Microeconomics III: Problem Set 8[a]

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[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS8, Ex. 1 (A):

PS8, Ex. 1 (A):

PS8, Ex. 1 (A):

PS8, Ex. 2 (A):

PS8, Ex. 2 (A):

PS8, Ex. 2.a (A):

PS8, Ex. 3 (A):

PS8, Ex. 3 (A):

PS8, Ex. 3.a (A):

PS8, Ex. 4:

PS8, Ex. 4:

PS8, Ex. 4.a:

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

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Consider the 'dating game'. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2's types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides.

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

PS8, Ex. 5.a: The dating game (Bayesian Nash Equilibria)

PS8, Ex. 6:

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Difficult. Consider the public goods game from lecture 7. Suppose now instead that there is two-sided incomplete information. In particular, the cost of writing the reference is uniformly distributed between 0 and 2:

$$c_i \sim u(0,2) \text{ for } i = 1,2$$

In this setting, we can show that the players optimally follow a 'cutoff' strategy. Thus, the equilibrium strategies take the form

$$s_1^*(c_1) = \begin{cases} \textit{Write} & \text{if} \quad c_1 \leq c_1^* \\ \textit{Don't} & \text{if} \quad c_1 > c_1^* \end{cases}$$

$$s_2^*(c_2) = \begin{cases} \textit{Write} & \text{if} \quad c_2 \leq c_2^* \\ \textit{Don't} & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = \textit{Write})$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that
$$1 - c_i^* = z_{-i}^* \tag{1}$$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*. *Hint* : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

(c) Use the result from (b) together with eq. (1) to find (c_1^*, c_2^*).

(d) What's the probability of under-investment (i.e. that nobody writes the reference)? What's the prob. of overinvestment (i.e. both write)?

PS8, Ex. 6.a:

Code examples

Matrix, no player names:

	L (q)	R (1-q)
T (p)		
B (1-p)		

Matrix, no colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)		
	B (1-p)		

Matrix, with colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	
	B (1-p)		