Microeconomics III: Problem Set 11[a]

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[a] Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

- PS12, Ex. 1 (A): Job-market signaling

- PS12, Ex. 2 (A): Farrell & Rabin (1996): "Cheap talk"
  - PS12, Ex. 2.a (A): Formulate a cheap talk game
  - PS12, Ex. 2.b (A): Find a separating PBE
  - PS12, Ex. 2.c (A): Discuss if PBE is reasonable?

- PS12, Ex. 3: Cheap talk games (extensive form)

- PS12, Ex. 4: Three-type job-applicant cheap talk game
  - PS12, Ex. 4.a: Fully separating PBE
  - PS12, Ex. 4.b: Partial pooling PBE
  - PS12, Ex. 4.c: Fully pooling PBE

PS12, Ex. 1 (A): Job-market signaling

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(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?

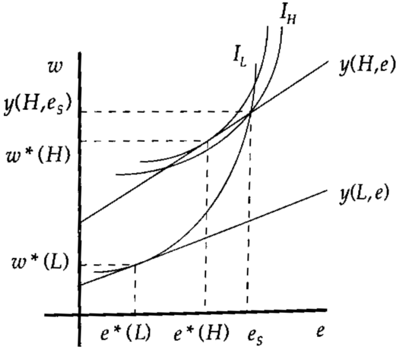


Figure 4.2.8.

Step 1: **Explain the graphs $\boldsymbol{I_L, I_H, y(L,e), y(H,e)}$.**

(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?

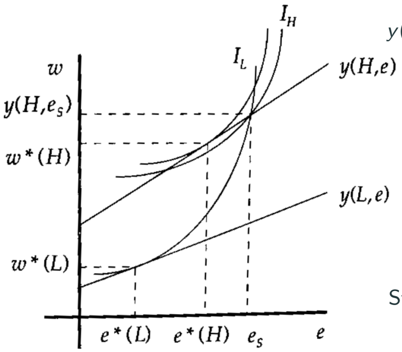


Figure 4.2.8.

Step 1: Explain $I_L, I_H, y(L,e), y(H,e)$:

For each type of worker $\eta \in L, H$:

I_η: The indifference curve over which the worker's utility is constant. I.e. how much the wage must increase to compensate for higher education.

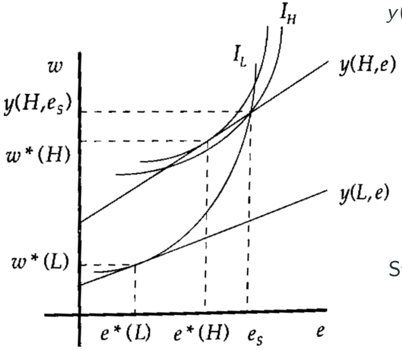
$y(\eta, e)$: The expected output of a worker with ability η and education e which is equal to the wage offered by the firms under competition. I.e. education is now productive and more so for the high-ability worker.

Under complete information, the optimal education is where a worker's indifference curve is tangent to her productivity.

Step 2: **Why can H not choose $e^*(H)$ in a separating equilibrium?**

PS12, Ex. 1 (A): Job-market signaling

(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?



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Under complete information, the optimal education is where a worker's indifference curve is tangent to her productivity.

Step 2: Why can H not choose $e^*(H)$?

In a separating equilibrium, the firms perfectly identify H and L by education choices. However, as $[e^*(H), w^*(H)]$ is above L's indifference curve, L would imitate H. Thus, H needs to increase education to e_s to credibly signal type H.

PS12, Ex. 2 (A): Farrell & Rabin (1996): ”Cheap talk”

PS12, Ex. 2 (A): Farrell & Rabin (1996): "Cheap talk"

(A) In their paper "Cheap Talk" published in the *Journal of Economic Perspectives* (1996), Joseph Farrell and Matthew Rabin describe the following situation: "Sally knows which one of two tasks is efficient to perform. Rayco [the firm] could hire Sally specifically to perform Job 1, specifically to perform Job 2, or as a highly paid manager who will choose which job to perform. If Rayco knew which task is efficient, it would still hire her to perform the task, but at a lower salary, because she has lost her informational advantage. Sally wants to be hired as manager, but prefers to be hired to do the right task and be more productive rather than to do the wrong task and be less productive." Payoffs in this situation are

		Job 1	Job 2	Manager
Sally's knowledge	Task 1 efficient	2, 5	1, -2	3, 3
	Task 2 efficient	1, -2	2, 5	3, 3

where the left number in each cell is Sally's payoff and the right number is the firm's payoff (note that this matrix does not describe the normal form of the game!)

(a) Formulate this strategic situation as a cheap talk game, assuming that the type space is equal to the message space ($T = M$).

(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

PS12, Ex. 2.a (A): Farrell & Rabin (1996). Formulate a cheap talk game

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
Task 2 efficient ($t = T_2$)	1, -2	2, 5	3, 3

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Task 2 efficient ($t = T_2$)	1, -2	2, 5	3, 3

(a) Formulate this strategic situation as a cheap talk game, assuming that the type space is equal to the message space ($T = M$).

The game is as follows:

1. Sally's type is realized: $t \in \{T_1, T_2\}$, where $t = T_1, T_2$ corresponds to efficiency with Task 1 and Task 2 respectively.
2. Sally observes her type and sends a cheap talk message $m(t) \in \{T_1, T_2\}$.
3. Rayco observes the message and chooses a job for Sally: $a(m) \in \{T_1, T_2, M\}$ where $a = T_1, T_2, M$ corresponds to giving Sally one of three jobs:
    - Job 1 (compatible with task 1 efficiency)
    - Job 2 (compatible with task 2 efficiency)
    - Manager (compatible with both but more expensive).

PS12, Ex. 2.b (A): Farrell & Rabin (1996). Find a separating PBE

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
Task 2 efficient ($t = T_2$)	1, -2	2, 5	3, 3

(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

PS12, Ex. 2.b (A): Farrell & Rabin (1996). Find a separating PBE

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
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(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

Step 1: **Go over the beliefs and actions in such a separating PBE. Does either type want to deviate?**

PS12, Ex. 2.b (A): Farrell & Rabin (1996). Find a separating PBE

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
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(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

Step 1: Go over the beliefs and actions in such a separating PBE. Does either type want to deviate?

In a PBE, the beliefs must correspond to the action of the senders.
Thus in a separating PBE where $m(t = T_1) = T_1$ and $m(t = T_2) = T_2$, beliefs are

$$\mu(t = T_1 | m = T_1) = 1 \text{ and } \mu(t = T_2 | m = T_2) = 1$$

This gives R's best responses.

$$a(m = T_1) = T_1 \text{ and } a(m = T_2) = T_2$$

Since no message yields the position *Manager*, neither type can imitate the other to get this position, thus, no type has an incentive to deviate.

Step 2: **Write up the separating PBE.**

PS12, Ex. 2.b (A): Farrell & Rabin (1996). Find a separating PBE

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
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Since no message yields the position *Manager*, neither type can imitate the other to get this position, thus, no type has an incentive to deviate.

Step 2: Write up the separating PBE:

$$\{ \underbrace{(T_1, T_2)}_{m(t=T_1),m(t=T_2)}, \underbrace{(T_1, T_2)}_{a(m=T_1),a(m=T_2)}, \underbrace{\mu(T_1|T_1) = 1}_{\mu(t=T_1|m=T_1)}, \underbrace{\mu(T_2|T_2) = 1}_{\mu(t=T_2|m=T_2)} \}$$

PS12, Ex. 2.c (A): Farrell & Rabin (1996). Discuss if PBE is reasonable?

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
Task 2 efficient ($t = T_2$)	1, -2	2, 5	3, 3

(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

PS12, Ex. 2.c (A): Farrell & Rabin (1996). Discuss if PBE is reasonable?

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
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Step 1: **Why does Sally not have an incentive to deviate?**

PS12, Ex. 2.c (A): Farrell & Rabin (1996). Discuss if PBE is reasonable?

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
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(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

Step 1: Why does Sally not have an incentive to deviate?

Sally has to say that she is efficient at one of the jobs. Since Rayco believe she is telling the truth, she has no choice but to tell the truth, as she would otherwise end up in a worse job for her.

PS12, Ex. 2.c (A): Farrell & Rabin (1996). Discuss if PBE is reasonable?

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
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(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

Step 1: Why does Sally not have an incentive to deviate?

Sally has to say that she is efficient at one of the jobs. Since Rayco believe she is telling the truth, she has no choice but to tell the truth, as she would otherwise end up in a worse job for her.

Step 2: **Under which circumstances would a pooling PBE exist where both types would get hired as *Manager*?**

	Job 1 ($a = T_1$)	Job 2 ($a = T_2$)	Manager ($a = Manager$)
Task 1 efficient ($t = T_1$)	2, 5	1, -2	3, 3
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(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

Step 1: Why does Sally not have an incentive to deviate?

Sally has to say that she is efficient at one of the jobs. Since Rayco believe she is telling the truth, she has no choice but to tell the truth, as she would otherwise end up in a worse job for her.

Step 2: Under which circumstances would a pooling PBE exist where both types would get hired as *Manager*?

There is a pooling equilibrium where all types of Sally choose the same message ($m = T_1$ or $m = T_2$), thus, don't send any informative signal on what type they are. In this case, if nature distributes types somewhat equally, i.e. $\mathbb{P}(t = T_1) = p \in \left[\frac{2}{7}; \frac{5}{7}\right]$, then the best response for Rayco would be $a(m = T_1) = a(m = T_2) = Manager$ and no type of Sally would want to deviate. If nature is very likely to draw type T_1 $\left(p > \frac{5}{7}\right)$ then Rayco always offers T_1 as $\mathbb{E}[u_R(a = T_1)] > 3$ no matter the signals. Likewise, if nature is unlikely to draw T_1 $\left(p < \frac{2}{7}\right)$ then $\mathbb{E}[u_R(a = T_2)] > 3$.

I.e. the separating PBE would be realistic if each type made a strategy in isolation. However, as Sally constitutes all types, she can decide on a tactic for all type of senders. No type would want to deviate from the pooling PBE $\left(\text{for } p \in \left[\frac{2}{7}; \frac{5}{7}\right]\right)$.

PS12, Ex. 3: Cheap talk games (extensive form)

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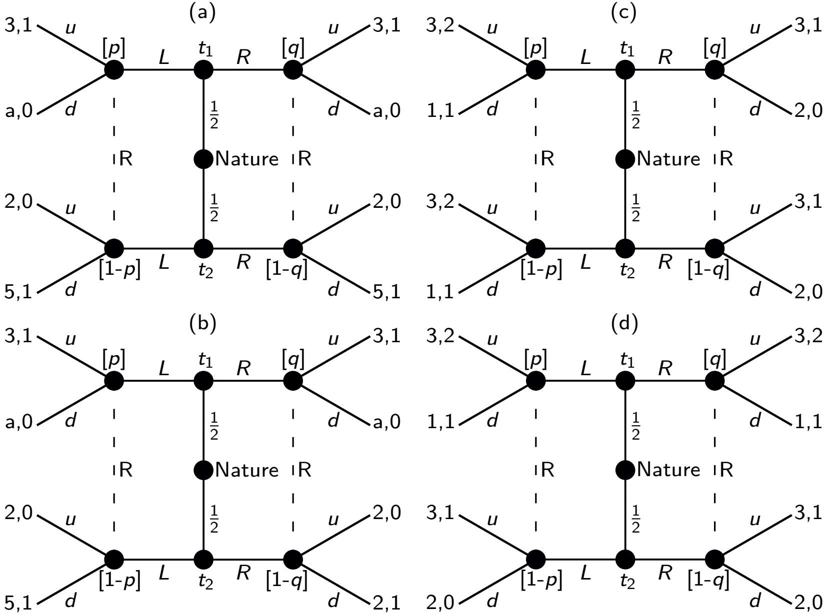
Consider the games (a)-(d) on the next page. (i) Which of these are cheap talk games? (ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

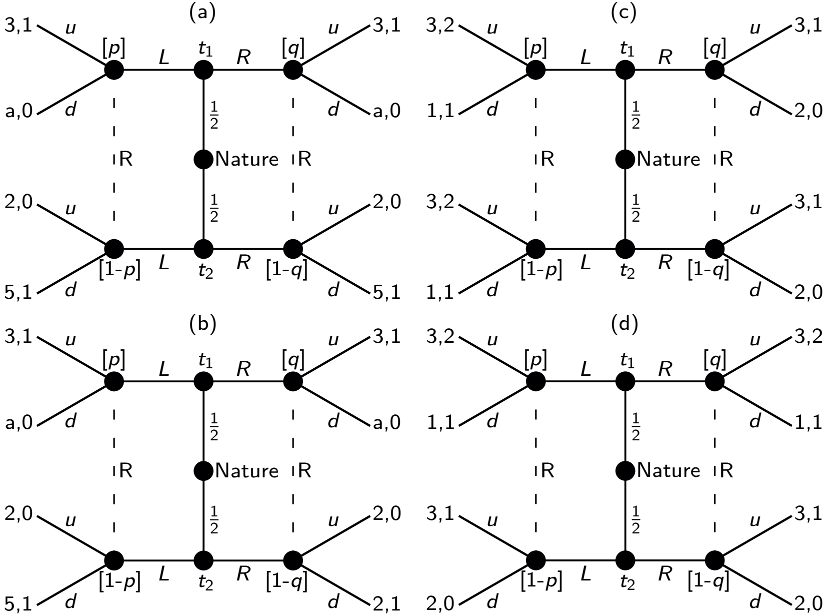
PS12, Ex. 3.i: Cheap talk games (extensive form)

(ii) Which of these are cheap talk games?



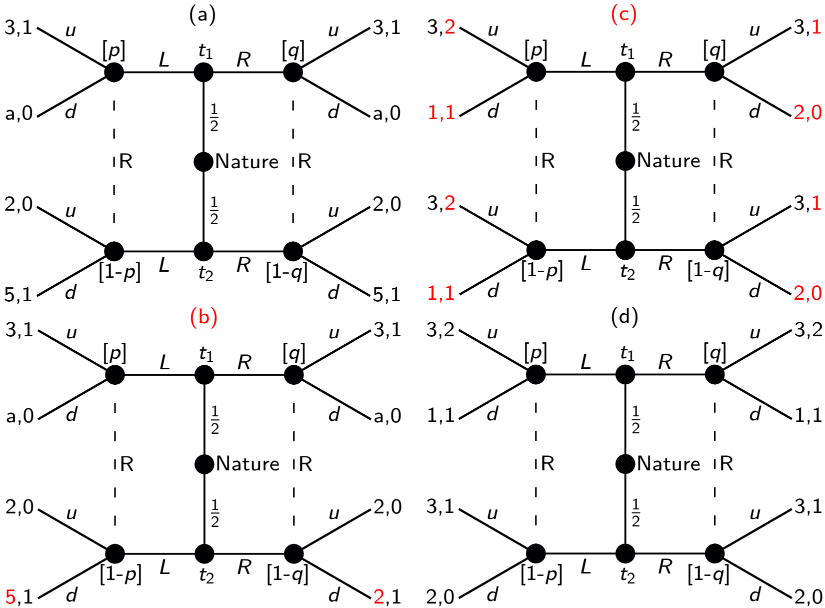
PS12, Ex. 3.i: Cheap talk games (extensive form)

(ii) In cheap talk games, payoffs don't depend on the message: $u_S(t_i, a_k), u_R(t_i, a_k)$.



PS12, Ex. 3.i: Cheap talk games (extensive form)

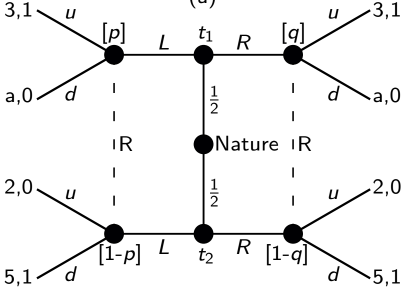
(ii) In (a) and (d) signals are "cheap" as all payoffs are the same for L and R:



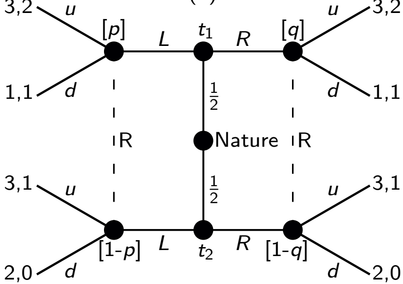
PS12, Ex. 3.ii: Cheap talk games (extensive form)

(ii) For those which are cheap talk games [(a) and (d)], find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

(a)

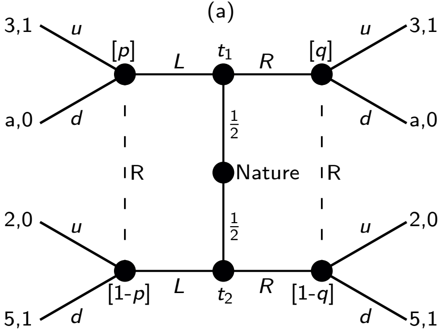


(d)



PS12, Ex. 3.ii.a: Cheap talk games (extensive form)

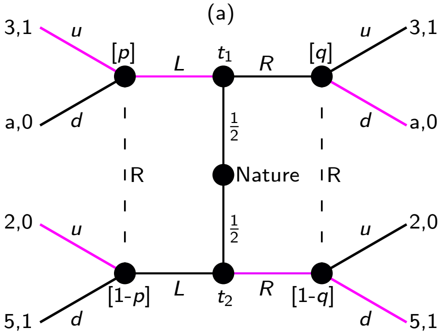
(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: **For the separating strategy (*L*,*R*), go over SR3, SR2R, and SR2S. PBE?**

Step 2: For the separating strategy (*R*,*L*), go over SR3, SR2R, and SR2S. PBE?

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



1. SR3: R: Beliefs given S's strategy: $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u,\ a^*(R) = d$.

   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$$
   t_2 will never deviate as
   $$u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$$

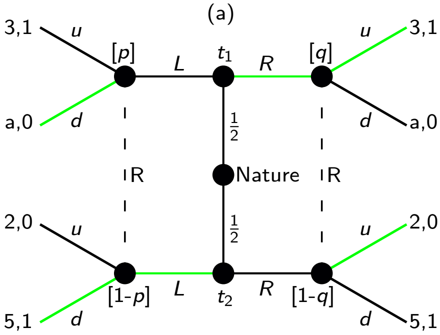
   PBE: No deviation if $a \leq 3$ (pink).

Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: **For the separating strategy (*R*,*L*), go over SR3, SR2R, and SR2S. PBE?**

PS12, Ex. 3.ii.a: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



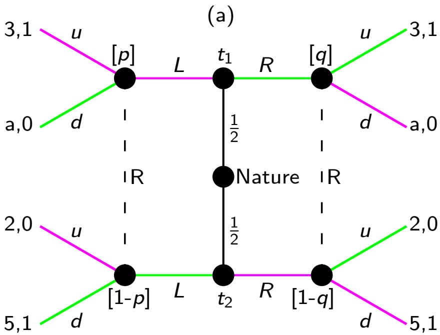
Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: **Write up the set of separating PBE.**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$
   SR2R: R: $a^*(L) = u, \ a^*(R) = d$.
   SR2S: t_1 will not deviate if
   $u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$
   t_2 will never deviate as
   $u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$
   PBE: No deviation if $a \leq 3$ (pink).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$
   SR2R: R: $a^*(L) = d, \ a^*(R) = u$.
   SR2S: t_1 will not deviate if
   $u_S(R, u|t_1) = 3 \geq a = u_S(L, d|t_1)$
   t_2 will never deviate as
   $u_S(L, d|t_2) = 5 > 2 = u_S(R, u|t_2)$
   PBE: No deviation if $a \leq 3$ (green).

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the set of separating PBE.

Step 4: **Is S's "cheap talk" informative? How is/isn't that possible?**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$
   SR2R: R: $a^*(L) = u,\ a^*(R) = d$.
   SR2S: t_1 will not deviate if
   $$u_S(L,u|t_1) = 3 \geq a = u_S(R,d|t_1)$$
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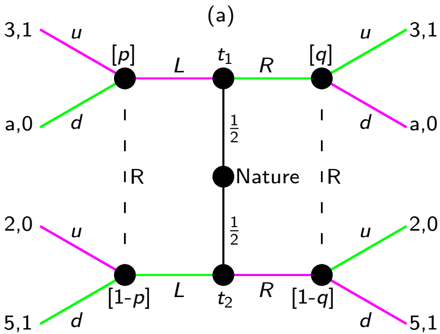
   PBE: No deviation if $a \leq 3$ (pink).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$
   SR2R: R: $a^*(L) = d,\ a^*(R) = u$.
   SR2S: t_1 will not deviate if
   $$u_S(R,u|t_1) = 3 \geq a = u_S(L,d|t_1)$$
   t_2 will never deviate as
   $$u_S(L,d|t_2) = 5 > 2 = u_S(R,u|t_2)$$
   PBE: No deviation if $a \leq 3$ (green).
3. Two separating PBE exist for $a \leq 3$:
   $$PBE = \left\{ \begin{array}{c} (L,R),(u,d), p=1, q=0 \\ (R,L),(d,u), p=0, q=1 \end{array} \right\}$$

PS12, Ex. 3.ii.a: Cheap talk games (extensive form)



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the set of separating PBE.

Step 4: S's "cheap talk" is informative as: (1) t_1 and t_2 prefer different actions, (2) R prefers different actions for different sender types, and (3) S's types and R's preferences are aligned.

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u,\ a^*(R) = d$.

   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$$
   t_2 will never deviate as
   $$u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$$
   PBE: No deviation if $a \leq 3$ (pink).

2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

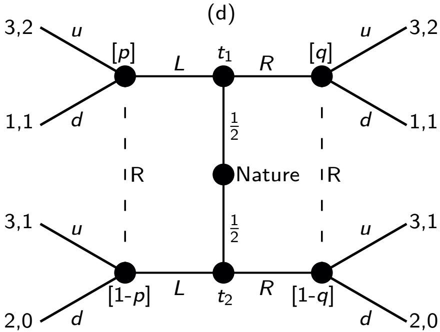
   SR2R: R: $a^*(L) = d,\ a^*(R) = u$.

   SR2S: t_1 will not deviate if
   $$u_S(R, u|t_1) = 3 \geq a = u_S(L, d|t_1)$$
   t_2 will never deviate as
   $$u_S(L, d|t_2) = 5 > 2 = u_S(R, u|t_2)$$
   PBE: No deviation if $a \leq 3$ (green).

3. Two separating PBE exist for $a \leq 3$:
   $$PBE = \left\{ \begin{array}{c} (L, R), (u, d), p = 1, q = 0 \\ (R, L), (d, u), p = 0, q = 1 \end{array} \right\}$$

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

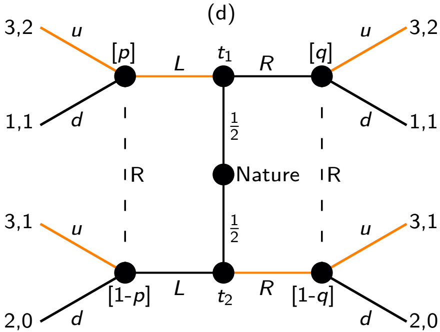
(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: **For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?**

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: **For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?**

1. SR3: R: Beliefs given S's strategy: $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

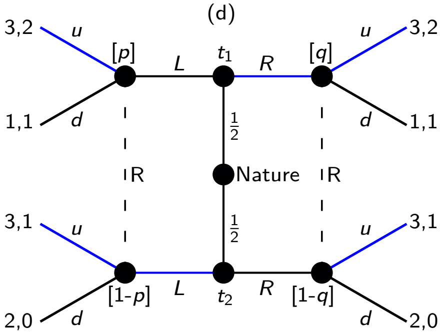
   SR2R: R: $a^*(L) = u$, $a^*(R) = u$.

   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$
   t_2 will never deviate as
   $$u_S(R, u|t_2) = 3 > 3 = u_S(L, u|t_2)$$

   PBE: No deviation → PBE (orange).

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: **Write up the full set of separating PBE.**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u, \ a^*(R) = u$.

   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$
   t_2 will never deviate as
   $$u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$$

   PBE: No deviation $\rightarrow$ PBE (orange).

2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

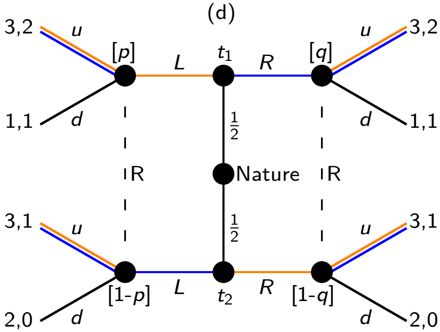
   SR2R: R: $a^*(L) = u, \ a^*(R) = u$.

   SR2S: t_1 will not deviate as
   $$u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$$
   t_2 will not deviate as
   $$u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$$

   PBE: No deviation $\rightarrow$ PBE (blue).

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the full set of separating PBE.

Step 4: **Explain. Why are the signals non-informative "cheap talk"?**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u, \ a^*(R) = u$.

   SR2S: t_1 will not deviate if

   $u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$

   t_2 will never deviate as

   $u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$

   PBE: No deviation $\rightarrow$ PBE (orange).

2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

   SR2R: R: $a^*(L) = u, \ a^*(R) = u$.

   SR2S: t_1 will not deviate as

   $u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$

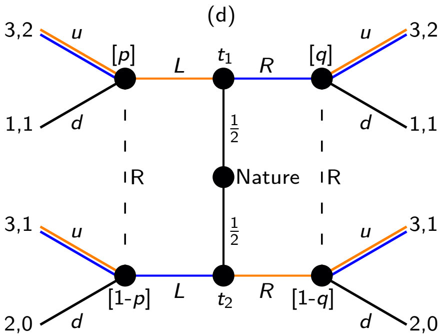
   t_2 will not deviate as

   $u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$

   PBE: No deviation $\rightarrow$ PBE (blue).

3. $\left\{ \begin{array}{c} (L, R), (u, u), p = 1, q = 0 \\ (R, L), (u, u), p = 0, q = 1 \end{array} \right\}$

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the full set of separating PBE.

Step 4: Explain. Why are the signals non-informative "cheap talk"?

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$
   SR2R: R: $a^*(L) = u,\ a^*(R) = u.$
   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$
   t_2 will never deviate as
   $$u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$$
   PBE: No deviation $\rightarrow$ PBE (orange).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$
   SR2R: R: $a^*(L) = u,\ a^*(R) = u.$
   SR2S: t_1 will not deviate as
   $$u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$$
   t_2 will not deviate as
   $$u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$$
   PBE: No deviation $\rightarrow$ PBE (blue).
3. $\left\{ \begin{array}{c} (L,R),(u,u),p=1,q=0 \\ (R,L),(u,u),p=0,q=1 \end{array} \right\}$
4. Two "weak separating PBE": Both t_1 and t_2 is indifferent between signals as R will always choose u.

PS12, Ex. 4: Three-type job-applicant cheap talk game

PS12, Ex. 4: Three-type job-applicant cheap talk game

Consider the following job-applicant cheap talk game based on Farrell-Rabin (1996). Suppose that there are three types of potential applicants (high ability, medium ability, and low ability) and the firm can place the applicant in one of threeH possible positions (highly qualified, medium qualified, low qualified). The applicant is equally likely to be each of the three types (probability 1/3). Payoff are represented below, where for each cell, the left entry gives the payoff of the applicant, and the right entry gives the payoff of the firm, conditional on the firm's action and the applicant's type. Notice: this matrix does not show the normal form game! It merely gives you the payoffs for each type-job combination, but does not incorporate the cheap talk message.

The game is as follows: first, the applicant's type is realized: $t \in \{L, M, H\}$, where $t = L$ corresponds to low ability etc. The applicant observes his type and sends a cheap talk message $m \in \{L, M, H\}$. The firm observes the message and chooses a job for the applicant: $a \in \{L, M, H\}$, where $a = L$ corresponds to giving the applicant the low qualified job etc.

	Highly qualified	Medium qualified	Low qualified
High ability	3, 3	0, 0	0, 0
Medium ability	1, 0	2, 2	0, 0
Low ability	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

(b) Show that a partial pooling PBE does exist, where $m(H) = H$ and $m(M) = m(L) = M$. What are the firm's beliefs? Solve for each case.

(c) (If time permits) Does a fully pooling PBE exist, $m(H) = m(M) = m(L) = H$?

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

Step 1: **According to the firm, what would the ideal separating PBE look like?**

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

Step 1: According to the firm, what would the ideal separating PBE look like?

The firm prefers to give each candidate the job corresponding to their type.

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

Step 1: According to the firm, what would the ideal separating PBE look like?

The firm prefers to give each candidate the job corresponding to their type.

Step 2: **So, in order to show that no fully separating PBE exist, look for a type who would like to get a job that does not correspond to one's type, and go from there.**

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

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The firm prefers to give each candidate the job corresponding to their type.

Step 2: So, in order to show that no fully separating PBE exist, look for a type who would like to get a job that does not correspond to one's type, and go from there.

Type H and M prefer their corresponding jobs and will send the corresponding message to let themselves be identified. However, type L would like to get a type M job. Since messaging has no cost, in a separating equilibrium he would have an incentive to deviate and use the message M.

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

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Type H and M prefer their corresponding jobs and will send the corresponding message to let themselves be identified. However, type L would like to get a type M job. Since messaging has no cost, in a separating equilibrium he would have an incentive to deviate and use the message M.

Step 3: **To formally show that no such PBE exist, look at the PBE where the applicant sending the message H will be given the job H and so forth and show that the firm or one of the applicant types wants to deviate.**

(Each message could be paired with any job, as long as each applicant type sends a different message and are correctly identified, but the argument stays the same).

PS12, Ex. 4.a: Three-type: Fully separating PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

Step 1: According to the firm, what would the ideal separating PBE look like?

The firm prefers to give each candidate the job corresponding to their type.

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Type H and M prefer their corresponding jobs and will send the corresponding message to let themselves be identified. However, type L would like to get a type M job. Since messaging has no cost, in a separating equilibrium he would have an incentive to deviate and use the message M.

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(Each message could be paired with any job, as long as each applicant type sends a different message and are correctly identified, but the argument stays the same).

Since $u_S(a = L) < max[u_S(a = M), u_S(a = H)]$ for all types, no fully separating PBE can exist, as no type will send the message L to indicate type L, thus, letting the firm place one in the low ability position.

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$. What are the firm's beliefs about the applicant if he receives the message $m = H$ or $m = M$ (on the equilibrium path), or if he receives the message $m = L$ (off the equilibrium path)? In each case, solve for the firm's optimal action given its beliefs.

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$.

Step 1: **On the equilibrium path, find R's beliefs when receiving the messages $m = H$, $m = M$ and the corresponding best responses for the receiver.**

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$.

Step 1: On the equilibrium path, find R's beliefs when receiving the messages $m = H$, $m = M$ and the corresponding best responses for the receiver.

In a PBE, the receiver needs to have beliefs corresponding to what happens in the game. This yields the following beliefs and best responses on the equilibrium path:

$$\mu(t = H|m = H) = 1,\ \mu(t = M|m = M) = \mu(t = L|m = M) = \frac{1}{2}$$

$$a^*(m = H) = H,\ a^*(m = M) = M$$

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

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$$a^*(m = H) = H, \ a^*(m = M) = M$$

Step 2: **Off the equilibrium path, find beliefs for $m = L$ and the corresponding best responses for R, which will uphold the PBE, i.e. such that S will not deviate.**

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$.

Step 1: On the equilibrium path, find R's beliefs when receiving the messages $m = H$, $m = M$ and the corresponding best responses for the receiver.

In a PBE, the receiver needs to have beliefs corresponding to what happens in the game. This yields the following beliefs and best responses on the equilibrium path:

$$\mu(t = H|m = H) = 1, \ \mu(t = M|m = M) = \mu(t = L|m = M) = \frac{1}{2}$$

$$a^*(m = H) = H, \ a^*(m = M) = M$$

Step 2: Off the equilibrium path, find beliefs for $m = L$ and the corresponding best responses for R, which will uphold the PBE, i.e. such that S will not deviate.

We covered in (a) that no sender wants to get $a = L$, so find the beliefs which allows for $a(m = L) = L$. Consider $\mu(t = L|m = L) = 1$ where R believes that if someone is to deviate, it's a low type and the best response is $a(m = L) = L$.

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$.

Step 1: On the equilibrium path, find R's beliefs when receiving the messages $m = H$, $m = M$ and the corresponding best responses for the receiver.

In a PBE, the receiver needs to have beliefs corresponding to what happens in the game. This yields the following beliefs and best responses on the equilibrium path:

$$\mu(t = H|m = H) = 1,\ \mu(t = M|m = M) = \mu(t = L|m = M) = \frac{1}{2}$$

$$a^*(m = H) = H,\ a^*(m = M) = M$$

Step 2: Off the equilibrium path, find beliefs for $m = L$ and the corresponding best responses for R, which will uphold the PBE, i.e. such that S will not deviate.

We covered in (a) that no sender wants to get $a = L$, so find the beliefs which allows for $a(m = L) = L$. Consider $\mu(t = L|m = L) = 1$ where R believes that if someone is to deviate, it's a low type and the best response is $a(m = L) = L$.

Step 3: **Write up this partially separating PBE.**

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$.

Step 1: On the equilibrium path, find R's beliefs when receiving the messages $m = H$, $m = M$ and the corresponding best responses for the receiver.

In a PBE, the receiver needs to have beliefs corresponding to what happens in the game. This yields the following beliefs and best responses on the equilibrium path:

$$\mu(t = H|m = H) = 1,\ \mu(t = M|m = M) = \mu(t = L|m = M) = \frac{1}{2}$$
$$a^*(m = H) = H,\ a^*(m = M) = M$$

Step 2: Off the equilibrium path, find beliefs for $m = L$ and the corresponding best responses for R, which will uphold the PBE, i.e. such that S will not deviate.

We covered in (a) that no sender wants to get $a = L$, so find the beliefs which allows for $a(m = L) = L$. Consider $\mu(t = L|m = L) = 1$ where R believes that if someone is to deviate, it's a low type and the best response is $a(m = L) = L$.

Step 3: Write up this partially separating PBE:

$$\{ \underbrace{(H, M, M)}_{m(H),m(M),m(L)}, \underbrace{(H, M, L)}_{a(H),a(M),a(L)}, \underbrace{\mu(H|H) = 1}_{\mu(t|m=H)}, \underbrace{\mu(M|M) = \mu(L|M) = \frac{1}{2}}_{\mu(t|m=M)}, \underbrace{\mu(L|L) = 1}_{\mu(t|m=L)} \}$$

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

Step 1: **Suggest a fully pooling PBE.**

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

Step 1: Suggest a fully pooling PBE:

$$\{ \underbrace{(H, H, H)}_{m(H), m(M), m(L)}, \underbrace{(H, L, L)}_{a(H), a(M), a(L)}, \underbrace{\mu(H|H) = \mu(M|H) = \mu(L|H) = \frac{1}{3}}_{\mu(t|m=H)}, \underbrace{\mu(L|M) = 1}_{\mu(t|m=M)}, \underbrace{\mu(L|L) = 1}_{\mu(t|m=L)} \}$$

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

Step 1: Suggest a fully pooling PBE:

$$\{ \underbrace{(H, H, H)}_{m(H), m(M), m(L)}, \underbrace{(H, L, L)}_{a(H), a(M), a(L)}, \underbrace{\mu(H|H) = \mu(M|H) = \mu(L|H) = \frac{1}{3}}_{\mu(t|m=H)}, \underbrace{\mu(L|M) = 1}_{\mu(t|m=M)}, \underbrace{\mu(L|L) = 1}_{\mu(t|m=L)} \}$$

Step 2: **For the above PBE, explain the messages being send and the responding actions.**

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

Step 1: Suggest a fully pooling PBE:

$$\{\underbrace{(H, H, H)}_{m(H), m(M), m(L)}, \underbrace{(H, L, L)}_{a(H), a(M), a(L)}, \underbrace{\mu(H|H) = \mu(M|H) = \mu(L|H) = \frac{1}{3}}_{\mu(t|m=H)}, \underbrace{\mu(L|M) = 1}_{\mu(t|m=M)}, \underbrace{\mu(L|L) = 1}_{\mu(t|m=L)}\}$$

Step 2: For the above PBE, explain the messages being send and the responding actions:

In this PBE, every type of applicants sends the signal $m = H$. Off the equilibrium path, the receiver believes that anyone who plays $m = M$ or $m = L$ will be of type L.

Step 3: **Does this PBE seem realistic?**

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	$a = H$	$a = M$	$a = M$
$t = H$	3, 3	0, 0	0, 0
$t = M$	1, 0	2, 2	0, 0
$t = L$	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).

Step 1: Suggest a fully pooling PBE:

$$\{ \underbrace{(H, H, H)}_{m(H), m(M), m(L)}, \underbrace{(H, L, L)}_{a(H), a(M), a(L)}, \underbrace{\mu(H|H) = \mu(M|H) = \mu(L|H) = \frac{1}{3}}_{\mu(t|m=H)}, \underbrace{\mu(L|M) = 1}_{\mu(t|m=M)}, \underbrace{\mu(L|L) = 1}_{\mu(t|m=L)} \}$$

Step 2: For the above PBE, explain the messages being send and the responding actions:

In this PBE, every type of applicants sends the signal $m = H$. Off the equilibrium path, the receiver believes that anyone who plays $m = M$ or $m = L$ will be of type L.

Step 3: Does this PBE seem realistic?

No, both type M and L would want something other than $a = H$, thus, the receiver should believe that someone who deviates and sends the signal $m = M$ is type M or L (each with probability $\frac{1}{2}$), and therefore, offer $a(M) = M$ in response which would be a Pareto improvement.