Microeconomics III: Problem Set 7[a]

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[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

Kahoot!

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

PS7, Ex. 3 (A): A single stage game NE (finitely repeated game)

PS7, Ex. 4: Credible punishment (twice-repeated game)

PS7, Ex. 5: Trigger strategy (infinitely repeated game)

PS7, Ex. 6: Tit-for-tat strategy (infinitely repeated game)

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

PS7, Ex. 8: Trigger strategy (infinitely repeated game)

PS7, Ex. 9: Optimal punishment strategy (infinitely repeated game)

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Kahoot!

Kahoot: A exercises

Form a group for each table:

- Get prepared to answer the three A exercises as a team (5 min).

Link to Kahoot: https://bit.ly/36DvwLD (to pep up your exam reading)

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

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In this course we normally consider games in which there is 'perfect recall': players can always remember what they themselves have done in the past.

We have seen an example in class of a game with 'imperfect recall' where the player forgets his own actions. But what would a game where he forgets the opponent's actions look like? Construct a game with two players. The timing is as follows: Player 1 moves first, then Player 2, and then Player 2 again. Everytime they move, the players choose one of two actions: $\{L, R\}$.

Draw the game tree and construct the information sets such that (a) Player 2 observes Player 1's action the first time he moves, but (b) when Player 2 moves the second time, he has forgotten what Player 1 chose. However, he recalls his own action.

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

In this course we normally consider games in which there is 'perfect recall': players can always remember what they themselves have done in the past.

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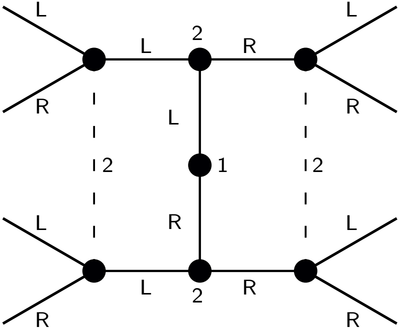

PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

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Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

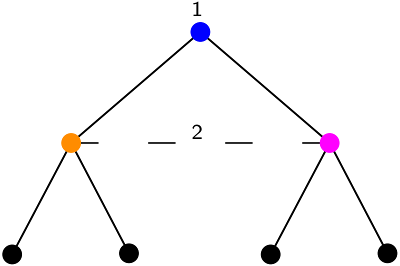
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.

Example of violation of condition 1:



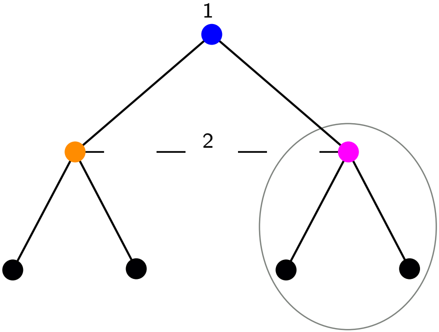
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.

Example of violation of condition 1:



The purple decision node to the right is not a singleton information set (nor is the orange decision node to the left).

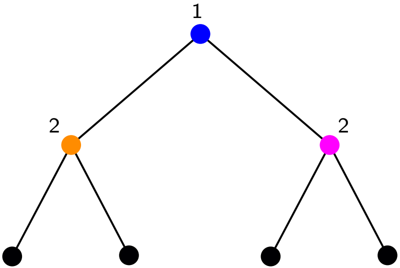
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.
2. **It includes all following decision and terminal nodes following *n* in the game tree**, but no nodes that do not follow *n*.

Example of violation of the first part of condition 2:



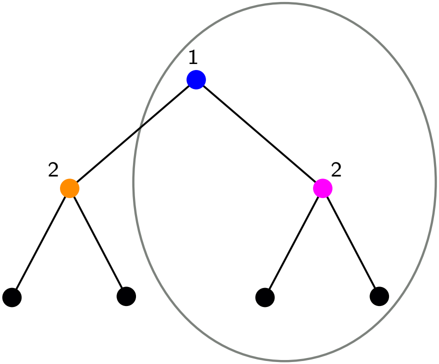
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. **It includes all following decision and terminal nodes following n in the game tree**, but no nodes that do not follow n.

Example of violation of the first part of condition 2:



For a subgame containing the blue decision node n, all following decision nodes must be included.

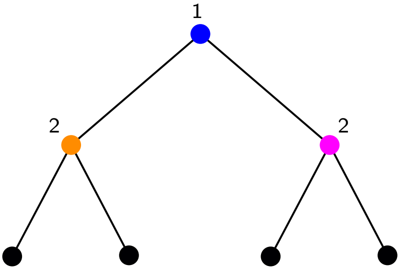
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. It includes all following decision and terminal nodes following n in the game tree, **but no nodes that do not follow n.**

Example of violation of the second part of condition 2:



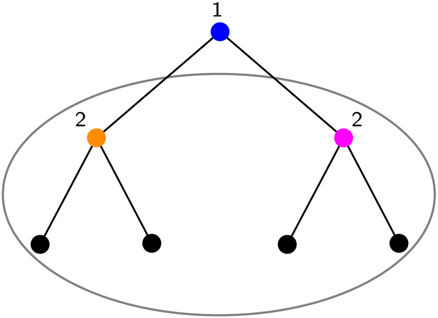
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.
2. It includes all following decision and terminal nodes following *n* in the game tree, **but no nodes that do not follow *n*.**

Example of violation of the second part of condition 2:



Regardless of whether the orange or the purple node is chosen as the first decision node *n*, the other decision node does not follow *n*, and therefore cannot be part of the subgame.

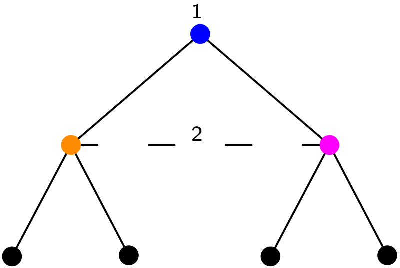
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. It includes all following decision and terminal nodes following n in the game tree, but no nodes that do not follow n.
3. It does not "cut" any information set: if a decision node n' follows n in the game tree, then all other nodes in the information set including n' must also follow n (and so be included in the subgame).

Example of violation of condition 3:



PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. It includes all following decision and terminal nodes following n in the game tree, but no nodes that do not follow n.
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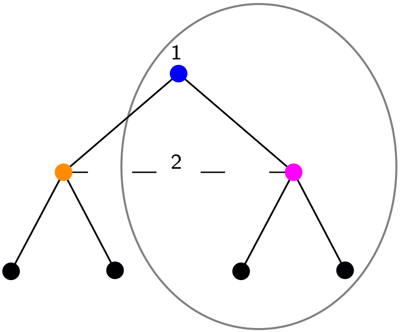
Example of violation of condition 3:



The orange decision node to the left is part of the same information set as the purple node to the right, so it must be included in the same subgame.

PS7, Ex. 3 (A): A single stage game NE (finitely repeated game)

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Let G be the following game:

		Player 2	
		C	D
Player 1	A	27, -3	0, 0
	B	6, 6	-2, 7

Consider the repeated game G(T), where G is repeated T times and the outcomes of each round are observed by both players before the next round.

(a) If T = 2, is there a Subgame Perfect Nash Equilibrium such that (B,C) is played during the 1^{st} round?

(b) What if T = 42?

PS7, Ex. 3.a (A): A single stage game NE (finitely repeated game)

(a) If T = 2, is there a Subgame Perfect Nash Equilibrium such that (B,C) is played during the 1st round?

Player 2

Player 1		C	D
	A	27, -3	0, 0
	B	6, 6	-2, 7

No. Since there is only one NE (A,D) which is not (B,C), that NE will be played in both games.

Explanation:
In the last round, a NE from the stage game must be played. In this case there is only one NE, which is (A,D). Knowing that (A,D) will be played no matter what in the 2nd round, no player has an incentive to cooperate in the 1st turn. Player A will play his dominant strategy A and player B will play her dominant strategy D.

PS7, Ex. 3.b (A): A single stage game NE (finitely repeated game)

(b) If T = 42, is there a Subgame Perfect Nash Equilibrium such that (B,C) is played during the 1st round?

		Player 2	
		C	D
Player 1	A	27, -3	0, 0
	B	6, 6	-2, 7

PS7, Ex. 3.b (A): A single stage game NE (finitely repeated game)

(b) If $T = 42$, is there a Subgame Perfect Nash Equilibrium such that (B,C) is played during the 1st round?

Player 2

Player 1		C	D
	A	27, -3	0, 0
	B	6, 6	-2, 7

No. Since there is only one NE (A,D) which is not (B,C), that NE will be played in every turn of any finite game G(T).

Explanation:

In the last round, an NE from the stage game must be played. In this case there is only one NE, which is (A,D). Knowing that (A,D) will be played no matter what in the last round, no player has an incentive to cooperate in the round before that. This keeps applying until the players reach the 1st stage of the game. Thus, the NE (A,D) will be played in every turn of any finite game G(T).

PS7, Ex. 4: Credible punishment (twice-repeated game)

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Consider the two times repeated game where the stage game is:

Player 2

Player 1		X	Y	Z
	A	6, 6	0, 8	0, 0
	B	7, 1	2, 2	1, 1
	C	0, 0	1, 1	4, 5

(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1^{st} stage is (B,Y). Make sure to write down the full equilibrium.

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1^{st} stage is (C,Z). Make sure to write down the full equilibrium.

(c) Can you find a subgame perfect Nash equilibrium such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

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Player 1 \ Player 2	X	Y	Z
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B	7, 1	2, 2	1, 1
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(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

Information so far:

1. Stage game NE: $\{(B,Y),(C,Z)\}$

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

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Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
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(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

(Step b) Knowing the NE, is a SPNE possible with (B,Y) in the 1st stage?

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

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(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

(Step b) Knowing the NE, is a SPNE possible with (B,Y) in the 1st stage?

As (B,Y) is a NE it is a possible outcome in any stage. We can choose the strategies such that (B,Y) will be the outcome of the 1st stage, and then either of the NE can be the outcome of the 2nd stage.

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
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(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

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(Step c) Write up a possible SPNE.

Information so far:

1. Stage game NE: $\{(B,Y),(C,Z)\}$

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
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(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

(Step b) Knowing the NE, is a SPNE possible with (B,Y) in the 1st stage?

As (B,Y) is a NE it is a possible outcome in any stage. We can choose the strategies such that (B,Y) will be the outcome of the 1st stage, and then either NE can be the outcome of the 2nd stage.

(Step c) Write up a possible SPNE.

Keep in mind that you need to write up a 2nd stage strategy for each of the possible outcomes of the 1st stage (3·3 matrix, so 9 possible outcomes).

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.a: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(a) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (B,Y). Make sure to write down the full equilibrium.

(Step a) Find the NE in the stage game.

(Step b) Knowing the NE, is a SPNE possible with (B,Y) in the 1st stage?

As (B,Y) is a NE it is a possible outcome in any stage. We can choose the strategies such that (B,Y) will be the outcome of the 1st stage, and then either NE can be the outcome of the 2nd stage.

(Step c) Write up a possible SPNE.

Keep in mind that you need to write up a 2nd stage strategy for each of the possible outcomes of the 1st stage (3·3 matrix, so 9 possible outcomes).

Information so far:

1. Stage game NE: $\{(B,Y),(C,Z)\}$
2. Write up one of 2^9 possible SPNE:

$$\left\{ \begin{array}{c} (BBBBBBBBBB, YYYYYYYYYY) \\ (BCBBBBBBBB, YZYYYYYYYY) \\ \vdots \\ (BCCCCCCCCC, YZZZZZZZZZ) \end{array} \right\}$$

PS7, Ex. 4.b: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1$^{\text{st}}$ stage is (C,Z). Make sure to write down the full equilibrium.

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.b: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (C,Z). Make sure to write down the full equilibrium.

(Step a) Knowing the NE, is a SPNE possible with (C,Z) in the 1st stage?

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.b: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (C,Z). Make sure to write down the full equilibrium.

(Step a) Knowing the NE, is a SPNE possible with (C,Z) in the 1st stage?

Yes, similarly to question (a), any NE can be played in either round.

Information so far:

1. Stage game NE: $\{(B,Y),(C,Z)\}$

PS7, Ex. 4.b: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 2

Player 1	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (C,Z). Make sure to write down the full equilibrium.

(Step a) Knowing the NE, is a SPNE possible with (C,Z) in the 1st stage?

Yes, similarly to question (a), any NE can be played in either round.

(Step b) Write up a possible SPNE.

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$

PS7, Ex. 4.b: Credible punishment (twice-repeated game)

Consider the two times repeated game where the stage game is:

Player 1 \ Player 2	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(b) Find a subgame perfect Nash equilibrium such that the outcome of the 1st stage is (C,Z). Make sure to write down the full equilibrium.

(Step a) Knowing the NE, is a SPNE possible with (C,Z) in the 1st stage?

Yes, similarly to question (a), any NE can be played in either round.

(Step b) Write up a possible SPNE.

Information so far:

1. Stage game NE: $\{(B, Y), (C, Z)\}$
2. Write up one of 2^9 possible SPNE:

$$\left\{ \begin{array}{c} (CBBBBBBBBB, ZYYYYYYYYY) \\ (CCBBBBBBBB, ZZYYYYYYYY) \\ \vdots \\ (CCCCCCCCCC, ZZZZZZZZZZ) \end{array} \right\}$$

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

		Player 2		
		X	Y	Z
Player 1	A	6, 6	0, 8	0, 0
	B	7, 1	2, 2	1, 1
	C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: $\delta=1$).

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: $\delta = 1$).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

a. Total payoffs are (10,11) for:

- t=1: (A,X) (not a stage game NE)
- t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

- t=1: Play (A,X) .
- t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .

t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

a. Total payoffs are (10,11) for:
   - t=1: (A,X) (not a stage game NE)
   - t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:
   - t=1: Play (A,X) .
   - t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .

t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

$$U_1(PS, PS) \geq U_1(OD_1, PS) \Leftrightarrow 6 + 4 \geq 7 + 2$$

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)
t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .
t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

$$U_1(PS, PS) \geq U_1(OD_1, PS) \Leftrightarrow 6 + 4 \geq 7 + 2$$

c. P2: Check *PS* vs. $OD_2 = (Y, Y)$:

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .

t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

$$U_1(PS, PS) \geq U_1(OD_1, PS) \Leftrightarrow 6 + 4 \geq 7 + 2$$

c. P2: Check *PS* vs. $OD_2 = (Y, Y)$:

$$U_2(PS, PS) \geq U_2(PS, OD_2) \Leftrightarrow 6 + 5 \geq 8 + 2$$

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

As *PS* is a best response to *PS* for both players, *(PS,PS)* is a SPNE.

(Step d) Write up the full SPNE.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .

t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

$$U_1(PS, PS) \geq U_1(OD_1, PS) \Leftrightarrow 6 + 4 \geq 7 + 2$$

c. P2: Check *PS* vs. $OD_2 = (Y, Y)$:

$$U_2(PS, PS) \geq U_2(PS, OD_2) \Leftrightarrow 6 + 5 \geq 8 + 2$$

PS7, Ex. 4.c: Credible punishment (twice-repeated game)

	X	Y	Z
A	6, 6	0, 8	0, 0
B	7, 1	2, 2	1, 1
C	0, 0	1, 1	4, 5

(c) Can you find a SPNE such that the total payoffs that the players receive are 10 for player 1 and 11 for player 2? If yes, write down the full equilibrium.

(Step a) Find out which combination of outcomes would yield the payoff (10,11), under the restriction that the last stage must be a NE (and perfect patience: δ=1).

(Step b) Now, look for a punishment strategy *PS* which if followed will lead to this combination.

(Step c) Check if either player is better off from his optimal deviation strategy *OD* than from the punishment strategy *PS*.

As *PS* is a best response to *PS* for both players, (PS, PS) is a SPNE.

(Step d) Write up the full SPNE.

a. Total payoffs are (10,11) for:

t=1: (A,X) (not a stage game NE)

t=2: (C,Z) (a stage game NE)

b. Punishment Strategy *PS*:

t=1: Play (A,X) .

t=2: If (A,X) was played in t=1, play (C,Z). Otherwise, play (B,Y).

c. P1: Check *PS* is better than his optimal deviation $OD_1 = (B, B)$:

$$U_1(PS, PS) \geq U_1(OD_1, PS) \Leftrightarrow 6 + 4 \geq 7 + 2$$

c. P2: Check *PS* vs. $OD_2 = (Y, Y)$:

$$U_2(PS, PS) \geq U_2(PS, OD_2) \Leftrightarrow 6 + 5 \geq 8 + 2$$

d. SPNE:

$\{(ACBBBBBBBB, XZYYYYYYYY)\}$

PS7, Ex. 5: Trigger strategy (infinitely repeated game)

PS7, Ex. 5: Trigger strategy (infinitely repeated game)

Consider the situation of two flatmates. They both prefer having a clean kitchen, but cleaning is a tedious task, so that it is individually rational not to clean regardless of what the other does. This results in the following game G:

Player 1 \ Player 2	Cl	DCl
Cl	4, 4	0, 6
DCl	5, 0	1, 1

Now consider the situation where the two flatmates have to decide every day whether to clean or not, i.e. consider the infinitely repeated game $G(\infty, \delta)$

(a) Define trigger strategies such that the outcome of all stages will be (Clean,Clean).

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

PS7, Ex. 5.a: Trigger strategy (infinitely repeated game)

Consider the infinitely repeated game $G(\infty, \delta)$ with the stage game:

Player 1 \ Player 2	CI	DCI
CI	4, 4	0, 6
DCI	5, 0	1, 1

(a) Define trigger strategies such that the outcome of all stages will be (Clean,Clean).

A trigger strategy is defined as the player will play the same option in every game (the carrot), unless the opponent does something (the trigger), then he will play something else (the stick).

1 Define the carrot, the trigger and the stick.

PS7, Ex. 5.a: Trigger strategy (infinitely repeated game)

Consider the infinitely repeated game $G(\infty, \delta)$ with the stage game:

Player 1 \ Player 2	Cl	DCl
Cl	4, 4	0, 6
DCl	5, 0	1, 1

(a) Define trigger strategies such that the outcome of all stages will be (Clean,Clean).

A trigger strategy is defined as the player will play the same option in every game (the carrot), unless the opponent does something (the trigger), then he will play something else (the stick).

1 Define the carrot, the trigger and the stick.

2 Write up the trigger strategy

1. Carrot: Playing Clean
2. Trigger: if the other player doesn't play Clean
3. Stick: Playing Don't Clean

PS7, Ex. 5.a: Trigger strategy (infinitely repeated game)

Consider the infinitely repeated game $G(\infty, \delta)$ with the stage game:

Player 1 \ Player 2	Cl	DCl
Cl	4, 4	0, 6
DCl	5, 0	1, 1

(a) Define trigger strategies such that the outcome of all stages will be (Clean,Clean).

A trigger strategy is defined as the player will play the same option in every game (the carrot), unless the opponent does something (the trigger), then he will play something else (the stick).

1 Define the carrot, the trigger and the stick.

2 Write up the trigger strategy

1. Carrot: Playing Clean
2. Trigger: if the other player doesn't play Clean
3. Stick: Playing Don't Clean
4. Trigger strategy: In the 1^{st} turn, play Clean. In every subsequent turn, if outcome from every previous turn was (Clean,Clean), play Clean, otherwise play Don't Clean.

PS7, Ex. 5.b: Trigger strategy (infinitely repeated game)

Player 2

Player 1	CI	DCI
CI	4, 4	0, 6
DCI	5, 0	1, 1

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

(Step a) *On the equilibrium path:* Define the payoff for staying with the trigger strategy, and for deviating, then write up the inequality and isolate δ to find for what values of δ Player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

1. $U_2(CI, CI) = 4$
2. $U_2(CI, DCI) = 6$
3. $U_2(DCI, DCI) = 1$
4. Algebra of infinite sequences: $\sum_{t=1}^{\infty} a \cdot \delta^{t-1} = \frac{a}{1-\delta}$ $\sum_{t=2}^{\infty} a \cdot \delta^{t-1} = \frac{a\delta}{1-\delta}$

PS7, Ex. 5.b: Trigger strategy (infinitely repeated game)

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

(Step a) *On the equilibrium path:* Define the payoff for staying with the trigger strategy, and for deviating, then write up the inequality and isolate δ to find for what values of δ Player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

(Step b) *Off the equilibrium path:*

1. $U_2(CI, CI) = 4$
2. $U_2(CI, DCI) = 6$
3. $U_2(DCI, DCI) = 1$
4. Algebra of infinite sequences:
   $\sum_{t=1}^{\infty} a \cdot \delta^{t-1} = \frac{a}{1-\delta}$
   $\sum_{t=2}^{\infty} a \cdot \delta^{t-1} = \frac{a\delta}{1-\delta}$
5. On the equilibrium path:

$$4 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$

$$4\delta^0 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$

$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq 6 + \sum_{\boldsymbol{t=2}}^{\infty} 1 \cdot \delta^{t-2} \Rightarrow$$

$$\frac{4}{1-\delta} \geq 6 + \frac{\delta}{1-\delta} \Rightarrow \delta \geq \frac{2}{5}$$

PS7, Ex. 5.b: Trigger strategy (infinitely repeated game)

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

(Step a) *On the equilibrium path:* Define the payoff for staying with the trigger strategy, and for deviating, then write up the inequality and isolate δ to find for what values of δ Player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

(Step b) *Off the equilibrium path:* Check if the trigger strategy is credible if a player deviated from the equilibrium path by playing "don't clean" in the previous round.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. Algebra of infinite sequences:
   $\sum_{t=1}^{\infty} a \cdot \delta^{t-1} = \frac{a}{1-\delta}$
   $\sum_{t=2}^{\infty} a \cdot \delta^{t-1} = \frac{a\delta}{1-\delta}$
5. On the equilibrium path:

$$4 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$
$$4\delta^0 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$
$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq 6 + \sum_{\boldsymbol{t=2}}^{\infty} 1 \cdot \delta^{t-2} \Rightarrow$$
$$\frac{4}{1-\delta} \geq 6 + \frac{\delta}{1-\delta} \Rightarrow \delta \geq \frac{2}{5}$$

PS7, Ex. 5.b: Trigger strategy (infinitely repeated game)

(b) Find the lowest value of δ such that the trigger strategies from (b) constitute a SPNE in $G(\infty, \delta)$. Recall: you have to check for deviations both on and off the equilibrium path.

(Step a) *On the equilibrium path:* Define the payoff for staying with the trigger strategy, and for deviating, then write up the inequality and isolate δ to find for what values of δ Player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

(Step b) *Off the equilibrium path:* Check if the trigger strategy is credible if a player deviated from the equilibrium path by playing "don't clean" in the previous round.

The best response to "don't clean" is to also play "don't clean". As (DCl,DCl) is the stage game NE, this is a credible punishment as there is no incentive to deviate from this eternal punishment.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. Algebra of infinite sequences:
   $\sum_{t=1}^{\infty} a \cdot \delta^{t-1} = \frac{a}{1-\delta}$
   $\sum_{t=2}^{\infty} a \cdot \delta^{t-1} = \frac{a\delta}{1-\delta}$
5. On the equilibrium path:

$$4 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$
$$4\delta^0 + 4\delta + 4\delta^2 + ... \geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow$$
$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq 6 + \sum_{\boldsymbol{t=2}}^{\infty} 1 \cdot \delta^{t-2} \Rightarrow$$
$$\frac{4}{1-\delta} \geq 6 + \frac{\delta}{1-\delta} \Rightarrow \delta \geq \frac{2}{5}$$

6. Neither player will deviate for $\delta \geq \frac{2}{5}$

PS7, Ex. 6: Tit-for-tat strategy (infinitely repeated game)

PS7, Ex. 6: Tit-for-tat strategy (infinitely repeated game)

Consider again the the infinitely repeated game $G(\infty, \delta)$ with the stage game:

		Player 2	
		Cl	DCl
Player 1	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(a) Define a tit-for-tat strategy such that the outcome of all stages will be (Clean, Clean).

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean''

(c) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Start by playing 'Do not clean', then play 'tit-for-tat' forever after that'.

(d) Argue informally that 'tit-for-tat' is a NE for the appropriate values of δ. In particular, think about whether there are other deviations that would be better for the players.

When we say "against", it doesn't mean that the other player is playing the "against" strategy. It means to compare the two strategies, in this case "on the equilibrium path", so if the other player is playing "tit-for-tat"

PS7, Ex. 6.a: Tit-for-tat strategy (infinitely repeated game)

Player 1 \ Player 2	Cl	DCl
Cl	4, 4	0, 6
Dcl	5, 0	1, 1

(a) Define a tit-for-tat strategy such that the outcome of all stages will be (Clean, Clean).

A tit-for-tat strategy is defined as a strategy where the player plays the carrot option, if it's the 1st round or the other player played the carrot option in the last round, otherwise the player will play the stick option.

(Step a) Define the carrot and the stick.

PS7, Ex. 6.a: Tit-for-tat strategy (infinitely repeated game)

		Player 2	
		Cl	DCl
Player 1	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(a) Define a tit-for-tat strategy such that the outcome of all stages will be (Clean, Clean).

A tit-for-tat strategy is defined as a strategy where the player plays the carrot option, if it's the 1st round or the other player played the carrot option in the last round, otherwise the player will play the stick option.

1. Carrot: Playing Clean
2. Stick: Playing Don't Clean

(Step a) Define the carrot and the stick.

(Step b) Write up the tit-for-tat strategy

PS7, Ex. 6.a: Tit-for-tat strategy (infinitely repeated game)

Player 1		Player 2: Cl	Player 2: DCl
	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(a) Define a tit-for-tat strategy such that the outcome of all stages will be (Clean, Clean).

A tit-for-tat strategy is defined as a strategy where the player plays the carrot option, if it's the 1^{st} round or the other player played the carrot option in the last round, otherwise the player will play the stick option.

(Step a) Define the carrot and the stick.

(Step b) Write up the tit-for-tat strategy

1. Carrot: Playing Clean
2. Stick: Playing Don't Clean

PS7, Ex. 6.b: Tit-for-tat strategy (infinitely repeated game)

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean''

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$

PS7, Ex. 6.b: Tit-for-tat strategy (infinitely repeated game)

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean''

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating.

(Step b) Write up the inequality and isolate δ to find for what values of δ neither player would deviate.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

PS7, Ex. 6.b: Tit-for-tat strategy (infinitely repeated game)

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean''

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating.

(Step b) Write up the inequality and isolate δ to find for what values of δ neither player would deviate, you only need to check P2 as P2 has the highest incentive to deviate.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

PS7, Ex. 6.b: Tit-for-tat strategy (infinitely repeated game)

(b) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Always play 'Do not clean''

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating.

(Step b) Write up the inequality and isolate δ to find for what values of δ player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

$$
\begin{aligned}
4 + 4\delta + 4\delta^2 + ... &\geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow \\
4\delta^0 + 4\delta + 4\delta^2 + ... &\geq 6 + 1\delta + 1\delta^2 + ... \Rightarrow \\
\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} &\geq 6 + \sum_{\boldsymbol{t=2}}^{\infty} 1 \cdot \delta^{t-1} \Rightarrow \\
\frac{4}{1-\delta} &\geq 6 + \frac{\delta}{1-\delta} \Rightarrow \\
\delta &\geq \frac{2}{5}
\end{aligned}
$$

5. Neither player will deviate for $\delta \geq \frac{2}{5}$

PS7, Ex. 6.c: Tit-for-tat strategy (infinitely repeated game)

(c) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Start by playing 'Do not clean', then play 'tit-for-tat' forever after that'.

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating. Then write up the inequality and isolate δ to find for what values of δ player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

PS7, Ex. 6.c: Tit-for-tat strategy (infinitely repeated game)

(c) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Start by playing 'Do not clean', then play 'tit-for-tat' forever after that'.

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating. Then write up the inequality and isolate δ to find for what values of δ player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

In the case where the P2 deviates, the outcome in round 1 will be (clean,don't clean), in the next round, following his tit-for-tat strategy, P1 will play don't clean. P2 will switch to his tit-for-tat strategy and play clean. The outcome in round 2 will be (Don't clean,clean) and in round 3 the (clean, don't clean), continuing this pattern.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

$$4 + 4\delta + ... \geq 6 + 0\delta + 6\delta^2 + 0\delta^3 + 6\delta^4 ... \Rightarrow$$

$$4 + 4\delta + ... \geq 6\delta^0 + 6\delta^2 + 6\delta^4 + ... \Rightarrow$$

$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq \sum_{t=1}^{\infty} 6 \cdot \delta^{2(t-1)} \Rightarrow$$

$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq \sum_{t=1}^{\infty} 6 \cdot (\delta^2)^{t-1} \Rightarrow$$

$$\frac{4}{1-\delta} \geq \frac{6}{1-\delta^2} \Rightarrow$$

$$-2\delta^2 + 3\delta - 1 \geq 0$$

PS7, Ex. 6.c: Tit-for-tat strategy (infinitely repeated game)

(c) Check for which δ tit-for-tat is optimal on the equilibrium path against the following strategy: 'Start by playing 'Do not clean', then play 'tit-for-tat' forever after that'.

(Step a) Define the payoff for staying with the tit-for-tat strategy, and for deviating. Then write up the inequality and isolate δ to find for what values of δ player 2 wouldn't deviate, you only need to check P2 as P2 has the highest incentive to deviate.

In the case where the P2 deviates, the outcome in round 1 will be (clean,don't clean), in the next round, following his tit-for-tat strategy, P1 will play don't clean. P2 will switch to his tit-for-tat strategy and play clean. The outcome in round 2 will be (Don't clean,clean) and in round 3 the (clean, don't clean), continuing this pattern.

1. $U_2(Cl, Cl) = 4$
2. $U_2(Cl, DCl) = 6$
3. $U_2(DCl, DCl) = 1$
4. On the equilibrium path:

$$4 + 4\delta + ... \geq 6 + 0\delta + 6\delta^2 + 0\delta^3 + 6\delta^4 ... \Rightarrow$$

$$\sum_{t=1}^{\infty} 4 \cdot \delta^{t-1} \geq \sum_{t=1}^{\infty} 6 \cdot (\delta^2)^{t-1} \Rightarrow$$

$$\frac{4}{1-\delta} \geq \frac{6}{1-\delta^2} \Rightarrow$$

$$-2\delta^2 + 3\delta - 1 \geq 0$$

1. This is a 2nd degree polynomial which is equal to 0 at $\delta = \frac{1}{2}$ and $\delta = 1$. In between it is positive. I.e. neither player will deviate to the proposed strategy for $\delta \in \left[\frac{1}{2}, 1\right]$.

PS7, Ex. 6.d: Tit-for-tat strategy (infinitely repeated game)

Consider again the the infinitely repeated game $G(\infty, \delta)$ with the stage game:

		Player 2	
		Cl	DCl
Player 1	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(d) Argue informally that 'tit-for-tat' is a NE for the appropriate values of δ. In particular, think about whether there are other deviations that would be better for the players.

PS7, Ex. 6.d: Tit-for-tat strategy (infinitely repeated game)

Consider again the the infinitely repeated game $G(\infty, \delta)$ with the stage game:

Player 2

Player 1		Cl	DCl
	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(d) Argue informally that 'tit-for-tat' is a NE for the appropriate values of δ. In particular, think about whether there are other deviations that would be better for the players.

For $\delta \geq \frac{1}{2}$ we have shown that tit-for-tat is better than the two deviations. If one of the players were to apply the trigger strategy or "always play clean", the outcome would be the same as for playing tit-for-that, which is (clean,clean) in every round.

Of the two deviations, for $\delta \geq \frac{1}{2}$ the "play don't clean then tit for tat" dominates the "always play don't clean". This is seen by looking at the payoff of the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round (1$^{\text{st}}$ round is the same). $1 + \frac{1}{2} \cdot 1 \leq 0 + 6 \cdot \frac{1}{2}$ the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round is essentially repeated forever, so if the payoff for the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round is higher, then the sum of the payoffs are higher.

Could other deviations be better? What is required for a strategy to be part of a NE?

PS7, Ex. 6.d: Tit-for-tat strategy (infinitely repeated game)

Consider again the the infinitely repeated game $G(\infty, \delta)$ with the stage game:

		Player 2	
		Cl	DCl
Player 1	Cl	4, 4	0, 6
	Dcl	5, 0	1, 1

(d) Argue informally that 'tit-for-tat' is a NE for the appropriate values of δ. In particular, think about whether there are other deviations that would be better for the players.

For $\delta \geq \frac{1}{2}$ we have shown that tit-for-tat is better than the two deviations. If one of the players were to apply the trigger strategy or "always play clean", the outcome would be the same as for playing tit-for-that, which is (clean,clean) in every round.

Of the two deviations, for $\delta \geq \frac{1}{2}$ the "play don't clean then tit for tat" dominates the "always play don't clean". This is seen by looking at the payoff of the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round (1$^{\text{st}}$ round is the same). $1 + \frac{1}{2} \cdot 1 \leq 0 + 6 \cdot \frac{1}{2}$ the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round is essentially repeated forever, so if the payoff for the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ round is higher, then the sum of the payoffs are higher.

The final piece of the puzzle is to realize that all other plausible deviations are combinations of the two deviations we have already examined. Thus, for $\delta \geq \frac{1}{2}$ no deviation can give a strictly higher payoff and 'tit-for-tat' is best-response *on* the equilibrium path which is the requirement for being part of a NE.

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Exercise 2.13 in Gibbons (p. 135): Recall the static Bertrand duopoly model (with homogeneous products) from Problem 1.7: the firms name prices simultaneously; demand for firm i's product is $a - p_i$ if $p_i < p_j$, is 0 if $p_i > p_j$, and is $(a - p_i)/2$ if $p_i = p_j$; marginal costs are $c < a$. Consider the infinitely repeated game based on this stage game. Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (\textit{price} - \textit{cost}) \cdot \textit{demand}$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (\mathit{price} - \mathit{marginal\ cost}) \cdot \mathit{demand}$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (\mathit{price} - \mathit{marginal\ cost}) \cdot \mathit{demand}$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (\mathit{price} - \mathit{marginal\ cost}) \cdot \mathit{demand}$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase:

I.e. to split the monopoly market (LHS) vs the best deviation which is to slightly underbid, i.e. $p = p^M - \varepsilon \approx p^M$ (RHS).

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase:

To split the monopoly market (LHS) vs the best deviation which is to slightly underbid, i.e. $p = p^M - \varepsilon \approx p^M$ (RHS):

$$\sum_{t=0}^{\infty} \frac{1}{2}\pi^M \delta^t \geq \pi^M + \sum_{t=1}^{\infty} \frac{1}{2} 0 \cdot \delta^t \Leftrightarrow$$

$$\frac{\frac{1}{2}\pi^M}{1-\delta} \geq \pi^M \Leftrightarrow$$

$$\frac{1}{2} \geq (1-\delta) \Leftrightarrow$$

$$1 \geq 2 - 2\delta \Leftrightarrow$$

$$\delta \geq \frac{1}{2}$$

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

c: The TS is a NE in the "normal" phase for $\delta \geq \frac{1}{2}$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase.

Step d: Check that the trigger strategy (TS) is a NE in the "punishment" phase.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

c: The TS is a NE in the "normal" phase for $\delta \geq \frac{1}{2}$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase.

Step d: Check that the trigger strategy (TS) is a NE in the "punishment" phase:

Given that $p = c$ is a NE in the stage game, it must also be a NE in the "punishment" phase, i.e. it's a best response for both firms, given the other firm's price of $p = c$.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

c: The TS is a NE in the "normal" phase for $\delta \geq \frac{1}{2}$

PS7, Ex. 7: Bertrand duopoly (infinitely repeated game)

Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

Step a: Recall the stage game price level.

Step b: Suggest a trigger strategy that can sustain the monopoly price level p^M.

Step c: Check that the trigger strategy (TS) is a NE in the "normal" phase.

Step d: Check that the trigger strategy (TS) is a NE in the "punishment" phase:

Given that $p = c$ is a NE in the stage game, it must also be a NE in the "punishment" phase, i.e. it's a best response for both firms, given the other firm's price of $p = c$.

Thus, the trigger strategies gives a SPNE where the firms can act together as a monopolist if the they are sufficiently patient, i.e. for $\delta \geq \frac{1}{2}$.

Information so far:

1. Players: Firm i, $i \in 1, 2$
2. Strategies: $S_i = \{p_i | p_i \in \mathbb{R}_+\}$
3. Payoffs:

$$\pi_i(p_i, p_j) = (price - marginal\ cost) \cdot demand$$

$$= \begin{cases} (p_i - c)(a - p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(p_i - c)(a - p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

a: Stage game NE: $p_1^* = p_2^* = c$

b: Play p^M in $t = 0$ or if it was played in all previous rounds ("normal").

Play $p = c$ if there was a deviation in any previous round ("punishment").

c: The TS is a NE in the "normal" phase for $\delta \geq \frac{1}{2}$

d: TS is SPNE for $\delta \geq \frac{1}{2}$

PS7, Ex. 8: Trigger strategy (infinitely repeated game)

PS7, Ex. 8: Trigger strategy (infinitely repeated game)

The next exercises use the following game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

Suppose that the Players play the infinitely repeated game $G(\infty)$ and that they would like to support as a SPNE the 'collusive' outcome in which (L, L) is played every round.

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(Step a) Write up the trigger strategy.

PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(Step a) Write up the trigger strategy.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."
2. Stage game NE: (M, M).

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

(Step b) Explain.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."
2. Stage game NE: (M, M).

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

(Step b) Explain.

For a trigger strategy to constitute a SPNE, the threat of (eternal and unchangeable) punishment must be credible, i.e. must be a stage game NE.

Thus, $x_1 = x_2 = M$ is a necessary (but not sufficient) condition for the trigger strategies to constitute a SPNE.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."
2. Unique stage game NE: (M, M).

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Player i's payoff from playing TS:

$$10 + 10\delta + 10\delta^2 + ... = \sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$.

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Player i's payoff from playing TS:

$$10 + 10\delta + 10\delta^2 + ... = \sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Player i's payoff from playing ODS:

$$15 + 7\delta + 7\delta^2 + ... = 15 + \sum_{t=2}^{\infty} 7\delta^{t-1} = 15 + \frac{7\delta}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$:

$$U_i\,(ODS, TS) > U_i\,(TS, TS)$$

$$\Rightarrow 15 + \frac{7\frac{4}{7}}{1-\frac{4}{7}} > \frac{10}{1-\frac{4}{7}}, \qquad \text{for } \delta = \frac{4}{7}$$

$$\Rightarrow \frac{73}{3} > \frac{70}{3} \qquad Q.E.D.$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
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PS7, Ex. 9: Optimal punishment strategy (infinitely repeated game)

PS7, Ex. 9: Optimal punishment strategy (infinitely repeated game)

We continue analyzing $G(\infty)$. As in ~~Lecture 8~~ (Lecture 6, slides 50-68), consider the strategy profile (OP, OP), where OP stands for optimal punishment...

[See the lecture slides and the full description of the exercise in the problem set.]

Stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 9.a: Optimal punishment strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q_D, rather than deviate again).

PS7, Ex. 9.a: Optimal punishment strategy (infinitely repeated game)

Consider $G(\infty)$ with stage game G, underlining (Q^D, Q^P) in the 'tough' stage:

	L	M	<u>H</u>
L	10, 10	3, 15	0, 7
<u>**M**</u>	15, 3	7, 7	<u>**-4, 5**</u>
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q^D, rather than deviate again).

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the 'punishment path' (Q^D, Q^P).

PS7, Ex. 9.a: Optimal punishment strategy (infinitely repeated game)

Consider $G(\infty)$ with stage game G, underlining (Q^D, Q^P) in the 'mild' stage:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q^D, rather than deviate again).

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the 'punishment path' (Q^D, Q^P).

The 'mild' stage (from the 2nd round of punishment):

- P1 earns 3 from (L, M) in this and all subsequent rounds.
- He can deviate by playing M and earn 7, but then the punishment will start over again and P1 will earn -4 in the 'tough' stage (or alternatively 0 by deviating again).
- Both -4 and 0 is less than 3, which is why this punishment structure provides P1 with an incentive to stay in the 'mild stage'.

PS7, Ex. 9.b: Optimal punishment strategy (infinitely repeated game)

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the punishment path.

The 'mild' stage (from the 2nd round of punishment):

- P1 earns 3 from (L, M) in this and all subsequent rounds.
- He can deviate by playing M and earn 7, but then the punishment will start over again and P1 will earn -4 in the 'tough' stage (or alternatively 0 by deviating again).
- Both -4 and 0 is less than 3, which is why this punishment structure provides P1 with an incentive to stay in the 'mild stage'.

(b) ***How does your answer relate to the following quote from Wikipedia?***
The "carrot and stick" approach is an idiom that refers to a policy of offering a combination of rewards and punishment to induce behavior. It is named in reference to a cart driver dangling a carrot in front of a mule and holding a stick behind it. The mule would move towards the carrot because it wants the reward of food, while also moving away from the stick behind it, since it does not want the punishment of pain, thus drawing the cart.

PS7, Ex. 9.b: Optimal punishment strategy (infinitely repeated game)

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the punishment path.

The 'mild' stage (from the 2nd round of punishment):

- P1 earns 3 from (L, M) in this and all subsequent rounds.
- He can deviate by playing M and earn 7, but then the punishment will start over again and P1 will earn -4 in the 'tough' stage (or alternatively 0 by deviating again).
- Both -4 and 0 is less than 3, which is why this punishment structure provides P1 with an incentive to stay in the 'mild stage'.

(b) How does your answer relate to the following quote from Wikipedia?
The "carrot and stick" approach is an idiom that refers to a policy of offering a combination of rewards and punishment to induce behavior. It is named in reference to a cart driver dangling a carrot in front of a mule and holding a stick behind it. The mule would move towards the carrot because it wants the reward of food, while also moving away from the stick behind it, since it does not want the punishment of pain, thus drawing the cart.
In other words: Which stage is "the carrot" and which is "the stick"? Explain.

PS7, Ex. 9.b: Optimal punishment strategy (infinitely repeated game)

(b) How does your answer relate to the following quote from Wikipedia?

The "carrot and stick" approach is an idiom that refers to a policy of offering a combination of rewards and punishment to induce behavior. It is named in reference to a cart driver dangling a carrot in front of a mule and holding a stick behind it. The mule would move towards the carrot because it wants the reward of food, while also moving away from the stick behind it, since it does not want the punishment of pain, thus drawing the cart.

The 'tough' stage (the 1st round of punishment):

- Is the "the stick" that harshly punishes any deviation. During the 'mild' stage, the threat of a future 'tough punishment' should discourage deviation from the 'punishment path'.

The 'mild' stage (from the 2nd round of punishment):

PS7, Ex. 9.b: Optimal punishment strategy (infinitely repeated game)

(b) How does your answer relate to the following quote from Wikipedia?

The "carrot and stick" approach is an idiom that refers to a policy of offering a combination of rewards and punishment to induce behavior. It is named in reference to a cart driver dangling a carrot in front of a mule and holding a stick behind it. The mule would move towards the carrot because it wants the reward of food, while also moving away from the stick behind it, since it does not want the punishment of pain, thus drawing the cart.

The 'tough' stage (the 1st round of punishment):

- Is the "the stick" that harshly punishes any deviation. During the 'mild' stage, the threat of a future 'tough punishment' should discourage deviation from the 'punishment path'.

The 'mild' stage (from the 2nd round of punishment):

- Is "the carrot" as the promise of a future 'mild punishment' should be regarded as a reward for accepting the punishment in the 'tough' stage without deviating from the 'punishment path'.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

We continue analyzing $G(\infty)$. Complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$. We checked the first three points of the road map in Lecture 6 (slide 56). The last two points consist of checking that it is optimal for Player 2 to punish Player 1 after a deviation. In particular, you need to check that Player 2 has no profitable deviation when he is in the first and in the second round of punishing Player 1.

[The roadmap is summed up on the next two slides]

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

We continue analyzing $G(\infty)$. Complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$. We checked the first three points of the road map in Lecture 6 (slide 56). The last two points consist of checking that it is optimal for Player 2 to punish Player 1 after a deviation. In particular, you need to check that Player 2 has no profitable deviation when he is in the first and in the second round of punishing Player 1.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1st round of being punished.
3. Subsequent rounds of being punished.

Underlining (Q^D, Q^P) in the 1st round of punishment (the 'tough' stage):

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

Underlining (Q^D, Q^P) in subsequent rounds of punishment (the 'mild' stage):

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

We continue analyzing $G(\infty)$. Complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$. We checked the first three points of the road map in Lecture 6 (slide 56). The last two points consist of checking that it is optimal for Player 2 to punish Player 1 after a deviation. In particular, you need to check that Player 2 has no profitable deviation when he is in the first and in the second round of punishing Player 1.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1st round of being punished.
3. Subsequent rounds of being punished.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.
5. When he is in subsequent rounds ('mild' stage) of punishing Player 1.

Remember to use $\delta = 4/7$.

Underlining (Q^D, Q^P) in the 1st round of punishment (the 'tough' stage):

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

Underlining (Q^D, Q^P) in subsequent rounds of punishment (the 'mild' stage):

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.

First, calculate Player 2's expected utility from sticking to Q^P for $\delta = 4/7$ (i.e. from not deviating).

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

If Player 2 does not deviate from Q^P, his expected utility is:

$$U_2(\underbrace{Q^D}_{s_1}; \underbrace{Q^P}_{s_2}) = 5 + 15\delta + 15\delta^2 + \ldots = 5 + \sum_{t=2}^{\infty} 15\delta^{t-1} = 5 + \frac{15\delta}{1-\delta} = 25, \text{ for } \delta = \frac{4}{7}$$

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

If Player 2 does not deviate from Q^P, his expected utility is:

$$U_2(\underbrace{Q^D}_{s_1}; \underbrace{Q^P}_{s_2}) = 5 + 15\delta + 15\delta^2 + \ldots = 5 + \sum_{t=2}^{\infty} 15\delta^{t-1} = 5 + \frac{15\delta}{1-\delta} = 25, \text{ for } \delta = \frac{4}{7}$$

What is Player 2's expected utility from his best possible deviation when he is in the 1st round of punishing Player 1?

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

If Player 2 does not deviate from Q^P, his expected utility is:

$$U_2(\underbrace{Q^D}_{s_1}; \underbrace{Q^P}_{s_2}) = 5 + 15\delta + 15\delta^2 + ... = 5 + \sum_{t=2}^{\infty} 15\delta^{t-1} = 5 + \frac{15\delta}{1-\delta} = 25, \text{ for } \delta = \frac{4}{7}$$

If Player 2 deviates from Q^P to play M, from the next round Player 1 will instead force Player 2 to take the punishment and play according to Q^D forever:

$$U_2(\underbrace{M, Q^P}_{s_1'}; \underbrace{M, Q^D}_{s_2'}) = 7 + \delta U_2(Q^P; Q^D) = 7 + \delta\left(-4 + \sum_{t=3}^{\infty} 3\delta^{t-1}\right) = 7 + \delta \underbrace{\left(-4 + \frac{3\delta}{1-\delta}\right)}_{=0 \text{ for } \delta = 4/7}$$

$$= 7, \text{ for } \delta = 4/7$$

Does Player 2 have an incentive to deviate when he is in the 1st round of punishing Player 1?

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

4. When he is in the 1st round (the 'tough' stage) of punishing Player 1.

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

If Player 2 does not deviate from Q^P, his expected utility is:

$$U_2(\underbrace{Q^D}_{s_1}; \underbrace{Q^P}_{s_2}) = 5 + 15\delta + 15\delta^2 + ... = 5 + \sum_{t=2}^{\infty} 15\delta^{t-1} = 5 + \frac{15\delta}{1-\delta} = 25, \text{ for } \delta = \frac{4}{7}$$

If Player 2 deviates from Q^P to play M, from the next round Player 1 will instead force Player 2 to take the punishment and play according to Q^D forever:

$$U_2(\underbrace{M, Q^P}_{s_1'}; \underbrace{M, Q^D}_{s_2'}) = 7 + \delta U_2(Q^P; Q^D) = 7 + \delta \left(-4 + \sum_{t=3}^{\infty} 3\delta^{t-1} \right) = 7 + \delta \underbrace{\left(-4 + \frac{3\delta}{1-\delta} \right)}_{=0 \text{ for } \delta = 4/7}$$

$$= 7, \text{ for } \delta = 4/7$$

As $U_2(Q^D; Q^P) = \mathbf{25} > \mathbf{7} = U_2(M, Q^P; M, Q^D)$, **Player 2 has no incentive to deviate.**

I.e. in the 1st round of punishing Player 1, Player 2 expects higher utility from playing according to Q^P (25) than from deviating (7) for $\delta = 4/7$.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

5. When he is in subsequent rounds ('mild' stage) of punishing Player 1.

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

5. When he is in subsequent rounds ('mild' stage) of punishing Player 1.

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

In each subsequent round, the outcome from (Q^D, Q^P) is (L, M) with payoffs $(3, 15)$.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

5. When he is in subsequent rounds ('mild' stage) of punishing Player 1.

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

In each subsequent round, the outcome from (Q^D, Q^P) is (L, M) with payoffs $(3, 15)$.

Does player 2 have an incentive to deviate during this 'mild' stage?

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

5. When he is in subsequent rounds ('mild' stage) of punishing Player 1.

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

In each subsequent round, the outcome from (Q^D, Q^P) is (L, M) with payoffs $(3, 15)$.

From the 2$^{\text{nd}}$ round of punishing Player 1, Player 2 expects to earn 15 in every round which is the highest possible payoff in the stage game.

I.e. in the 'mild' stage there is no incentive to deviate from Q^P for any value of $\delta \geq 0$.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1$^{\text{st}}$ round of being punished.
3. Subsequent rounds of being punished.

In this exercise we have checked that Player 2 will not deviate:

4. When he is in the 1$^{\text{st}}$ round of punishing Player 1.
5. When he is in subsequent rounds of punishing Player 1.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. **Round 1, or if (L, L) was played in all previous rounds.**
2. The 1^{st} round of being punished.
3. Subsequent rounds of being punished.

In this exercise we have checked that Player 2 will not deviate:

4. When he is in the 1^{st} round of punishing Player 1.
5. When he is in subsequent rounds of punishing Player 1.

Condition 1 secures that (OP, OP) is optimal ***on*** the equilibrium path.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. **The 1$^{\text{st}}$ round of being punished.**
3. **Subsequent rounds of being punished.**

In this exercise we have checked that Player 2 will not deviate:

4. **When he is in the 1$^{\text{st}}$ round of punishing Player 1.**
5. **When he is in subsequent rounds of punishing Player 1.**

Condition 1 secures that (OP, OP) is optimal ***on*** the equilibrium path.

Conditions 2-5 secure that (OP, OP) is optimal ***off*** the equilibrium path.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1st round of being punished.
3. Subsequent rounds of being punished.

In this exercise we have checked that Player 2 will not deviate:

4. When he is in the 1st round of punishing Player 1.
5. When he is in subsequent rounds of punishing Player 1.

Condition 1 secures that (OP, OP) is optimal ***on*** the equilibrium path.

Conditions 2-5 secure that (OP, OP) is optimal ***off*** the equilibrium path.

As all conditions hold, we can conclude that (OP, OP) is a SPNE for $\delta = 4/7$.