PS4, Ex. 1 (A): MSNE and best-response functions

PS4, Ex. 1 (A): MSNE and best-response functions

1. (A) Find all equilibria (pure and mixed) in the following games, first analytically and then through plotting the best-response functions.

(a)

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

(b)

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

Hint: Find the probabilities q for which Player 1 is indifferent, e.g. $u_1(T, q) = u_1(B, q)$. and the probabilities p for which Player 2 is indifferent, e.g. $u_2(L, p) = u_2(R, p)$.

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Highlight the best responses in pure strategies.

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 2

Player 1		L (q)	L (1-q)
	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

For which values of q is Player 1 indifferent?

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$=$$

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 1		Player 2	
		L (q)	L (1-q)
	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

Write up all NE (pure and mixed).

$$NE = (p^*, q^*) =$$

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

Write up all NE (pure and mixed).

The players have symmetric payoffs, thus:

$$NE = (p^*, q^*) = \{(0,0); (1,1); ...\}$$

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

The players have symmetric payoffs, thus:

$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Write up Player 1's best-response (BR) function, $p^*(q)$

$$BR_1(q) = \{$$

PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

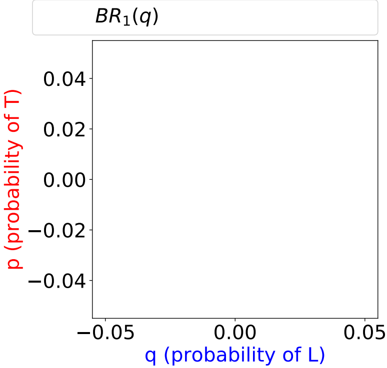
The players have symmetric payoffs, thus:

$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Plot Player 1's best-response (BR) function, $p^*(q)$

Write up and plot the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 4/7 \\ p \in [0,1] & \text{if} \quad q = 4/7 \\ p = 1 & \text{if} \quad q > 4/7 \end{cases}$$



PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

The players have symmetric payoffs, thus:

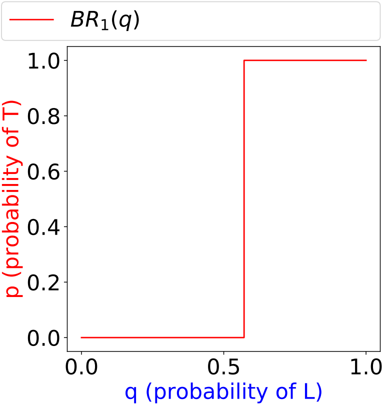
$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Write up Player 2's BR function, $q^*(p)$

Write up and plot the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 4/7 \\ p \in [0,1] & \text{if} \quad q = 4/7 \\ p = 1 & \text{if} \quad q > 4/7 \end{cases}$$

$$BR_2(p) = \{$$



PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 2

Player 1		L (q)	L (1-q)
	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

The players have symmetric payoffs, thus:

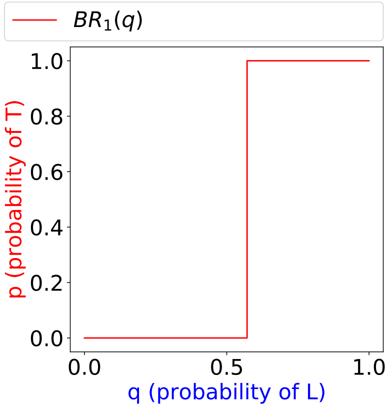
$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Plot Player 2's BR function, $q^*(p)$

Write up and plot the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 4/7 \\ p \in [0,1] & \text{if} \quad q = 4/7 \\ p = 1 & \text{if} \quad q > 4/7 \end{cases}$$

$$BR_2(p) = \begin{cases} q = 0 & \text{if} \quad p < 4/7 \\ q \in [0,1] & \text{if} \quad p = 4/7 \\ q = 1 & \text{if} \quad p > 4/7 \end{cases}$$



PS4, Ex. 1.a (A): MSNE and best-response functions

(a) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
	B (1-p)	0, 0	4, 4

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1|T] = E[u_1|B]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

The players have symmetric payoffs, thus:

$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Write up and plot the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 4/7 \\ p \in [0,1] & \text{if} \quad q = 4/7 \\ p = 1 & \text{if} \quad q > 4/7 \end{cases}$$

$$BR_2(p) = \begin{cases} q = 0 & \text{if} \quad p < 4/7 \\ q \in [0,1] & \text{if} \quad p = 4/7 \\ q = 1 & \text{if} \quad p > 4/7 \end{cases}$$