PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

Highlight the best responses in pure strategies.

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Player 1 \ Player 2	L (q)	R (1-q)
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For which values of q is Player 1 indifferent?

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Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1(T)|q] = E[u_1(B)|q]$$
$$=$$

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Player 2

Player 1		L (q)	R (1-q)
	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1(T)|q] = E[u_1(B)|q]$$

$$q = q + 2(1-q) \Leftrightarrow q = 1$$

For which values of p is Player 2 indifferent?

Find p such that Player 2 expect to have equal payoffs from playing L and R:

$$E[u_2(L)|q] = E[u_2(R)|q]$$

$$=$$

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(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

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Find q such that Player 1 expects to have equal payoffs from playing T and B:

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Find p such that Player 2 expect to have equal payoffs from playing L and R:

$$E[u_2(L)|q] = E[u_2(R)|q]$$

$$p = 1 - p \Leftrightarrow p = \frac{1}{2}$$

Write up all NE (pure and mixed).

$$NE = (p^*, q^*) =$$

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

Player 1 \ Player 2	L (q)	R (1-q)
T (p)	1, 1	0, 0
B (1-p)	1, 0	2, 1

Player 1 is indifferent for:

$$E[u_1(T)|q] = E[u_1(B)|q]$$

$$q = q + 2(1-q) \Leftrightarrow q = 1$$

Player 2 is indifferent for:

$$E[u_2(L)|q] = E[u_2(R)|q]$$

$$p = 1 - p \Leftrightarrow p = \frac{1}{2}$$

The pure and mixed strategy NE are:

$$NE : \left\{ (0,0); (1,1); \left(p \in \left[\frac{1}{2}, 1 \right), q = 1 \right) \right\}$$

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(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

		Player 2	
		L (q)	L (1-q)
Player 1	T (p)	3, 3	0, 0
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Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1(T)|q] = E[u_1(B)|q]$$

$$3q = 4(1-q) \Leftrightarrow q = \frac{4}{7}$$

The players have symmetric payoffs, thus:

$$NE = (p^*, q^*) = \left\{(0,0); (1,1); \left(\frac{4}{7}, \frac{4}{7}\right)\right\}$$

Write up Player 1's best-response (BR) function, $p^*(q)$

$$BR_1(q) = \{$$

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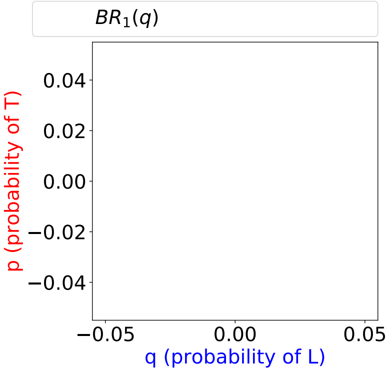
Plot Player 1's best-response (BR) function, $p^*(q)$

Write up and plot the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 4/7 \\ p \in [0,1] & \text{if} \quad q = 4/7 \\ p = 1 & \text{if} \quad q > 4/7 \end{cases}$$



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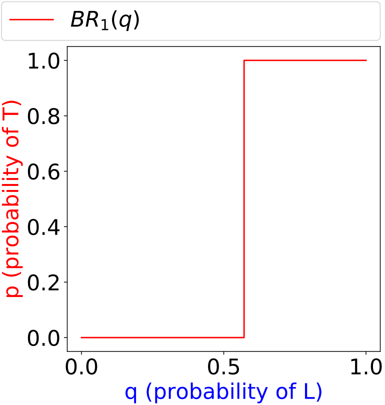

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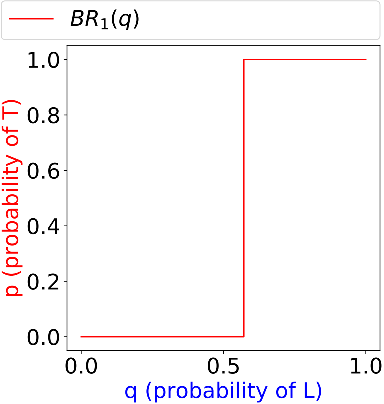

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