Microeconomics III: Problem Set 8[a]

Thor Donsby Noe (thor.noe@econ.ku.dk) & Christopher Borberg (christopher.borberg@econ.ku.dk)

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Department of Economics, University of Copenhagen

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[a] Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS8, Ex. 1 (A):

PS8, Ex. 2 (A): Bayes Rule

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

PS8, Ex. 3 (A):

PS8, Ex. 4:

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

PS8, Ex. 6: Static public goods game (two-sided incomplete information)

PS8, Ex. 1 (A):

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

Consider the following game G:

		Player 2		
		X	Y	Z
Player 1	A	6, 6	0, 8	0, 0
	B	7, 1	2, 2	1, 1
	C	0, 0	1, 1	4, 5

(a) Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game - exam answer)

- The following is how I personally would write my answer at the exam, it cannot be guaranteed that writing your answer in this way would yield fill points.

(a) Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

Trigger strategies such that the outcome of all stages of the game is (A,X) are possible using respectively B,Y or C,Z as the threats. Since the threats B,Y will make the SPNE possible for the smallest δ, I will use B,Y in the trigger strategies i define:

1. Trigger strategy $p1$: In the 1st turn, play A. In every subsequent turn, if outcome from every previous turn was (A,X), play A, otherwise play B.
2. Trigger strategy $p2$: In the 1st turn, play X. In every subsequent turn, if outcome from every previous turn was (A,X), play X, otherwise play Y.

Player 2 has the highest incentive to deviate, so I only examine player 2's incentive to deviate. In order to find the lowest δ to secure cooperation I set up the inequality for which the payoff for cooperation is higher than the payoff for deviating:

$$\frac{6}{1-\delta} \geq 8 + \frac{2\delta}{1-\delta} \Rightarrow 6 \geq 8 - 8\delta + 2\delta \Rightarrow \quad \delta \geq$$

$\delta = \frac{1}{3}$ is the smallest value for which the strategies constitute a SPNE.

PS8, Ex. 2 (A): Bayes Rule

PS8, Ex. 2 (A): Bayes Rule:

- Review the intuition from the ‘Doctor’ example in lecture 7, and then use Bayes’ rule to solve the following problem:
- A cab was involved in a hit and run accident at night. 85% of the cabs in the city are Green and 15% are Blue. A witness later recalls that the cab was Blue, and we know that this witness’ memory is reliable 80% of the time. the statement from the witness, calculate the probability that the cab involved in the accident was actually Blue.

PS8, Ex. 2 (A):

We wanna find the chance that the car is blue, given that the witness says it's blue. This is the samee as the odds that the car will be blue and the witness says it's blue, divided by the uncontional chance the witness says it blue.

- P(B) The chance that the car is blue: $\frac{15}{100}$
- P(obs B|B): The chance that the witness says the car is blue, given that it is blue: $\frac{80}{100}$
- P(obs B|G): The chance that the witness says the car is blue, given that it is green: $\frac{20}{100}$
- P(obs B): The unconditional chance that the witness says the car is blue, so the amount of chance the witness would look at a blue car and say it's blue, plus the chance the witness would look at a green car and say it's blue: P(obs B|B)*P(B)+P(obs B|G)*P(G)$= \frac{80}{100} * \frac{15}{100} + \frac{20}{100} * \frac{85}{100} = \frac{29}{100}$
- P(B|obs B):$\frac{P(ObsB|B)*P(B)}{P(obsB)} = \frac{\frac{80}{100} * \frac{15}{100}}{\frac{29}{100}} = 0.414$

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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1. The timing is as follows where p is a commonly known distribution:
    1.1 Nature draws all players' type according to p.
    1.2 Each player i learns her own type t_i.
    1.3 Players form their beliefs about the type profile.
    1.4 Players simultaneously choose actions and payoffs are realized.

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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    1.1 Nature draws all players' type according to p.
    1.2 Each player i learns her own type t_i.
    1.3 Players form their beliefs about the type profile.
    1.4 Players simultaneously choose actions and payoffs are realized.
2. The static Bayesian game consists of:
    2.1 Players: $Player\ 1, ..., Player\ n$
    2.2 Type spaces: $T_1 = \{t_{11}, ..., t_{1K}\}, ...$
    2.3 Beliefs: $\mathbb{P}_1[t_2 = t_{21}] = \cdot, ...$
    2.4 Action spaces: $A_1 = \{\cdot\}, ...$
    2.5 Strategy spaces: $S_1 = \{s_1(t_1), \cdot\} = \{(s_1|t_{11}, ..., s_1|t_{1K}), \cdot\}, ...$
    2.6 Type-dependent payoff matrices.

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    2.6 Type-dependent payoff matrices.
3. Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for a player i (the player with the smallest strategy space). For each strategy $s_i(t_i)$:
    3.1 Write up the best-response of the other player(s): $s_j^*(t_j) \equiv BR_j\left(s_i(t_i)|t_j\right)$.
    3.2 If $s_i(t_i) = BR_i\left(s_j(t_j)|t_i\right) \equiv s_i^*(t_i)$ then $\left(s_i^*(t_i), s_j^*(t_j)\right)$ is a BNE.

Recipe for a static Bayesian game (Bayesian Nash Equilibria)

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3. Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for a player i (the player with the smallest strategy space). For each strategy $s_i(t_i)$:
    3.1 Write up the best-response of the other player(s): $s_j^*(t_j) \equiv BR_j(s_i(t_i)|t_j)$.
    3.2 If $s_i(t_i) = BR_i(s_j(t_j)|t_i) \equiv s_i^*(t_i)$ then $\left(s_i^*(t_i), s_j^*(t_j)\right)$ is a BNE.
4. In BNE, a strategy must maximize expected utility given the strategy of the other player(s) and the probability of them being each type, i.e. no type of any player has an incentive to deviate as in equilibrium player i's strategy is a best response to player j's strategy given player i's beliefs:

$$\max_{s_i} \sum_{j \neq i} \sum_{t_{jk} \in T_j} \mathbb{P}_i[t_j = t_{jk}] \cdot u_i\left(s_i(t_i), s_j^*(t_j)|t_i\right)$$

PS8, Ex. 3 (A):

PS8, Ex. 3 (A):

PS8, Ex. 3.a (A):

a a only affects p_2's payoff, but also p_1's strategy. For a=2 p_2 will have R as a dominant strategy, for a=-2 p_2 will have L as a dominant strategy.

b This can be modelled as a Bayesian game since p_2 has two types (he either has L or R as a dominant strategy) and p_1 has a belief about the distribution of these types (Each happen $\frac{1}{2}$ the time.).

Players: (p_1, p_2)

A. sets: $A_1 = (U, D), A_2 = (L, R)$

Type space: $T_1 = (t)$ [one type], $T_2 = (2, -2)$

Beliefs: $p_1(t_2 = 2) = p_1(t_2 = -2) = \frac{1}{2}$

The expected payoff matrix:

		Player 2			
		LL	LR	RL	RR
Player 1	U	2,1	1,-$\frac{1}{2}$	1, $\frac{3}{2}$	0, 0
	D	0, 1	$\frac{1}{2}$, -$\frac{1}{2}$	$\frac{1}{2}$, $\frac{3}{2}$	1, 0

c The BNE is: (U,RL)

PS8, Ex. 4:

PS8, Ex. 4:

Game 1:

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2:

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

PS8, Ex. 4.a:

Game 1 ($p = \frac{1}{2}$):

	L	R
T	1, 1	0, 0
B	0, 0	0, 0

Game 2 ($p = \frac{1}{2}$):

	L	R
T	0, 0	0, 0
B	0, 0	2, 2

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

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Consider the ‘dating game’. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2’s types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2’s type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

[Hints on the next slide.]

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

Consider the ‘dating game’. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2’s types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2’s type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

Hint: Write up the Bayesian game (players, type spaces, beliefs, action spaces, strategy spaces, and the type-dependent payoff matrices.)

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

Hints:

1. Check for equilibria where player 1 plays *Football* and *Opera* respectively.
2. In equilibrium, a strategy should maximize expected payoff given the strategy of the other player and the probability of each type.

PS8, Ex. 5.a: The dating game (Bayesian Nash Equilibria)

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

1. Players: P1, P2.
2. Type spaces: $T_1 = \{t\},\ T_2 = \{t_1, t_2\}$
3. Beliefs: $\mathbb{P}_1(T_2 = t_1) = \mathbb{P}_1(T_2 = t_2) = \frac{1}{2},\ \mathbb{P}_2(T_1 = t) = 1$
4. Action space: $A_i = \{Football, Opera\},$ for $i \in 1, 2$
5. Strategy spaces: $S_1 = \{F, O\},\ S_2 = \{FF, FO, OF, OO\}$
6. Type-dependent payoff matrices:

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*.

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.a: $BR_2(F) = (FO)$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.a: $BR_2(F) = (FO)$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e.

$BNE_1 = \{F, FO\}$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

- 1.a: Write up player 2's best-response.
- 1.b: Does player 1 have an incentive to deviate and play *Opera*?
- 1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

- 2.a: Write up player 2's best-response.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best-response.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

- 1.a: Write up player 2's best-response.
- 1.b: Does player 1 have an incentive to deviate and play *Opera*?
- 1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

- 2.a: Write up player 2's best-response.
- 2.b: Does player 1 have an incentive to deviate and play *Football*?

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

- 1.a: Write up player 2's best-response.
- 1.b: Does player 1 have an incentive to deviate and play *Opera*?
- 1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

- 2.a: Write up player 2's best-response.
- 2.b: Does player 1 have an incentive to deviate and play *Football*?

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2 \text{ plays } FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$
$u_1(O|P2 \text{ plays } FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2 \text{ plays } OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2 \text{ plays } OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best-response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:

1.a: Write up player 2's best-response.

1.b: Does player 1 have an incentive to deviate and play *Opera*?

1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:

2.a: Write up player 2's best-response.

2.b: Does player 1 have an incentive to deviate and play *Football*?

2.c: If no, it's a BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e. $BNE_2 = \{O, OF\}$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 ($p = \frac{1}{2}$)

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 ($p = \frac{1}{2}$)

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:
- 1.a: Write up player 2's best-response.
- 1.b: Does player 1 have an incentive to deviate and play *Opera*?
- 1.c: If no, it's a BNE.

Step 2: Check for a BNE where player 1 plays *Opera*:
- 2.a: Write up player 2's best-response.
- 2.b: Does player 1 have an incentive to deviate and play *Football*?
- 2.c: If no, it's a BNE.

Step 3: Write up the set of all BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$

$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e. $BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e. $BNE_2 = \{O, OF\}$

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

(b) Find the set of (pure strategy) Bayesian Nash Equilibria (BNE) of this game.

Step 1: Check for a BNE where player 1 plays *Football*:
  1.a: Write up player 2's best-response.
  1.b: Does player 1 have an incentive to deviate and play *Opera*?
  1.c: If no, it's a BNE.
Step 2: Check for a BNE where player 1 plays *Opera*:
  2.a: Write up player 2's best-response.
  2.b: Does player 1 have an incentive to deviate and play *Football*?
  2.c: If no, it's a BNE.
Step 3: Write up the set of all BNE.

1.a: $BR_2(F) = (FO)$

1.b: $u_1(F|P2\ plays\ FO) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 2$
$u_1(O|P2\ plays\ FO) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

1.c: No incentive to deviate, i.e.
$BNE_1 = \{F, FO\}$

2.a: $BR_2(O) = (OF)$

2.b: $u_1(O|P2\ plays\ OF) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$
$u_1(F|P2\ plays\ OF) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$

2.c: No incentive to deviate, i.e.
$BNE_2 = \{O, OF\}$

3: $BNE = \{(F, FO), (O, OF)\}$

PS8, Ex. 6: Static public goods game (two-sided incomplete information)

PS8, Ex. 6: Static public goods game (two-sided incomplete information)

Difficult. Consider the public goods game from lecture 7 (slides 34-40):

	W	D
W	$1-c_1, 1-c_2$	$1-c_1, 1$
D	$1, 1-c_2$	$0, 0$

Suppose now instead that there is two-sided incomplete information. In particular, the cost of writing the reference is uniformly distributed between 0 and 2:

$$c_i \sim u(0,2) \text{ for } i = 1,2$$

In this setting, we can show that the players optimally follow a 'cutoff' strategy. Thus, the equilibrium strategies take the form

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases}$$

$$s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that

$$1 - c_i^* = z_{-i}^* \qquad (1)$$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*. *Hint* : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

(c) Use the result from (b) together with equation (1) to find (c_1^*, c_2^*).

(d) What's the probability of underinvestment (i.e. that nobody writes the reference)? What's the probability of overinvestment (i.e. that both write the reference)?

PS8, Ex. 6.a: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that $1 - c_i^* = z_{-i}^* \quad (1)$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

Step 1: Write up the expected payoffs from playing W and D respectively.

PS8, Ex. 6.a: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that $1 - c_i^* = z_{-i}^* \quad (1)$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

Step 1: Write up the expected payoffs from playing W and D respectively:

$$E[s_i = W] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot (1 - c_i) + \mathbb{P}\left[s_{-i}^* = D\right] \cdot (1 - c_i) = 1 - c_i$$

$$E[s_i = D] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot 1 + \mathbb{P}\left[s_{-i}^* = D\right] \cdot 0 = \mathbb{P}\left[s_{-i}^* = W\right] = z_{-i}^*$$

PS8, Ex. 6.a: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \text{Write} & \text{if} \quad c_1 \leq c_1^* \\ \text{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \text{Write} & \text{if} \quad c_2 \leq c_2^* \\ \text{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that $1 - c_i^* = z_{-i}^* \quad (1)$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

Step 1: Write up the expected payoffs from playing W and D respectively:

$$E[s_i = W] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot (1 - c_i) + \mathbb{P}\left[s_{-i}^* = D\right] \cdot (1 - c_i) = 1 - c_i$$
$$E[s_i = D] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot 1 + \mathbb{P}\left[s_{-i}^* = D\right] \cdot 0 = \mathbb{P}\left[s_{-i}^* = W\right] = z_{-i}^*$$

Step 2: Use this to argue that equation (1) holds.

Hint: the 'cutoff' value c_i^* is where player i is indifferent between W and D.

PS8, Ex. 6.a: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that $1 - c_i^* = z_{-i}^* \quad (1)$

Hint: Calculate *i*'s expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

Step 1: Write up the expected payoffs from playing W and D respectively:

$$E[u_i|s_i = W] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot (1 - c_i) + \mathbb{P}\left[s_{-i}^* = D\right] \cdot (1 - c_i) = 1 - c_i$$

$$E[u_i|s_i = D] = \mathbb{P}\left[s_{-i}^* = W\right] \cdot 1 + \mathbb{P}\left[s_{-i}^* = D\right] \cdot 0 = \mathbb{P}\left[s_{-i}^* = W\right] = z_{-i}^*$$

Step 2: Use this to argue that equation (1) holds.

Hint: the 'cutoff' value c_i^* is where player i is indifferent between W and D.

For $c_i = c_i^*$ player *i*'s expected payoff is the same regardless of strategy, i.e.

$$E[u_i|s_i = W] = E[u_i|s_i = D] \Rightarrow$$

$$1 - c_i^* = z_{-i}^*$$

Q.E.D.

PS8, Ex. 6.b: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*.

Hint : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

PS8, Ex. 6.b: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*.

Hint : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

Step 1: Knowing that z_i^* is the probability that the other player plays W in equilibrium, write up z_i as the probability player i herself plays W in equilibrium.

PS8, Ex. 6.b: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*.

Hint : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

Step 1: Knowing that z_i^* is the probability that the other player plays W in equilibrium, write up z_i as the probability player i herself plays W in equilibrium.

1. $z_i^* = \mathbb{P}[s_i^* = write] = \mathbb{P}[c_i \leq c_i^*]$

PS8, Ex. 6.b: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*.

Hint : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

Step 1: Knowing that z_i^* is the probability that the other player plays W in equilibrium, write up z_i as the probability player i herself plays W in equilibrium.

Step 2: Use that c_i is uniformly distributed $c_i \sim u(0,2)$.

1. $z_i^* = \mathbb{P}[s_i^* = \textit{write}] = \mathbb{P}[c_i \leq c_i^*]$

PS8, Ex. 6.b: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*.

Hint : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

Step 1: Knowing that z_i^* is the probability that the other player plays W in equilibrium, write up z_i as the probability player i herself plays W in equilibrium.

Step 2: Use that c_i is uniformly distributed $c_i \sim u(0,2)$.

1. $z_i^* = \mathbb{P}[s_i^* = write] = \mathbb{P}[c_i \leq c_i^*]$
2. $z_i^* = \mathbb{P}[c_i \leq c_i^*] = \frac{c_i^*}{2}$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Step 2: Substitute in z_i^* from (b).

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player *i* instead of $-i$.

Step 2: Substitute in z_i^* from (b).

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = \textit{write}] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = \textit{write}] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$
2. $\frac{c_i^*}{2} = 1 - c_{-i}^*$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Step 2: Substitute in z_i^* from (b).

Step 3: Use symmetry in the distribution of the costs to find the cutoff value c_i^*.

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$
2. $\frac{c_i^*}{2} = 1 - c_{-i}^*$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Step 2: Substitute in z_i^* from (b).

Step 3: Use symmetry in the distribution of the costs to find the cutoff value c_i^*.

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$
2. $\frac{c_i^*}{2} = 1 - c_{-i}^*$
3. Due to symmetry, they must have the same 'cutoff' value c_i^*:

$$\begin{aligned} \frac{c_i^*}{2} &= 1 - c_i^* \\ c_i^* &= 2 - 2c_i^* \\ c_i^* &= \frac{2}{3} \end{aligned}$$

PS8, Ex. 6.c: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(c) Use the result from question (b) together with equation (1) to find (c_1^*, c_2^*).

Step 1: Though the drawn costs might differ, take advantage of the symmetry in the distribution of the costs to write up equation (1) for player i instead of $-i$.

Step 2: Substitute in z_i^* from (b).

Step 3: Use symmetry in the distribution of the costs to find the cutoff value c_i^*.

Information so far:

(a) $z_{-i}^* = \mathbb{P}[s_{-i}^* = write] = 1 - c_i^* \quad (1)$

(b) $z_i^* = \mathbb{P}[s_i^* = write] = \frac{c_i^*}{2}$

1. $z_i^* = 1 - c_{-i}^*$
2. $\frac{c_i^*}{2} = 1 - c_{-i}^*$
3. Due to symmetry, they must have the same 'cutoff' value c_i^*:
$$\begin{aligned} \frac{c_i^*}{2} &= 1 - c_i^* \\ c_i^* &= 2 - 2c_i^* \\ c_i^* &= \frac{2}{3} \end{aligned}$$
4. Hence, $(c_1^*, c_2^*) = \left(\frac{2}{3}, \frac{2}{3}\right)$

PS8, Ex. 6.d: Static public goods game (two-sided incomplete information)

$$s_1^*(c_1) = \begin{cases} \mathit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \mathit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases} \qquad s_2^*(c_2) = \begin{cases} \mathit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \mathit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

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$$= (1 - z^*)^2 = (1 - (1 - c^*))^2 = \left(1 - \left(1 - \frac{2}{3}\right)\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\mathbb{P}[Both\ write] = (z^*)^2 = (1 - c^*)^2 = \left(1 - \frac{2}{3}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$