PS11: Signaling games in general

Players:

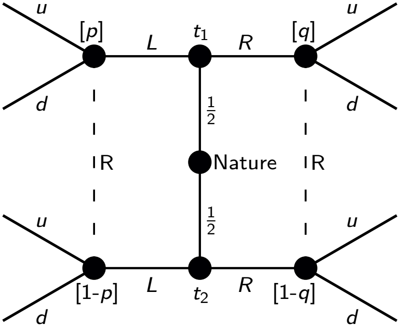
- 2 players: Sender (S) and receiver (R). E.g. firm and consumer, or employer and employee (Spence).

Timing:

1. Nature chooses the sender's type from $T = \{t_1, ...\}$.
2. S: The sender realizes her type and sends a signal from $M = \{m_1, ...\}$, typically either L (left) or R (right).
3. R: The receiver observes m (but not the type t!) and forms his beliefs:
   $$p = \mu(t_1|L) \text{ and } q = \mu(t_1|R)$$
   Consequently, for S having two possible types:
   $1 - p = \mu(t_2|L)$ and $1 - q = \mu(t_2|R)$
4. R: The receiver chooses an action from $A = \{a_1, ...\}$, e.g. *up* or *down*.
5. Payoffs are realized.

Four possible equilibria for two types:

- Pooling on L or pooling on R.
- Separating: t_1 plays L and t_2 plays R or the other way around.



Cookbook: For each possible equilibrium go over signaling requirements 3 and 2:

SR3: R: Find the beliefs p, q given S's eq. strategy. (Only consider beliefs that are consistent with S's eq. strategy.)

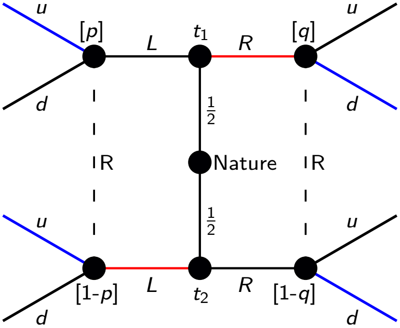
SR2R: R: Given beliefs, find $a(m_j|\mu(t_1|m_j))$.

SR2S: S: Does t_1 or t_2 want to deviate?

PBE: No deviation $\rightarrow$ PBE. Pooling on L: Find off-eq. $a(R|q)$ $\rightarrow$ possibly two different PBE for different q.

PS11, Ex. 3.b: Notation (separating PBE)

(b) Consider a possible separating PBE where t_1 sends message R, t_2 sends message L, and where the receiver chooses u if and only if he receives message L. Can you write down payoffs for this game such that nobody has an incentive to deviate?



SR3: R: Find the beliefs of R given S's equilibrium strategy. (In equilibrium, we only consider beliefs of R that are consistent with S's eq. strategy.)

SR2R: R: Find R's optimal strategy given beliefs about S's strategy.

SR2S: S: Check whether S wants to deviate.

PBE: Write up the conditions such that SR2R and SR2S hold (no incentive to deviate) for the following **PBE**:

$$\{(\underbrace{R}_{m(t_1)}, \underbrace{L}_{m(t_2)}), (\underbrace{u}_{a(L)}, \underbrace{d}_{a(R)}), \underbrace{p=0}_{\mu(t_1|L)}, \underbrace{q=1}_{\mu(t_1|R)}\}$$

Note: For the sender's strategy, we always write the message of t_1 first and t_2 second. Whereas for the receiver's strategy, we always write the action in response to L first and to R second.

SR3: In the separating PBE, R has beliefs:

$$\mu(t_1|L) = p^* = 0$$

$$\mu(t_1|R) = q^* = 1$$

SR2R: $\mathbb{E}[u_R(L, u|p{=}0)] \geq \mathbb{E}[u_R(L, d|p{=}0)]$
$\mathbb{E}[u_R(R, d|q{=}1)] \geq \mathbb{E}[u_R(R, u|q{=}1)]$

SR2S: $u_S(R, d|t_1) \geq u_S(L, a(L)|t_1)$
$u_S(L, u|t_2) \geq u_S(R, a(R)|t_2)$