

# Microeconomics III, Ex. Class 4: Session 1

Thor Donsby Noe (thor.noe@econ.ku.dk) September 11 2019

Department of Economics, University of Copenhagen

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# Welcome

## About me

Thesis on the electricity market:

- auction design

Research Assistant: Estimating the blue Net National Product

- valuation of public goods

Erasmus student at Universitat de Barcelona - School of Economics.

public policy, taxation, regulation, privatization, institutions

Student assistant at LO (now FH - Danish Trade Union Confederation)

labour economics, family economics, industrial organization



Motivation

# Relevance of game theory

# From the course description:

- This course furthers the introduction of game theory and its applications in economic models.
- The student who successfully completes the course will learn the basics of game theory and will be enabled to work further with advanced game theory.

# Relevance of game theory

## From the course description:

- This course furthers the introduction of game theory and its applications in economic models.
- The student who successfully completes the course will learn the basics of game theory and will be enabled to work further with advanced game theory.
- The student will also learn how economic problems involving strategic situations can be *modeled* using game theory, as well as how these models are *solved*.
- 4. The course intention is that the student becomes able to work with modern economic theory, for instance within the areas of industrial organization, macroeconomics, international economics, labor economics, public economics, political economics and financial economics.

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# Game theory in current master's courses (2019/2020)

#### Courses where game theory is central:

- Mechanism Design
- Contract Theory
- Auctions Theory and Practice, Incentives and Organizations
- Industrial Organization
- Advanced Industrial Organization
- Strategic Management
- Advanced Strategic Management
- Behavioral Finance (F)
- Foundations of Behavioral Economics
- Behavioral and Experimental Economics (summerschool)
- Science of Behavior Change

## Courses where game theory plays a part:

- Public Finance (taxation)
- Labour Economics
- Health Economics
- Political Economics
- Advanced Development Economics -Micro Aspects

Strategic, logical thinking is also useful for macroeconomists and econometricians

# Overview of the course

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#### Form:

- 12 lectures + conclusion
- 12 problem sets in 12 exercise classes
- 3 mandatory assignments delivered by mail or in my pigeon box in the hall of building 26 no later than:
   Oct. 4, Nov. 1, Nov. 29 (Fridays)

Question lesson on December 18, 10:00-12:00 in 7-0-28

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#### Course content

- 1. Static games with complete information (PS 1-3)
- 2. Dynamic games with complete information (PS 4-6)
- 3. Static games with incomplete information (PS 3, 8-10)
- 4. Dynamic games with incomplete information (PS 6-7, 10-11)
- 5. Psychological Game Theory (PS 12)

## Exam

#### Exam form:

 Two hours without aids on Peter Bangs Vej 36

#### Content:

- Cook-book solutions trained in the problem sets
- Reflection and discussion

#### Exam form

 Two hours without aids on Peter Bangs Vej 36

#### Content

- Cook-book solutions from the problem sets
- Reeflection and discussion

Important to complete all problem sets but also to attend the lectures and read the curriculum:

- Last winter 33% failed the exam
- General lack in the ability to interpret, explain, and give examples from the real world

We will train this in class as well

# Learning outcome

From the course description:

## Knowledge:

- Formally state the definition of a game and explain the key differences between games of different types.
- In detail account for the equilibrium (solution) concepts that are relevant for these games (Nash Equilibrium, Subgame Perfect Nash Equilibrium, Bayes-Nash Equilibrium, Perfect Bayesian Equilibrium).
- Identify a number of special games and particular issues associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

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#### Skills:

- Explicitly solve for the equilibria of these games.
- 2. **Explain** the relevant steps in the reasoning of the solution.
- 3. *Interpret* the outcomes of the analysis.
- Apply equilibrium *refinements* and *discuss* the solution concepts

## Competencies:

- Analyze strategic situations by modeling them as formal games.
- Set up, prove, analyze and apply the theories and methods used in the course in an independent manner.
- 3. **Evaluate and discuss** the crucial assumptions underlying the theory.

Exam example

# Example from the exam Autumn 2018

- 1.a "The reason that players cannot achieve a good outcome in the prisoner's dilemma is that they cannot communicate." True or false? Explain in 2-3 sentences.
- 1.c "Iterated Elimination of Strictly Dominated Strategies never eliminates a Nash Equilibrium" True or false? Explain in 2-3 sentences.
- 1.d You are writing your dating app profile and want to signal that you are adventurous. Give an example of a signal that is not credible and an example that is credible and explain the reasons why.

Take 5 min to discuss the questions with your neighbor(s)

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- True! Nash equilibrium is a refinement. However, this does not hold for weakly dominated strategies
- 1.d Credible?
  Not credible?
  Difference?

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- False! Even if players could communicate their best response would still be to play Fink (confess).
- 1.c True! Nash equilibrium is a refinement. However, this does not hold for weakly dominated strategies
- 1.d Credible signals: Show a picture of you skydiving, swimming with sharks etc. Not credible: Write that you are adventurous or only claim that you have been skydiving etc. There needs to be a differential cost that makes it affordable for those with a hidden desirable trait (being adventurous), not affordable for those without this trait.

There is a cost to the signal: Those with the desirable trait are more likely to send the signal. Those who exhibit the signal are more desirable.

# PS1, Ex. 1: Basics of game theory

What is game theory and why do we do it? To answer this, briefly discuss the following questions:

- (a) What are the ingredients of a (normal form) game?
- (b) How do we analyze games?
- (c) Why do you think it is practical to analyze problems as games?

Take 5 min. to discuss it with your neighbor(s)

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- (c) Why do you think it is practical to analyze problems as games?

- (a) A normal form game consists of:
  - 1. The set of players i
  - 2. The possible strategy sets  $S_i \in \{s_1, s_2, ..., s_n\}$  for each player i
  - 3. Each players utility (payoff) function  $u_i(s_1, s_2, ..., s_n)$

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- (b) Define a solution concept, state the assumptions it relies on, and its possible limitations.

E.g. Iterative Elimination of Strictly Dominated Strategies (IESDS) requires common knowledge of rationality but is not always sufficient to find the Nash Equilibrium.

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- (a) A normal form game consists of:
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- (b) Define a solution concept, state the assumptions it relies on, and its possible limitations.
   E.g. Iterative Elimination of Strictly Dominated Strategies (IESDS) requires common knowledge of rationality but is not always sufficient to find the Nash Equilibrium.
- (c) Complex situations can be analyzed in an unambiguous way through modelling them as games and applying logic.

# PS1, Ex. 2: IESDS

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

Take 5 min. to solve it on your own or with your neighbor(s)

	$t_1$	t <sub>2</sub>	t <sub>3</sub>
$s_1$	5, 0	3, 3	1, 1
<i>s</i> <sub>2</sub>	3, 4	2, 2	3, 1
<i>s</i> <sub>3</sub>	2, 2	1, 1	0, 5

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	$t_1$	t <sub>2</sub>	t <sub>3</sub>
<b>s</b> <sub>1</sub>	<b>5</b> , 0	<b>3</b> , 3	1, 1
<b>s</b> <sub>2</sub>	3, 4	<b>2</b> , 2	<b>3</b> , 1
<b>s</b> <sub>3</sub>	<b>2</b> , 2	<b>1</b> , 1	<mark>0</mark> , 5

Player 1:  $s_3$  is strictly dominated by  $s_1$  as well as  $s_2$  and can be eliminated, giving us the reduced form game:

	$t_1$	t <sub>2</sub>	<i>t</i> <sub>3</sub>
$s_1$	5, 0	3, 3	1, 1
<b>s</b> <sub>2</sub>	3, 4	2, 2	3, 1

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

	$t_1$	t <sub>2</sub>	<i>t</i> <sub>3</sub>
<b>s</b> <sub>1</sub>	<b>5</b> , 0	<b>3</b> , 3	<b>1</b> , 1
<b>s</b> <sub>2</sub>	3, 4	<b>2</b> , 2	3, 1
<del>5</del> 3	<del>2, 2</del>	<del>1, 1</del>	<del>0, 5</del>

Player 1:  $s_3$  is strictly dominated by  $s_1$  as well as  $s_2$  and can be eliminated, giving us the reduced form game:

Player 2:  $t_3$  is strictly dominated by  $t_2$  and can be eliminated.

Giving us a new reduced form game:

	$t_1$	t <sub>2</sub>
<i>s</i> <sub>1</sub>	5, 0	3, 3
<b>s</b> <sub>2</sub>	3, 4	2, 2

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Player 2:  $t_3$  is strictly dominated by  $t_2$  and can be eliminated.

Giving us a new reduced form game:

Player 1:  $s_2$  is strictly dominated by  $s_1$  and is eliminated. Reduced form game:

Solve this game by Iterative Elimination of Strictly Dominated Strategies (IESDS):

Player 1:  $s_3$  is strictly dominated by  $s_1$  as well as  $s_2$  and can be eliminated, giving us the reduced form game:

Player 2:  $t_3$  is strictly dominated by  $t_2$  and can be eliminated.

Giving us a new reduced form game:

$$\begin{array}{c|cccc} & t_1 & t_2 \\ \hline s_1 & 5, 0 & 3, 3 \\ \hline s_2 & 3, 4 & 2, 2 \\ \hline \end{array}$$

Player 1:  $s_2$  is strictly dominated by  $s_1$  and is eliminated. Reduced form game:

Player 2:  $t_1$  is strictly dominated by  $t_2$  and is eliminated. I.e. IESDS provides the unique strategy profile  $(s_1, t_2)$ , implying that this is also the Nash Equilibrium:

$$\begin{array}{c|c} & t_2 \\ \hline s_1 & 3, 3 \end{array}$$

#### \_\_\_\_

PS1, Ex. 3: The Travelers' Dilemma

The Travelers' Dilemma:

"An airline loses two suitcases belonging to two different travelers. Both suitcases look identical and contain identical items. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of \$100 per suitcase, and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value no less than \$0 and no larger than \$100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount

However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with the following: \$1 extra will be paid to the traveler who wrote down the lower value and a \$1 fine imposed on the person who wrote down the higher amount."

- (a) Write down the normal form of this game: players, strategy sets, payoffs
- (b) Can you solve this game by IESDS?
- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

Take 10 min. to find the answers on your own or with your neighbor(s)

- (a) Write down the normal form of this game: players, strategy sets, payoffs
- (b) Can you solve this game by IESDS?
- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

(a) A normal form game consists of:

3. Payoffs for player  $i \neq i$ :

- 1. The set of players: Traveler 1 and Traveler 2.
- 2. Strategy sets:  $S_i = \{0; 0.01; ...; 99: 99; 100\}$  for i = 1; 2

$$u_i(s_i, s_j) = \left\{ egin{array}{ll} s_i & ext{if} & s_i = s_j \ s_i + 1 & ext{if} & s_i < s_j \ s_j - 1 & ext{if} & s_i > s_j \end{array} 
ight.$$

- (a) Write down the normal form of this game: players, strategy sets, payoffs
- (b) Can you solve this game by IESDS?
- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

- (a) A normal form game consists of:
  - 1. The set of players: Traveler 1 and Traveler 2.
  - 2. Strategy sets:

$$S_i = \{0; 0.01; ...; 99: 99; 100\}$$
 for  $i = 1; 2$ 

3. Payoffs for player  $i \neq j$ :

$$u_i(s_i, s_j) = \begin{cases} s_i & \text{if} \quad s_i = s_j \\ s_i + 1 & \text{if} \quad s_i < s_j \\ s_j - 1 & \text{if} \quad s_i > s_j \end{cases}$$

(b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.

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- (b) Can you solve this game by IESDS?
- (c) What number do you think each traveler will write down? Why? An informal discussion of the reasoning will suffice.

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- (b) No, as no strategy is always dominated by one other strategy no matter what the other traveler plays.
- (c) Given common knowledge of rationality each traveler will avoid getting "underbid" by the other, i.e. the Nash Equilibrium is  $s_i, s_j = (0, 0)$  as there is no incentive to deviate.

# PS1, Ex. 4: IESDS

Solve these games by iterative elimination of strictly dominated strategies:

Take 5 min. to solve them on your own or with your neighbor(s)

Solve these games by iterative elimination of strictly dominated strategies:

	$t_1$	t <sub>2</sub>	t <sub>3</sub>
$s_1$	5, 0	2, 3	1, 1
<i>s</i> <sub>2</sub>	2, 4	2, 2	3, 1
<del>5</del> 3	2, 2	1, 1	0, 5

	$t_1$	t <sub>2</sub>	t <sub>3</sub>	
$s_1$	5, 0	2, 3	1, 1	
<b>s</b> <sub>2</sub>	2, 4	2, 2	3, 1	
<b>s</b> 3	2, 2	1, 1	1, 5	

- Player 1: s<sub>3</sub> is strictly dominated by s<sub>1</sub> and is eliminated
- Player 2: After eliminating s<sub>3</sub>, t<sub>3</sub> is strictly dominated by t<sub>2</sub> and is eliminated, giving us the reduced form game:

	$t_1$	$t_2$
<i>s</i> <sub>1</sub>	5, 0	2, 3
<b>s</b> <sub>2</sub>	2, 4	2, 2

# PS1, Ex. 5: The higher number wins

Mikael and Jonas are playing a game instead of working. The game has the following rules: Both secretly pick a (natural) number between 1 and 5. Then they reveal the numbers to each other. If both have picked the same number, nobody gets anything. If Jonas' number is higher than Mikael's number, Mikael has to pay Jonas 1 kr. If Mikael's number is higher than Jonas', Jonas has to pay 10 kr. to Mikael

- (a) Does this game seem fair to you?
- (b) Write the game in bimatrix form.
- (c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.
- (d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

Take 10 min. to answer the questions on your own or with your neighbor(s)

- (a) Does this game seem fair to you?
- (b) Write the game in bimatrix form.
- (c) Are there any strictly dominated strategies? Solve the game by iterated elimination of strictly dominated strategies.
- (d) What is the outcome of the game if both Mikael and Jonas are rational, know that the other is rational, know that the other knows that they are rational etc.?

	"1"	1	"2"		"3"		"4"	"5"
"1"	0, 0		-1, 1	Ī	-1, 1		-1, 1	-1, 1
"2"	10, -10		0, 0		-1, 1		-1, 1	-1, 1
"3"	10, -10		10, -10	Ī	0, 0		-1, 1	-1, 1
"4"	10, -10		10, -10		10, -10		0, 0	-1, 1
"5"	10, -10		10, -10		10, -10		10, -10	0, 0

# PS1, Ex. 6: Three player game

We can also write games with more than two players. Consider the game below where player 1 chooses the bi-matrix (A or B), player 2 chooses the row (C or D), and player 3 chooses the column (E or F). In each cell, the first number gives the payoff of Player 1, the second number the payoff of Player 2, and the third number the payoff of Player 3.

Find the pure strategy profiles that survive iterated elimination of strictly dominated strategies. (10 min.)

	Е	F
С	<b>0</b> , 2, 2	<b>2</b> , 1, 1
D	0, 1, 1	3, 0, 0
Α		

	Е	F
С	<b>1</b> , 0, 1	<b>3</b> , 1, 2
D	1, 1, 0	<b>5</b> , 2, 1

В

 $1^{\text{st}}$  step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

В

	E	F
С	<mark>0</mark> , 2, 2	<b>2</b> , 1, 1
D	0, 1, 1	<b>3</b> , 0, 0
	Α	

	Е	F
С	<b>1</b> , 0, 1	<b>3</b> , 1, 2
D	1, 1, 0	5, 2, 1

В

1st step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

 $2^{nd}$  step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.



В

1<sup>st</sup> step: Player 1: A is strictly dominated by B, thus, matrix A can be eliminated:

2<sup>nd</sup> step: Player 2: After matrix A is eliminated, C is strictly dominated by D and we eliminate C.

 $3^{rd}$  step: Player 3: After matrix A and row C is eliminated, E (payoff = 0) is strictly dominated by F (payoff = 1) and we eliminate E.

The unique pure strategy profile that survives IESDS is (B; D; F).

Preparation for exercise classes

### Preparation for all exercise classes

To get through all problem sets you need to show up prepared:

- Ideally: Read in the curriculum and participate in the lecture. Print the problem set and try to solve it.
- Bare minimum: Read through the lecture slides and the problem set.
- "In all problem sets, there will be two types of exercises, A and B. When you show up for class, you are expected to have done all A-exercises and have read and understood all B-exercises. A-exercises will not be solved on the whiteboard. Instead, you will get the final answer, and have the opportunity to discuss the solution with your TA."