

PS5, Ex. 4: The Mutated Seabass (backwards induction)

Consider a game where two evil organizations, rather prosaically named A and B, are battling for world domination. The battle takes the form of a three-stage game. Organization A is on the verge of acquiring a new powerful weapon, the *mutated seabass*. In stage 1 of the game, they decide whether to acquire the weapon or not. Their choice is observed by organization B. In stage 2, organization B decides whether to attack organization A. If an attack occurs, the game stops. If no attack occurs, it moves to stage 3, where organization A decides whether or not to attack organization B. The payoffs are as follows. If no-one attacks the other, the payoffs to both organizations are 0. If B attacks A, then the payoffs to both organizations are .1. The same if A attacks B, without having acquired the seabass weapon. If, on the other hand, A acquires the weapon, the payoffs from A attacking B are 2 to A and -2 to B.

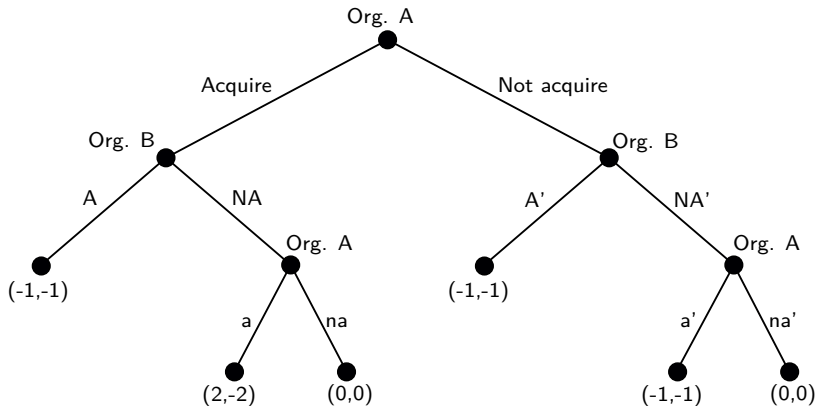
- (a) Draw the game tree that corresponds to the game. What are the strategies of the players?
- (b) What is the backwards induction outcome?
- (c) What is the intuition for the outcome? What role do you think it plays that B observes if A acquires the weapon or not?

PS5, Ex. 4.a: The Mutated Seabass (backwards induction)

(a) Draw the game tree that corresponds to the game. What are the strategies of the players?

$S_A = \{(Acquire, a, a'), (Acquire, a, na'), (Acquire, na, a'), (Acquire, na, na'),$
 $(Not\ acquire, a, a'), (Not\ acquire, a, na'), (Not\ acquire, na, a'), (Not\ acquire, na, na')\}$

$S_B = \{(A, A'), (A, NA'), (NA, A'), (NA, NA')\}$



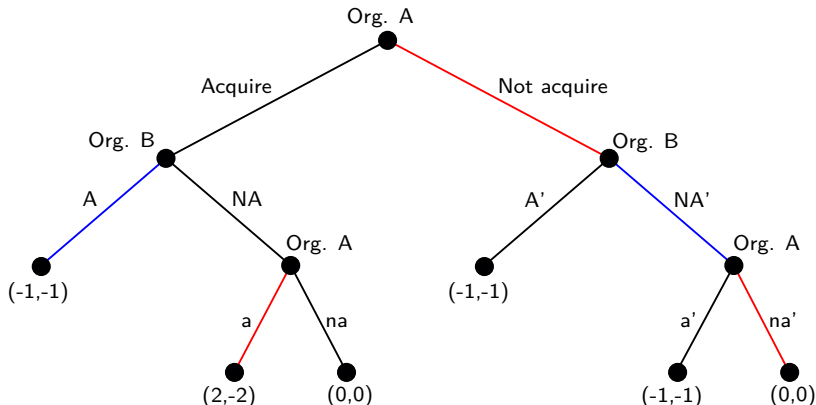
PS5, Ex. 4.b: The Mutated Seabass (backwards induction)

(b) What is the backwards induction outcome?

3rd stage: Org. A will choose to attack if having acquired the weapon and not attack if not having acquired the weapon.

2nd stage: Org. B will choose to attack if Org. A has acquired the weapon and not attack if they have not acquired the weapon.

1st stage: Org. A will choose to not acquire the weapon in order to signal peaceful intentions to Org. B, i.e. giving the payoffs (0, 0).



$$SPNE = \{S_A, S_B\} = \{(Not\ acquire, a, na'), (A, NA')\}$$

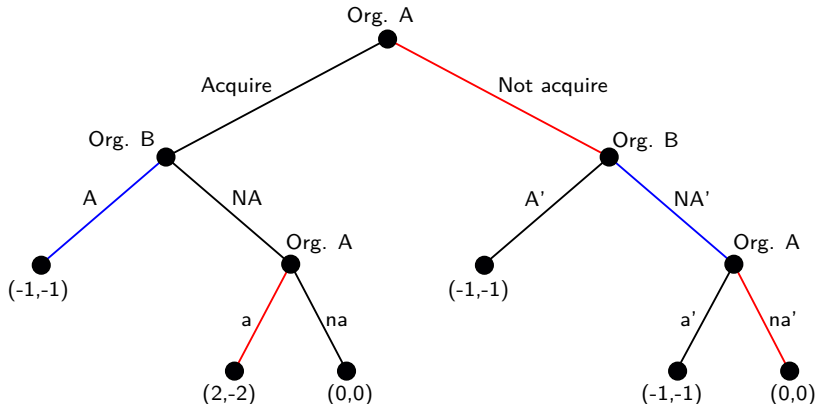
PS5, Ex. 4.c: The Mutated Seabass (backwards induction)

(c) What is the intuition for the outcome?

3rd stage: Org. A does only benefit from attacking if having acquired the weapon.

2nd stage: Org. B will only choose to attack if Org. A has acquired the weapon.

1st stage: Not acquiring the weapon is a credible signal that Org. A will not attack.



What role do you think it plays that B observes if A acquires the weapon or not?

I.e. what is the outcome if Organization A cannot send a signal in the 1st stage?

PS5, Ex. 4.c: The Mutated Seabass (backwards induction)

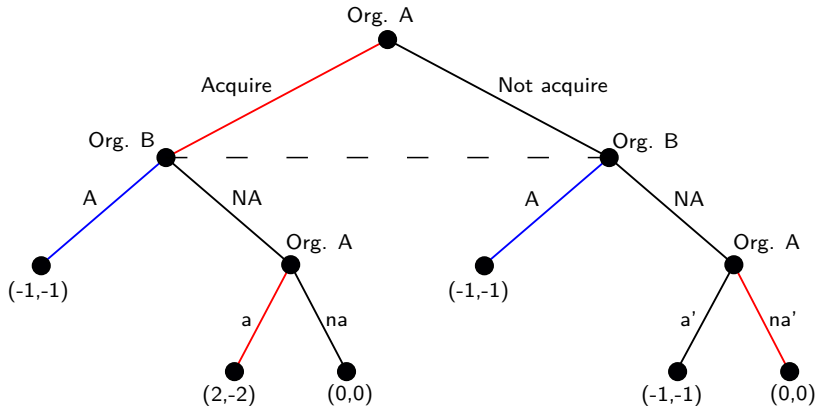
(c) What role do you think it plays that B observes if A acquires the weapon or not?

I.e. what is the outcome if Organization A cannot send a signal in the 1st stage?

3rd stage: [unchanged] Org. A will choose to attack if having acquired the weapon and not attack if not having acquired the weapon.

2nd stage: Knowing that Org. A will attack if having acquired the weapon, Org. B chooses to attack first, giving the payoffs $(-1, -1)$ regardless of stage one.

1st stage: Org. A cannot affect the outcome, but acquires it in case Org. B deviates.



$$SPNE = \{S_A, S_B\} = \{(Acquire, a, na'), A\}$$