PS3, Ex. 3 (A): NE proof using IEWDS

3. (A) We have seen in the lectures that IESDS never eliminates a Nash Equilibrium. However, we saw in Problem Set 2 that this is not true if we do iterated elimination of weakly dominated strategies (IEWDS.) Go through the proof in the slides from lecture 2 and identify the step that is no longer true if we replace IESDS by IEWDS. That is, explain why the proof is no longer true when we replace 'strict domination' by 'weak domination'.

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The proof that all NE survive IESDS holds by contradiction. We **highlight** where the contradiction breaks down using IEWDS instead:

- Let (s_1^*, s_2^*) be a NE.
- Say we carry out **IEWDS** and s_1^* is the first NE strategy to be eliminated (in round n of elimination).
- Then there must be a strategy $s_1^{'} \neq s_1^*$ that **weakly** dominates s_1^*, i.e.

$$\forall s_2 \in S_2^n : \ u_1(s_1^*, s_2) \underbrace{\leq}_{\textbf{Weak}} u_1(s_1^{'}, s_2) \ \ (1)$$

and the inequality holds strictly for at least one strategy $s_2^{'} \in S_2^n$ where S_2^n is the set of player-2 strategies that have not been eliminated in rounds $1, ..., n-1$.

- Since $s_2^* \in S_2^n$, inequality (1) also means

$$u_1(s_1^*, s_2^*) \underbrace{\leq}_{\textbf{Weak}} u_1(s_1^{'}, s_2^*)$$

- ~~**But**~~ (s_1^*, s_2^*) is a NE, so by definition

$$\forall s_1 \in S_1 : \ u_1(s_1^*, s_2^*) \geq u_1(s_1, s_2^*)$$

- **No** contradiction!

Conclusion: for a NE (s_1^*, s_2^*) IEWDS can eliminate s_1^* if $s_1^{'}, s_2^{'}$ exist such that:

$$\text{for } s_1^{'} \in S_1^n : \ u_1(s_1^*, s_2^*) = u_1(s_1^{'}, s_2^*)$$

and

$$\text{for } s_2^{'} \in S_2^n : \ u_1(s_1^*, s_2^{'}) < u_1(s_1^{'}, s_2^{'})$$