



## Microeconomics III: Problem Set 9<sup>a</sup>

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<sup>a</sup>Slides created for exercise class 3 and 4, with reservation for possible errors.

PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

PS8, Ex. 2 (A): Mixed-Strategy BNE

**PS8, Ex. 1 (A): Mixed-Strategy NE  
and Pure-Strategy BNE**

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## PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

Consider this static game, where  $k \in \mathbb{N}$ :

	C	D
A	0, 2	2, 3
B	3, 1	$k$ , 8

- (a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).
- (b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

*[Hints on the next slide. Try to independently write down the approach/criteria for a mixed-strategy NE and a pure-strategy BNE respectively.]*

## PS8, Ex. 1 (A): Mixed-Strategy NE and Pure-Strategy BNE

Consider this static game, where  $k \in \mathbb{N}$ :

	C ( $q$ )	D ( $1-q$ )
A ( $p$ )	0, 2	2, 3
B ( $1-p$ )	3, 1	$k$ , 8

(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

**Hint:** To find a mixed-strategy NE (MSNE):

Find the probabilities  $q$  for which Player 1 is indifferent, i.e.  $u_1(A, q) = u_1(B, q)$  and the probabilities  $p$  for which Player 2 is indifferent, i.e.  $u_2(C, p) = u_2(D, p)$ .

(b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

**Hint:** Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type,  $t_2$ ). For each possible strategy  $s_2(t_2)$ :

1. Given the different types  $t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$ , write up the best response of player 1:  $s_1^*(t_{1,k}) \equiv BR_1(s_2(t_2)|t_{1,k})$ .
2. If it also holds that  $s_2(t_2) = BR_2(s_1^*(t_{1,k})|t_2) \equiv s_2^*(t_2)$  then  $(s_1^*(t_{1,k}), s_2^*(t_2))$  is a BNE.

## PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where  $k \in \mathbb{N}$ :

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

***First, find all pure-strategy NE given  $k$ .***

## PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where  $k \in \mathbb{N}$ :

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

P2:  $C$  is strictly dominated by  $D$ , thus  $D$  is played in any NE, pure or mixed.

P1: For P2 playing  $D$  consider:

$k = 1$ :

$k = 2$ :

$k \geq 3$ :

## PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where  $k \in \mathbb{N}$ :

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

P2:  $C$  is strictly dominated by  $D$ , thus  $D$  is played in any NE, pure or mixed.

P1: For P2 playing  $D$  consider:

$k = 1$ : One PSNE:  $\{(A, D)\}$

$k = 2$ : Two PSNE:  $\{(A, D); (B, D)\}$

$k \geq 3$ : One PSNE:  $\{(B, D)\}$

***Then find all mixed-strategy NE given  $k$ .***



## PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where  $k \in \mathbb{N}$ :

	C (q)	D (1-q)
A (p)	0, 2	2, 3
B (1-p)	3, 1	k, 8

(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

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P1: For P2 playing  $D$  consider:

$k = 1$  : One PSNE:  $\{(A, D)\}$

$k = 2$  : Two PSNE:  $\{(A, D); (B, D)\}$

$k \geq 3$  : One PSNE:  $\{(B, D)\}$

For P1 to mix, she has to be indifferent between  $A$  and  $B$ , thus we only need to look at:

$k = 2$  :

## PS8, Ex. 1.a (A): Mixed-Strategy NE

Consider this static game, where  $k \in \mathbb{N}$ :

	C (q)	D (1-q)
A (p)	0, 2	2, 3
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(a) For all possible values of  $k$ , find all Nash Equilibria (pure and mixed).

P2:  $C$  is strictly dominated by  $D$ , thus  $D$  is played in any NE, pure or mixed.

P1: For P2 playing  $D$  consider:

$k = 1$ : One PSNE:  $\{(A, D)\}$

$k = 2$ : Two PSNE:  $\{(A, D); (B, D)\}$

$k \geq 3$ : One PSNE:  $\{(B, D)\}$

For P1 to mix, she has to be indifferent between  $A$  and  $B$ , thus we only need to look at:

$k = 2$ : One MSNE:  $\{(p \in (0, 1), q = 0)\}$

Consider this static game, where  $k \in \mathbb{N}$ :

	C	D
A	0, 2	2, 3
B	3, 1	$k$ , 8

- (b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

*[Hint for BNE on next slide.]*

## PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where  $k \in \mathbb{N}$ :

	C	D
A	0, 2	2, 3
B	3, 1	$k$ , 8

- (b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

**Hint:** Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type,  $t_2$ ). For each possible strategy  $s_2(t_2)$ :

Step 1: Given the different types

$t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$ , write up the best response of player 1:  
 $s_1^*(t_{1,k}) \equiv BR_1(s_2(t_2)|t_{1,k})$ .

Step 2: If it also holds that

$s_2(t_2) = BR_2(s_1^*(t_{1,k})|t_2) \equiv s_2^*(t_2)$   
then  $(s_1^*(t_{1,k}), s_2^*(t_2))$  is a BNE.

## PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where  $k \in \mathbb{N}$ :

	C	D
A	0, 2	2, 3
B	3, 1	$k$ , 8

- (b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

**Hint:** Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type,  $t_2$ ). For each possible strategy  $s_2(t_2)$ :

Step 1: Given the different types

$t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$ , write up the best response of player 1:  
 $s_1^*(t_{1,k}) \equiv BR_1(s_2(t_2)|t_{1,k})$ .

Step 2: If it also holds that

$s_2(t_2) = BR_2(s_1^*(t_{1,k})|t_2) \equiv s_2^*(t_2)$   
then  $(s_1^*(t_{1,k}), s_2^*(t_2))$  is a BNE.

As  $C$  is strictly dominated, player 2 only has the viable strategy  $s_2(t_2) = D$ :

1. Best response of player 1,  $s_1^*(t_{1,k})$ :

$$BR_1(D|t_{1,k}) = (s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}) = (A, B)$$

## PS8, Ex. 1.b (A): Pure-Strategy BNE

Consider this static game, where  $k \in \mathbb{N}$ :

	C	D
A	0, 2	2, 3
B	3, 1	$k$ , 8

- (b) Now assume that player 1 knows  $k$ , but player 2 only knows that  $k = 1$  with probability  $\frac{1}{2}$  and  $k = 3$  with probability  $\frac{1}{2}$ . Find the Bayesian Nash Equilibrium of this game.

**Hint:** Find Bayesian Nash Equilibria (BNE) by going through the possible strategies for player 2 (the player with only one type,  $t_2$ ). For each possible strategy  $s_2(t_2)$ :

Step 1: Given the different types

$t_{1,k} \in T_1 = \{t_{1,k=1}, t_{1,k=3}\}$ , write up the best response of player 1:  
 $s_1^*(t_{1,k}) \equiv BR_1(s_2(t_2)|t_{1,k})$ .

Step 2: If it also holds that

$s_2(t_2) = BR_2(s_1^*(t_{1,k})|t_2) \equiv s_2^*(t_2)$   
then  $(s_1^*(t_{1,k}), s_2^*(t_2))$  is a BNE.

As  $C$  is strictly dominated, player 2 only has the viable strategy  $s_2(t_2) = D$ :

1. Best response of player 1,  $s_1^*(t_{1,k})$ :

$$BR_1(D|t_{1,k}) = (s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}) = (A, B)$$

2. As  $D = BR_2((A, B)|t_2) \equiv s_2^*(t_2)$  we have a unique BNE:

$$((s_1^*|t_{1,k=1}, s_1^*|t_{1,k=3}), s_2^*(t_2)) = \{((A, B), D)\}$$

## **PS8, Ex. 2 (A): Mixed-Strategy BNE**

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## PS8, Ex. 2 (A): Mixed-Strategy BNE

Consider the same set-up as exercise 3.4 in Gibbons, but now with the following bi-matrices for Game 1 and Game 2 respectively:

G1:

	L	R
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L	R
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

*Exercise 3.4 in Gibbons (p. 169). Find all the pure-strategy Bayesian Nash equilibria in the following static Bayesian game:*

- Nature determines whether the payoffs are as in Game 1 or as in Game 2, each game being equally likely.*
- Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not.***
- Player 1 chooses either  $U$  or  $D$ ; player 2 simultaneously chooses either  $L$  or  $R$ .*
- Payoffs are given by the game drawn by nature.*



## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:		L	R
U		1, 1	0, 0
D		0, 0	2, 0

G2:		L	R
U		0, 0	0, 0
D		0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

Step 1: **Find the player 1 strategies  $s_1'$  for which player 2 will want to mix.**

## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:		L	R
U		1, 1	0, 0
D		0, 0	2, 0

G2:		L	R
U		0, 0	0, 0
D		0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

Step 1: Find the player 1 strategies  $s'_1$  for which player 2 will want to mix.

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2:  $U$  is weakly dominated by  $D$ , thus, P1 will play  $D$  in G2 as long as P2 plays  $R$  with positive probability.

I.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:		L	R
U		1, 1	0, 0
D		0, 0	2, 0

G2:		L	R
U		0, 0	0, 0
D		0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

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I.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

$BR_2((D, D)) = R$  but P2 is indifferent between  $L$  and  $R$  for  $s_1' = (U, D)$ .

## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L ( $q$ )	R ( $1-q$ )
U	1, 1	0, 0
D	0, 0	2, 0

G1:

	L ( $q$ )	R ( $1-q$ )
U	0, 0	0, 0
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Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

- Step 1: Find the player 1 strategies  $s'_1$  for which player 2 will want to mix.
- Step 2: **Find the values of  $q$  (the probability that player 2 plays  $L$ ) such that player 1 will actually play  $s'_1$ .**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2:  $U$  is weakly dominated by  $D$ , thus, P1 will play  $D$  in G2 as long as P2 plays  $R$  with positive probability.

I.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

$BR_2((D, D)) = R$  but P2 is indifferent between  $L$  and  $R$  for  $s'_1 = (U, D)$ .

## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:

	L ( $q$ )	R ( $1-q$ )
U	1, 1	0, 0
D	0, 0	2, 0

G2:

	L ( $q$ )	R ( $1-q$ )
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

Step 1: Find the player 1 strategies  $s'_1$  for which player 2 will want to mix.

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1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2:  $U$  is weakly dominated by  $D$ , thus, P1 will play  $D$  in G2 as long as P2 plays  $R$  with positive probability.

I.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

$BR_2((D, D)) = R$  but P2 is indifferent between  $L$  and  $R$  for  $s'_1 = (U, D)$ .

2. P1 wants to play  $U$  in G1 if:

$$\mathbb{E}[u_1|U] \geq \mathbb{E}[u_1|D] \Leftrightarrow$$

$$q \geq 2(1-q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

## PS8, Ex. 2 (A): Mixed-Strategy BNE

Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not:

G1:	L ( $q$ )	R ( $1-q$ )
U	1, 1	0, 0
D	0, 0	2, 0

G2:	L ( $q$ )	R ( $1-q$ )
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

- Step 1: Find the player 1 strategies  $s'_1$  for which player 2 will want to mix.
- Step 2: Find the values of  $q$  (the probability that player 2 plays  $L$ ) such that player 1 will actually play  $s'_1$ .
- Step 3: **Write up the mixed-strategy BNE where P1 plays a pure strategy and P2 randomizes between  $L$  and  $R$ .**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2:  $U$  is weakly dominated by  $D$ , thus, P1 will play  $D$  in G2 as long as P2 plays  $R$  with positive probability.

I.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

$BR_2((D, D)) = R$  but P2 is indifferent between  $L$  and  $R$  for  $s'_1 = (U, D)$ .

2. P1 wants to play  $U$  in G1 if:

$$\mathbb{E}[u_1|U] \geq \mathbb{E}[u_1|D] \Leftrightarrow$$

$$q \geq 2(1 - q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

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G2:

	L ( $q$ )	R ( $1-q$ )
U	0, 0	0, 0
D	0, 0	1, 1

Find all mixed-strategy Bayesian Nash Equilibria of the following form: Player 1 plays a pure strategy, and Player 2 randomizes between  $L$  and  $R$ .

- Step 1: Find the player 1 strategies  $s'_1$  for which player 2 will want to mix.
- Step 2: Find the values of  $q$  (the probability that player 2 plays  $L$ ) such that player 1 will actually play  $s'_1$ .
- Step 3: **Write up the mixed-strategy BNE where P1 plays a pure strategy and P2 randomizes between  $L$  and  $R$ .**

1. As P1 learns whether nature has drawn G1 or G2 her strategy space is:

$$S_1 = \{(U, U), (U, D), (D, D), (D, U)\}$$

G2:  $U$  is weakly dominated by  $D$ , thus, P1 will play  $D$  in G2 as long as P2 plays  $R$  with positive probability.

i.e. P1 either plays  $(U, D)$  or  $(D, D)$ .

$BR_2((D, D)) = R$  but P2 is indifferent between  $L$  and  $R$  for  $s'_1 = (U, D)$ .

2. P1 wants to play  $U$  in G1 if:

$$\mathbb{E}[u_1|U] \geq \mathbb{E}[u_1|D] \Leftrightarrow$$

$$q \geq 2(1 - q) \Leftrightarrow 3q \geq 2 \Leftrightarrow q \geq \frac{2}{3}$$

3. For  $q$  being the probability that P2 plays  $L$ , the mixed-strategy BNE is:

$$BNE' = \left\{ \left( (U, D), q \geq \frac{2}{3} \right) \right\}$$