Microeconomics III: Problem Set 11[a]

Thor Donsby Noe (thor.noe@econ.ku.dk) & Christopher Borberg (christopher.borberg@econ.ku.dk)
December 4 2019

Department of Economics, University of Copenhagen

[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS12, Ex. 1 (A): Job-market signaling

PS12, Ex. 1 (A): Job-market signaling

(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?

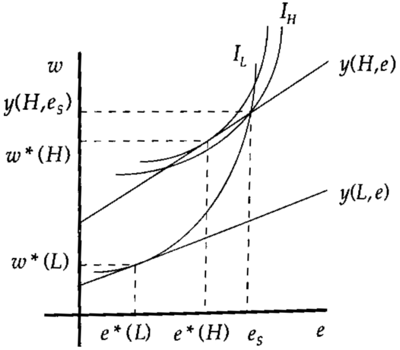


Figure 4.2.8.

Step 1: **Explain the graphs $\boldsymbol{I_L, I_H, y(L,e), y(H,e)}$.**

PS12, Ex. 1 (A): Job-market signaling

(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?

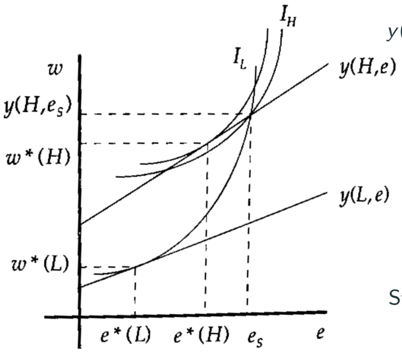


Figure 4.2.8.

Step 1: Explain $I_L, I_H, y(L,e), y(H,e)$:

For each type of worker $\eta \in L, H$:

- I_η: The indifference curve over which the worker's utility is constant. I.e. how much the wage must increase to compensate for higher education.
- $y(\eta, e)$: The expected output of a worker with ability η and education e which is equal to the wage offered by the firms under competition.
  I.e. education is now productive and more so for the high-ability worker.

Under complete information, the optimal education is where a worker's indifference curve is tangent to her productivity.

Step 2: **Why can H not choose $e^*(H)$ in a separating equilibrium?**

PS12, Ex. 1 (A): Job-market signaling

(A) Consider figure 4.2.8 in Gibbons (p. 201). Remind yourselves about the separating equilibrium related to the figure. Why can the high type not choose $e^*(H)$ in a separating equilibrium?

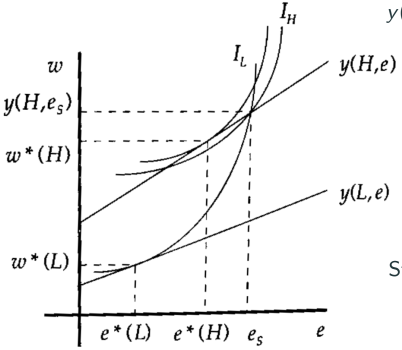


Figure 4.2.8.

Step 1: Explain $I_L, I_H, y(L,e), y(H,e)$:

For each type of worker $\eta \in L, H$:

- I_η: The indifference curve over which the worker's utility is constant. I.e. how much the wage must increase to compensate for higher education.
- $y(\eta, e)$: The expected output of a worker with ability η and education e which is equal to the wage offered by the firms under competition.
  I.e. education is now productive and more so for the high-ability worker.

Under complete information, the optimal education is where a worker's indifference curve is tangent to her productivity.

Step 2: Why can H not choose $e^*(H)$?

In a separating equilibrium, the firms perfectly identify H and L by education choices. However, as $[e^*(H), w^*(H)]$ is above L's indifference curve, L would imitate H. Thus, H needs to increase education to e_s to credibly signal type H.

PS12, Ex. 2 (A): Farrell & Rabin (1996): ”Cheap talk”

PS12, Ex. 2 (A): Farrell & Rabin (1996): "Cheap talk"

(A) In their paper "Cheap Talk" published in the *Journal of Economic Perspectives* (1996), Joseph Farrell and Matthew Rabin describe the following situation: "Sally knows which one of two tasks is efficient to perform. Rayco [the firm] could hire Sally specifically to perform Job 1, specifically to perform Job 2, or as a highly paid manager who will choose which job to perform. If Rayco knew which task is efficient, it would still hire her to perform the task, but at a lower salary, because she has lost her informational advantage. Sally wants to be hired as manager, but prefers to be hired to do the right task and be more productive rather than to do the wrong task and be less productive." Payoffs in this situation are

		Job 1	Job 2	Manager
Sally's knowledge	Task 1 efficient	2, 5	1, -2	3, 3
	Task 2 efficient	1, -2	2, 5	3, 3

where the left number in each cell is Sally's payoff and the right number is the firm's payoff (note that this matrix does not describe the normal form of the game!)

(a) Formulate this strategic situation as a cheap talk game, assuming that the type space is equal to the message space ($T = M$).

(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

PS12, Ex. 2.a (A): Farrell & Rabin (1996). Formulate a cheap talk game

		Job 1	Job 2	Manager
Sally's knowledge	Task 1 efficient	2, 5	1, -2	3, 3
	Task 2 efficient	1, -2	2, 5	3, 3

(a) Formulate this strategic situation as a cheap talk game, assuming that the type space is equal to the message space ($T = M$).

PS12, Ex. 2.b (A): Farrell & Rabin (1996). Find a separating PBE

Sally's knowledge		Job 1	Job 2	Manager
	Task 1 efficient	2, 5	1, -2	3, 3
	Task 2 efficient	1, -2	2, 5	3, 3

(b) Show that a separating equilibrium exists where Sally truthfully reveals which job is efficient, and the firm then places Sally in that specific job.

PS12, Ex. 2.c (A): Farrell & Rabin (1996). Discuss if PBE is reasonable?

Sally's knowledge		Job 1	Job 2	Manager
	Task 1 efficient	2, 5	1, -2	3, 3
	Task 2 efficient	1, -2	2, 5	3, 3

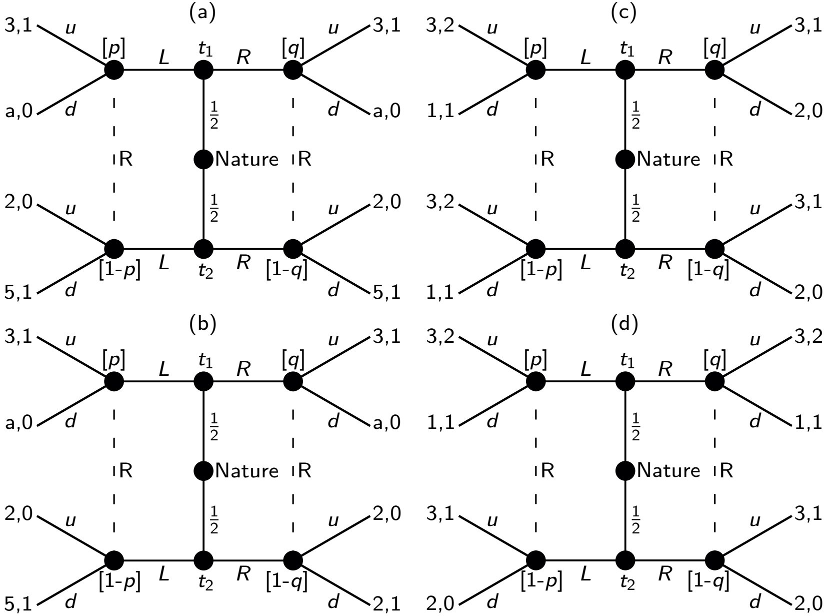
(c) Discuss whether or not this separating equilibrium seems reasonable. Why isn't Sally able to convince the firm to give her the manager position?

PS12, Ex. 3: Cheap talk games (extensive form)

PS12, Ex. 3: Cheap talk games (extensive form)

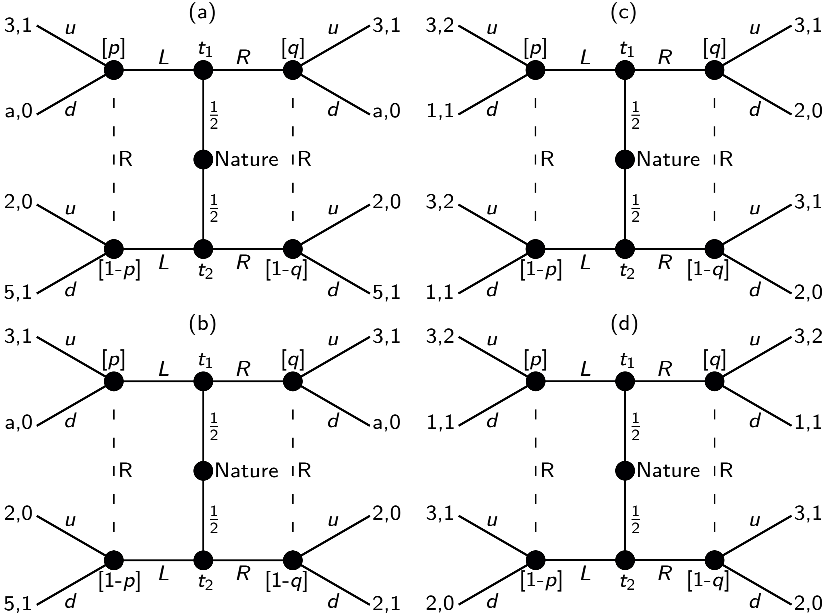
Consider the games (a)-(d) on the next page. (ii) Which of these are cheap talk games? (ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

(ii) Which of these are cheap talk games?



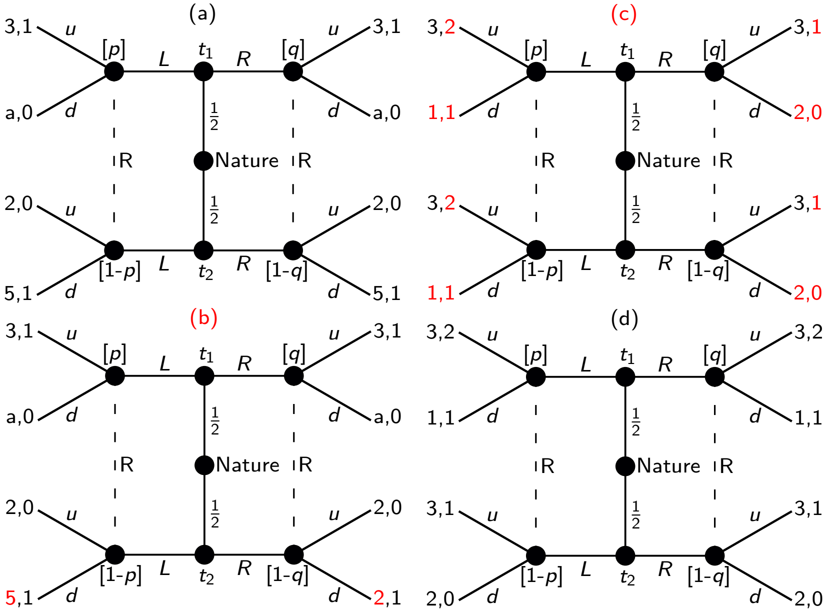
PS12, Ex. 3.i: Cheap talk games (extensive form)

(ii) In cheap talk games, payoffs don't depend on the message: $u_S(t_i, a_k), u_R(t_i, a_k)$.



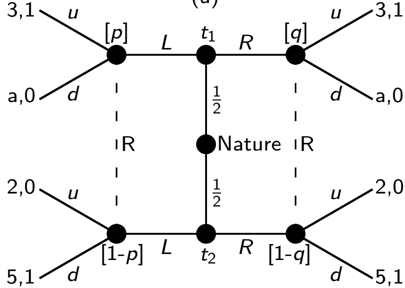
PS12, Ex. 3.i: Cheap talk games (extensive form)

(ii) (a) and (d) are cheap talk games as messages don't affect $u_S(t_i, a_k), u_R(t_i, a_k)$:

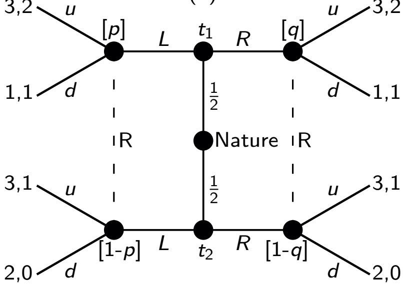


(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

(a)

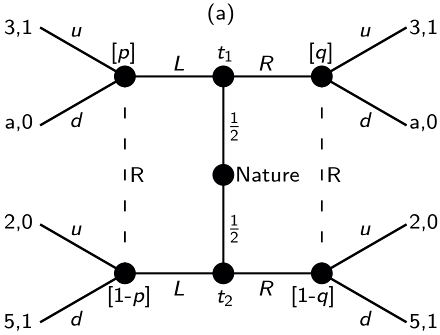


(d)



PS12, Ex. 3.ii.a: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

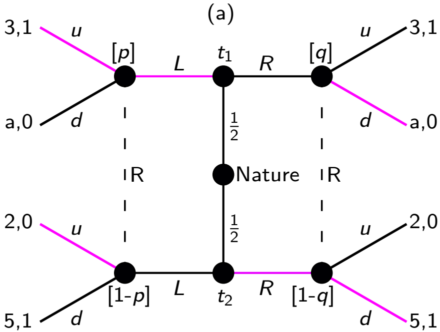


Step 1: **For the separating strategy (*L*,*R*), go over SR3, SR2R, and SR2S. PBE?**

Step 2: For the separating strategy (*R*,*L*), go over SR3, SR2R, and SR2S. PBE?

PS12, Ex. 3.ii.a: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u, \ a^*(R) = d$.

   SR2S: t_1 will not deviate if

   $u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$

   t_2 will never deviate as

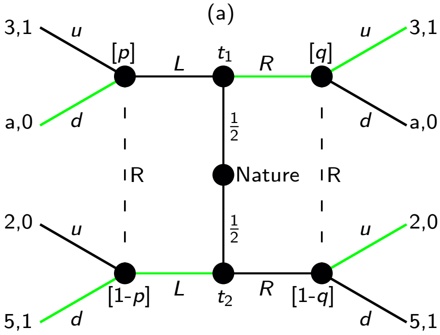
   $u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$

   PBE: No deviation if $a \leq 3$ (pink).

Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: **For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?**

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: **Write up the set of separating PBE.**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u, \ a^*(R) = d$.

   SR2S: t_1 will not deviate if

   $u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$

   t_2 will never deviate as

   $u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$

   PBE: No deviation if $a \leq 3$ (pink).

2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

   SR2R: R: $a^*(L) = d, \ a^*(R) = u$.

   SR2S: t_1 will not deviate if

   $u_S(R, u|t_1) = 3 \geq a = u_S(L, d|t_1)$

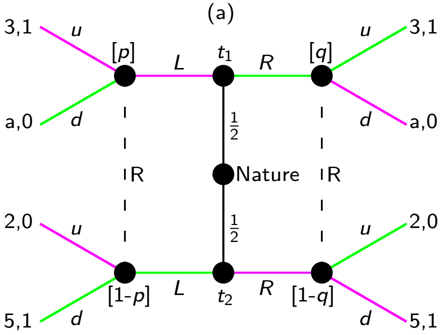
   t_2 will never deviate as

   $u_S(L, d|t_2) = 5 > 2 = u_S(R, u|t_2)$

   PBE: No deviation if $a \leq 3$ (green).

PS12, Ex. 3.ii.a: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



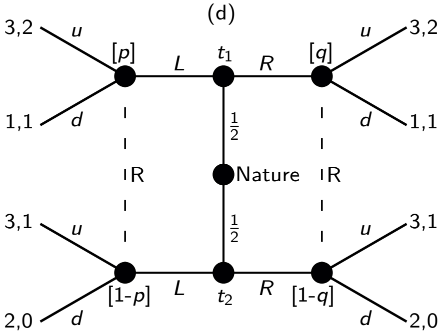
Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the set of separating PBE.

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$
   SR2R: R: $a^*(L) = u, \; a^*(R) = d$.
   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq a = u_S(R, d|t_1)$$
   t_2 will never deviate as
   $$u_S(R, d|t_2) = 5 > 2 = u_S(L, u|t_2)$$
   PBE: No deviation if $a \leq 3$ (pink).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$
   SR2R: R: $a^*(L) = d, \; a^*(R) = u$.
   SR2S: t_1 will not deviate if
   $$u_S(R, u|t_1) = 3 \geq a = u_S(L, d|t_1)$$
   t_2 will never deviate as
   $$u_S(L, d|t_2) = 5 > 2 = u_S(R, u|t_2)$$
   PBE: No deviation if $a \leq 3$ (green).
3. Two separating PBE exist for $a \leq 3$:
   $$PBE = \left\{ \begin{array}{c} (L, R), (u, d), p = 1, q = 0 \\ (R, L), (d, u), p = 0, q = 1 \end{array} \right\}$$

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.

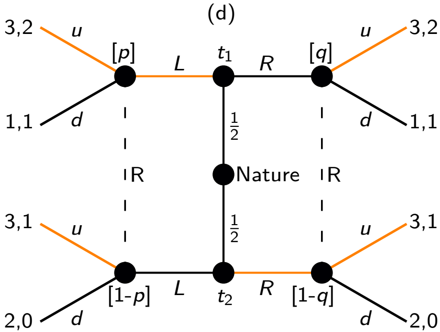


Step 1: **For the separating strategy (*L,R*), go over SR3, SR2R, and SR2S. PBE?**

Step 2: For the separating strategy (*R,L*), go over SR3, SR2R, and SR2S. PBE?

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: **For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?**

1. SR3: R: Beliefs given S's strategy: $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

SR2R: R: $a^*(L) = u,\ a^*(R) = u$.

SR2S: t_1 will not deviate if

$$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$

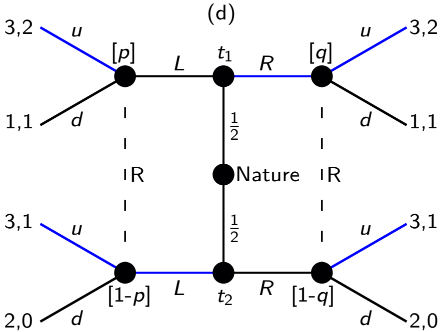
t_2 will never deviate as

$$u_S(R, u|t_2) = 3 > 3 = u_S(L, u|t_2)$$

PBE: No deviation → PBE (orange).

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

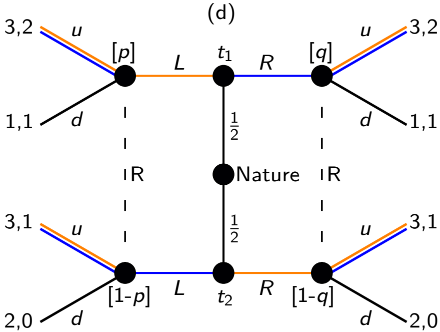
Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: **Write up the full set of separating PBE.**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$
   SR2R: R: $a^*(L) = u, \; a^*(R) = u$.
   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$
   t_2 will never deviate as
   $$u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$$
   PBE: No deviation $\rightarrow$ PBE (orange).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$
   SR2R: R: $a^*(L) = u, \; a^*(R) = u$.
   SR2S: t_1 will not deviate as
   $$u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$$
   t_2 will not deviate as
   $$u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$$
   PBE: No deviation $\rightarrow$ PBE (blue).

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)

(ii) For those which are cheap talk games, find a separating equilibrium if such an equilibrium exists, or show that no separating equilibrium exists.



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the full set of separating PBE.

Step 4: **Explain. Why can the signals be considered "cheap talk"?**

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u,\ a^*(R) = u$.

   SR2S: t_1 will not deviate if

   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$

   t_2 will never deviate as

   $$u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$$

   PBE: No deviation $\rightarrow$ PBE (orange).

2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

   SR2R: R: $a^*(L) = u,\ a^*(R) = u$.

   SR2S: t_1 will not deviate as

   $$u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$$

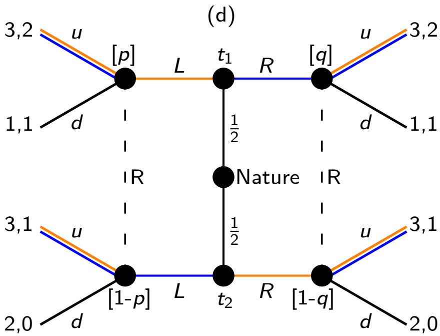
   t_2 will not deviate as

   $$u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$$

   PBE: No deviation $\rightarrow$ PBE (blue).

3. $\left\{ \begin{array}{l} (L, R), (u, d), p = 1, q = 0 \\ (R, L), (d, u), p = 0, q = 1 \end{array} \right\}$

PS12, Ex. 3.ii.d: Cheap talk games (extensive form)



Step 1: For the separating strategy (L,R), go over SR3, SR2R, and SR2S. PBE?

Step 2: For the separating strategy (R,L), go over SR3, SR2R, and SR2S. PBE?

Step 3: Write up the full set of separating PBE.

Step 4: Explain. Why can the signals be considered "cheap talk"?

1. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 1$ and $\mu(t_1|R) = q = 0$

   SR2R: R: $a^*(L) = u$, $a^*(R) = u$.

   SR2S: t_1 will not deviate if
   $$u_S(L, u|t_1) = 3 \geq 3 = u_S(R, u|t_1)$$
   t_2 will never deviate as
   $$u_S(R, u|t_2) = 3 \geq 3 = u_S(L, u|t_2)$$

   PBE: No deviation → PBE (orange).
2. SR3: R: Beliefs given S's strategy:
   $\mu(t_1|L) = p = 0$ and $\mu(t_1|R) = q = 1$

   SR2R: R: $a^*(L) = u$, $a^*(R) = u$.

   SR2S: t_1 will not deviate as
   $$u_S(R, u|t_1) = 3 \geq 3 = u_S(L, u|t_1)$$
   t_2 will not deviate as
   $$u_S(L, u|t_2) = 3 \geq 3 = u_S(R, u|t_2)$$

   PBE: No deviation → PBE (blue).
3. $$\left\{ \begin{array}{c} (L, R), (u, d), p = 1, q = 0 \\ (R, L), (d, u), p = 0, q = 1 \end{array} \right\}$$
4. Two "weak separating PBE": S is indifferent between signals which are "cheap talk" as R choose u anyway.

PS12, Ex. 4: Three-type job-applicant cheap talk game

PS12, Ex. 4: Three-type job-applicant cheap talk game

Consider the following job-applicant cheap talk game based on Farrell-Rabin (1996). Suppose that there are three types of potential applicants (high ability, medium ability, and low ability) and the firm can place the applicant in one of three possible positions (highly qualified, medium qualified, low qualified). The applicant is equally likely to be each of the three types (probability 1/3). Payoff are represented below, where for each cell, the left entry gives the payoff of the applicant, and the right entry gives the payoff of the firm, conditional on the firm's action and the applicant's type. Notice: this matrix does not show the normal form game! It merely gives you the payoffs for each type-job combination, but does not incorporate the cheap talk message.
The game is as follows: first, the applicant's type is realized: $t \in \{L, M, H\}$, where $t = L$ corresponds to low ability etc. The applicant observes his type and sends a cheap talk message $m \in \{L, M, H\}$. The firm observes the message and chooses a job for the applicant: $a \in \{L, M, H\}$, where $a = L$ corresponds to giving the applicant the low qualified job etc.

	Highly qualified	Medium qualified	Low qualified
High ability	3, 3	0, 0	0, 0
Medium ability	1, 0	2, 2	0, 0
Low ability	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

(b) Show that a partial pooling PBE does exist, where $m(H) = H$ and $m(M) = m(L) = M$. What are the firm's beliefs? Solve for each case.

(c) (If time permits) Does a fully pooling PBE exist, $m(H) = m(M) = m(L) = M$?

PS12, Ex. 4.a: Three-type: Fully separating PBE

	Highly qualified	Medium qualified	Low qualified
High ability	3, 3	0, 0	0, 0
Medium ability	1, 0	2, 2	0, 0
Low ability	1, 0	2, 0	1, 1

(a) Show that no fully separating PBE exist, where each type of applicant sends a different message. What is the intuition behind this result?

PS12, Ex. 4.b: Three-type: Partial pooling PBE

	Highly qualified	Medium qualified	Low qualified
High ability	3, 3	0, 0	0, 0
Medium ability	1, 0	2, 2	0, 0
Low ability	1, 0	2, 0	1, 1

(b) Show that a partial pooling PBE does exist, where the high-ability applicant sends the message $m = H$, and the other two types send the message $m = M$. What are the firm's beliefs about the applicant if he receives the message $m = H$ or $m = M$ (on the equilibrium path), or if he receives the message $m = L$ (off the equilibrium path)? In each case, solve for the firm's optimal action given its beliefs.

PS12, Ex. 4.c: Three-type: Fully pooling PBE

	Highly qualified	Medium qualified	Low qualified
High ability	3, 3	0, 0	0, 0
Medium ability	1, 0	2, 2	0, 0
Low ability	1, 0	2, 0	1, 1

(c) (If time permits) Does a fully pooling PBE exist where all types send the message $m = H$? If so, describe the players' equilibrium strategies and beliefs, and discuss whether this pooling PBE looks more or less reasonable than the partial pooling PBE from (b).