Microeconomics III: Problem Set 8[a]

Thor Donsby Noe (thor.noe@econ.ku.dk) & Christopher Borberg (christopher.borberg@econ.ku.dk)
November 13 2019

Department of Economics, University of Copenhagen

[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS8, Ex. 1 (A):

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game:

Consider the following game G:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(a) Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

PS8, Ex. 1 (A): Trigger strategy in infinitely repeated game - exam answer)

- The following is how I personally would write my answer at the exam, it cannot be guaranteed that writing your answer in this way would yield fill points.

(a) Suppose that G is repeated infinitely many times, so that we have G(1, ∞). Define trigger strategies such that the outcome of all stages is (A,X). Find the smallest value of δ such that these strategies constitute a SPNE.

Trigger strategies such that the outcome of all stages of the game is (A,X) are possible using respectively B,Y or C,Z as the threats. Since the threats B,Y will make the SPNE possible for the smallest δ, I will use B,Y in the trigger strategies i define:

1. Trigger strategy $p1$: In the 1st turn, play A. In every subsequent turn, if outcome from every previous turn was (A,X), play A, otherwise play B.
2. Trigger strategy $p2$: In the 1st turn, play X. In every subsequent turn, if outcome from every previous turn was (A,X), play X, otherwise play Y.

Player 2 has the highest incentive to deviate, so I only examine player 2's incentive to deviate. In order to find the lowest δ to secure cooperation I set up the inequality for which the payoff for cooperation is higher than the payoff for deviating:

$$\frac{6}{1-\delta} \geq 8 + \frac{2\delta}{1-\delta} \Rightarrow 6 \geq 8 - 8\delta + 2\delta \Rightarrow \quad \delta \geq \frac{1}{3} \tag{1}$$

$\delta = \frac{1}{3}$ is the smallest value for which the strategies constitute a SPNE.

PS8, Ex. 2 (A): Bayes Rule

PS8, Ex. 2 (A): Bayes Rule:

- Review the intuition from the ‘Doctor’ example in lecture 7, and then use Bayes’ rule to solve the following problem:
- A cab was involved in a hit and run accident at night. 85% of the cabs in the city are Green and 15% are Blue. A witness later recalls that the cab was Blue, and we know that this witness’ memory is reliable 80% of the time. Given the statement from the witness, calculate the probability that the cab involved in the accident was actually Blue.

PS8, Ex. 2 (A):

- P(B) The odds that the car is blue: $\frac{15}{100}$
- P(obs B given B): The odds that the witness says the car is blue, given that it is blue: $\frac{80}{100}$
- P(obs B given G): The odds that the witness says the car is blue, given that it is green: $\frac{20}{100}$
- P(obs B): The odds that the witness says the car is blue: P(obs B given B)*P(B)+P(obs B given G)*P(G)= $\frac{80}{100} * \frac{15}{100} + \frac{20}{100} * \frac{85}{100} = \frac{29}{100}$
- P(B given obs B): $\frac{P(ObsBgivenB) * P(B)}{P(obsB)} = \frac{\frac{85}{100} * \frac{85}{100}}{\frac{29}{100}} = 0.414$

PS8, Ex. 3 (A):

PS8, Ex. 3 (A):

PS8, Ex. 3.a (A):

PS8, Ex. 4:

PS8, Ex. 4:

PS8, Ex. 4.a:

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

Consider the ‘dating game’. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2’s types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2’s type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

PS8, Ex. 5: The dating game (Bayesian Nash Equilibria)

Consider the 'dating game'. In this game, player 1 wants to meet with player 2, even if player 2 is of type t_2, who does not want to meet with player 1. Player 2's types have equal probability.

Now suppose we construct an alternative version of the game, where player 1 has self respect, and only wants to meet up with player 2 if player 2 also wants to meet up (i.e. if player 2 is of type t_1). Otherwise, player 1 prefers to *miscoordinate* (i.e. not to meet up) as well. Player 1 still has the same preference for going to football versus the opera.

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

Hint: Write up the Bayesian game (players, type spaces, beliefs, action spaces, strategy spaces, and the type-dependent payoff matrices.)

(b) Find the set of (pure strategy) Bayesian Nash Equilibria of this game.

Hints:

1. Check for equilibria where player 1 plays *Football* and *Opera* respectively.
2. In equilibrium, a strategy should maximize expected payoff given the strategy of the other player and the probability of each type.

PS8, Ex. 5.a: The dating game (Bayesian Nash Equilibria)

(a) Write up this new game. Use the same payoffs as before (0,1, and 2) such that the payoff matrix when player 2's type is t_1 is the same as in the slides (Lecture 7, slides 22-26).

1. Players: P1, P2.
2. Type spaces: $T_1 = \{t\}, \ T_2 = \{t_1, t_2\}$
3. Beliefs: $\mathbb{P}_1(T_2 = t_1) = \mathbb{P}_1(T_2 = t_2) = \frac{1}{2}, \ \mathbb{P}_2(T_1 = t) = 1$
4. Action space: $A_i = \{Football, Opera\}$, for $i \in 1, 2$
5. Strategy spaces: $S_1 = \{F, O\}, \ S_2 = \{FF, FO, OF, OO\}$
6. Type-dependent payoff matrices:

Type t_1 $(p = \frac{1}{2})$

	F	O
F	2, 1	0, 0
O	0, 0	1, 2

Type t_2 $(p = \frac{1}{2})$

	F	O
F	0, 0	2, 2
O	1, 1	0, 0

PS8, Ex. 5.b: The dating game (Bayesian Nash Equilibria)

PS8, Ex. 6:

PS8, Ex. 6:

Difficult. Consider the public goods game from lecture 7. Suppose now instead that there is two-sided incomplete information. In particular, the cost of writing the reference is uniformly distributed between 0 and 2:

$$c_i \sim u(0,2) \text{ for } i = 1,2$$

In this setting, we can show that the players optimally follow a 'cutoff' strategy. Thus, the equilibrium strategies take the form

$$s_1^*(c_1) = \begin{cases} Write & \text{if} \quad c_1 \leq c_1^* \\ Don't & \text{if} \quad c_1 > c_1^* \end{cases}$$

$$s_2^*(c_2) = \begin{cases} Write & \text{if} \quad c_2 \leq c_2^* \\ Don't & \text{if} \quad c_2 > c_2^* \end{cases}$$

(a) Let $z_{-i}^* = \mathbb{P}(s_{-i}^* = Write)$, i.e. the probability that the other player plays 'Write' in equilibrium. Argue that
$$1 - c_i^* = z_{-i}^* \tag{2}$$

Hint: Calculate i's expected payoff from writing the reference and from not writing the reference, conditional on z_{-i}^*.

(b) A standard result on uniform distributions gives the following: if $x \sim u(0,2)$, then $\mathbb{P}(x < a) = \frac{a}{2}$. Use this to find z_i^*. *Hint* : Use the equilibrium strategy and your knowledge of the distribution of c_{-i}.

(c) Use the result from (b) together with eq. (2) to find (c_1^*, c_2^*).

(d) What's the probability of under-investment (i.e. that nobody writes the reference)? What's the prob. of overinvestment (i.e. both write)?

PS8, Ex. 6.a:

Code examples

Code examples

Matrix, no player names:

	L (q)	R (1-q)
T (p)		
B (1-p)		

Matrix, no colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)		
	B (1-p)		

Matrix, with colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	
	B (1-p)		