Microeconomics III: Problem Set 7[a]

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[a]Slides created for exercise class 3 and 4, with reservation for possible errors.

Outline

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

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In this course we normally consider games in which there is 'perfect recall': players can always remember what they themselves have done in the past.

We have seen an example in class of a game with 'imperfect recall' where the player forgets his own actions. But what would a game where he forgets the opponent's actions look like? Construct a game with two players. The timing is as follows: Player 1 moves first, then Player 2, and then Player 2 again. Everytime they move, the players choose one of two actions: $\{L, R\}$.

Draw the game tree and construct the information sets such that (a) Player 2 observes Player 1's action the first time he moves, but (b) when Player 2 moves the second time, he has forgotten what Player 1 chose. However, he recalls his own action.

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

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We have seen an example in class of a game with 'imperfect recall' where the player forgets his own actions. But what would a game where he forgets the opponent's actions look like? Construct a game with two players. The timing is as follows: Player 1 moves first, then Player 2, and then Player 2 again. Everytime they move, the players choose one of two actions: $\{L, R\}$.

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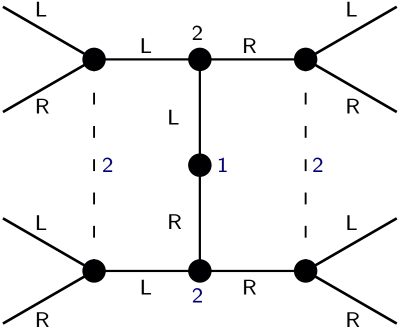

PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

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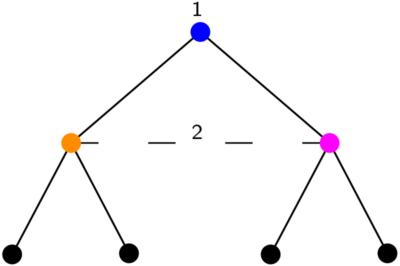
Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.

Example of violation of condition 1:



PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

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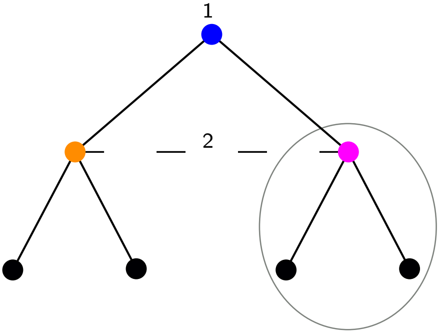


The purple decision node to the right is not a singleton information set (nor is the orange decision node to the left).

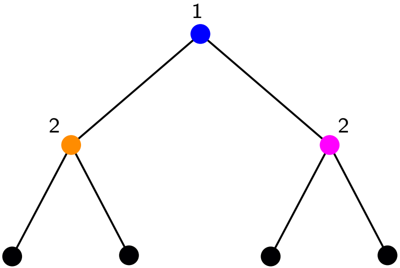
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.
2. **It includes all following decision and terminal nodes following *n* in the game tree**, but no nodes that do not follow *n*.

Example of violation of the first part of condition 2:



PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

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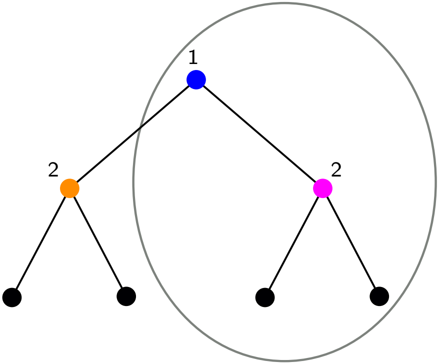


For a subgame containing the blue decision node n, all following decision nodes must be included.

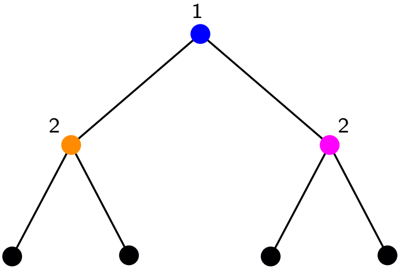
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. It includes all following decision and terminal nodes following n in the game tree, **but no nodes that do not follow n.**

Example of violation of the second part of condition 2:



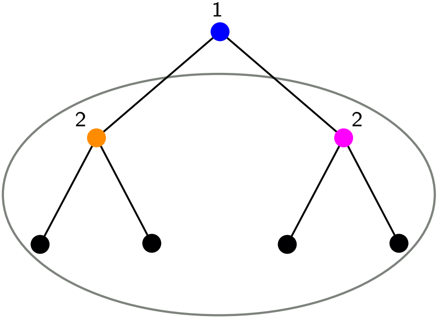
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node *n* that is a singleton information set.
2. It includes all following decision and terminal nodes following *n* in the game tree, **but no nodes that do not follow *n*.**

Example of violation of the second part of condition 2:



Regardless of whether the orange or the purple node is chosen as the first decision node *n*, the other decision node does not follow *n*, and therefore cannot be part of the subgame.

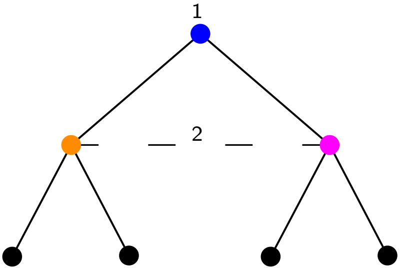
PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node n that is a singleton information set.
2. It includes all following decision and terminal nodes following n in the game tree, but no nodes that do not follow n.
3. It does not "cut" any information set: if a decision node n' follows n in the game tree, then all other nodes in the information set including n' must also follow n (and so be included in the subgame).

Example of violation of condition 3:



PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

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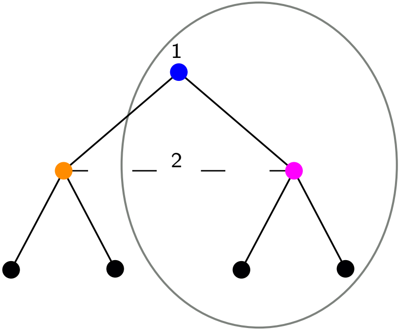
Example of violation of condition 3:



The orange decision node to the left is part of the same information set as the purple node to the right, so it must be included in the same subgame.

PS7, Ex. 3 (A):

PS7, Ex. 3 (A):

PS7, Ex. 3.a (A):

PS7, Ex. 4:

PS7, Ex. 4:

PS7, Ex. 4.a:

PS7, Ex. 5:

PS7, Ex. 5:

PS7, Ex. 5.a:

PS7, Ex. 6:

PS7, Ex. 6:

PS7, Ex. 6.a:

PS7, Ex. 7:

PS7, Ex. 7:

PS7, Ex. 7.a:

PS7, Ex. 8: Trigger strategy (infinitely repeated game)

The next exercises use the following game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

Suppose that the Players play the infinitely repeated game $G(\infty)$ and that they would like to support as a SPNE the 'collusive' outcome in which (L, L) is played every round.

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
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(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(Step a) Write up the trigger strategy.

PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
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(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives (x_1, x_2) forever after a deviation.

(Step a) Write up the trigger strategy.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

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(Step a) Find the PSNE in the stage game G.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."
2. Stage game NE: (M, M).

PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

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L	10, 10	3, 15	0, 7
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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.

(Step b) Explain.

Information so far:

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Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game G.
(Step b) Explain.

For a trigger strategy to constitute a SPNE, the threat of (eternal and unchangeable) punishment must be credible, i.e. must be a stage game NE.

Thus, $x_1 = x_2 = M$ is a necessary (but not sufficient) condition for the trigger strategies to constitute a SPNE.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. Otherwise, play x_i."
2. Unique stage game NE: (M, M).

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

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Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
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(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Player i's payoff from playing TS:

$$10 + 10\delta + 10\delta^2 + ... = \sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$.

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

Player i's payoff from playing TS:

$$10 + 10\delta + 10\delta^2 + ... = \sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Player i's payoff from playing ODS:

$$15 + 7\delta + 7\delta^2 + ... = 15 + \sum_{t=2}^{\infty} 7\delta^{t-1} = 15 + \frac{7\delta}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
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4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player j plays the Trigger Strategy (TS), write up Player i's Optimal Deviation Strategy (ODS).

(Step b) Given Player j plays TS, write up Player i's respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$:

$$U_i\left(ODS, TS\right) > U_i\left(TS, TS\right)$$

$$\Rightarrow 15 + \frac{7\frac{4}{7}}{1-\frac{4}{7}} > \frac{10}{1-\frac{4}{7}}, \qquad \text{for } \delta = \frac{4}{7}$$

$$\Rightarrow \frac{73}{3} > \frac{70}{3} \qquad Q.E.D.$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was (L, L), play L. If not, play M."
2. Optimal Deviation Strategy (ODS): "Always play M."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

PS7, Ex. 9: Optimal Punishment Strategy (infinitely repeated game)

We continue analyzing $G(\infty)$. As in ~~Lecture 8~~ (Lecture 6, slides 50-68), consider the strategy profile (OP, OP), where OP stands for optimal punishment...

[See the lecture slides and the full description of the exercise in the problem set.]

Stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 9.a: Optimal Punishment Strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game G:

	L	M	H
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q_D, rather than deviate again).

PS7, Ex. 9.a: Optimal Punishment Strategy (infinitely repeated game)

Consider $G(\infty)$ with stage game G, underlining (Q^D, Q^P) in the 'tough' stage:

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q^D, rather than deviate again).

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the 'punishment path' (Q^D, Q^P).

PS7, Ex. 9.a: Optimal Punishment Strategy (infinitely repeated game)

Consider $G(\infty)$ with stage game G, underlining (Q^D, Q^P) in the 'mild' stage:

	L	<u>M</u>	H
<u>L</u>	10, 10	<u>3, 15</u>	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

(a) Discuss how this increased leniency over time may give player 1 an incentive to accept his punishment (and actually play according to Q^D, rather than deviate again).

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the 'punishment path' (Q^D, Q^P).

The 'mild' stage (from the 2nd round of punishment):

- P1 earns 3 from (L, M) in this and all subsequent rounds.
- He can deviate by playing M and earn 7, but then the punishment will start over again and P1 will earn -4 in the 'tough' stage (or alternatively 0 by deviating again).
- Both -4 and 0 is less than 3, which is why this punishment structure provides P1 with an incentive to stay in the 'mild stage'.

PS7, Ex. 9.b: Optimal Punishment Strategy (infinitely repeated game)

The 'tough' stage (the 1st round of punishment):

- P1 earns -4 from (M, H) and 3 from (L, M) in all later rounds.
- He can deviate by playing L and earn 0, but then the punishment will start over again and P1 will therefore stay in the 'tough' stage.
- Both -4 and 0 is less than 3, which is why this punishment structure leaves P1 without an incentive to deviate from the punishment path.

The 'mild' stage (from the 2nd round of punishment):

- P1 earns 3 from (L, M) in this and all subsequent rounds.
- He can deviate by playing M and earn 7, but then the punishment will start over again and P1 will earn -4 in the 'tough' stage (or alternatively 0 by deviating again).
- Both -4 and 0 is less than 3, which is why this punishment structure provides P1 with an incentive to stay in the 'mild stage'.

(b) How does your answer relate to the following quote from Wikipedia?
The "carrot and stick" approach is an idiom that refers to a policy of offering a combination of rewards and punishment to induce behavior. It is named in reference to a cart driver dangling a carrot in front of a mule and holding a stick behind it. The mule would move towards the carrot because it wants the reward of food, while also moving away from the stick behind it, since it does not want the punishment of pain, thus drawing the cart.

PS7, Ex. 9.b: Optimal Punishment Strategy (infinitely repeated game)

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The 'tough' stage (the 1$^{\text{st}}$ round of punishment):

- The promise of a future 'mild punishment' should be regarded as a reward for accepting the punishment without deviating from the 'punishment path' and is therefore the carrot.

PS7, Ex. 9.b: Optimal Punishment Strategy (infinitely repeated game)

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The 'tough' stage (the 1st round of punishment):

- The promise of a future 'mild punishment' should be regarded as a reward for accepting the punishment without deviating from the 'punishment path' and is therefore the carrot.

The 'mild' stage (from the 2nd round of punishment):

- The threat of a future 'tough punishment' should be regarded as further punishment for deviating in the first place and is therefore the stick.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

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We continue analyzing $G(\infty)$. Complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$. We checked the first three points of the road map in Lecture 6. The last two points consist of checking that it is optimal for Player 2 to punish Player 1 after a deviation. In particular, you need to check that Player 2 has no profitable deviation when he is in the first and in the second round of punishing Player 1.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

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In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1st round of being punished.
3. Subsequent rounds of being punished.

Underlining (Q^D, Q^P) in the 1st round of punishment (the 'tough' stage):

	L	M	**H**
L	10, 10	3, 15	0, 7
M	15, 3	7, 7	**-4, 5**
H	7, 0	5, -4	-15, -15

Underlining (Q^D, Q^P) in subsequent rounds of punishment (the 'mild' stage):

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

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Check that Player 2 will not deviate:

4. When he is in the 1st round of punishing Player 1.
5. When he is in subsequent rounds of punishing Player 1.

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If Player 2 does not deviate from Q^P, his expected utility is:

$$U_2(Q^P; Q^D) = 5 + 15\delta + 15\delta^2 + \ldots = 5 + \sum_{t=2}^{\infty} 15\delta^{t-1} = 5 + \frac{15\delta}{1-\delta} = 25, \text{ for } \delta = \frac{4}{7}$$

If Player 2 deviates from Q^P to play M, from the next round Player 1 will instead force Player 2 to take the punishment and play according to Q^D forever:

$$U_2(M, Q^D; M, Q^P) = 7 + \delta U_2(Q^D; Q^P) = 7 + \delta\left(-4 + \sum_{t=3}^{\infty} 3\delta^{t-1}\right) = 7 + \delta\left(-4 + \frac{3\delta}{1-\delta}\right)$$

$$= 7, \text{ for } \delta = 4/7$$

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

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$$= 7, \text{ for } \delta = 4/7$$

In the 1st round of punishing Player 1, Player 2 expects higher utility from playing according to Q^P (25) than from deviating (7), i.e. Player 2 has no incentive to deviate.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

Check that Player 2 will not deviate:

5. When he is in subsequent rounds of punishing Player 1:

	L	**M**	H
L	10, 10	**3, 15**	0, 7
M	15, 3	7, 7	-4, 5
H	7, 0	5, -4	-15, -15

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

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Check that Player 2 will not deviate:

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L	10, 10	**3, 15**	0, 7
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In each subsequent round, the outcome from (Q^D, Q^P) is (L, M) with payoffs $(3, 15)$.

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H	7, 0	5, -4	-15, -15

In each subsequent round, the outcome from (Q^D, Q^P) is (L, M) with payoffs $(3, 15)$.

From the 2$^{\text{nd}}$ round of punishing Player 1, Player 2 expects to earn 15 in every round, i.e. he has no incentive to deviate.

PS7, Ex. 10: Is the punishment credible? (infinitely repeated game)

Use the roadmap to complete the proof that (OP, OP) is a SPNE when $\delta = 4/7$.

In the lecture it was checked that Player 1 will not deviate from (OP, OP) in:

1. Round 1, or if (L, L) was played in all previous rounds.
2. The 1^{st} round of being punished.
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In this exercise we have checked that Player 2 will not deviate:

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Condition 1 secures that (OP, OP) is optimal ***on*** the equilibrium path.

Condition 2-5 secure that (OP, OP) is optimal ***off*** the equilibrium path.

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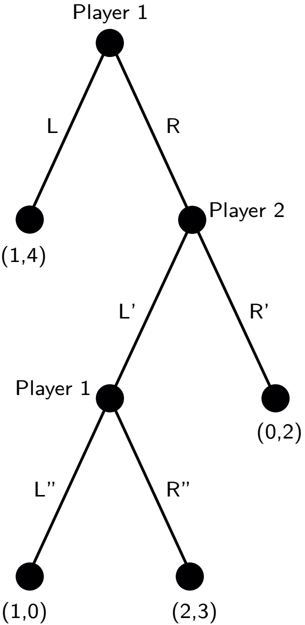
Condition 1 secures that (OP, OP) is optimal ***on*** the equilibrium path.

Condition 2-5 secure that (OP, OP) is optimal ***off*** the equilibrium path.

Therefore, we can conclude that (OP, OP) is a SPNE for $\delta = 4/7$.

Code examples

Code examples



Matrix, no player names:

	L (q)	R (1-q)
T (p)		
B (1-p)		

Matrix, no colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)		
	B (1-p)		

Matrix, with colors:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	
	B (1-p)		