Microeconomics III: Session 3

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Outline

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PS3, Ex.

PS3, Ex. 1 (A): Dominance and best response

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1. (A) Show that for each of the following two games, the only Nash equilibrium is in pure strategies. Describe the intuition for this result. What do these two games have in common?

		Player 2	
		L	R
Player 1	U	5, 5	1, 6
		6, 1	2, 2

(D, R) is a unique Pure Strategy Nash Equilibrium (PSNE). The game is a Prisoner's Dilemma as it fulfills:

$$T > R > P > S \Leftrightarrow 6 > 5 > 2 > 1$$

i.e. the **T**emptation to deviate (6) is greater than the **R**eward for cooperating on the socially optimal outcome (5) and the **P**unishment payoff (2) is greater than the "**S**ucker's" payoff (1).

PS3, Ex. 1 (A): Dominance and best response

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		Player 2		
		L	C	R
Player 1	U	1, 0	1, 2	0, 1
	D	0, 3	0, 1	2, 0

(U, C) is a unique Pure Strategy Nash Equilibrium (PSNE) as no other combination of (mixed or pure) strategies gives as high payoffs.

Iterated Elimination of Strictly Dominated Strategies (IESDS) leads to the same outcome as the best responses (eliminate R then D and lastly L).

As both games can be solved by IESDS they both have a unique PSNE.

PS3, Ex. 2 (A): Equilibrium selection

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2. (A) Solve for all pure strategy Nash equilibria. Which equilibrium do you find most reasonable?

		Player 2		
		a	b	c
Player 1	A	2, 2	0, 0	-1, 2
	B	0, 0	0, 0	0, 0
	C	2, -1	0, 0	1, 1

$PSNE = \{(A, a), (B, b), (C, c)\}.$

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For **risk neutral** players (A, a) is the most reasonable as it maximizes payoff for both players.

For **risk averse** players avoiding A and a eliminates the risk of a negative payoff. (C, c) is more reasonable than (B, b) as the payoffs are positive.

PS3, Ex. 3 (A): NE proof using IEWDS

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3. (A) We have seen in the lectures that IESDS never eliminates a Nash Equilibrium. However, we saw in Problem Set 2 that this is not true if we do iterated elimination of weakly dominated strategies (IEWDS.) Go through the proof in the slides from lecture 2 and identify the step that is no longer true if we replace IESDS by IEWDS. That is, explain why the proof is no longer true when we replace ‘strict domination’ by ‘weak domination’.

PS3, Ex. 3 (A): NE proof using IEWDS

The proof that all NE survive IESDS holds by contradiction. We **highlight** where the contradiction breaks down using IEWDS instead:

- Let (s_1^*, s_2^*) be a NE.
- Say we carry out **IEWDS** and s_1^* is the first NE strategy to be eliminated (in round n of elimination).
- Then there must be a strategy $s_1^{'} \neq s_1^*$ that **weakly** dominates s_1^*, i.e.

$$\forall s_2 \in S_2^n : \; u_1(s_1^*, s_2) \underbrace{\leq}_{\textbf{Weak}} u_1(s_1^{'}, s_2)$$

and the inequality holds strictly for at least one strategy $s_2^{'} \in S_2^n$ where S_2^n is the set of player-2 strategies that have not been eliminated in rounds 1, ..., $n-1$.

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and the inequality holds strictly for at least one strategy $s_2^{'} \in S_2^n$ where S_2^n is the set of player-2 strategies that have not been eliminated in rounds $1, \ldots, n-1$.

- Since $s_2^* \in S_2^n$, inequality (1) also means

$$u_1(s_1^*, s_2^*) \underbrace{\leq}_{\textbf{Weak}} u_1(s_1^{'}, s_2^*)$$

- ~~But~~ (s_1^*, s_2^*) is a NE, so by definition

$$\forall s_1 \in S_1 : \; u_1(s_1^*, s_2^*) \geq u_1(s_1, s_2^*)$$

- **No** contradiction!

Conclusion: for a NE (s_1^*, s_2^*) IEWDS can eliminate s_1^* if $s_1^{'}, s_2^{'}$ exist such that:

$$\text{for } s_1^{'} \in S_1^n : \; u_1(s_1^*, s_2^*) = u_1(s_1^{'}, s_2^*)$$

and

$$\text{for } s_2^{'} \in S_2^n : \; u_1(s_1^*, s_2^{'}) < u_1(s_1^{'}, s_2^{'})$$

PS3, Ex. 4 (A): Mixed strategy price competition

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4. (A). Consider price competition between two firms when some consumers are informed about prices and others are not. Firms have zero marginal cost and they set price simultaneously; for the sake of this example, assume each price can only take one of the following values: 80, 54, 38. The market consists of two consumers. The uninformed consumer will visit a firm at random (probabilities $\frac{1}{2}, \frac{1}{2}$) and buy from it, regardless of the price. The informed consumer will visit the firm with the lowest price and buy from it. If both firms set the same price, assume that the informed consumer picks a firm at random (probabilities $\frac{1}{2}, \frac{1}{2}$).

(a) Argue that this game can be represented by the following bimatrix.

	p_2=80	p_2=54	p_2=38
p_1=80	80, 80	40, 81	40, 57
p_1=54	81, 40	54, 54	27, 57
p_1=38	57, 40	57, 27	38, 38

(b) Show that there is no Nash equilibrium in pure strategies.

(c) Confirm that the following strategy profile is a Nash equilibrium: each firm plays price 80 with probability 0.232, price 54 with probability 0.361, and price 38 with probability 0.407.

(d) Why do you think the equilibrium is so different from the standard Bertrand pricing game (i.e. where competition drives price down to marginal cost)?

PS3, Ex. 4 (A): Mixed strategy price competition

(a) The game in normal form and bimatrix:

Players: *Firm 1, Firm 2*. Strategies: $p_i \in S_i = S = \{80, 54, 38\}$

Payoffs consist of *payoff from the informed consumer* + *payoff from the uninformed*. I.e. payoffs for player $i \neq j$:

$$u_i(p_i, p_j) = \begin{cases} p_i + \frac{1}{2} p_i & \text{if} \quad p_i < p_j \\ \frac{1}{2} p_i + \frac{1}{2} p_i & \text{if} \quad p_i = p_j \\ 0 + \frac{1}{2} p_i & \text{if} \quad p_i > p_j \end{cases} = \begin{cases} \frac{3}{2} p_i & \text{if} \quad p_i < p_j \\ p_i & \text{if} \quad p_i = p_j \\ \frac{1}{2} p_i & \text{if} \quad p_i > p_j \end{cases}$$

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Which can be represented as:

	p_j=80	p_j=54	p_j=38
p_i=80	80, -	$\frac{1}{2}80=40$, -	$\frac{1}{2}80=40$, -
p_i=54	$\frac{3}{2}54=81$, -	54, -	$\frac{1}{2}54=27$, -
p_i=38	$\frac{3}{2}80=57$, -	$\frac{3}{2}38=57$, -	38, -

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p_i=54	$\frac{3}{2}54=81$, -	54, -	$\frac{1}{2}54=27$, -
p_i=38	$\frac{3}{2}80=57$, -	$\frac{3}{2}38=57$, -	38, -

(b) Show that there is no Nash equilibrium in pure strategies:

		Firm 2		
		p_2=80	p_2=54	p_2=38
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PS3, Ex. 4 (A): Mixed strategy price competition

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(c) Confirm that the following strategy profile is a Nash equilibrium: each firm plays price 80 with probability 0.232, price 54 with probability 0.361, and price 38 with probability 0.407.

Remember: In an equilibrium in mixed strategies, a player is indifferent between all pure strategies that she is choosing with positive probability.

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Check that firm i is indifferent between all pure strategies when the opposing firm's strategy is given by the probability distribution $\widehat{p_j} = (0.232, 0.361)$:

$$u_i(p_i = 80, \widehat{p_j}) = 0.232 \cdot 80 + 0.361 \cdot 40 + (1 - 0.232 - 0.361) \cdot 40 = 49.280 \approx 49.3$$

$$u_i(p_i = 54, \widehat{p_j}) = 0.232 \cdot 81 + 0.361 \cdot 54 + (1 - 0.232 - 0.361) \cdot 27 = 49.275 \approx 49.3$$

$$u_i(p_i = 38, \widehat{p_j}) = 0.232 \cdot 57 + 0.361 \cdot 57 + (1 - 0.232 - 0.361) \cdot 38 = 49.267 \approx 49.3$$

There are rounding errors as the exact mixed strategy profile is $\widehat{p_j} = \left(\frac{193}{833}, \frac{8127}{22491}\right)$.

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In the standard Bertrand Oligopoly price competition would lead to the perfectly competitive outcome (price = marginal cost), here:

$$p_1^* = p_2^* = c = 0$$

Introduction of an uninformed consumer dampens the effect of price competition as a firm i can expect a revenue of at least $\frac{1}{2} p_i$ no matter what price p_i it sets.

Price competition could be increased by lowering the probability that an uninformed customer randomly picks the firm, i.e. through:

1. A higher share of informed customers.
2. More competing firms (moreover, increasing the pure price competition).

PS3, Ex. 5: Luxembourg as a rogue state

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Assume that Luxembourg has turned into a rogue state. It is close to acquiring nuclear weapons, which would threaten the stability in the whole region. The Vatican (V) and Denmark (D) are preparing an attack on Luxembourg's nuclear research facilities to stop or slow down its nuclear program. The probability that the attack will be a success is

$$p(s_V, s_D) = s_V + s_D - s_v s_D,$$

where $s_i \in [0, 1]$ is the share of its military capacity that country i ($i \in \{V, D\}$) uses in the attack. If the attack is successful then each country receives a payoff of 1. The cost of participating in the attack for country i is

$$c_i(s_i) = s_i^2$$

The objective of each country is to maximize its expected payoff from the attack minus the cost.

(a) Suppose that the Vatican and Denmark choose the shares of military capacity to use in the attack simultaneously and independently. Find the Nash equilibrium (NE) of this game.

(b) Find the social optimum (SO) under the condition that the two countries use the same share of their military capacity. I.e., find the $\bar{s}_V = \bar{s}_D = \bar{s}$ that maximizes aggregate payoff from the attack minus costs. Compare with the equilibrium from question (a) and give an intuitive explanation of your findings.

PS3, Ex. 5: Luxembourg as a rogue state

(a) Find the NE in the static game:

Expected payoff for player $i \neq j$:

$$u_i(s_i, s_j) = \underbrace{s_i + s_j - s_i s_j}_{\text{Probability of success}} - \underbrace{s_i^2}_{\text{Cost}}$$

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Find the best-response function for i:

$$FOC: \frac{\delta u_i}{\delta s_i} = 1 + 0 - s_j - 2s_i = 0$$

$$s_i = \frac{1 - s_j}{2}$$

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Take advantage of symmetry:

$$s_i = \frac{1 - s_i}{2}$$

$$2s_i = s_i = 1$$

$$s_i^* = \frac{1}{3}$$

i.e. $NE = \left\{(s_D^*, s_V^*) = (\frac{1}{3}, \frac{1}{3})\right\}$

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(b) Find the SO given shares are equal:

Expected payoff for i, $\bar{s}_D = \bar{s}_V = \bar{s}$:

$$u_i(\bar{s}) = \underbrace{\bar{s} + \bar{s} - \bar{s}\bar{s}}_{\text{Probability of success}} - \underbrace{\bar{s}^2}_{\text{Cost}}$$

$$= 2\bar{s} - 2\bar{s}^2$$

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Social planner target function:

$$\pi^S(\bar{s}) = \underbrace{2}_{\text{Countries}} (2\bar{s} - 2\bar{s}^2) = 4\bar{s} - 4\bar{s}^2$$

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Social planner target function:

$$\pi^S(\bar{s}) = \underbrace{2}_{\text{Countries}} (2\bar{s} - 2\bar{s}^2) = 4\bar{s} - 4\bar{s}^2$$

Find the social optimum (SO):

$$FOC: \frac{\delta \pi^S}{\delta s_i} = 4 - 8\bar{S} = 0$$

$$\bar{S} = \frac{4}{8} = \frac{1}{2} > \frac{1}{3}$$

i.e. the SO is higher than the NE as the positive externality is not rewarded, leading to an incentive to free ride.

PS3, Ex.

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		Player 2	
		A	B
Player 1	A		
	B		

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		A	B
Player 1	A	,	
	B		

	A	B
A		
B		