Recipe for a static Bayesian game (Bayesian Nash Equilibria)

1. The timing is as follows where p is a commonly known distribution:
    1.1 Nature draws all players' type according to p.
    1.2 Each player i learns her own type t_i.
    1.3 Players form their beliefs about the type profile.
    1.4 Players simultaneously choose actions and payoffs are realized.

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2. The static Bayesian game consists of:
   2.1 Players: $Player\ 1, ..., Player\ N$
   2.2 Type spaces: $T_1 = \{t_{11}, ..., t_{1K}\}, ...$
   2.3 Beliefs: $\mathbb{P}_1[t_2 = t_{21}] = \cdot, ...$
   2.4 Action spaces: $A_1 = \{a_1, ...\}, ...$
   2.5 Strategy spaces: $S_1 = \{(s_1(t_1)), ...\} = \{(a_1|t_{11}, ..., a_1|t_{1K}), ...\}, ...$
   2.6 Type-dependent payoff matrices.

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   2.6 Type-dependent payoff matrices.
3. Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for a player i (the player with the smallest strategy space). For each strategy $s_i(t_i)$:
   3.1 Write up the best response of the other player(s): $s_j^*(t_j) \equiv BR_j\left(s_i(t_i)|t_j\right)$.
   3.2 If $s_i(t_i) = BR_i\left(s_j^*(t_j)|t_i\right) \equiv s_i^*(t_i)$ then $\left(s_i^*(t_i), s_j^*(t_j)\right)$ is a BNE.

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(Bonus) Generally: In a BNE, strategies must maximize expected utility given the strategy of the other player(s) and the probability of them being each type. I.e. no type of any player has an incentive to deviate as in equilibrium player i's strategy is a best response to player j's strategy given player i's beliefs:

$$s_i^*(t_i) \equiv \max_{s_i} \sum_{j \neq i} \sum_{t_{jk} \in T_j} \mathbb{P}_i[t_j = t_{jk}] \cdot u_i\left(s_i(t_i), s_j^*(t_j)\right)$$