



## Microeconomics III: Problem Set 10<sup>a</sup>

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<sup>a</sup>Slides created for exercise class 3 and 4, with reservation for possible errors.

PS8, Ex. 1 (A): Asymmetric values (second-price sealed bid auction)

PS8, Ex. 2 (A): Crimea Through a Game-Theory

PS8, Ex. 3 (A): The 'Lemons' model (asymmetric information)

PS8, Ex. 4: A simple principal-agent model of corruption

PS8, Ex. 5: Extensive form games (Perfect Bayesian Equilibria)

PS8, Ex. 6: Extensive form game (Mixed-strategy Perfect Bayesian Equilibria)

PS8, Ex. 7: Dissolving a partnership (Perfect Bayesian Equilibria)

**PS8, Ex. 1 (A): Asymmetric values  
(second-price sealed bid auction)**

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## PS8, Ex. 1 (A): Asymmetric values (second-price sealed bid auction)

Suppose there are two bidders who have private but asymmetric values. In particular,  $v_1 \sim U(0, 1)$  and  $v_2 \sim U(0, 2)$ . Suppose the auction format is second-price sealed bid. When the values are private and symmetric, it is a weakly dominant strategy to bid one's value. Is this still true when the values are asymmetric?

## PS8, Ex. 1 (A): Asymmetric values (second-price sealed bid auction)

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## **PS8, Ex. 2 (A): Crimea Through a Game-Theory**

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Read through the New York Times article *Crime Through a Game-Theory Lens* by Tyler Cowen (co-author of the popular economics blog *Marginal Revolution*). Try to think about how you would set up models to describe the situations he writes about. *(This exercise is just for reflection, no answer will be provided).*

**PS8, Ex. 3 (A): The 'Lemons' model  
(asymmetric information)**

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## PS8, Ex. 3 (A): The 'Lemons' model (asymmetric information)

Consider the The 'Lemons' model of Akerlof. Suppose that used cars come in two types: high-quality “beauties” and low-quality “lemons”. Lemon-owners are willing to sell for \$800 but Beauty-owners will not sell for anything less than \$2000. Buyers will pay up to \$1200 for a lemon and up to \$2400 for a beauty.

- (a) Describe what would happen in the used-car market if buyers can distinguish between beauties and lemons.
- (b) What would happen if buyers cannot do so, and know that half of all used cars are lemons? Draw this as a dynamic game of incomplete information, where nature chooses the type of the car, the seller observes this and sets a price (any positive real number) and the buyer decides whether to buy or not.
- (c) Find a Perfect Bayesian Equilibrium of this model.

## PS8, Ex. 3.a (A): The 'Lemons' model (asymmetric information)

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## **PS8, Ex. 4: A simple principal-agent model of corruption**

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Suppose two lobbyists,  $i = 1, 2$ , are trying to persuade a policymaker to implement their preferred policy by making a costly effort  $e_i \in [0, 1]$ . The policymaker can only implement one of the policies, and will implement the policy of the lobbyist who makes the most effort (you can also think of the policymaker as being corrupt, and the effort being a bribe.) The point is, that the lobbyist has to make the effort *before* he learns if his policy is implemented.

The value to  $i$  of having his preferred policy implemented is  $v_i$ , where  $v_i \sim U(0, 1)$  independently (private values). The lobbyists know their own valuation, but not that of the other lobbyist.

- (a) Rewrite this as an auction. What is the difference to the auctions we have seen so far?
- (b) Check that there is a symmetric Bayesian Nash Equilibrium of the type  $b_i(v_i) = cv_i^2$ , and find  $c$ .

## PS8, Ex. 4.a: A simple principal-agent model of corruption

Suppose two lobbyists,  $i = 1, 2$ , are trying to persuade a policymaker to implement their preferred policy by making a costly effort  $e_i \in [0, 1]$ . The policymaker can only implement one of the policies, and will implement the policy of the lobbyist who makes the most effort (you can also think of the policymaker as being corrupt, and the effort being a bribe.) The point is, that the lobbyist has to make the effort *before* he learns if his policy is implemented.

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**PS8, Ex. 5: Extensive form games  
(Perfect Bayesian Equilibria)**

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**PS8, Ex. 6: Extensive form game  
(Mixed-strategy Perfect Bayesian  
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## PS8, Ex. 6: Mixed-strategy Perfect Bayesian Equilibria (extensive form game)

**PS8, Ex. 7: Dissolving a partnership  
(Perfect Bayesian Equilibria)**

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## PS8, Ex. 7: Dissolving a partnership (Perfect Bayesian Equilibria)

Difficult. Exercise 4.10 in Gibbons (p. 250). Two partners must dissolve their partnership. Partner 1 currently owns share  $s$  of the partnership, partner 2 owns share  $1 - s$ . The partners agree to play the following game: partner 1 names a price,  $p$ , for the whole partnership, and partner 2 then chooses either to buy 1's share for  $ps$  or to sell his or her share to 1 for  $p(1 - s)$ . Suppose it is common knowledge that the partners' valuations for owning the whole partnership are independently and uniformly distributed on  $[0, 1]$ , but that each partner's valuation is private information. What is the perfect Bayesian equilibrium?