Proof: The expected highest and second highest draw from a uniform distribution

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To find *seller's expected revenue* from a sealed bid auction (e.g. bidders simultaneously submit their bids in sealed envelopes without knowing the bids of others) with symmetric bidders with valuation drawn from a uniform distribution, there are two different approaches:

1. One approach is to derive each *bidder's expected payment* as a function of her valuation and then integrate this expression using the PDF to get the *ex-ante expected payment* of each bidder which can then be added together to find seller's expected revenue.
2. However, a more simple approach is to for N number of bidders to calculate the expected value of the highest (first-price sealed bid auction) or second-highest (second-price sealed bid auction). Plugging the value into the bid-function gives the seller's expected revenue.

Deriving the bid-function (best-response function) is a prerequisite for both approaches.

[Try to write up the PDF, CDF, and Mean for the uniform distribution $x \sim u(a, b)$, before going to the next slide.]

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For both approaches, we also utilize the standard results for a uniform distribution $x \sim u(a, b)$:

PDF: Probability density function: $f(x) = \frac{1}{b-a}$

CDF: Cumulative distribution function: $F(x) = \frac{x-a}{b-a} \Rightarrow \mathbb{P}(c > x) = \frac{c-a}{b-a}$

Mean: $\mu = \frac{a+b}{2} \Rightarrow \mathbb{E}(c < x) = \frac{a+x}{2}$

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First, consider the uniform distribution of x from 0 to 1: $x \sim u(0,1)$:

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Rule: For a uniform distribution of x from a to b: $x \sim u(a, b)$:

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Thus, for N number of bidders where each bidder i have her value v_i drawn from a uniform distribution $v_i \sim u(a, b)$:

The expected highest value Y for all bidders is

$$Y = max(v_1, v_2, ..., v_N) = a + N\frac{b-a}{N+1}$$

And the expected second-highest value for all bidders is

$$a + (N-1)\frac{b-a}{N+1}$$