PS4, Ex. 3: The Focal Point (plotting BR functions)

Thomas and Alice want to meet on a Friday night. There are two bars in their home town: “The Focal Point” and “The Other Place”. They have to decide independently where they go. If they meet in the same bar, they both get utility of 1. If they end up in different bars, they get utility of 0.

(a) Find all equilibria (pure and mixed). Which equilibrium do you consider the most realistic? Where would you go if you were one of them?

(b) Now assume that Thomas wants to meet Alice, but Alice does not want to meet Thomas. Thomas gets a payoff of 1 if he meets Alice, and -1 otherwise. Alice gets a payoff of -1 for meeting Thomas, and 1 otherwise. Find all equilibria (pure and mixed).

(c) Now assume again that Thomas and Alice both want to meet (so that payoffs are as in part (a)), but now there are N bars in town, where N can be very large. Show that there are $2^N - 1$ equilibria (pure and mixed). Say that the bars have names: “The First Bar in Town”, “The Second Bar in Town”, and so on. Which equilibrium is the most realistic?

PS4, Ex. 3.a: The Focal Point (plotting BR functions)

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(a) Find all equilibria (pure and mixed). Which equilibrium do you consider the most realistic? Where would you go if you were one of them?

		Thomas F (q)	Thomas O (1-q)
Alice	F (p)	1, 1	0, 0
Alice	O (1-p)	0, 0	1, 1

No PSNE. Alice is indifferent for:

$$E[u_A|Focal] = E[u_A|Other]$$

$$q = 1 - q \Leftrightarrow q = \frac{1}{2}$$

Taking advantage of symmetry:

$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{1}{2}, \frac{1}{2}\right)\right\}$$

Which is the most realistic?

$(\frac{1}{2}, \frac{1}{2})$ seems unlikely as expected payoffs are $\frac{1}{2}$ while being 1 for $(0,0)$ and $(1,1)$.

Where would you go?

I would go to the "The Focal Point" - it sounds like the place to meet.

PS4, Ex. 3.b: The Focal Point (plotting BR functions)

(b) Now assume that Thomas wants to meet Alice, but Alice does not want to meet Thomas. Find all NE.

		Thomas	
		F (q)	O (1-q)
Alice	F (p)	-1, 1	1, -1
	O (1-p)	1, -1	-1, 1

No PSNE. Alice is indifferent for:

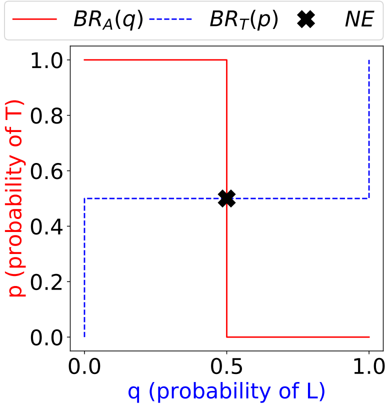
$$-q + (1-q) = q - (1-q) \Leftrightarrow q = \frac{1}{2}$$

Thomas is indifferent for:

$$p - (1-p) = -p + (1-p) \Leftrightarrow p = \frac{1}{2}$$

$$BR_A(q) = \begin{cases} p = 1 & \text{if} \quad q < 1/2 \\ p \in [0,1] & \text{if} \quad q = 1/2 \\ p = 0 & \text{if} \quad q > 1/2 \end{cases}$$

$$BR_T(p) = \begin{cases} q = 0 & \text{if} \quad p < 1/2 \\ q \in [0,1] & \text{if} \quad p = 1/2 \\ q = 1 & \text{if} \quad p > 1/2 \end{cases}$$



The only NE is the Mixed Strategy NE:

$$(p^*, q^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

PS4, Ex. 3.c: The Focal Point (plotting BR functions)

(c) Now assume again that Thomas and Alice both want to meet (so that payoffs are as in part (a)), but now there are N bars in town, where N can be very large. Show that there are $2^N - 1$ equilibria (pure and mixed). Say that the bars have names: "The First Bar in Town", "The Second Bar in Town", and so on.

For N=2: We have $3 = 2^N - 1$ equilibria:

$$(p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{1}{2}, \frac{1}{2}\right)\right\}$$

		Thomas	
		Bar_1 (q)	Bar_2 (1-q)
Alice	Bar_1 (p)	1, 1	0, 0
	Bar_2 (1-p)	0, 0	1, 1

For N=3: We have $7 = 2^N - 1$ equilibria, $(p_1^*, p_2^*, q_1^*, q_2^*)$:

$$\left\{(0,0,0,0); (0,1,0,1); (1,0,1,0); \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right); \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right); \left(0, \frac{1}{2}, 0, \frac{1}{2}\right); \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\right\}$$

		Thomas		
		Bar_1 (q_1)	Bar_2 (q_2)	Bar_3 (1-q_1-q_2)
Alice	Bar_1 (p_1)	1, 1	0, 0	0, 0
	Bar_2 (p_2)	0, 0	1, 1	0, 0
	Bar_3 (1-p_1-p_2)	0, 0	0, 0	1, 1

What about any N?

PS4, Ex. 3.c: The Focal Point (plotting BR functions)

(c) Which equilibrium is the most realistic?

For N=3: We have $7 = 2^N - 1$ equilibria, $(p_1^*, p_2^*, q_1^*, q_2^*)$:

$$\left\{(0,0,0,0); (0,1,0,1); (1,0,1,0); \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right); \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right); \left(0, \frac{1}{2}, 0, \frac{1}{2}\right); \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\right\}$$

Alice \ Thomas	Bar_1 (q_1)	Bar_2 (q_2)	Bar_3 $(1\text{-}q_1\text{-}q_2)$
Bar_1 (p_1)	1, 1	0, 0	0, 0
Bar_2 (p_2)	0, 0	1, 1	0, 0
Bar_3 $(1\text{-}p_1\text{-}p_2)$	0, 0	0, 0	1, 1

In the three PSNE, the expected payoffs are: $\Big(E[u_A|q_1^*, q_2^*)], E[u_T|p_1^*, p_2^*)]\Big) =$

$$\{(1-q_1-q_2, 1-p_1-p_2); (q_2, p_2); (q_1, p_1)\} \sim \{(1,1); (1,1); (1,1)\}$$

In the four MSNE, the expected payoffs are: $\Big(E[u_A|q_1^*, q_2^*)], E[u_T|p_1^*, p_2^*)]\Big) =$

$$\left\{\left(\frac{q_1+q_2}{2}, \frac{p_1+p_2}{2}\right); \left(\frac{1-q_2}{2}, \frac{1-p_2}{2}\right); \left(\frac{1-q_1}{2}, \frac{1-p_1}{2}\right); \left(\frac{1}{3}, \frac{1}{3}\right)\right\}$$

$$\sim \left\{\left(\frac{1}{2}, \frac{1}{2}\right); \left(\frac{1}{2}, \frac{1}{2}\right); \left(\frac{1}{2}, \frac{1}{2}\right); \left(\frac{1}{3}, \frac{1}{3}\right)\right\}$$

PSNE are the most realistic due to higher payoffs. Due to coordination issues, the expected payoffs are reciprocal to the number of actions that a MSNE is split between.