PS6, Ex. 7: To keep or split (imperfect information)

Consider the following 2×2 game where payoffs are monetary:

	L	R
T	3, 3	0, 4
B	4, 0	1, 1

Before this game is played, Player 1 can choose whether, after the game is played, players should keep their own payoffs or split the aggregate payoff evenly between them. Player 2 observes this choice.

(a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.

(b) Find the subgame perfect Nash Equilibria (SPNE).

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) and find the pure strategy Nash Equilibria (PSNE).

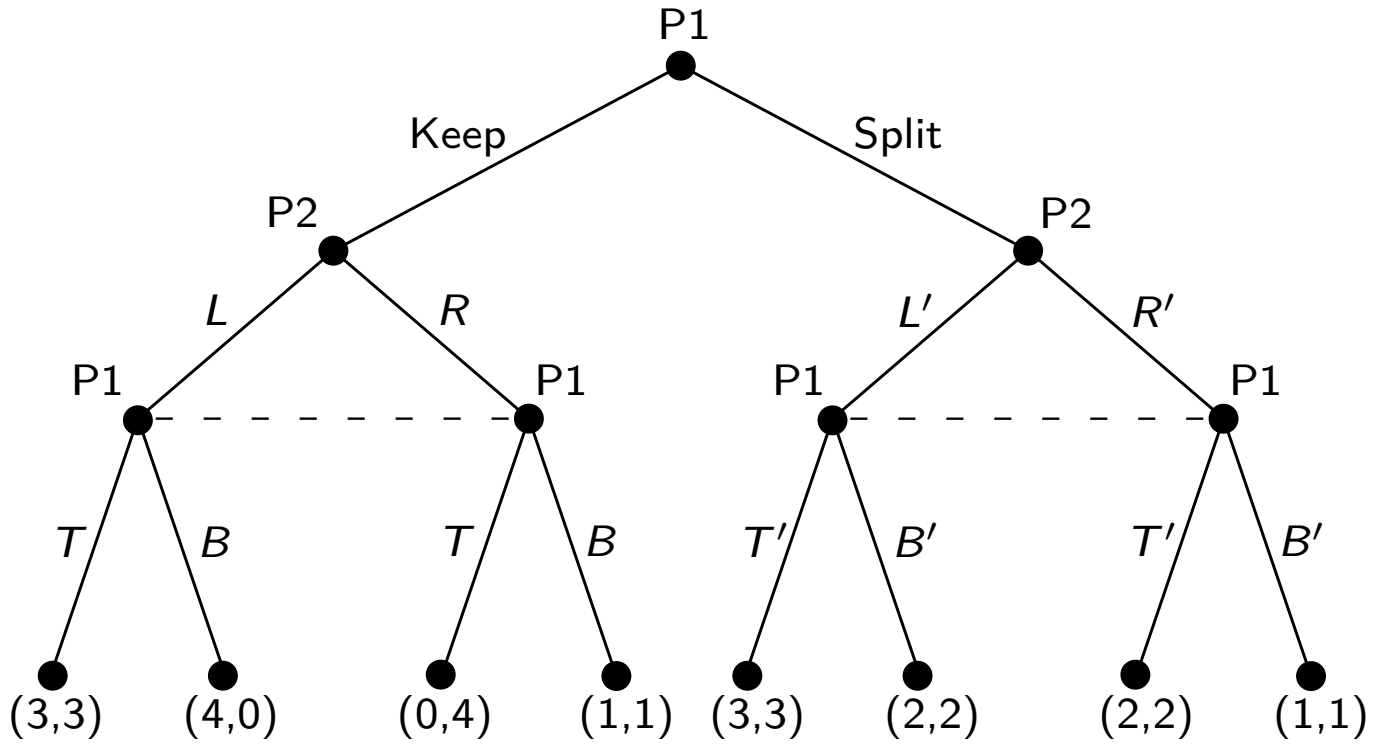
PS6, Ex. 7.a: To keep or split (imperfect information)

(a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.

1st stage: Player 1 chooses Keep or Split. Player 2 observes the choice.

2nd stage: Player 2 chooses L or R (L' or R'). The action is private information.

3rd stage: Player 1 chooses T or B (T' or B') without knowing what Player 2 did.

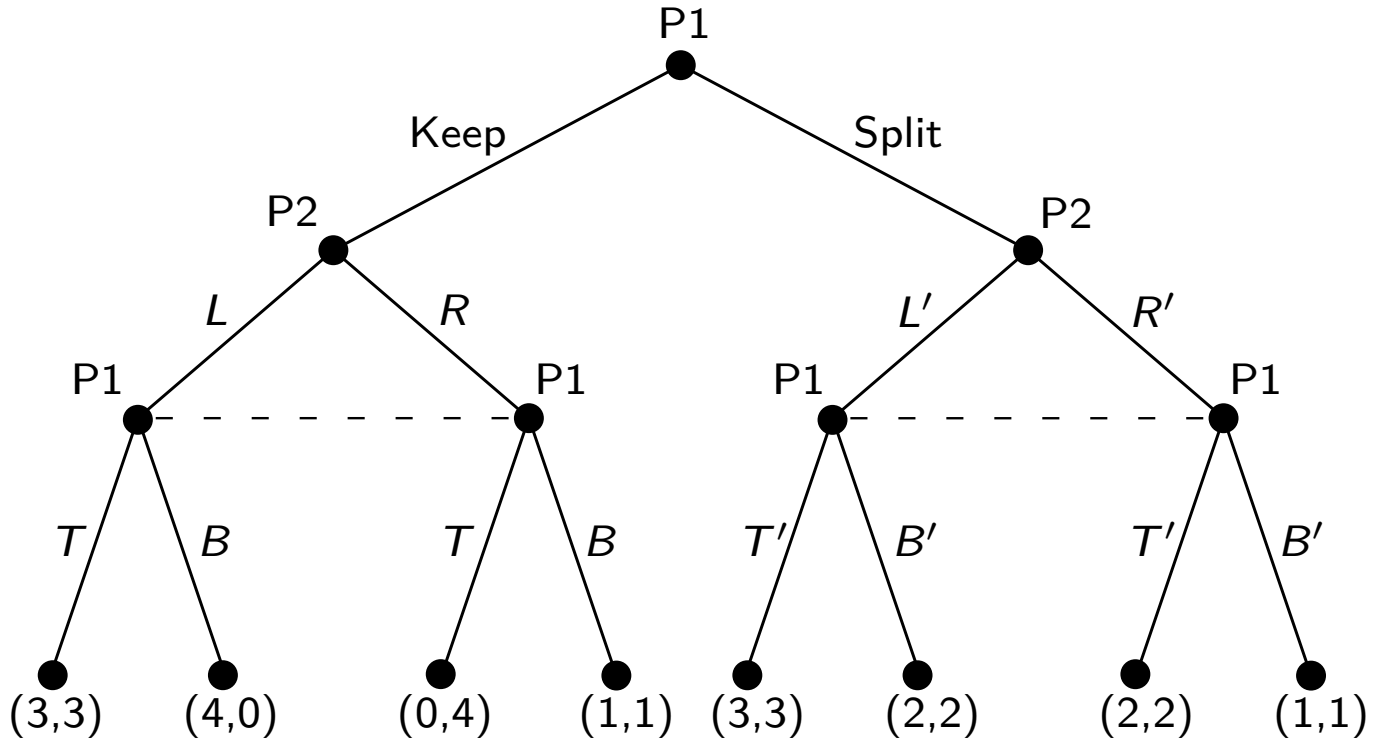


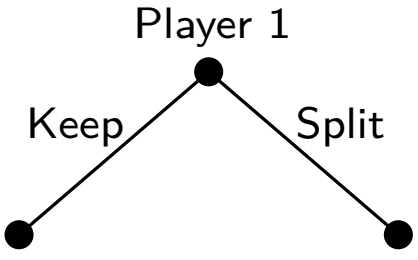
The order of stage 2 and 3 is arbitrary, but the 2nd stage must be private information.

(b) ***Find the subgame perfect Nash Equilibria (SPNE).***

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).

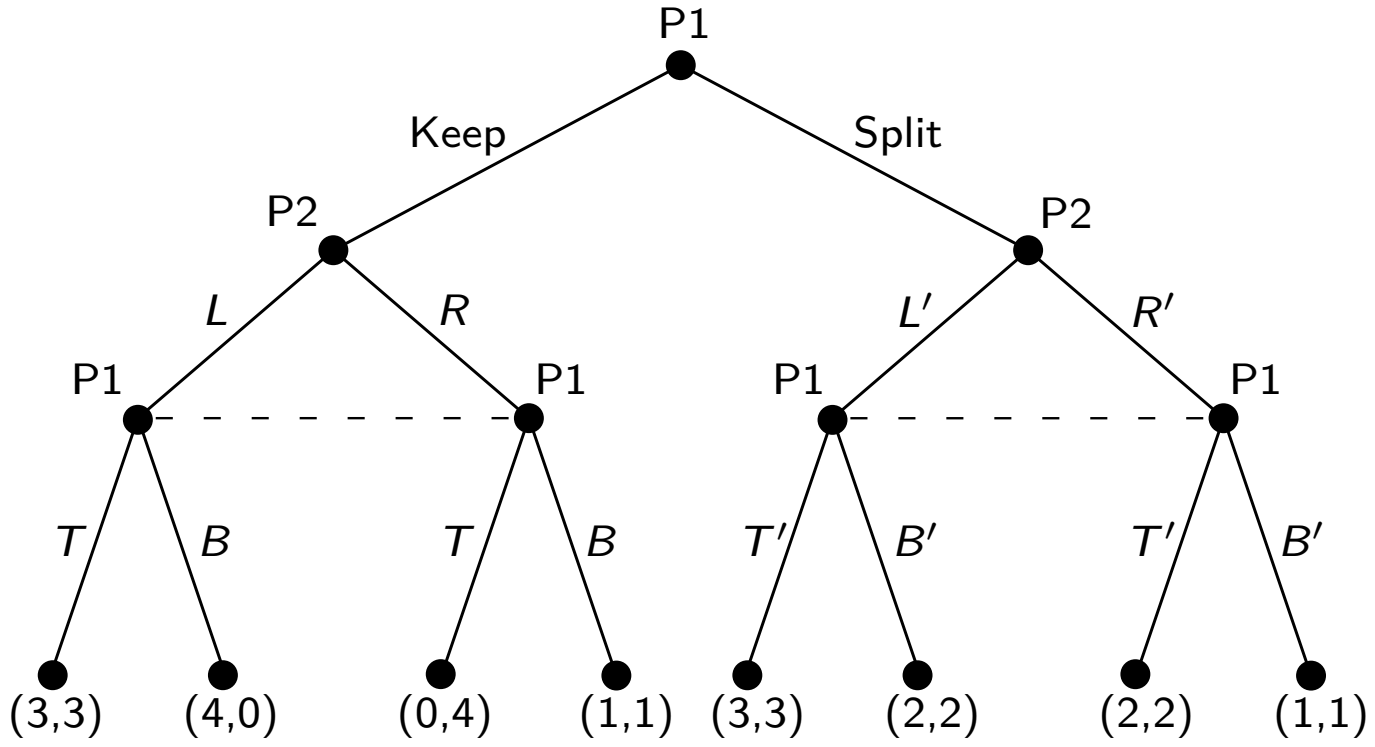


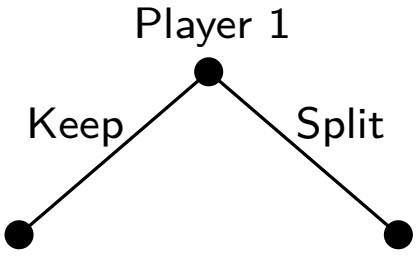


	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).

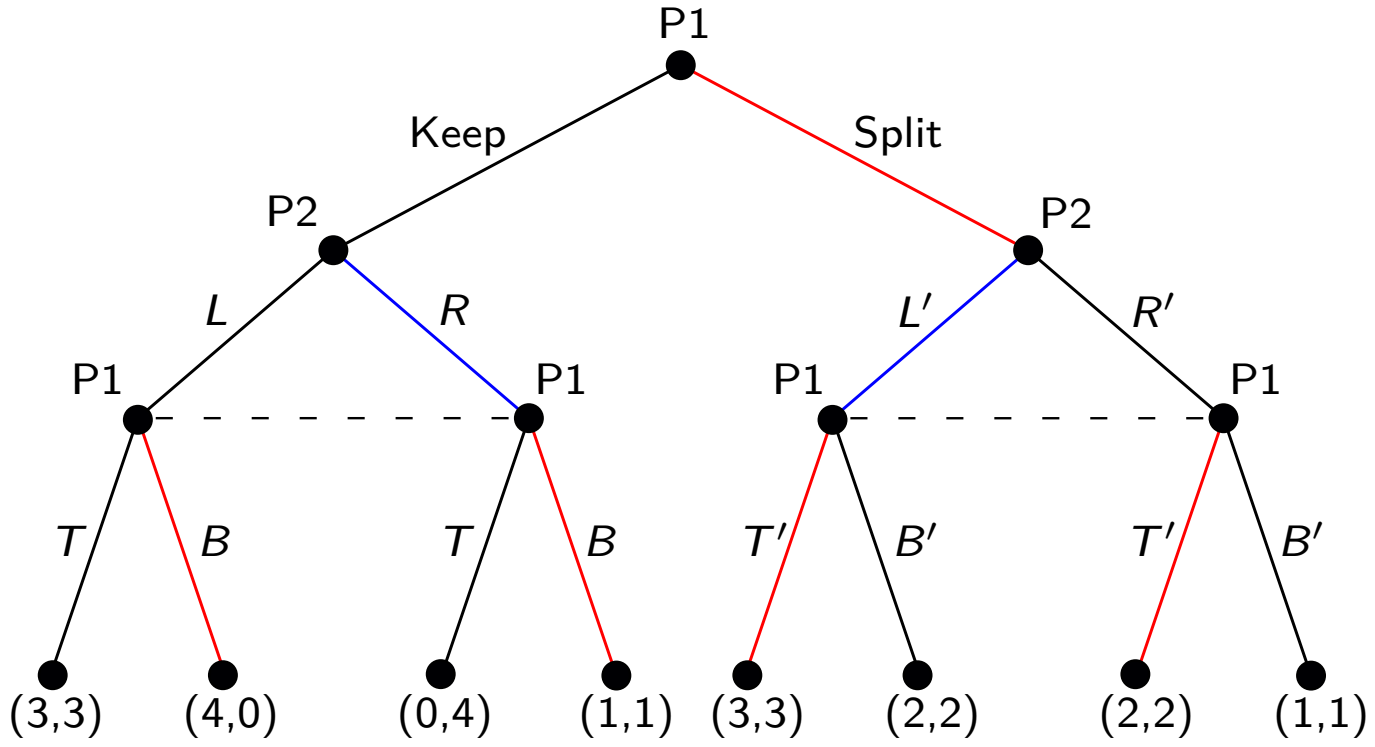


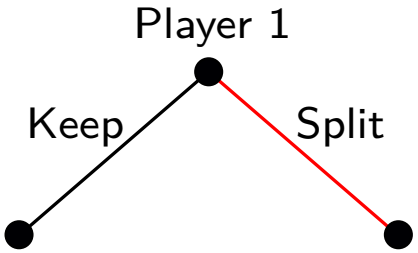


	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).





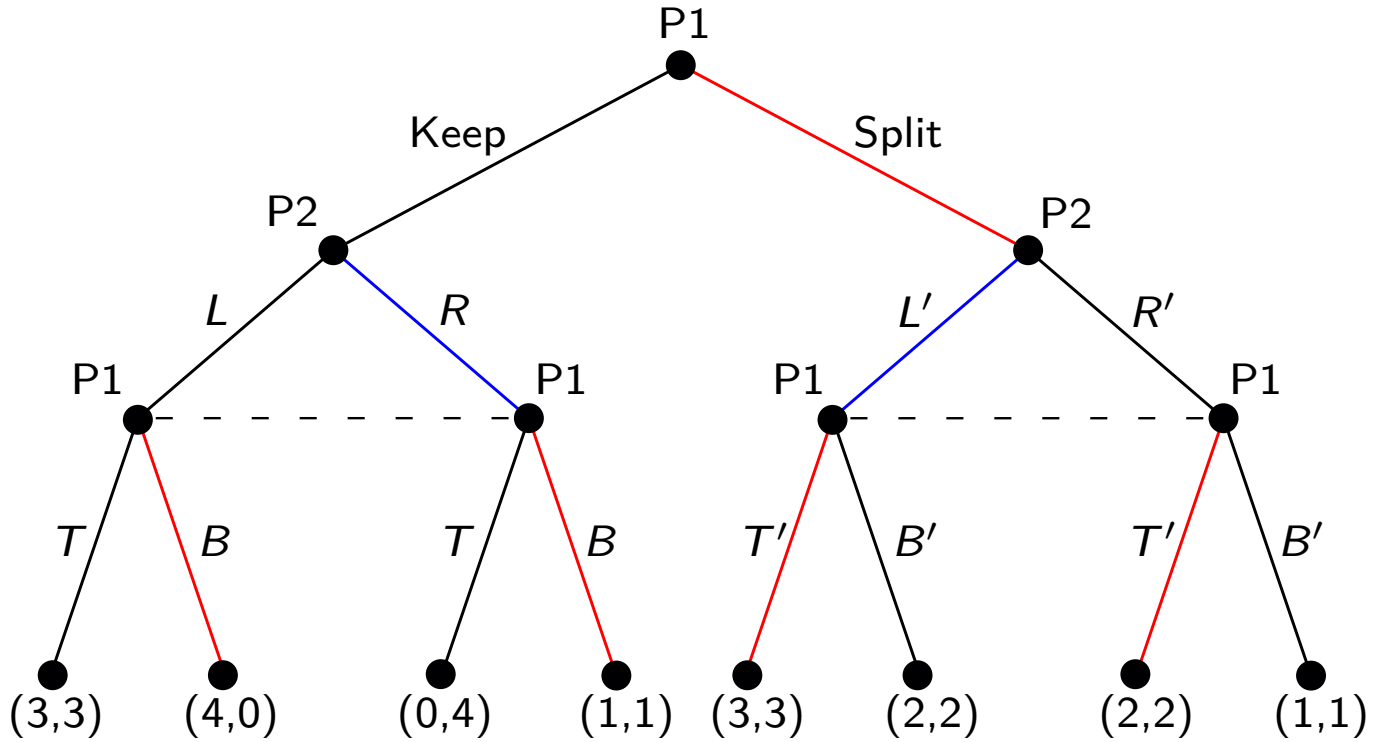
	L	R
T	3, 3	0, 4
B	4, 0	1, 1

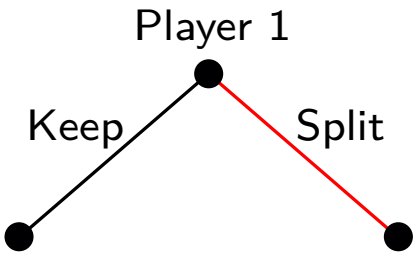
	L'	R'
T'	3, 3	2, 2
B'	2, 2	1, 1

Write up the full strategy profiles for the subgame perfect Nash Equilibria (SPNE).

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).





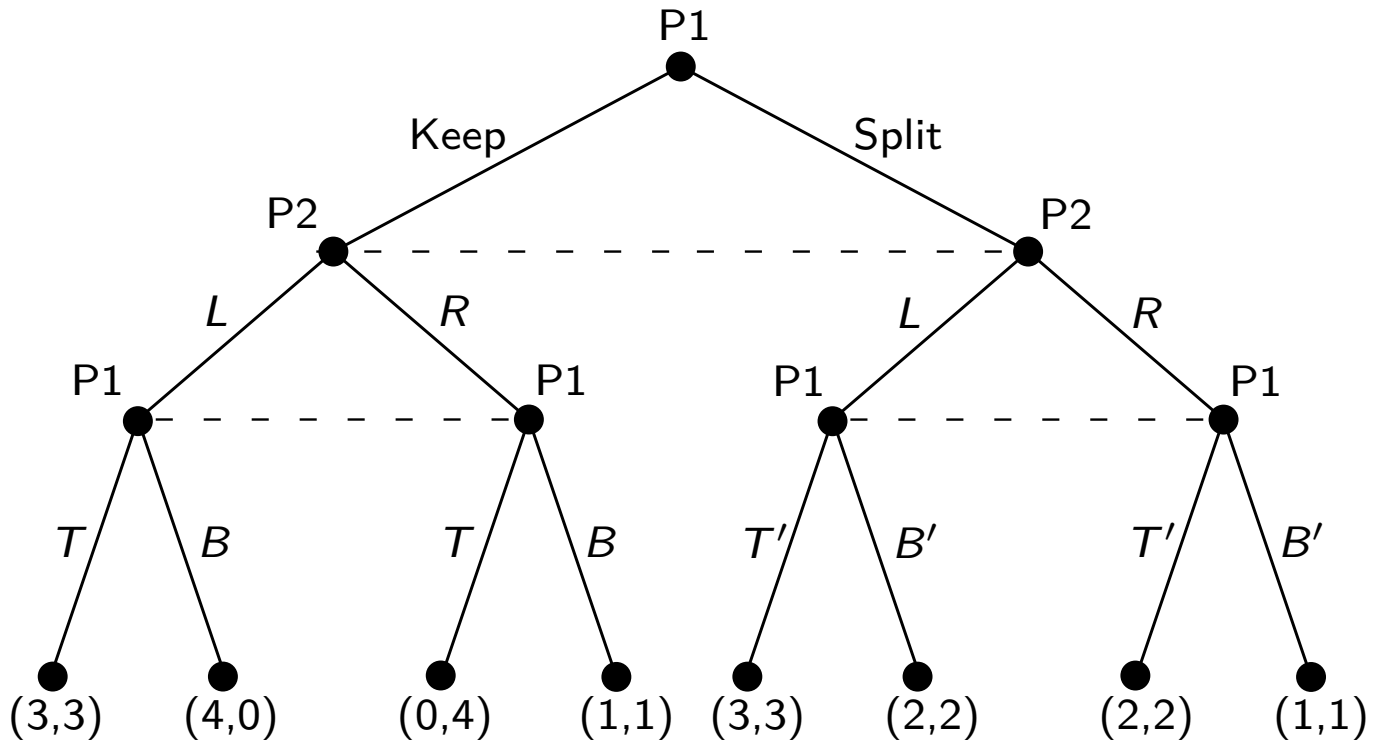
	L	R		L'	R'
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

$SPNE = \{(Split, B, T'), (R, L')\}$ with outcome (3,3).

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. ***Draw the game tree (again using information sets)*** and find the pure strategy Nash Equilibria (PSNE).

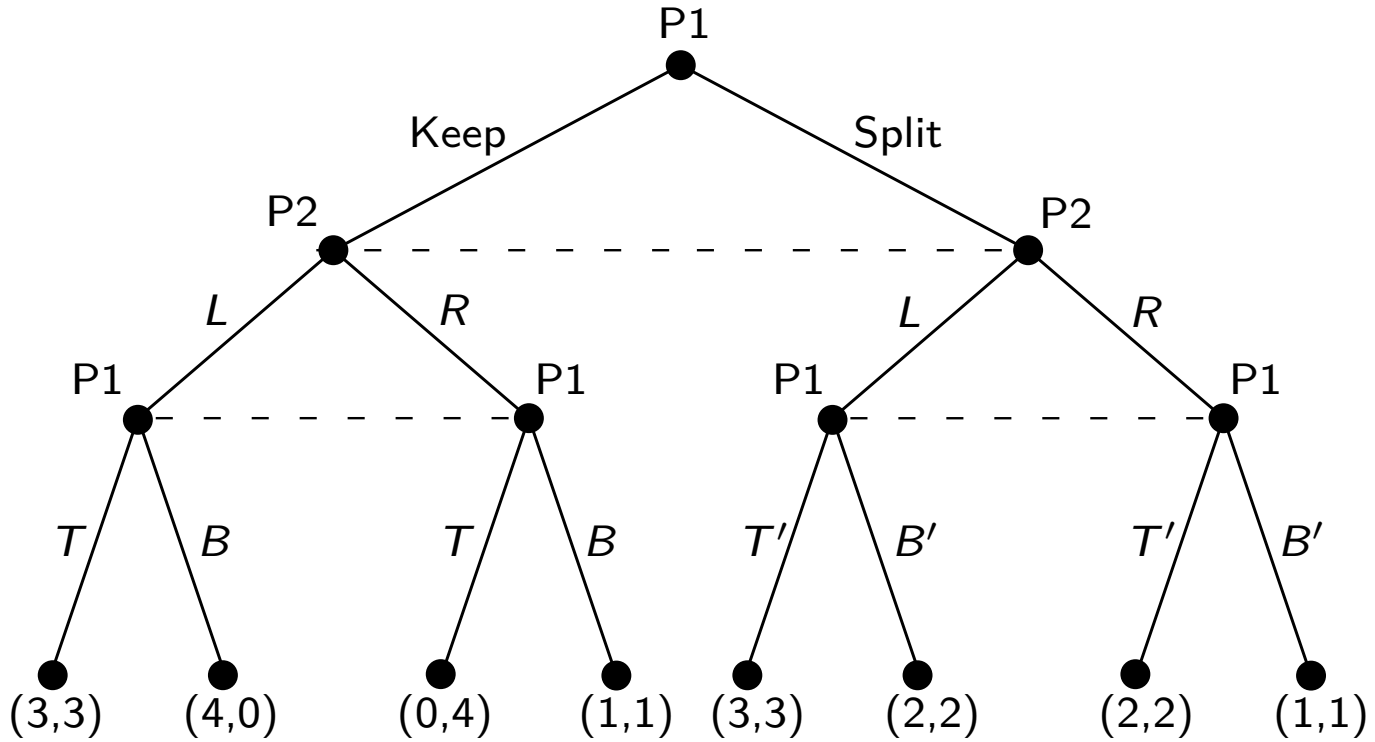
PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) ***and find the pure strategy Nash Equilibria (PSNE).***

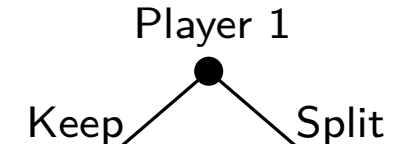


PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the pure strategy Nash Equilibria (PSNE).



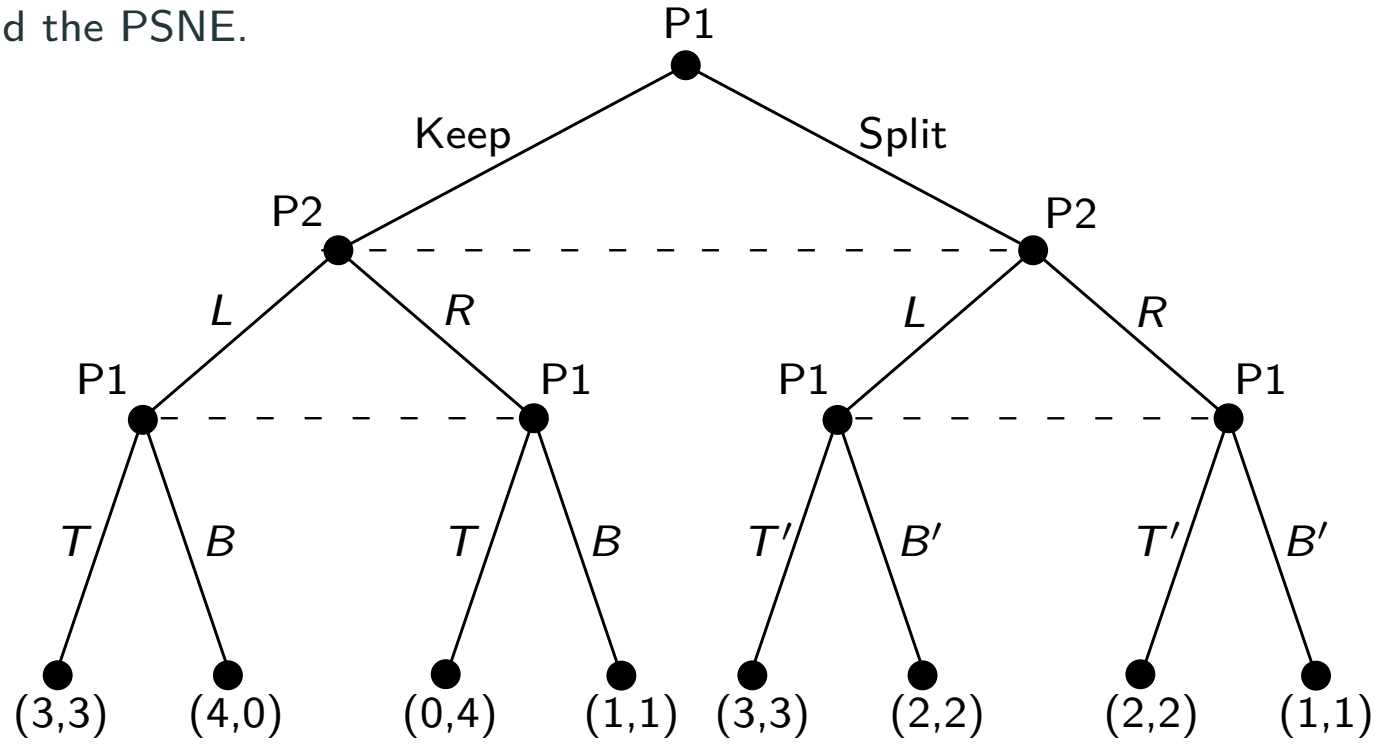
2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):



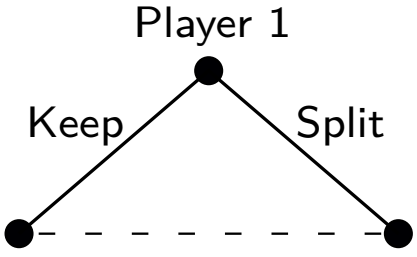
	L	R		L	R
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the PSNE.



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

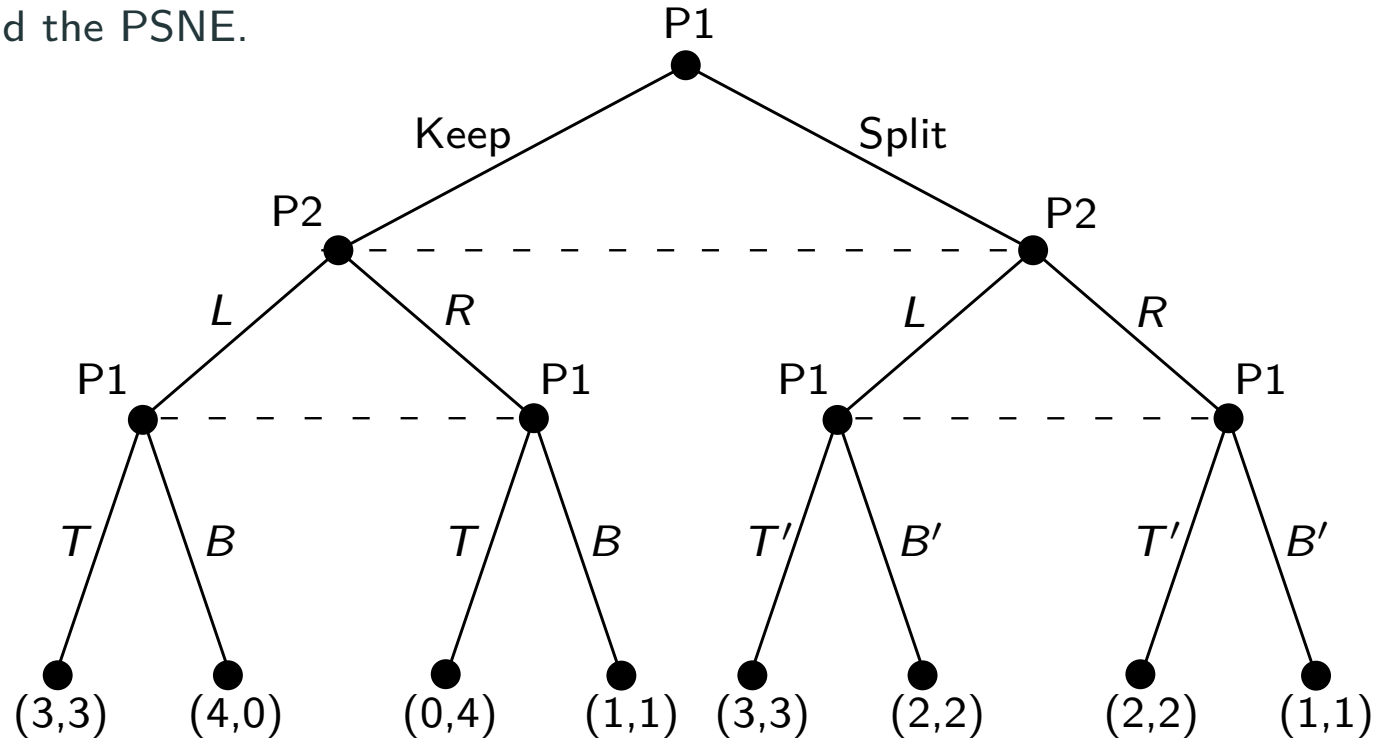
	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

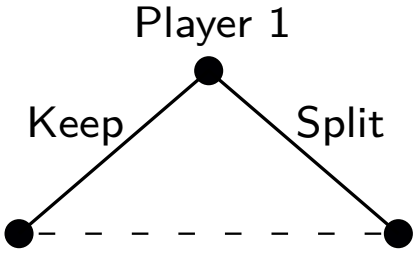
	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the PSNE.



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

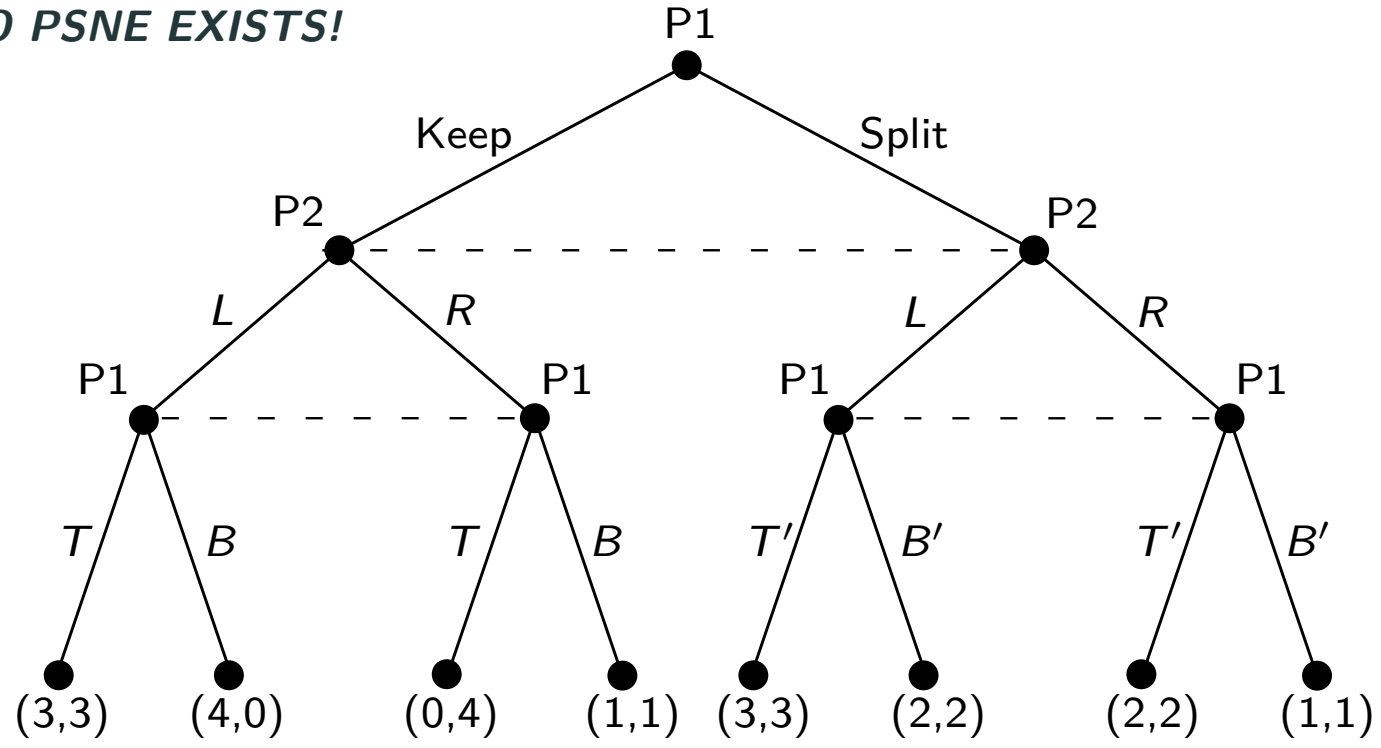
	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

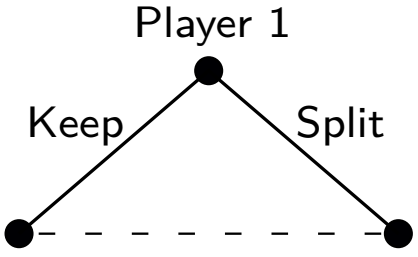
	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) ***NO PSNE EXISTS!***



2nd and 3rd stage in normal form:



	L	R
T	3, 3	0, 4
B	4, 0	1, 1

	L	R
T'	3, 3	2, 2
B'	2, 2	1, 1

Full game:

	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1