Microeconomics III: Problem Set 6[a]

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[a] Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

Kahoot!

PS6, Ex. 1 (A):

PS6, Ex. 2 (A):

PS6, Ex. 3: Dynamic games (imperfect information)

PS6, Ex. 4: The Mutated Seabass (imperfect information)

PS6, Ex. 5:

PS6, Ex. 6:

PS6, Ex. 7: To keep or split (imperfect information)

Code examples

Kahoot!

Kahoot: A exercises

Form a group for each table:

- Get prepared to answer the A exercises as a team (5 min).

PS6, Ex. 1 (A):

PS6, Ex. 1 (A):

PS6, Ex. 1 (A):

PS6, Ex. 2 (A):

PS6, Ex. 2 (A):

PS6, Ex. 2.a (A):

PS6, Ex. 3: Dynamic games (imperfect information)

PS6, Ex. 3: Dynamic games (imperfect information)

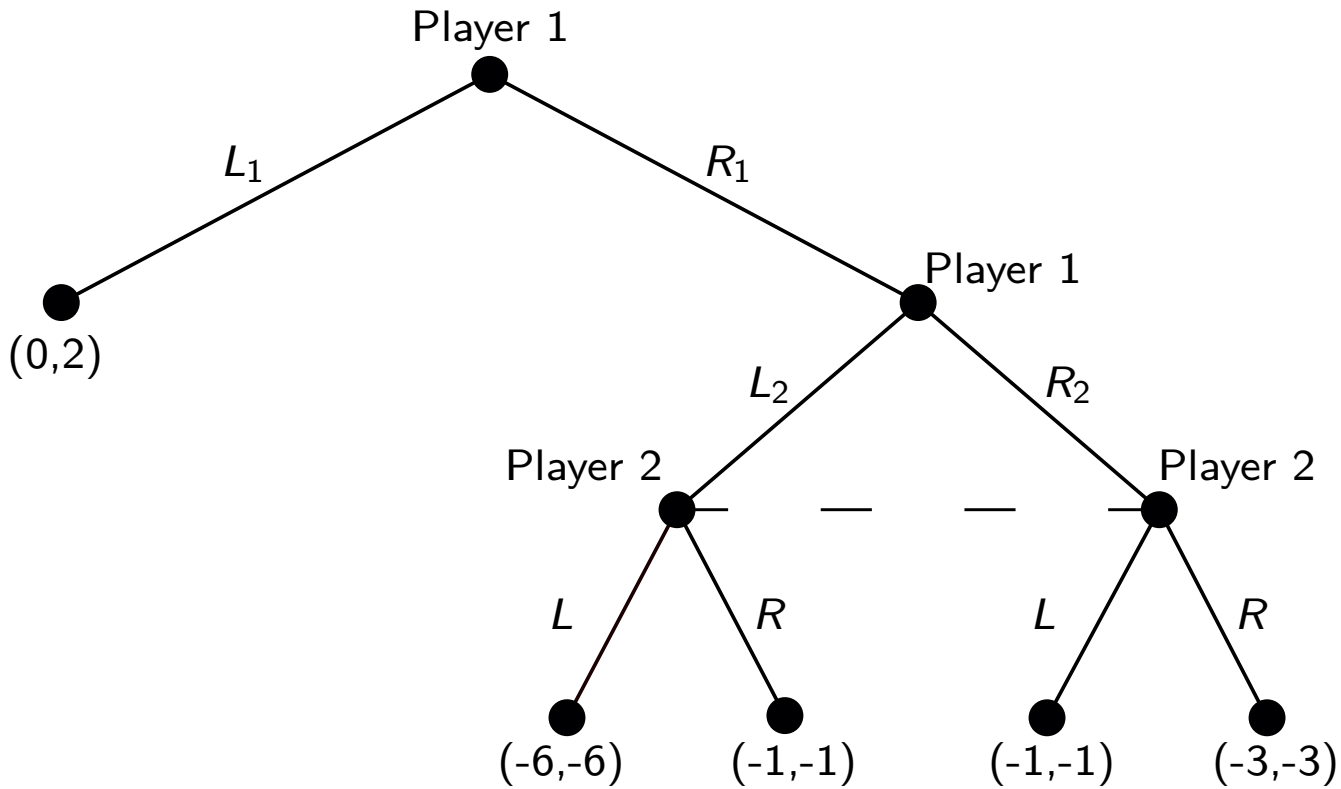
Find the SPNE in the four games.

Hints:

1. It becomes much easier to grasp dynamic games with imperfect information if you write the part with imperfect information in normal form (bi-matrix).
2. Be careful to cover all of the strategy profile (in every subgame!) when writing up the subgame perfect Nash Equilibria (SPNE).

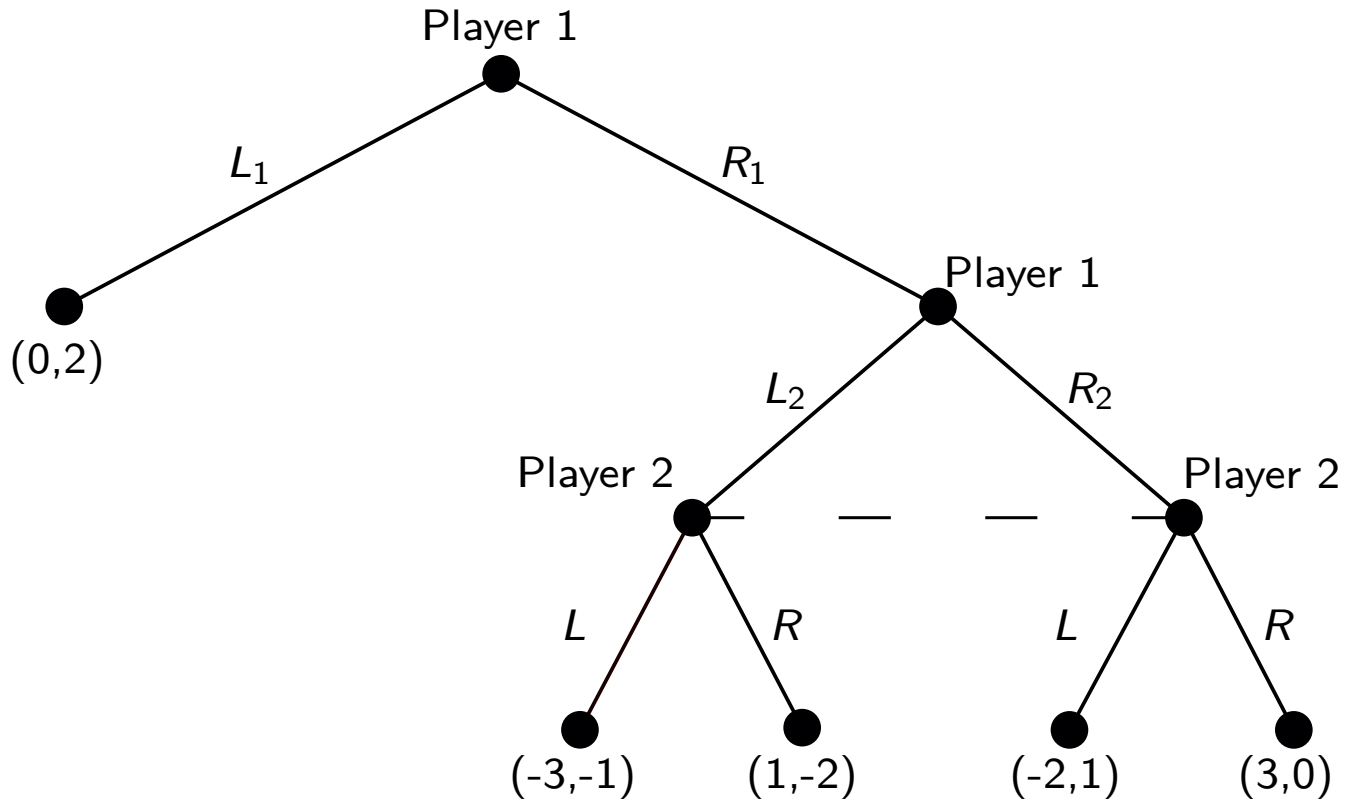
PS6, Ex. 3.a: Dynamic games (imperfect information)

(a) Find the SPNE in the following game:



PS6, Ex. 3.a: Dynamic games (imperfect information)

(a) Find the SPNE in the following game:

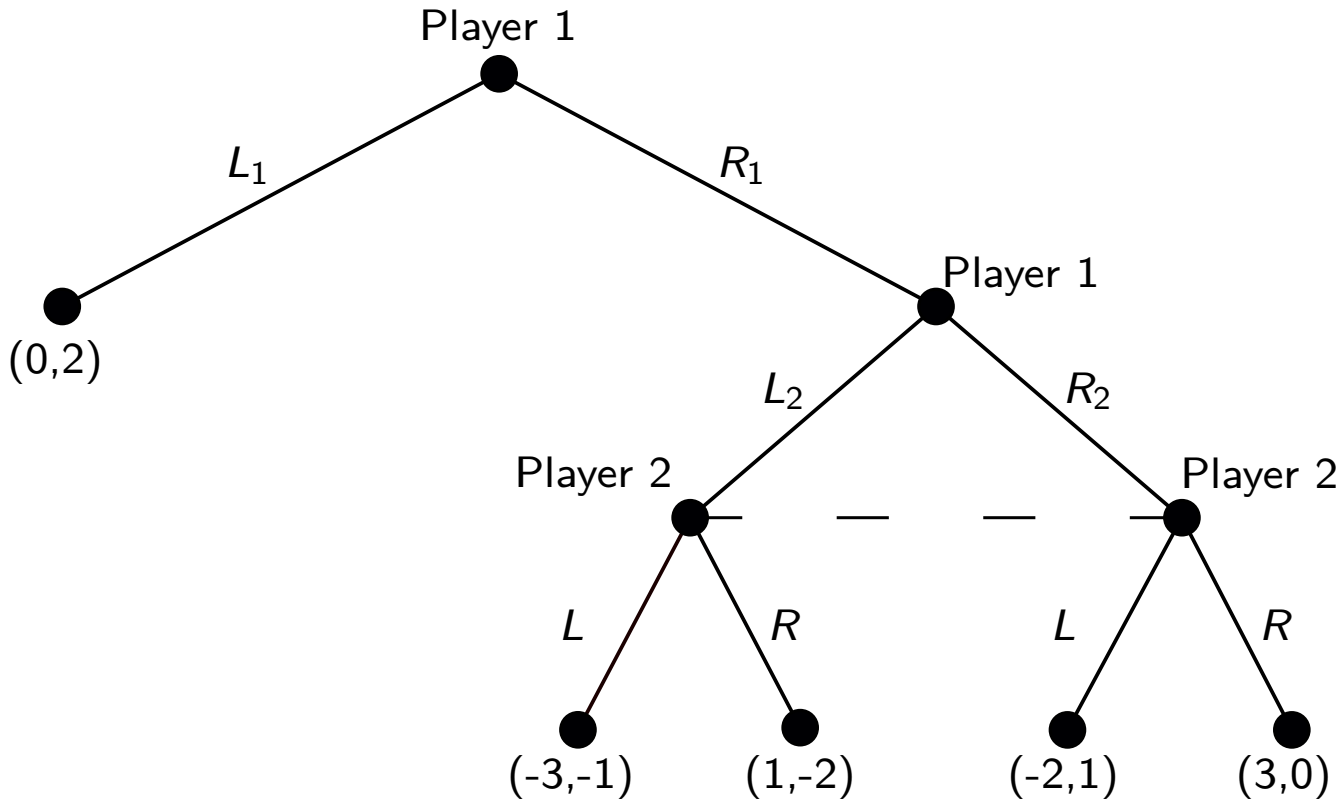


2nd and 3rd stage in normal form:

Player 1		Player 2: L	Player 2: R
	L_2	-3, -1	1, -2
	R_2	-2, 1	3, 0

PS6, Ex. 3.a: Dynamic games (imperfect information)

(a) Find the SPNE in the following game:



2nd and 3rd stage in normal form:

Player 1		Player 2: L	Player 2: R
	L_2	-3, -1	1, -2
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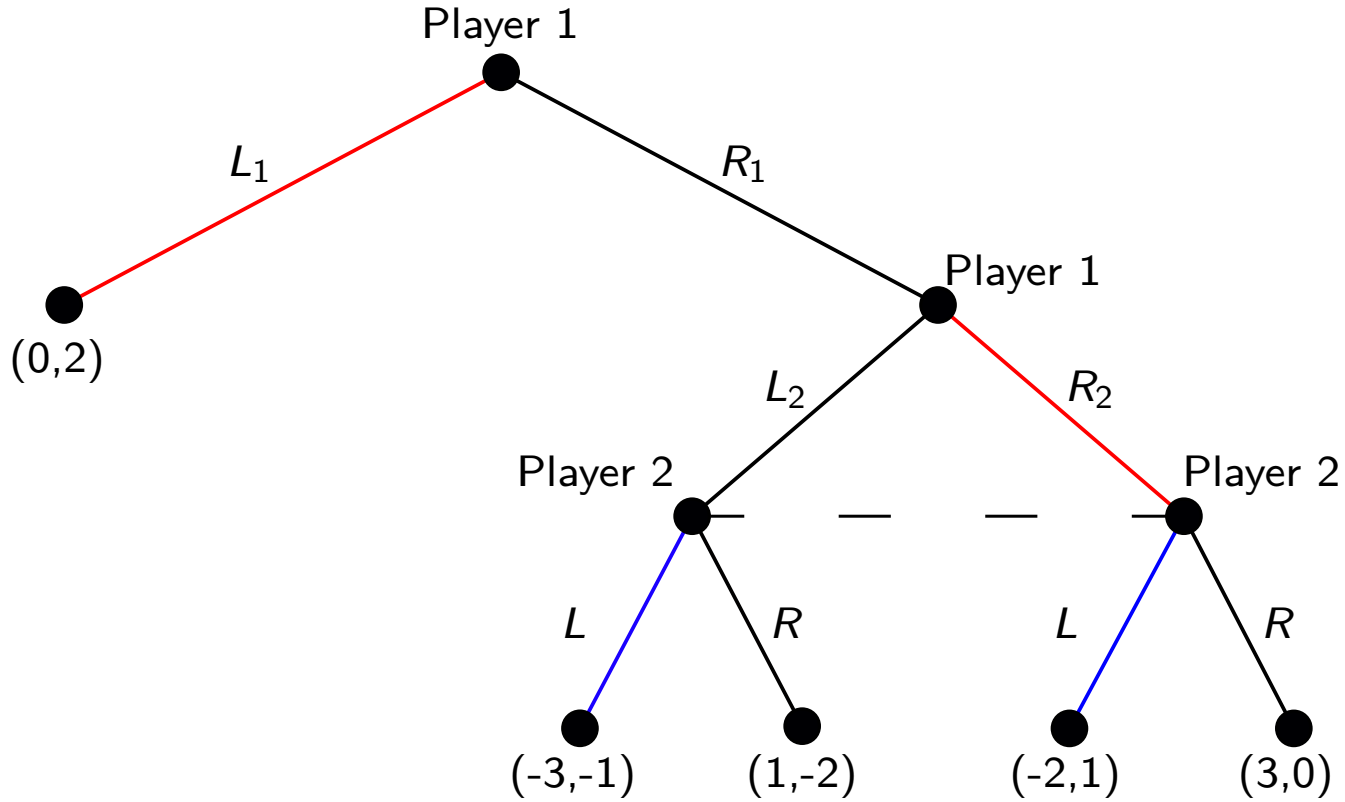
PS6, Ex. 3.a: Dynamic games (imperfect information)

(a) Find the SPNE in the following game:



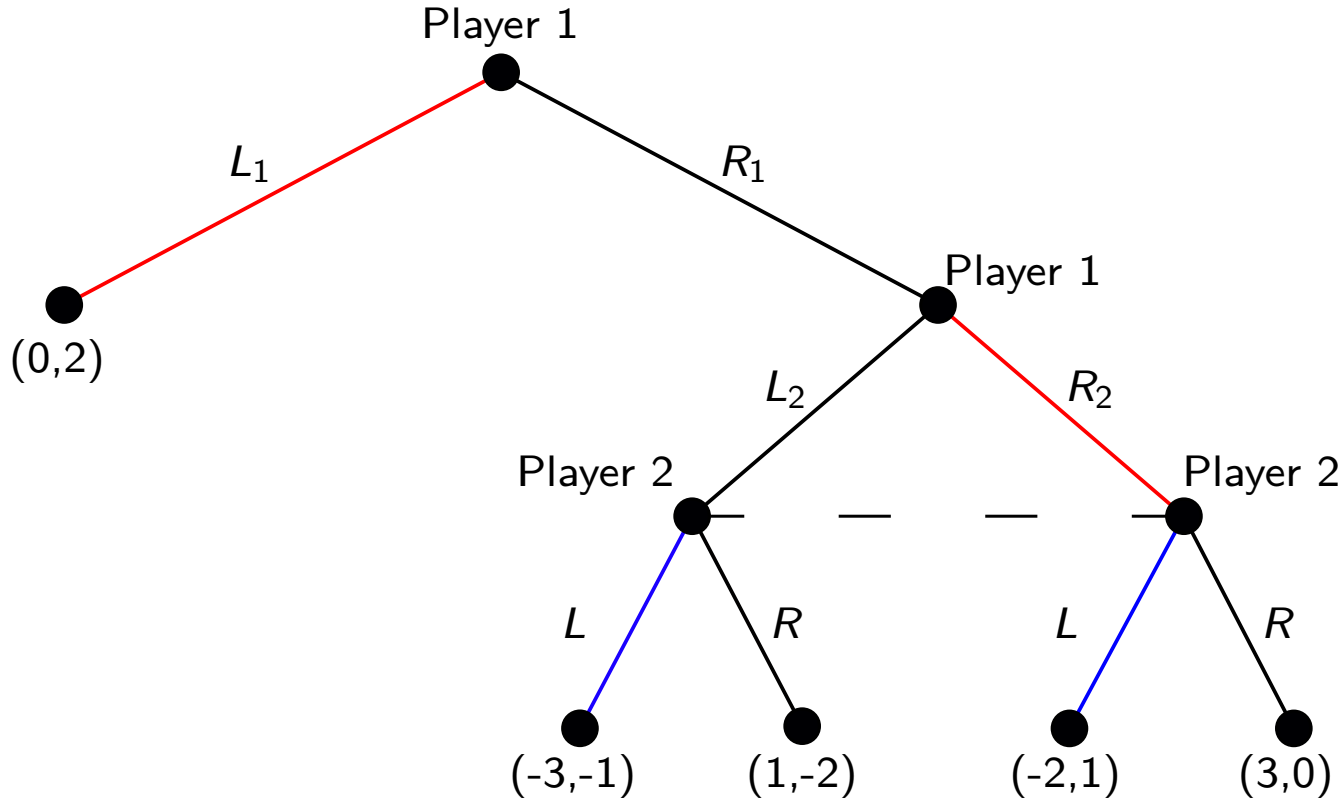
2nd and 3rd stage in normal form:

Player 1		Player 2: L	Player 2: R
	L_2	-3, -1	1, -2
	R_2	-2, 1	3, 0

Write up the SPNE!

PS6, Ex. 3.a: Dynamic games (imperfect information)

(a) Find the SPNE in the following game:



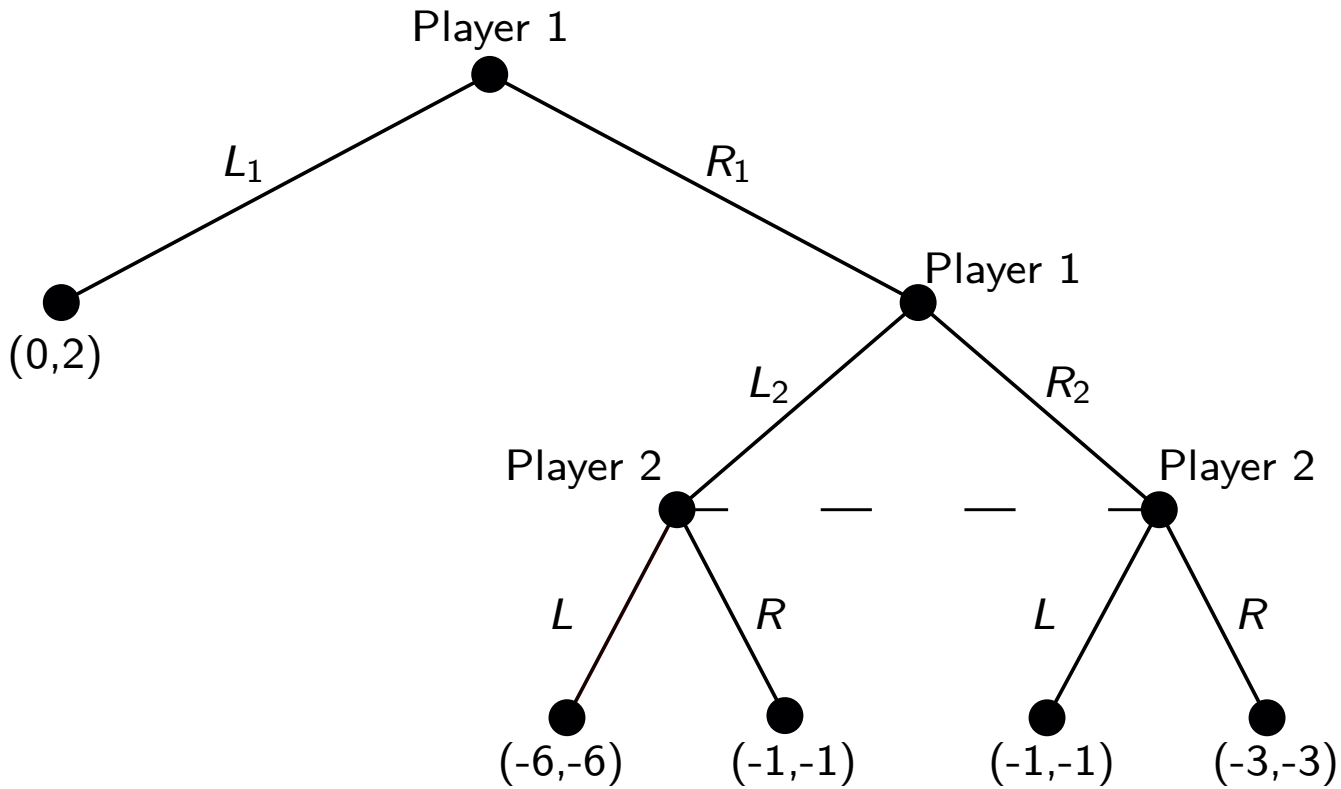
2nd and 3rd stage in normal form:

Player 1		Player 2: L	Player 2: R
	L_2	-3, -1	1, -2
	R_2	-2, 1	3, 0

$SPNE = \{s_1^*, s_2^*\} = \{(L_1, R_2), L\}$ with outcome (0,2).

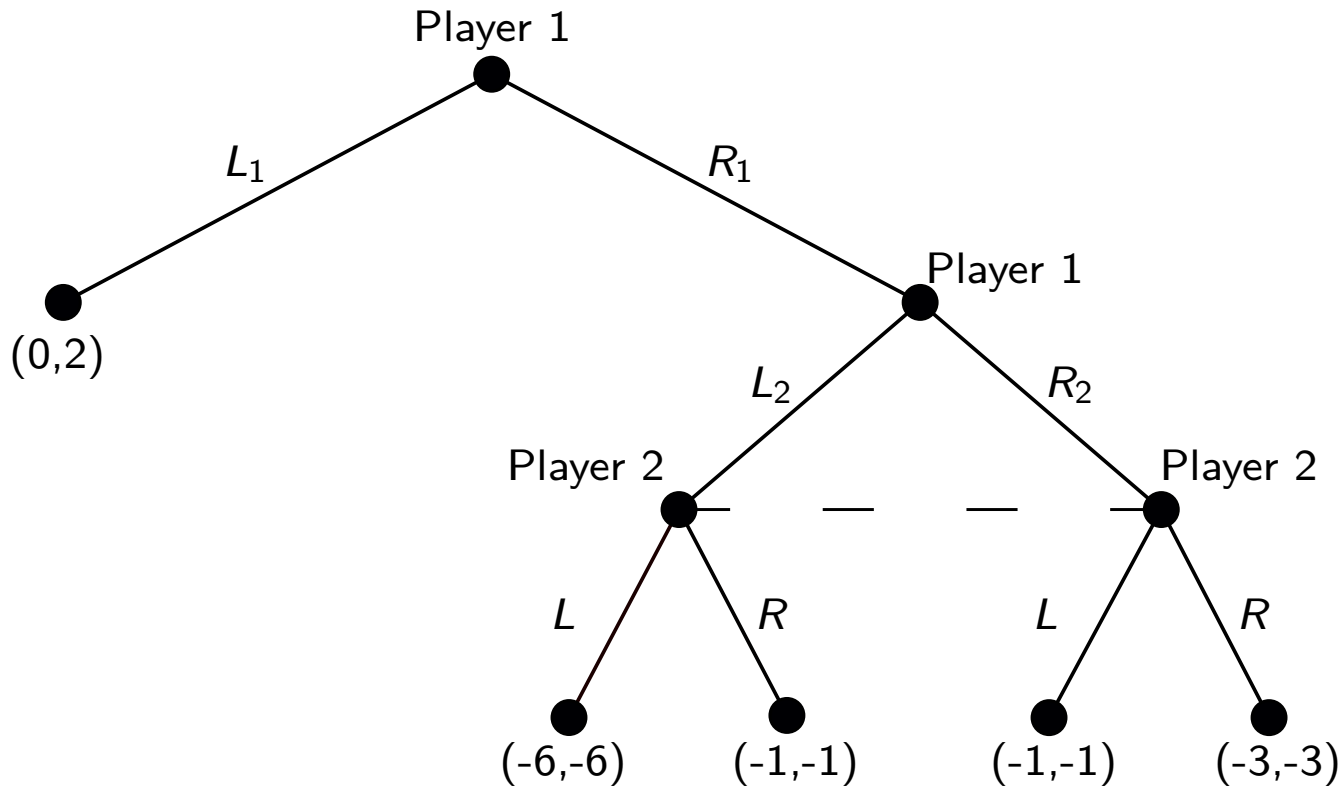
PS6, Ex. 3.b: Dynamic games (imperfect information)

(b) Find the SPNE in the following game:



PS6, Ex. 3.b: Dynamic games (imperfect information)

(b) Find the SPNE in the following game:

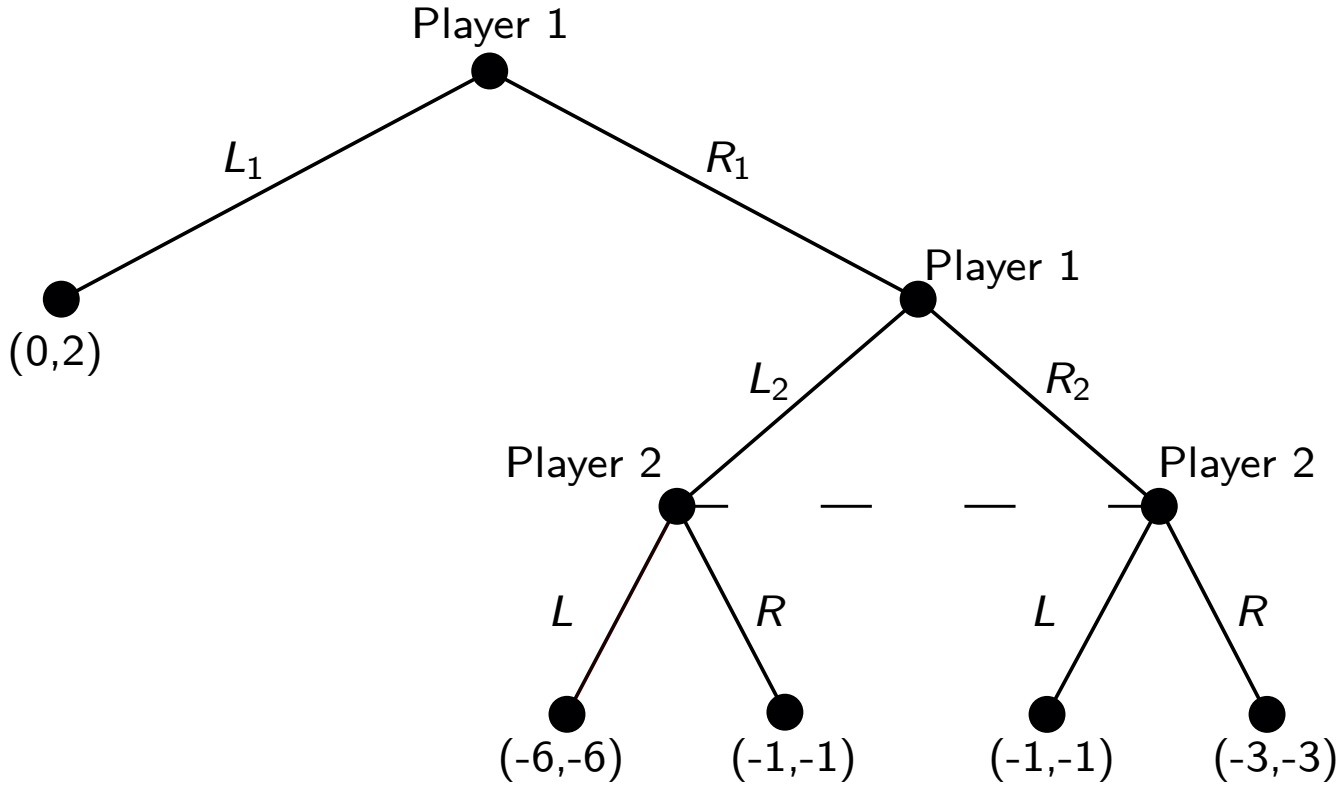


2nd and 3rd stage in normal form:

		Player 2	
		L	R
Player 1	L_2	-6, -6	-1, -1
	R_2	-1, -1	-3, -3

PS6, Ex. 3.b: Dynamic games (imperfect information)

(b) Find the SPNE in the following game:



2^{nd} and 3^{rd} stage in normal form:

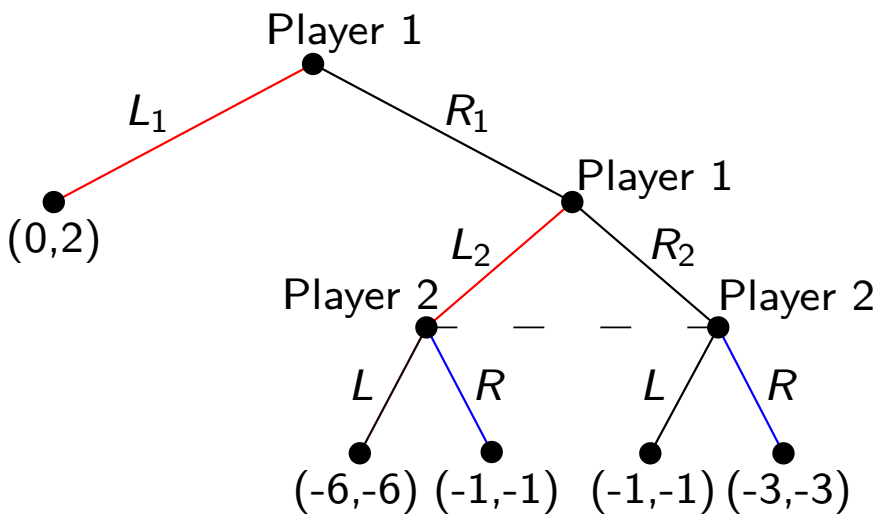
		Player 2: L	Player 2: R
Player 1	L_2	-6, -6	-1, -1
Player 1	R_2	-1, -1	-3, -3

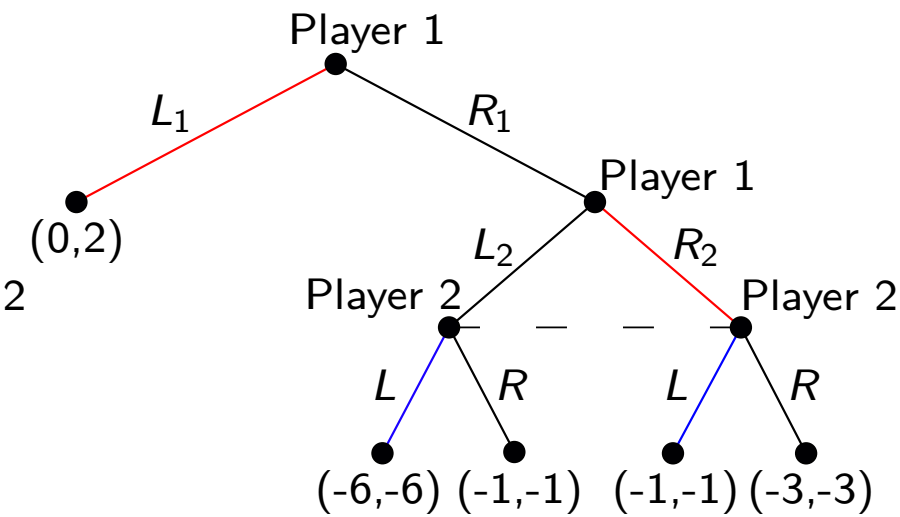
Two different pure strategy NE (PSNE) in the subgame. What now?

PS6, Ex. 3.b: Dynamic games (imperfect information)

(b) Find the SPNE in the following game:

R_1 is strictly dominated by L_1 and we have two subgame perfect solutions:



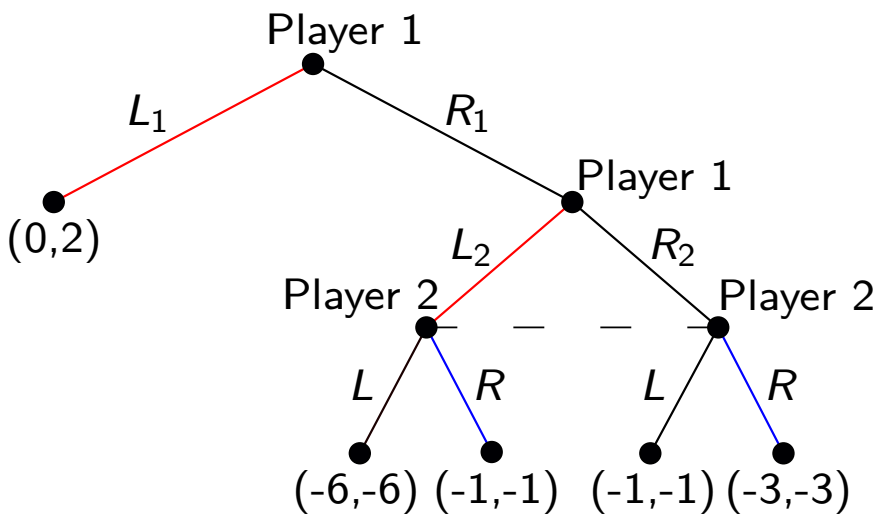


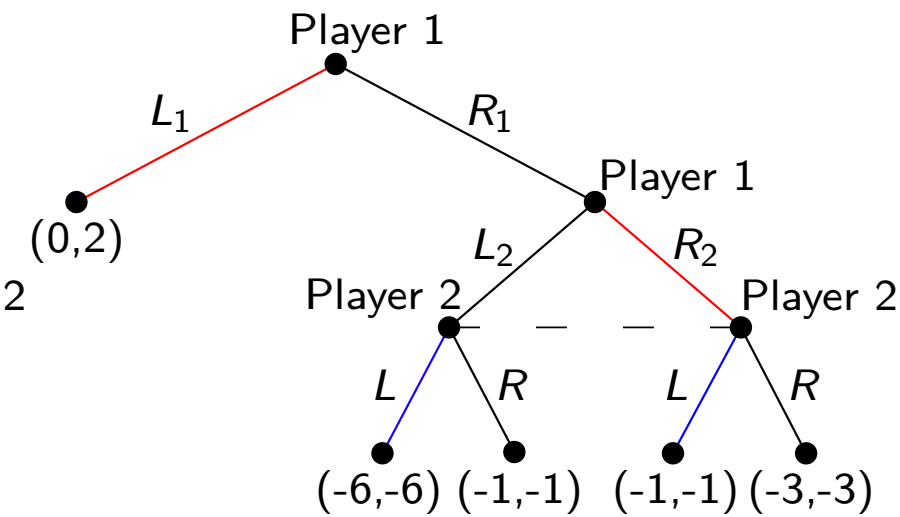
Write up the SPNE!

PS6, Ex. 3.b: Dynamic games (imperfect information)

(b) Find the SPNE in the following game:

R_1 is strictly dominated by L_1 and we have two subgame perfect solutions:

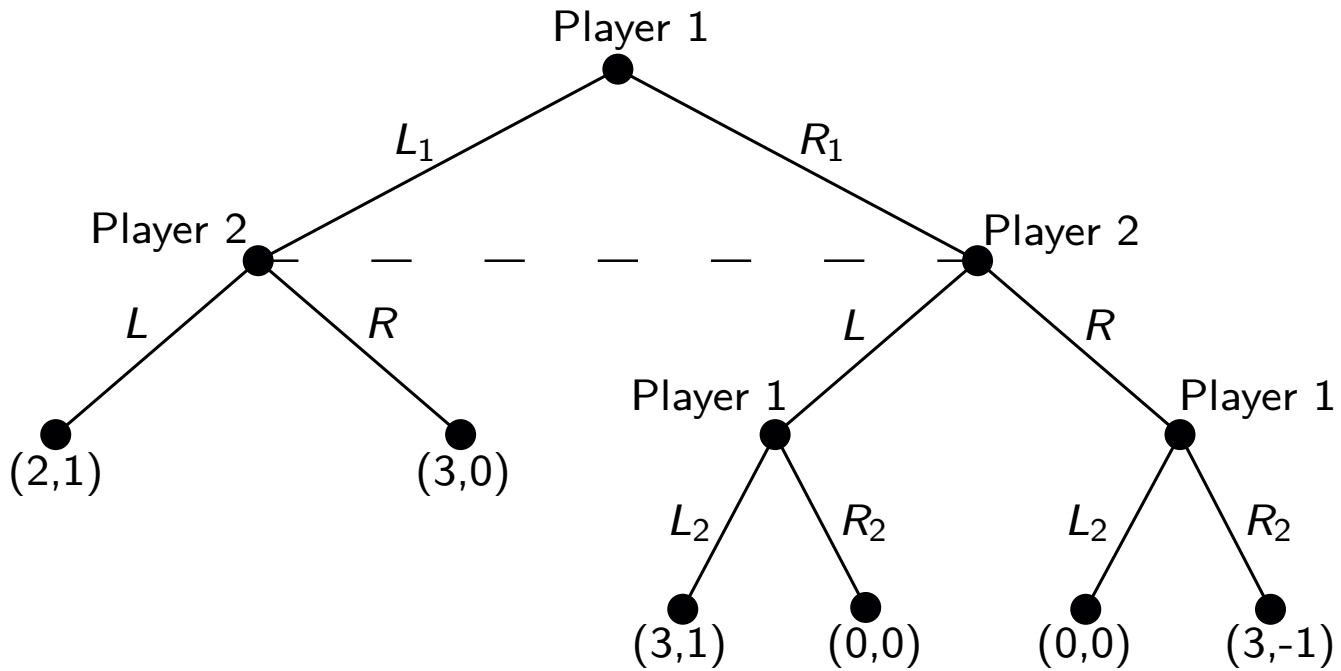




$SPNE = \{s_1^*, s_2^*\} = \{(L_1, L_2), R; (L_1, R_2), L\}$ both with outcome (0,2).

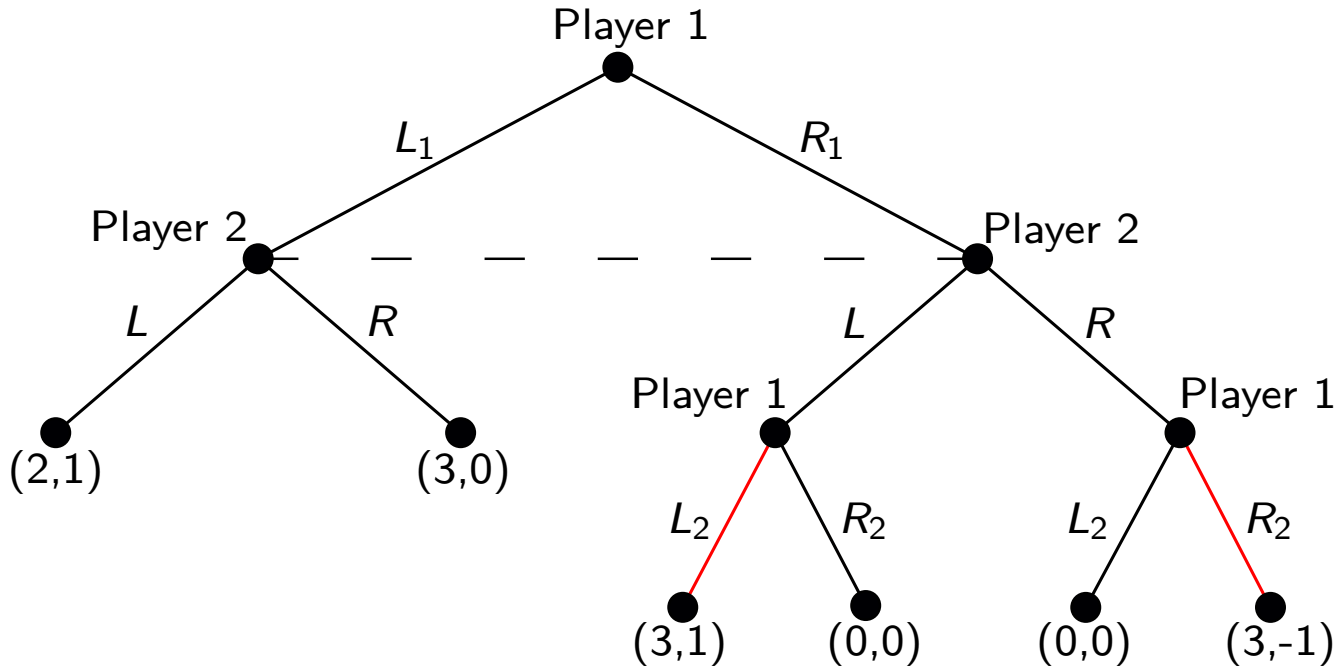
PS6, Ex. 3.c: Dynamic games (imperfect information)

(c) Find the SPNE in the following game:



PS6, Ex. 3.c: Dynamic games (imperfect information)

(c) Find the SPNE in the following game:

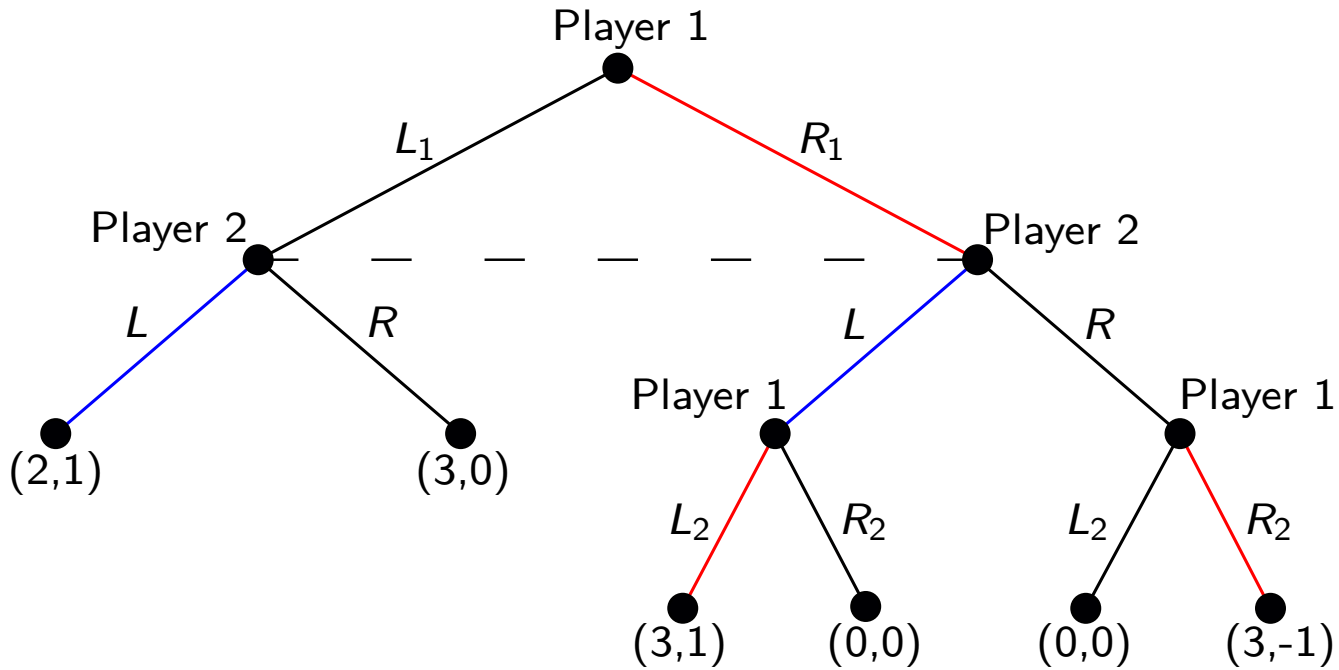


1st and 2nd stage in normal form (taking the 3rd stage as given):

Player 1		Player 2: L	Player 2: R
	L_1	2, 1	3, 0
	R_1	3, 1	3, -1

PS6, Ex. 3.c: Dynamic games (imperfect information)

(c) Find the SPNE in the following game:



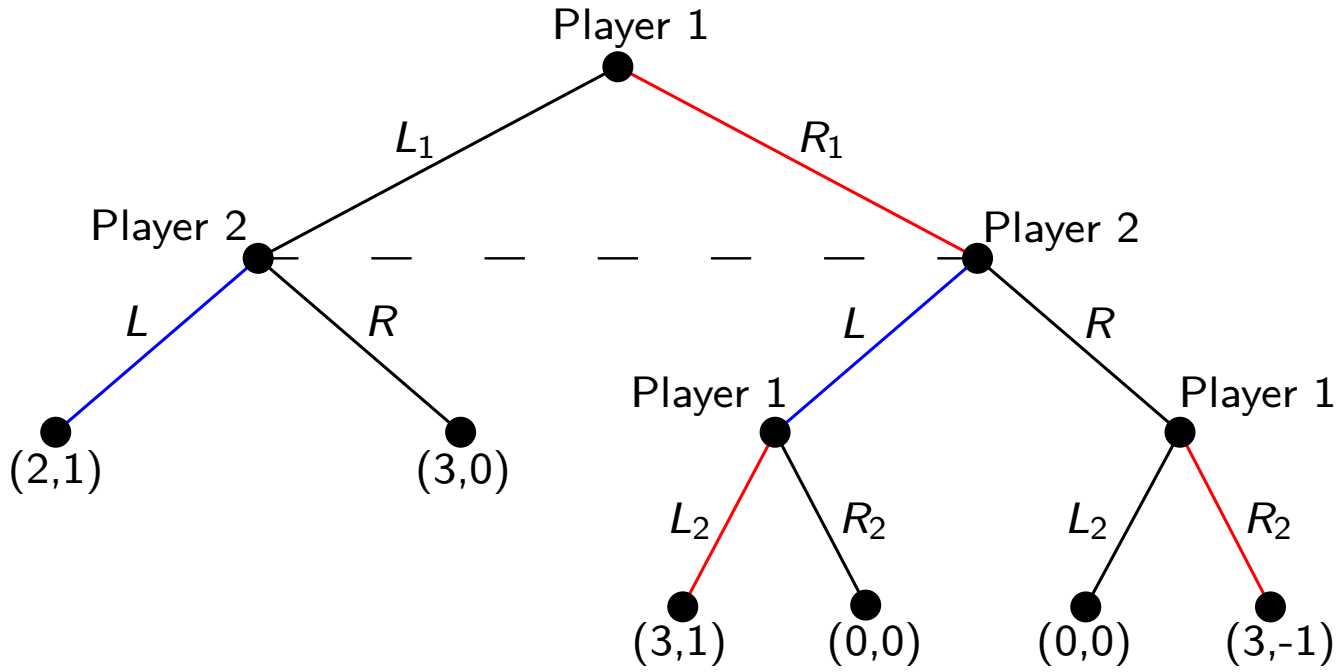
1st and 2nd stage in normal form (taking the 3rd stage as given):

Player 1		Player 2: L	Player 2: R
	L_1	2, 1	3, 0
	R_1	3, 1	3, -1

Consider how many subgames there are and write up the SPNE.

PS6, Ex. 3.c: Dynamic games (imperfect information)

(c) Find the SPNE in the following game:



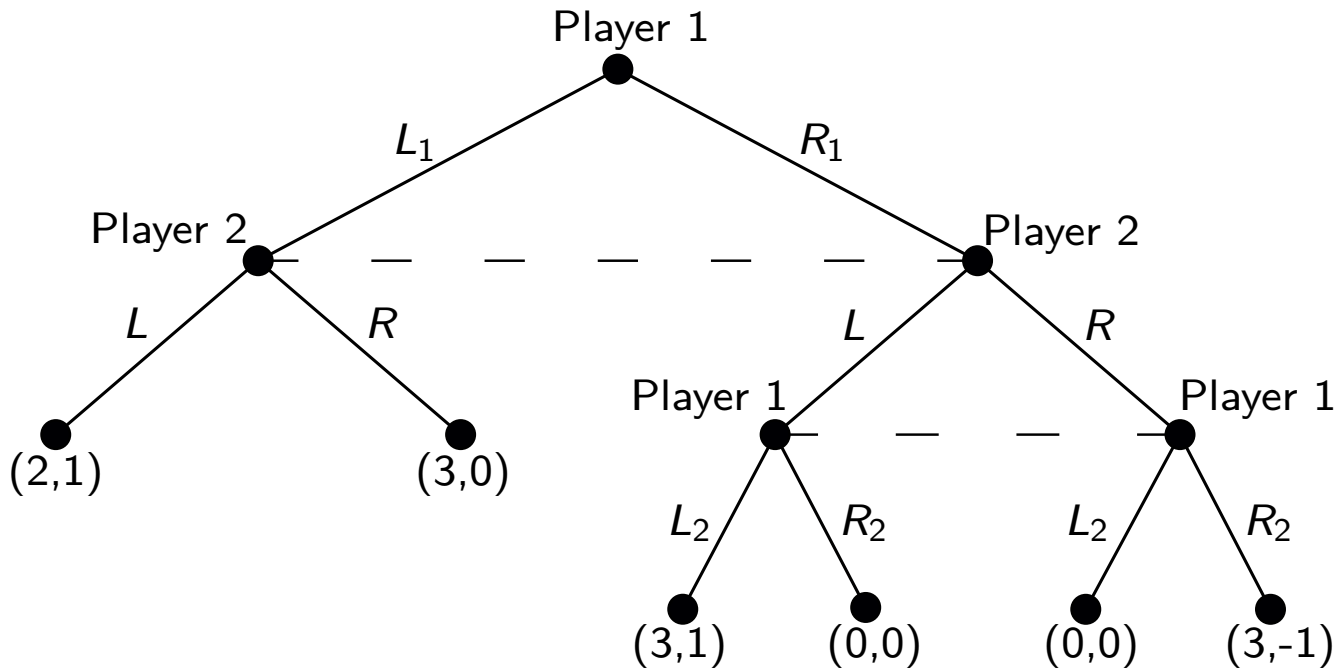
1st and 2nd stage in normal form (taking the 3rd stage as given):

Player 1	Player 2: L	Player 2: R
L_1	2, 1	3, 0
R_1	3, 1	3, -1

$SPNE = \{s_1^*, s_2^*\} = \{(R_1, L_2, R_2), L\}$ with outcome (3,1).

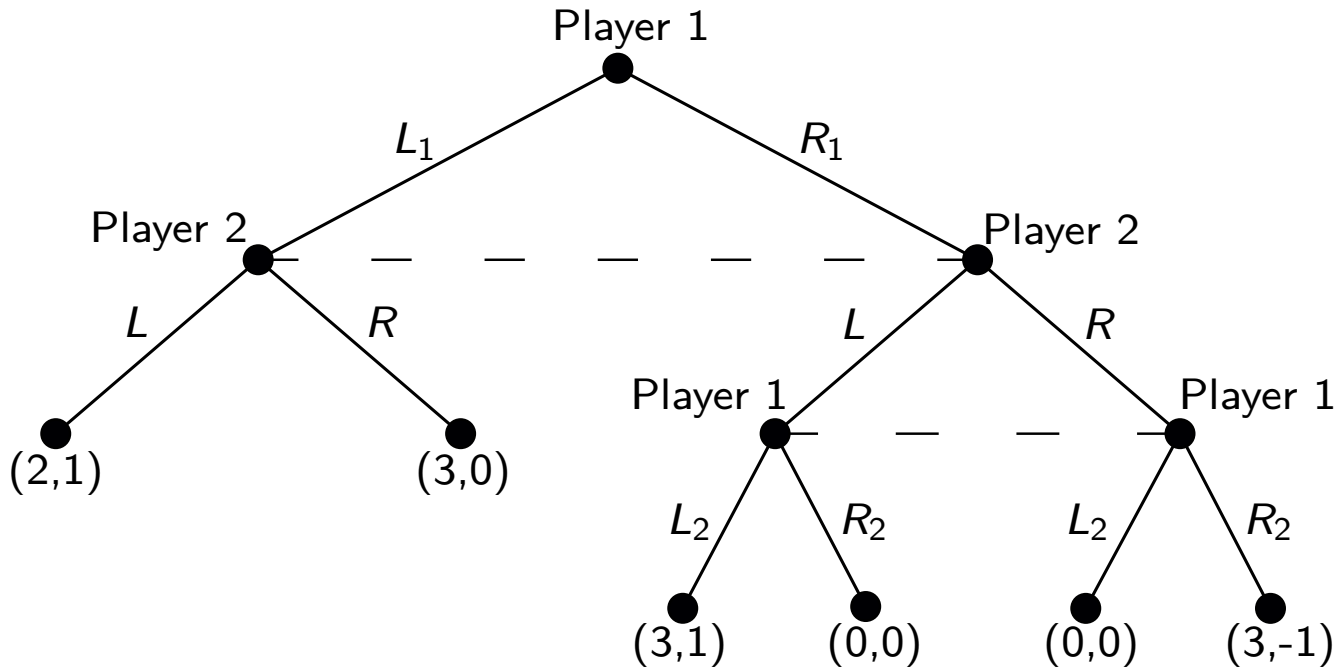
PS6, Ex. 3.d: Dynamic games (imperfect information)

(d) Find the SPNE in the following game:

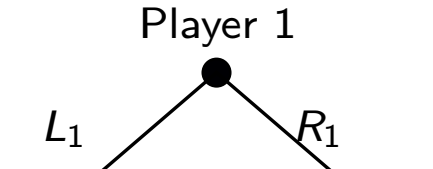


PS6, Ex. 3.d: Dynamic games (imperfect information)

(d) Find the SPNE in the following game:



2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):

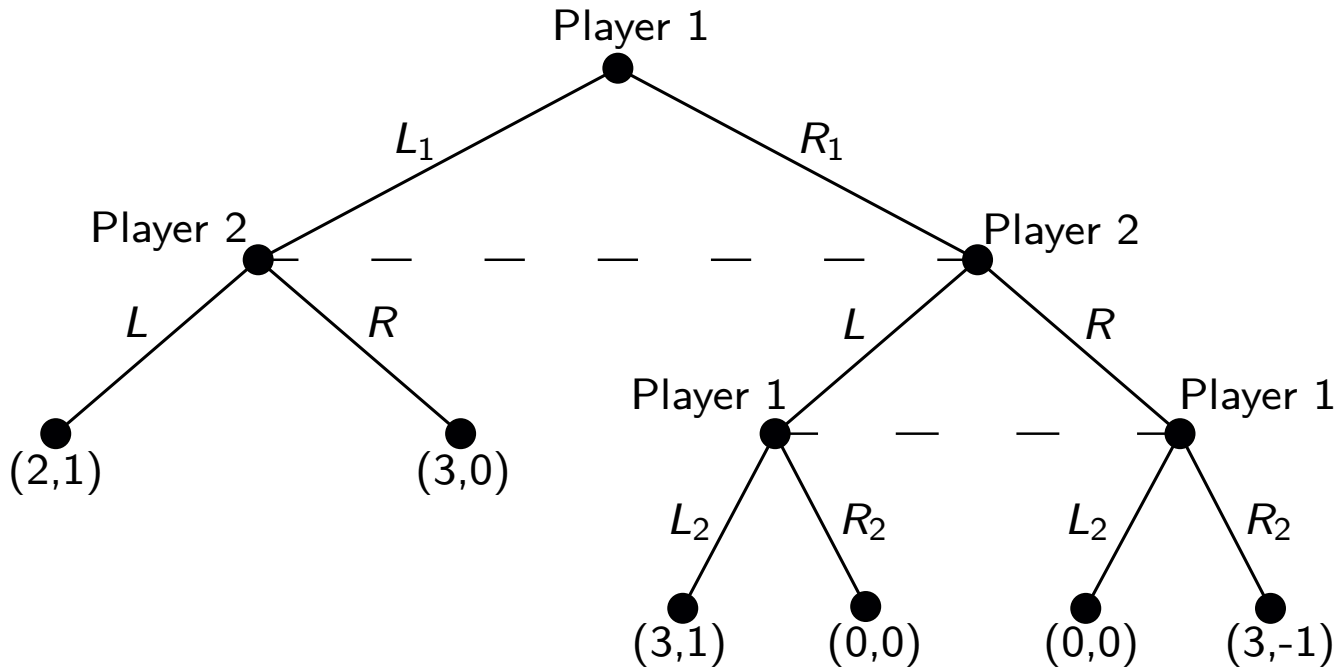


	L	R
L_2	2, 1	3, 0
R_2	2, 1	3, 0

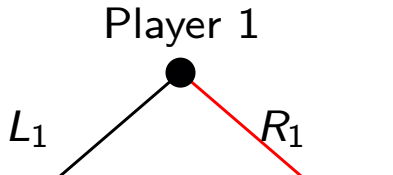
	L	R
L_2	3, 1	0, 0
R_2	0, 0	3, -1

PS6, Ex. 3.d: Dynamic games (imperfect information)

(d) Find the SPNE in the following game:



2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):

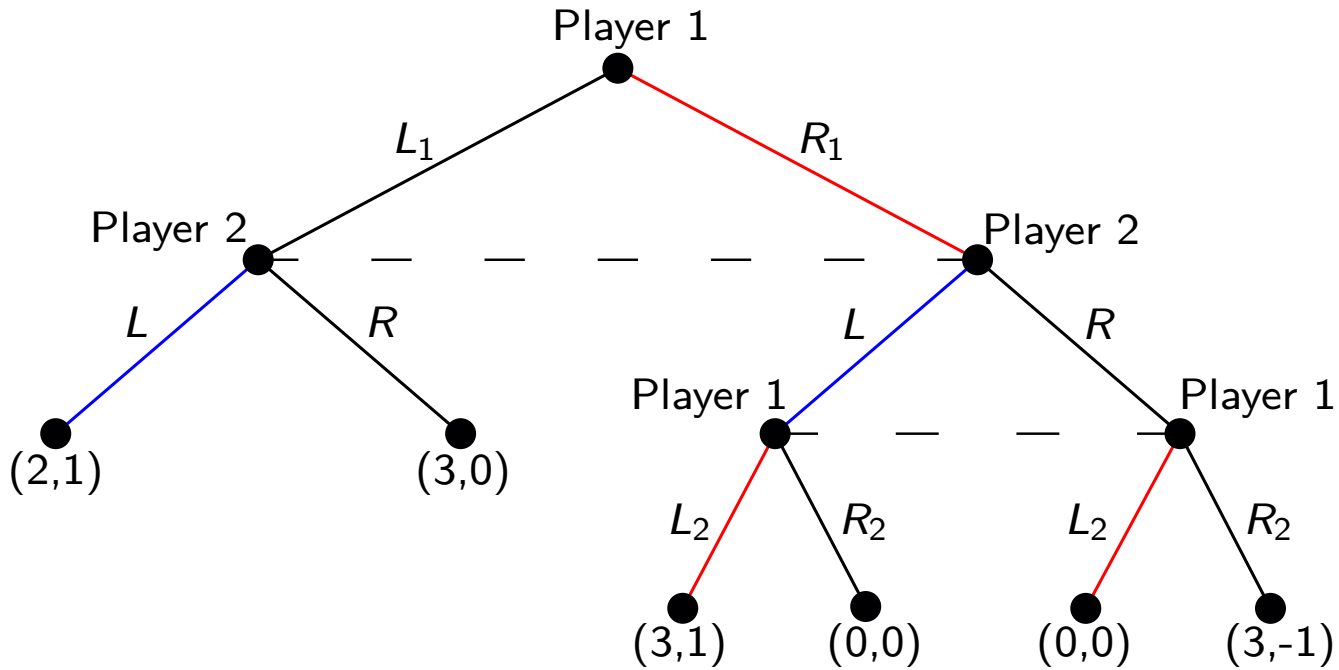


	L	R
L_2	2, 1	3, 0
R_2	2, 1	3, 0

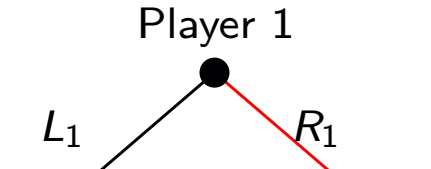
	L	R
L_2	3, 1	0, 0
R_2	0, 0	3, -1

PS6, Ex. 3.d: Dynamic games (imperfect information)

(d) Find the SPNE in the following game:



2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):



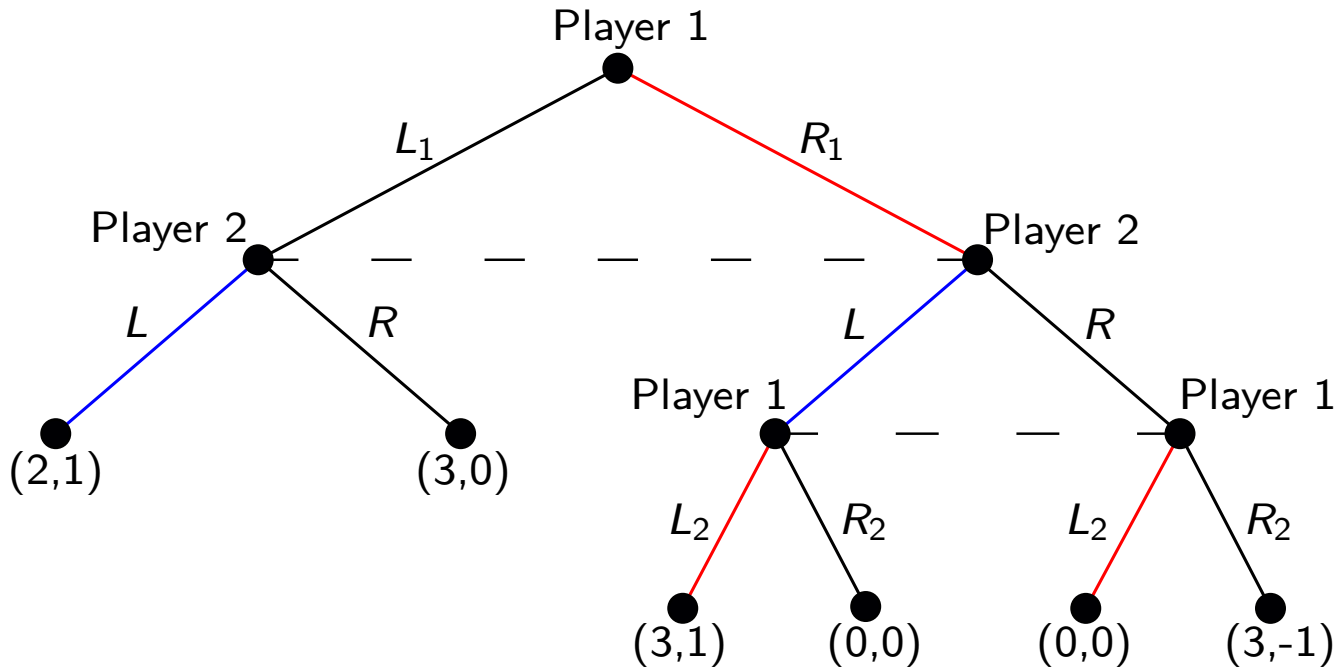
	L	R
L_2	2, 1	3, 0
R_2	2, 1	3, 0

	L	R
L_2	3, 1	0, 0
R_2	0, 0	3, -1

Write up the SPNE!

PS6, Ex. 3.d: Dynamic games (imperfect information)

(d) Find the SPNE in the following game:



2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):

	L	R
L_2	2, 1	3, 0
R_2	2, 1	3, 0

	L	R
L_2	3, 1	0, 0
R_2	0, 0	3, -1

$SPNE = \{s_1^*, s_2^*\} = \{(R_1, L_2), L\}$ with outcome (3,1).

PS6, Ex. 4: The Mutated Seabass (imperfect information)

PS6, Ex. 4:

Go back to exercise 4 in problem set 5. Write up the game tree for the situation in part (c), where the choice to acquire the weapon is not observed. Find the SPNE. What has changed?

Last class we actually solved this part and discussed it as an extension...

PS6, Ex. 5:

PS6, Ex. 5:

PS6, Ex. 5.a:

PS6, Ex. 6:

PS6, Ex. 6:

PS6, Ex. 6.a:

PS6, Ex. 7: To keep or split (imperfect information)

PS6, Ex. 7: To keep or split (imperfect information)

Consider the following 2×2 game where payoffs are monetary:

	L	R
T	3, 3	0, 4
B	4, 0	1, 1

Before this game is played, Player 1 can choose whether, after the game is played, players should keep their own payoffs or split the aggregate payoff evenly between them. Player 2 observes this choice.

(a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.

(b) Find the subgame perfect Nash Equilibria (SPNE).

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) and find the pure strategy Nash Equilibria (PSNE).

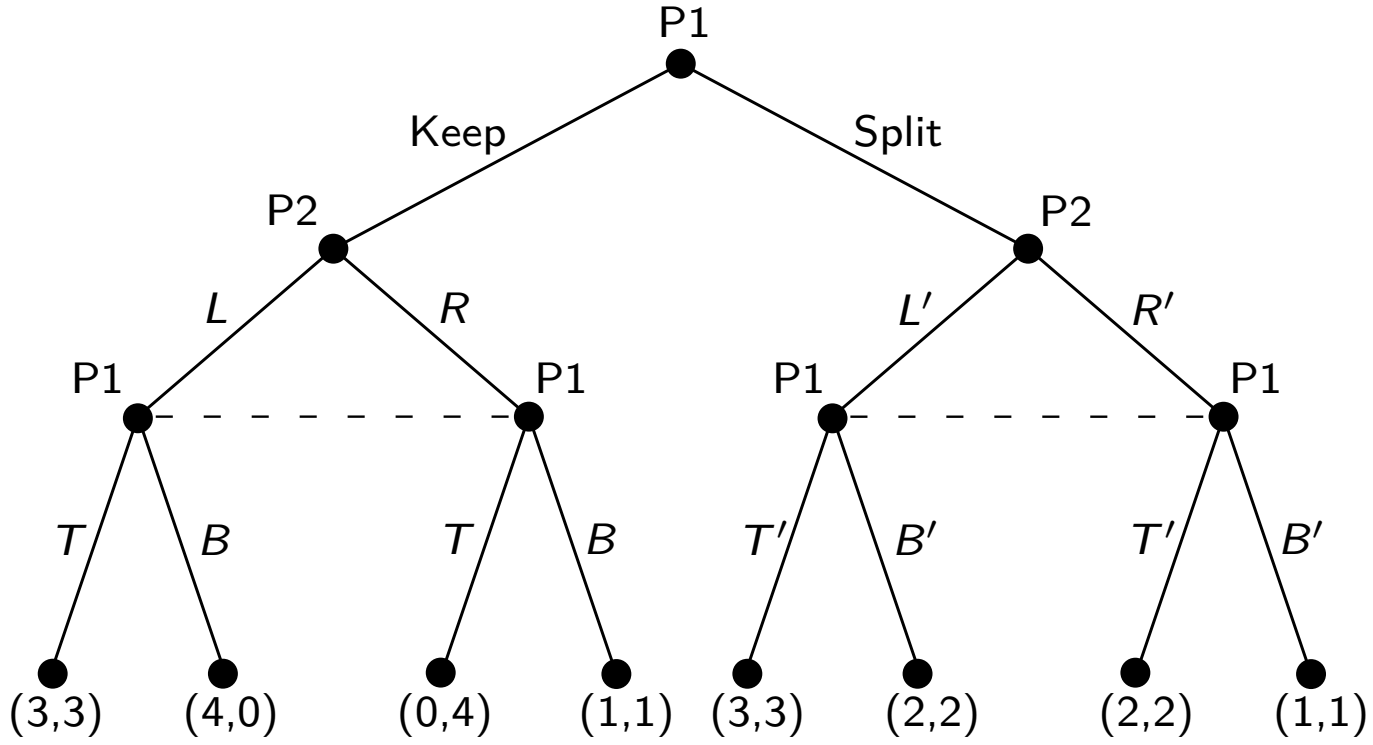
PS6, Ex. 7.a: To keep or split (imperfect information)

(a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.

1st stage: Player 1 chooses Keep or Split. Player 2 observes the choice.

2nd stage: Player 2 chooses L or R (L' or R'). The action is private information.

3rd stage: Player 1 chooses T or B (T' or B') without knowing what Player 2 did.

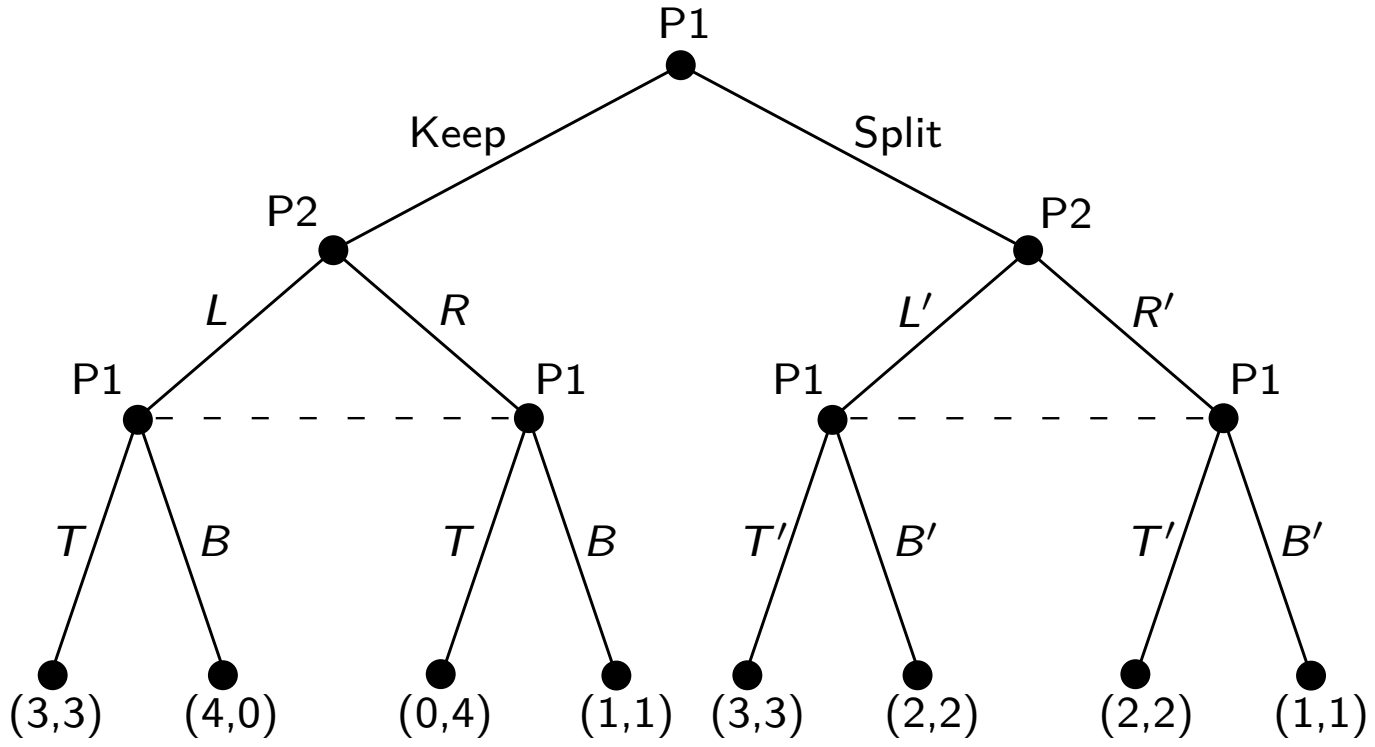


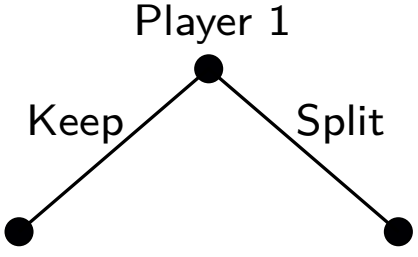
The order of stage 2 and 3 is arbitrary, but the 2nd stage must be private information.

(b) ***Find the subgame perfect Nash Equilibria (SPNE).***

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).



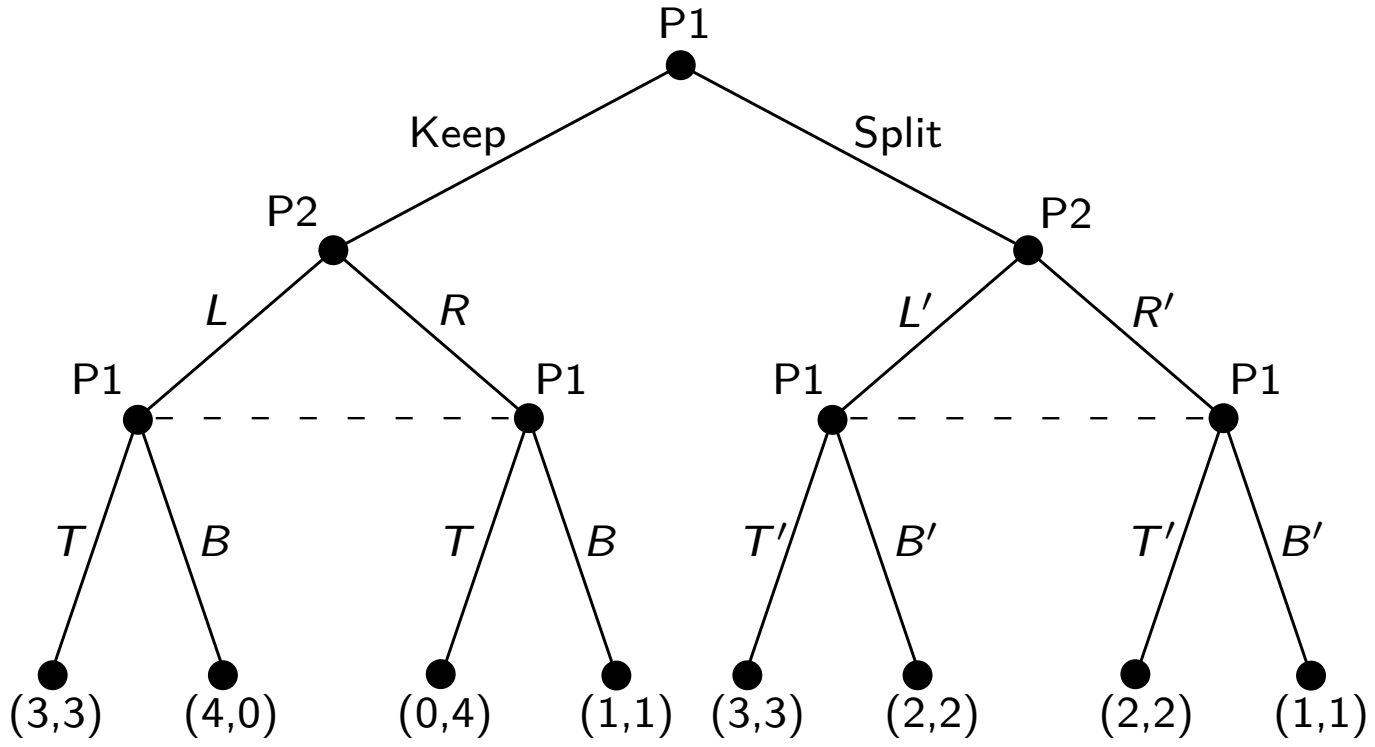


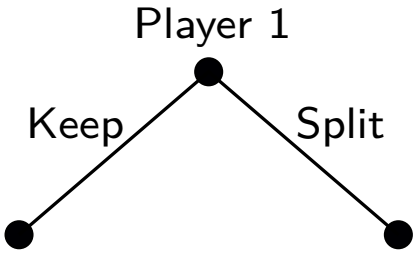
	L	R
T	3, 3	0, 4
B	4, 0	1, 1

	L'	R'
T'	3, 3	2, 2
B'	2, 2	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).

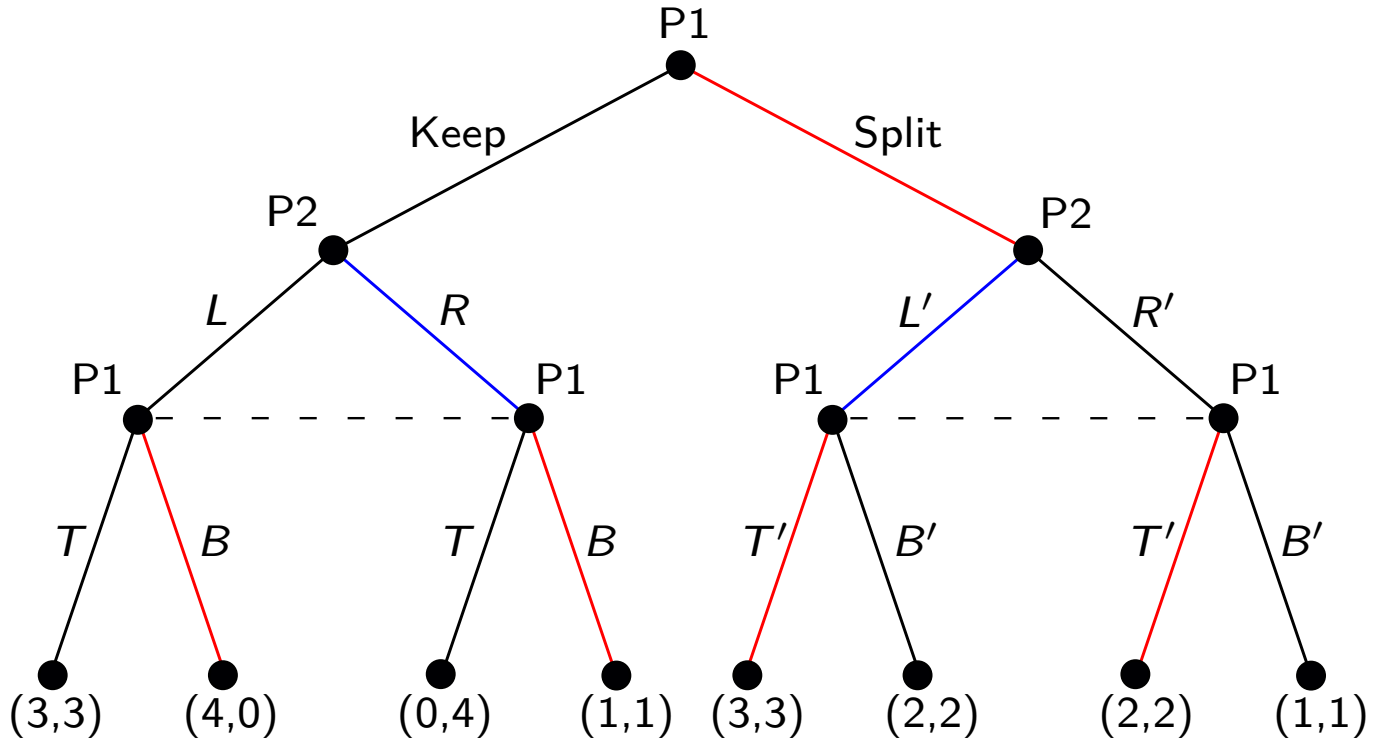


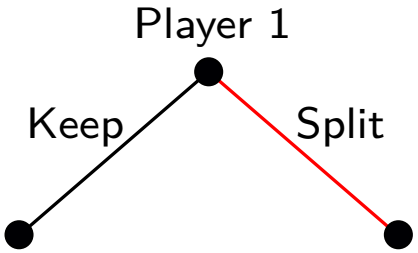


	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).



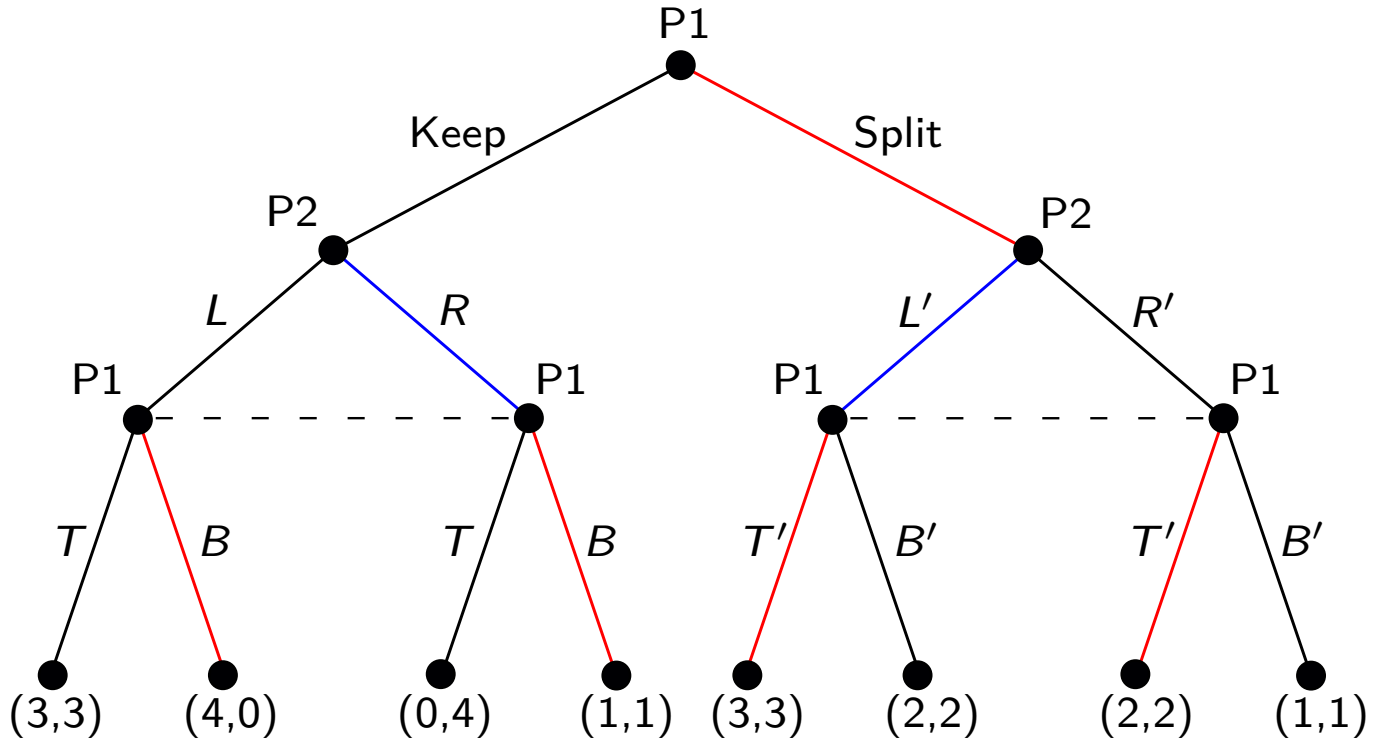


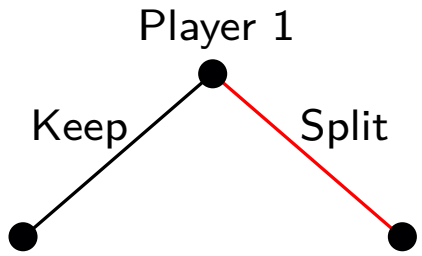
	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

Write up the full strategy profiles for the subgame perfect Nash Equilibria (SPNE).

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).





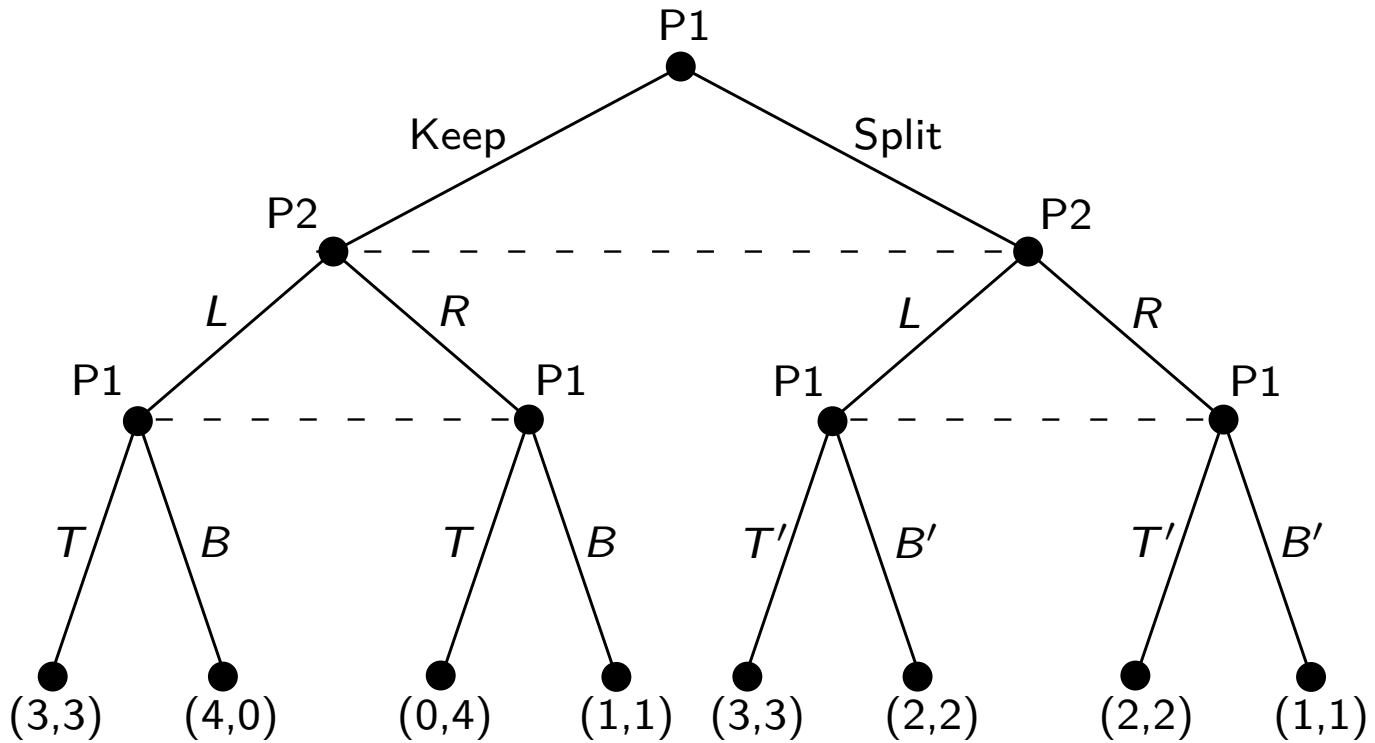
	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

$SPNE = \{(Split, B, T'), (R, L')\}$ with outcome (3,3).

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. ***Draw the game tree (again using information sets)*** and find the pure strategy Nash Equilibria (PSNE).

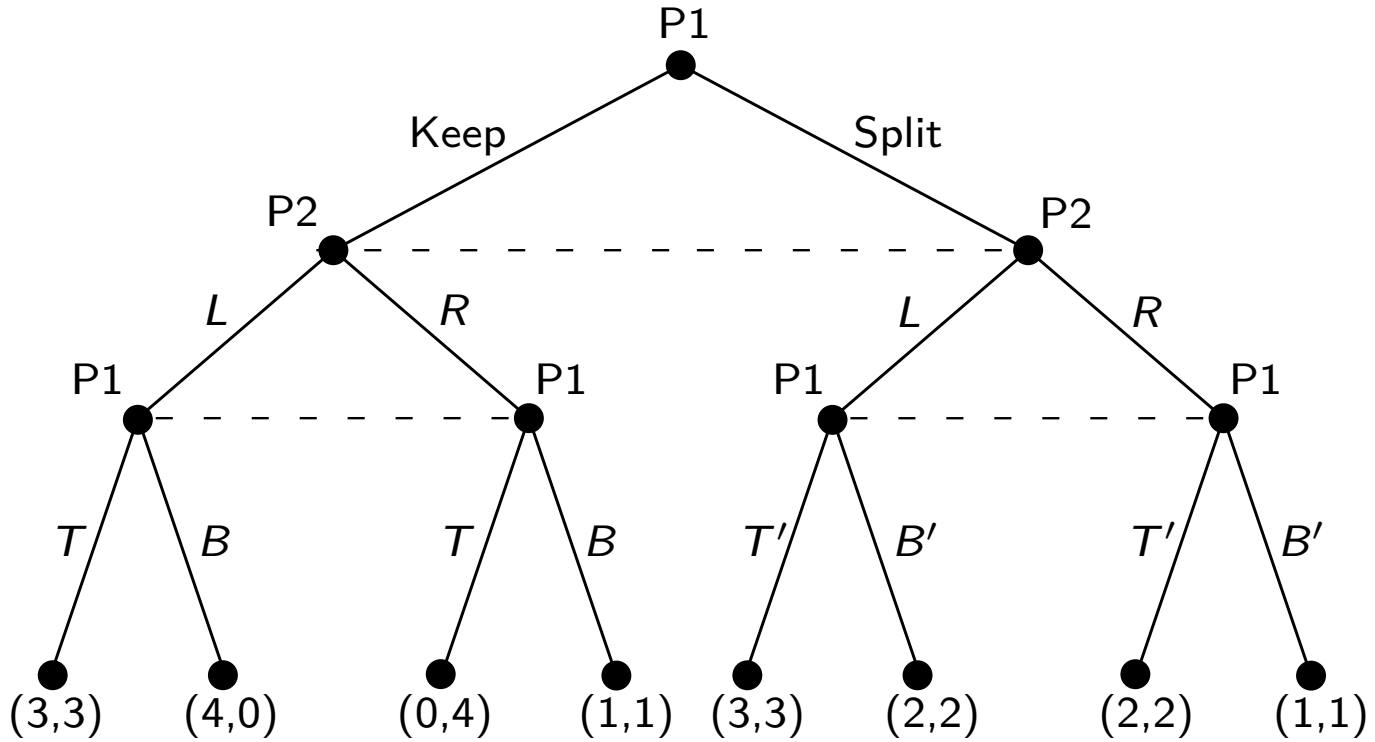
PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) ***and find the pure strategy Nash Equilibria (PSNE).***

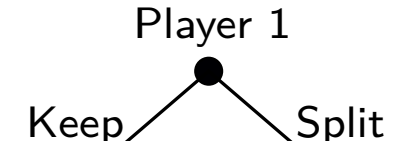


PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the pure strategy Nash Equilibria (PSNE).



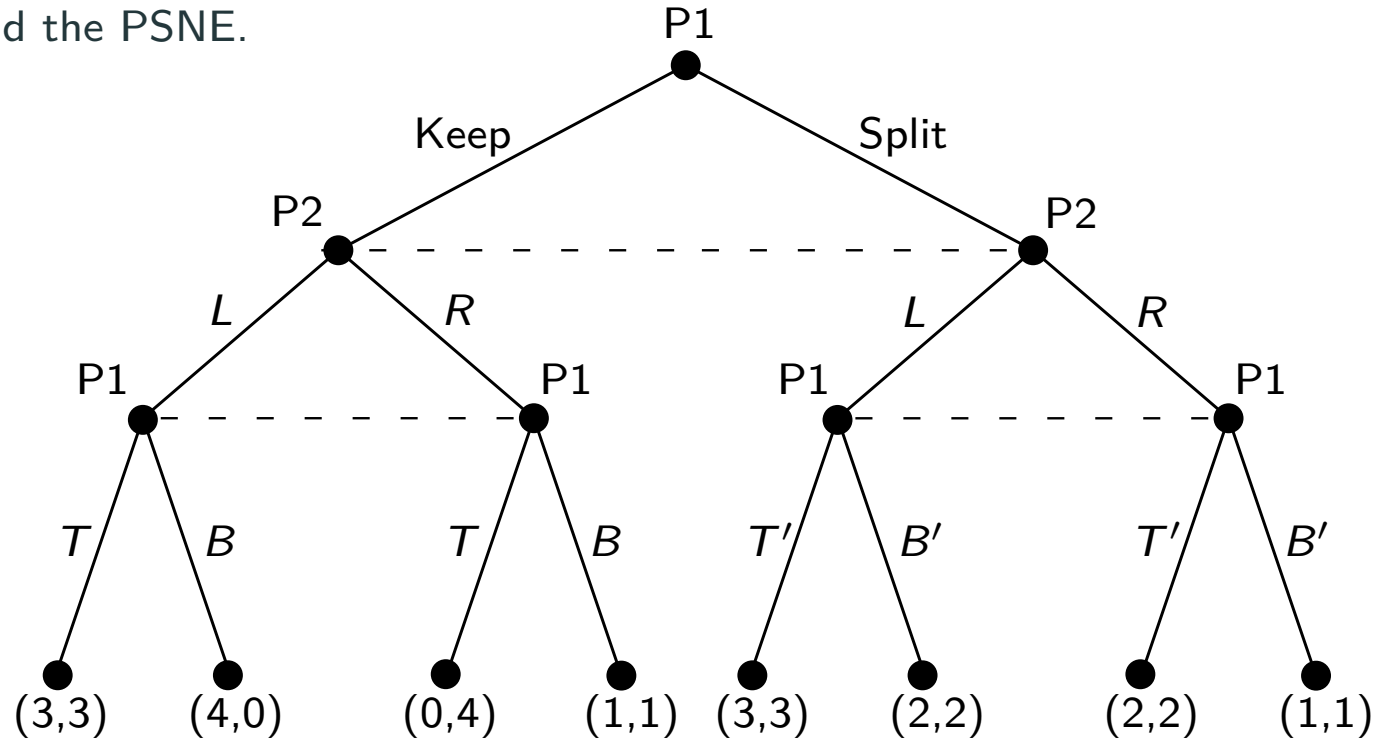
2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):



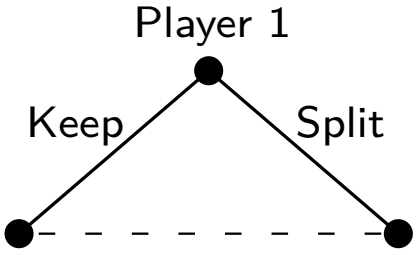
	L	R		L	R
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the PSNE.



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

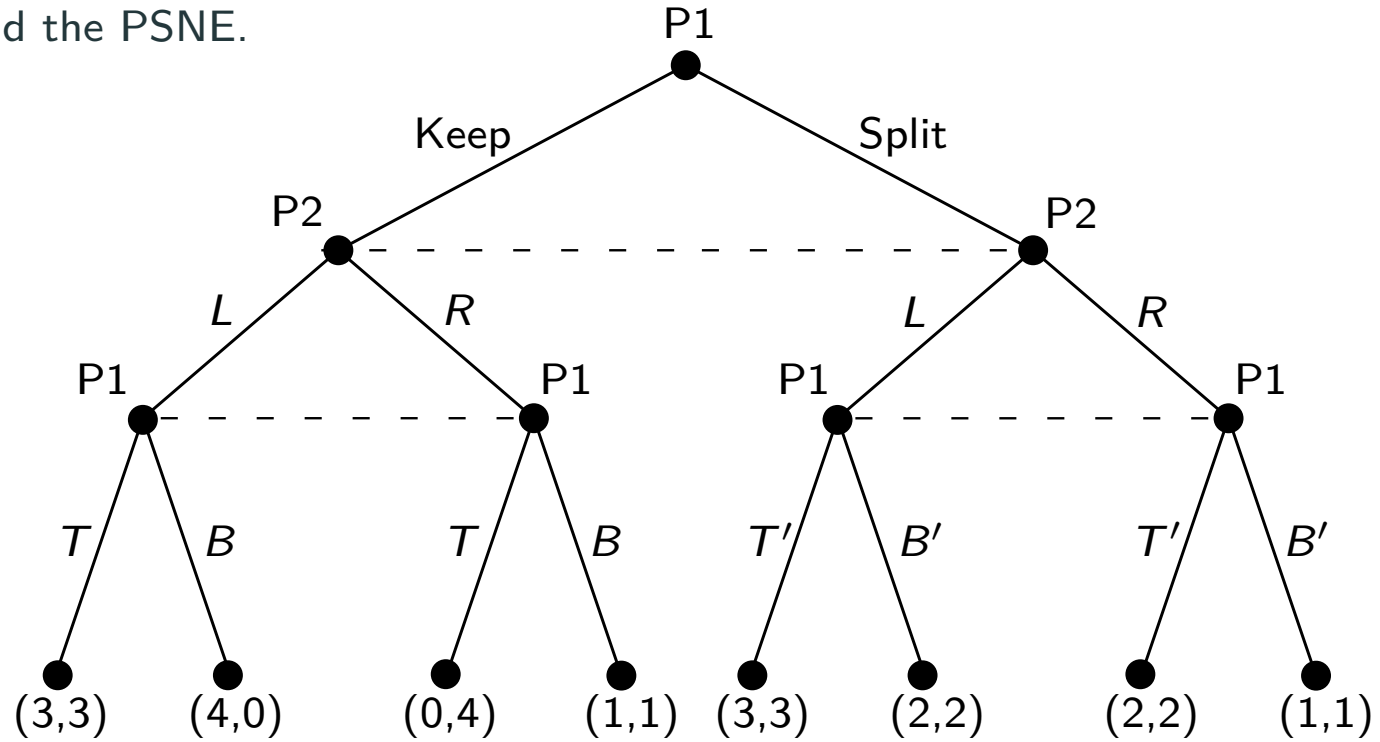
	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

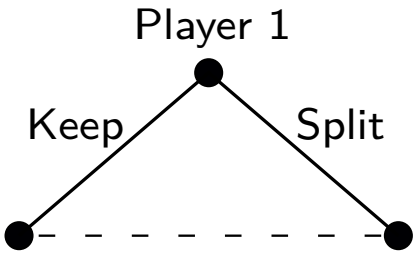
	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the PSNE.



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

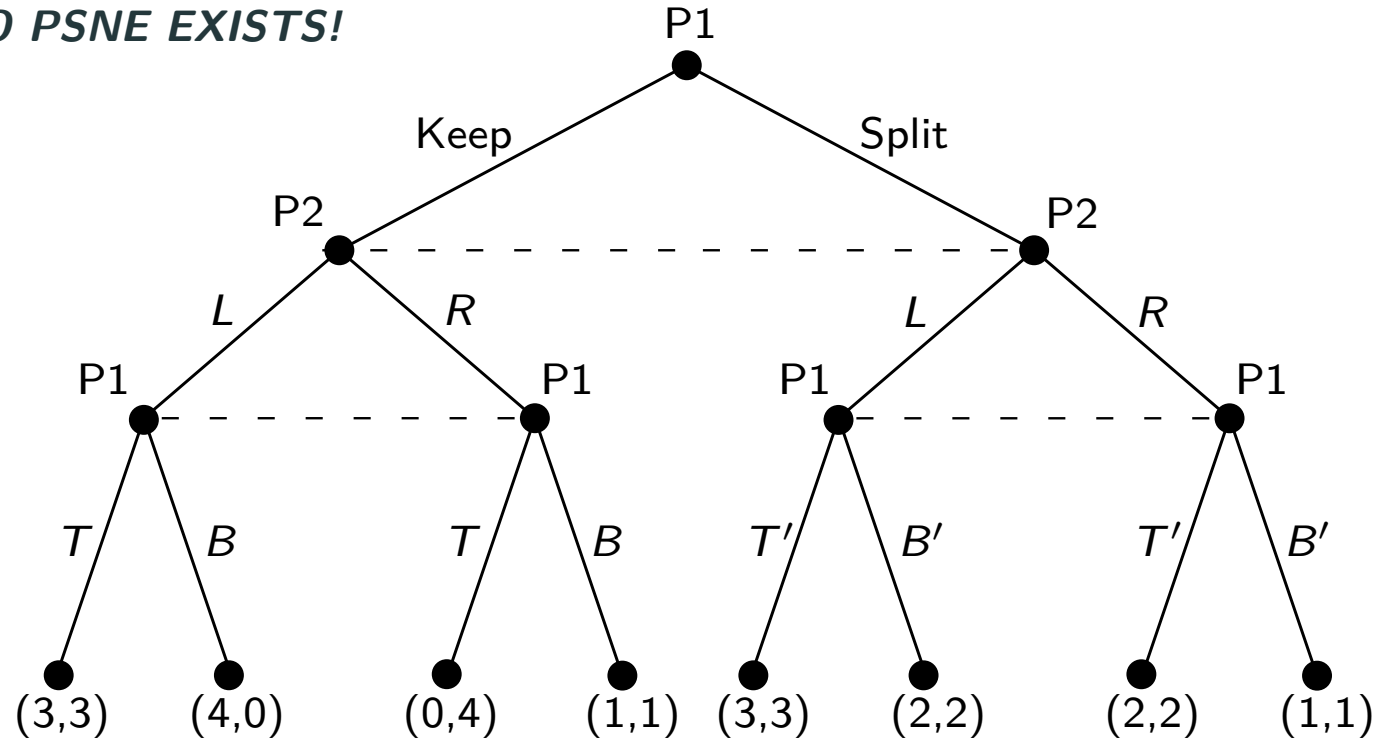
	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

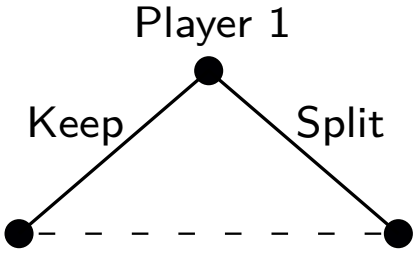
	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) ***NO PSNE EXISTS!***



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

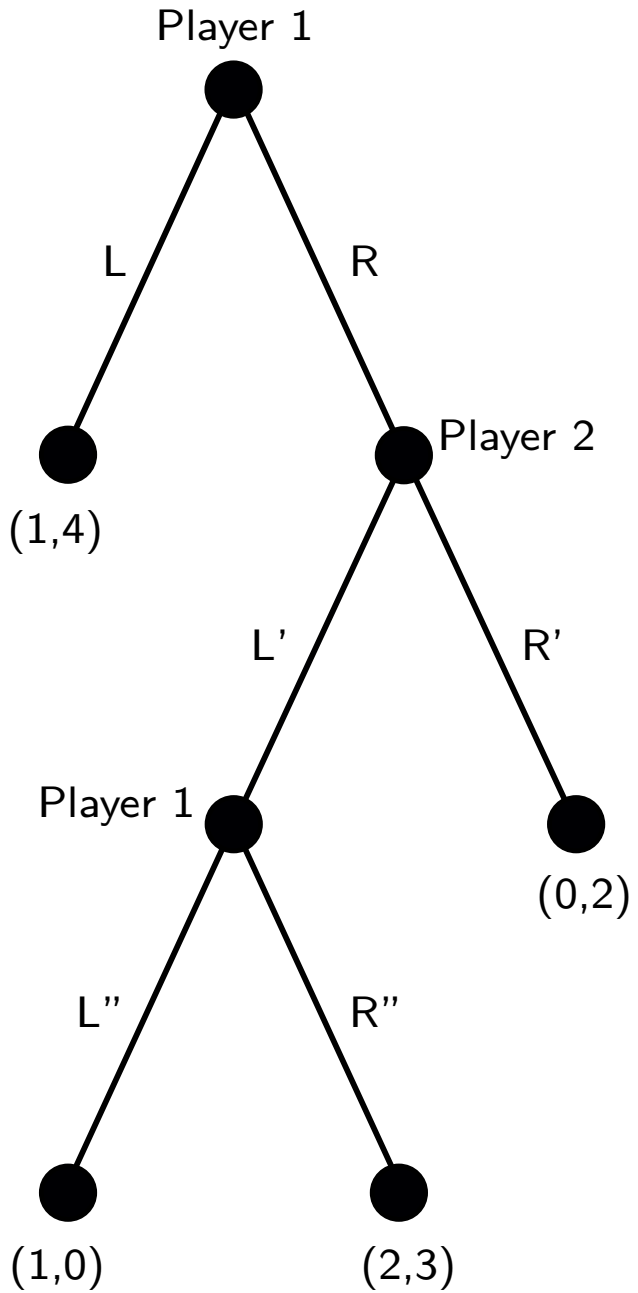
	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

Code examples

Code examples



Matrix, no player names:

	L (q)	R (1-q)
T (p)		
B (1-p)		

Matrix, no colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)		
	B (1-p)		

Matrix, with colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	
	B (1-p)		