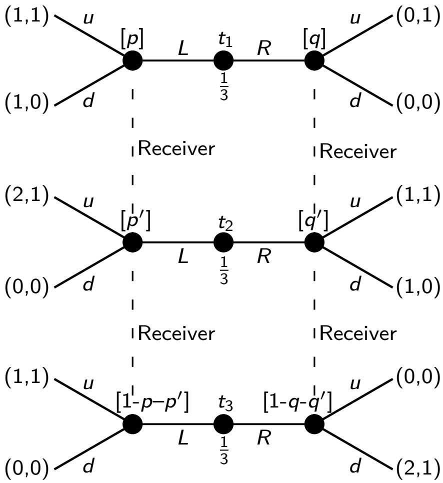


PS11, Ex. 5: Signaling games (pooling PBE)

Exercise 4.3.a in Gibbons (p. 246). Specify a pooling perfect Bayesian equilibria in which both Sender types play R in the following signaling game.



Exercise 4.3.b in Gibbons (p. 246). The following three-type signaling game begins with a move by nature, not shown in the tree, that yields one of the three types with equal probability Specify a pooling perfect Bayesian equilibria in which all three Sender types play L.



PS11, Ex. 6: Spence's education signaling model (PBE)

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Consider the following version of Spence's education signaling model, where a firm is hiring a worker. Workers are characterized by their type θ, which measures their ability. There are two worker types: $\theta \in \{\theta_L, \theta_H\}$. Nature chooses the worker's type, with $p_H = \mathbb{P}[\theta = \theta_H]$ and $p_H = \mathbb{P}[\theta = \theta_H] = 1 - p_H$.

The worker observes his own type, but the firm does not. The worker can choose his level of education: $e \in \mathbb{R}^+$. The cost to him of acquiring this education is $c_\theta(e) = e/\theta$. Education is observed by the firm, who then forms beliefs about the workers type: $\mu(\theta|e)$. We assume that the marginal productivity of a worker is equal to his ability and that the company is in competition such it pays the marginal productivity: $w(e) = \mathbb{E}[\theta|e]$. Thus, the payoff to a worker conditional on his type and education is $u_\theta(e) = w(e)c_\theta(e)$. Suppose for this exercise that $\theta_H = 3$ and $\theta_L = 1$.

(a) Find a separating pure strategy Perfect Bayesian Equilibrium.

(b) Find a pooling pure strategy Perfect Bayesian Equilibrium.