# Microeconomics III: Problem Set 7[a]

Thor Donsby Noe (thor.noe@econ.ku.dk) & Christopher Borberg (christopher.borberg@econ.ku.dk)
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Department of Economics, University of Copenhagen

[a] Slides created for exercise class 3 and 4, with reservation for possible errors.

# Outline

PS7, Ex. 1 (A): Imperfect recall (imperfect information)

PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

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Code examples

## PS7, Ex. 1 (A): Imperfect recall (imperfect information)

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In this course we normally consider games in which there is 'perfect recall': players can always remember what they themselves have done in the past.

We have seen an example in class of a game with 'imperfect recall' where the player forgets his own actions. But what would a game where he forgets the opponent's actions look like? Construct a game with two players. The timing is as follows: Player 1 moves first, then Player 2, and then Player 2 again. Everytime they move, the players choose one of two actions: $\{L, R\}$.

Draw the game tree and construct the information sets such that (a) Player 2 observes Player 1's action the first time he moves, but (b) when Player 2 moves the second time, he has forgotten what Player 1 chose. However, he recalls his own action.

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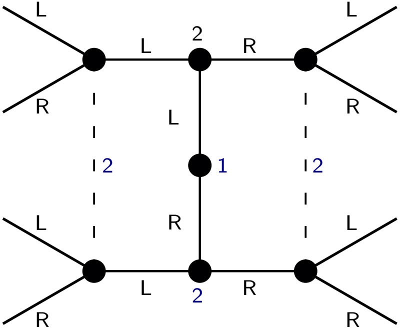

# PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

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Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

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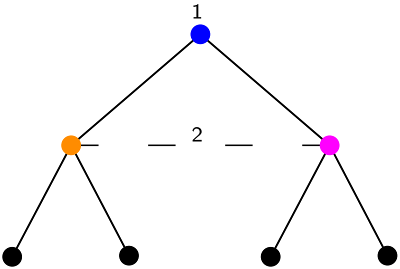
Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node $n$ that is a singleton information set.

Example of violation of condition 1:



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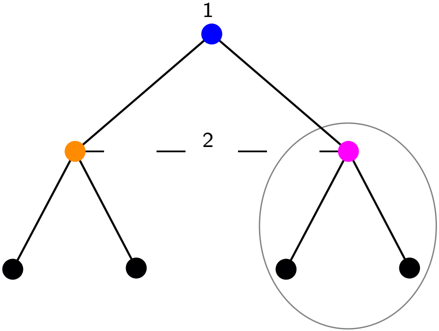



The purple decision node to the right is not a singleton information set (nor is the orange decision node to the left).

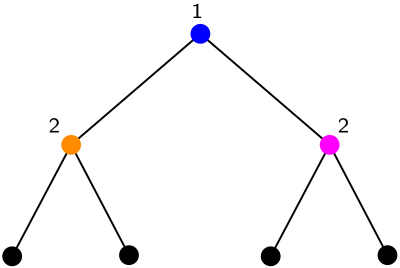
# PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node $n$ that is a singleton information set.
2. **It includes all following decision and terminal nodes following $n$ in the game tree**, but no nodes that do not follow $n$.

Example of violation of the first part of condition 2:



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Under imperfect information, a subgame must satisfy three properties:

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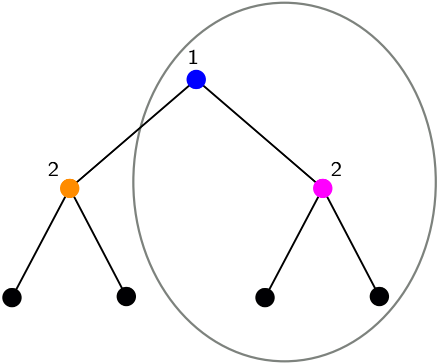



For a subgame containing the blue decision node $n$, all following decision nodes must be included.

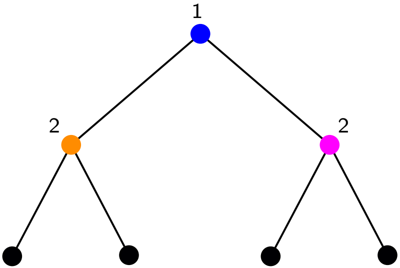
# PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node $n$ that is a singleton information set.
2. It includes all following decision and terminal nodes following $n$ in the game tree, **but no nodes that do not follow $n$.**

Example of violation of the second part of condition 2:



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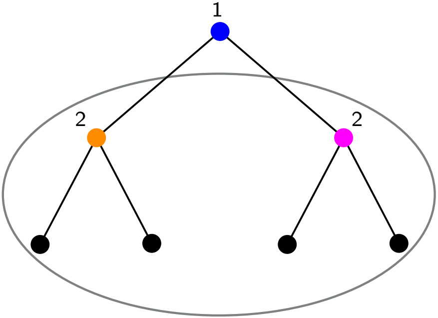
Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node $n$ that is a singleton information set.
2. It includes all following decision and terminal nodes following $n$ in the game tree, **but no nodes that do not follow $n$.**

Example of violation of the second part of condition 2:




Regardless of whether the orange or the purple node is chosen as the first decision node $n$, the other decision node does not follow $n$, and therefore cannot be part of the subgame.

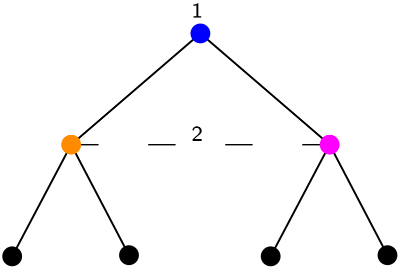
# PS7, Ex. 2 (A): Three conditions for a subgame (imperfect information)

Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

1. It begins at a decision node $n$ that is a singleton information set.
2. It includes all following decision and terminal nodes following $n$ in the game tree, but no nodes that do not follow $n$.
3. It does not "cut" any information set: if a decision node $n'$ follows $n$ in the game tree, then all other nodes in the information set including $n'$ must also follow $n$ (and so be included in the subgame).

Example of violation of condition 3:



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Recall that under imperfect information we have three conditions that define a subgame. Construct an example of a violation of each of the three conditions (pick different examples than those seen in the lectures).

Under imperfect information, a subgame must satisfy three properties:

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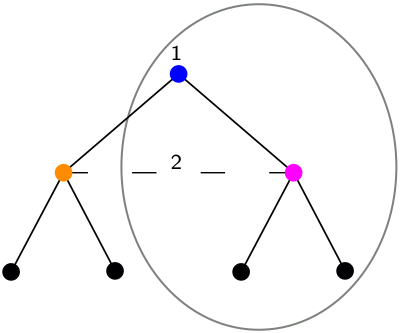
Example of violation of condition 3:




The orange decision node to the left is part of the same information set as the purple node to the right, so it must be included in the same subgame.

## PS7, Ex. 3 (A):

## PS7, Ex. 3 (A):

# PS7, Ex. 3.a (A):

## PS7, Ex. 4:

# PS7, Ex. 4:

# PS7, Ex. 4.a:

# PS7, Ex. 5:

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# PS7, Ex. 5.a:

# PS7, Ex. 6:

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# PS7, Ex. 6.a:

# PS7, Ex. 7:

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# PS7, Ex. 7.a:

## PS7, Ex. 8: Trigger strategy (infinitely repeated game)

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The next exercises use the following game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
| M | 15, 3 | 7, 7 | -4, 5 |
| H | 7, 0 | 5, -4 | -15, -15 |

Suppose that the Players play the infinitely repeated game $G(\infty)$ and that they would like to support as a SPNE the 'collusive' outcome in which $(L, L)$ is played every round.

(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives $(x_1, x_2)$ forever after a deviation.

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

# PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
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| L | 10, 10 | 3, 15 | 0, 7 |
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(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives $(x_1, x_2)$ forever after a deviation.

(Step a) Write up the trigger strategy.

# PS7, Ex. 8.a: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
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(a) Define a trigger strategy which delivers the collusive outcome in every period where no deviation has been made, and gives $(x_1, x_2)$ forever after a deviation.

(Step a) Write up the trigger strategy.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."

# PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
| M | 15, 3 | 7, 7 | -4, 5 |
| H | 7, 0 | 5, -4 | -15, -15 |

(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."

# PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game $G$.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."

## PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game $G$.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."
2. Stage game NE: $(M, M)$.

# PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game $G$.

(Step b) Explain.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."
2. Stage game NE: $(M, M)$.

# PS7, Ex. 8.b: Trigger strategy (infinitely repeated game)

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(b) A necessary (but not sufficient) condition for a SPNE is $x_1 = x_2 = M$. Explain why.

(Step a) Find the PSNE in the stage game $G$.

(Step b) Explain.

For a trigger strategy to constitute a SPNE, the threat of (eternal and unchangeable) punishment must be credible, i.e. must be a stage game NE.

Thus, $x_1 = x_2 = M$ is a necessary (but not sufficient) condition for the trigger strategies to constitute a SPNE.

Information so far:

1. Trigger strategy for Player $i \in 1, 2$: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $x_i$."
2. Unique stage game NE: $(M, M)$.

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
| M | 15, 3 | 7, 7 | -4, 5 |
| H | 7, 0 | 5, -4 | -15, -15 |

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

Information so far:

1. Trigger strategy: "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. Otherwise, play $M$."

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
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| L | 10, 10 | 3, 15 | 0, 7 |
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player $j$ plays the trigger strategy, write up Player $i$'s respective payoffs from playing "trigger" and from her best deviation.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
| M | 15, 3 | 7, 7 | -4, 5 |
| H | 7, 0 | 5, -4 | -15, -15 |

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

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(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
2. Optimal Deviation Strategy (ODS): "Always play $M$."

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

(Step b) Given Player $j$ plays TS, write up Player $i$'s respective payoffs from playing TS and ODS.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
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(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

(Step b) Given Player $j$ plays TS, write up Player $i$'s respective payoffs from playing TS and ODS.

Player $i$'s payoffs from playing TS:

$$\sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
2. Optimal Deviation Strategy (ODS): "Always play $M$."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$.

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
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(Step b) Given Player $j$ plays TS, write up Player $i$'s respective payoffs from playing TS and ODS.

Player $i$'s payoffs from playing TS:

$$\sum_{t=1}^{\infty} 10\delta^{t-1} = \frac{10}{1-\delta}$$

Player $i$'s payoffs from playing ODS:

$$15 + \sum_{t=2}^{\infty} 7\delta^{t-1} = 15 + \frac{7\delta}{1-\delta}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
2. Optimal Deviation Strategy (ODS): "Always play $M$."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

# PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

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(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

(Step b) Given Player $j$ plays TS, write up Player $i$'s respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$.

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
2. Optimal Deviation Strategy (ODS): "Always play $M$."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

## PS7, Ex. 8.c: Trigger strategy (infinitely repeated game)

Consider $G(\infty)$, i.e. the infinitely repeated game with stage game $G$:

| | L | M | H |
|---|---|---|---|
| L | 10, 10 | 3, 15 | 0, 7 |
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| H | 7, 0 | 5, -4 | -15, -15 |

(c) Suppose $\delta = 4/7$. Show by finding a profitable deviation that the above trigger strategy is not a SPNE.

(Step a) Given Player $j$ plays the Trigger Strategy (TS), write up Player $i$'s Optimal Deviation Strategy (ODS).

(Step b) Given Player $j$ plays TS, write up Player $i$'s respective payoffs from playing TS and ODS.

(Step c) Show that the deviation is preferred for $\delta = 4/7$:

$$U_i\left(ODS, TS; \delta = \frac{4}{7}\right) > U_i\left(TS, TS; \delta = \frac{4}{7}\right)$$

$$15 + \frac{7\frac{4}{7}}{1-\frac{4}{7}} > \frac{10}{1-\frac{4}{7}}$$

$$\frac{73}{3} > \frac{70}{3}$$

Information so far:

1. Trigger Strategy (TS): "If $t = 1$ or if the outcome in all previous stages was $(L, L)$, play $L$. If not, play $M$."
2. Optimal Deviation Strategy (ODS): "Always play $M$."
3. $U_i(TS, TS) = \frac{10}{1-\delta}$
4. $U_i(ODS, TS) = 15 + \frac{7\delta}{1-\delta}$

## PS7, Ex. 9:

# PS7, Ex. 9:

## PS7, Ex. 9.a:

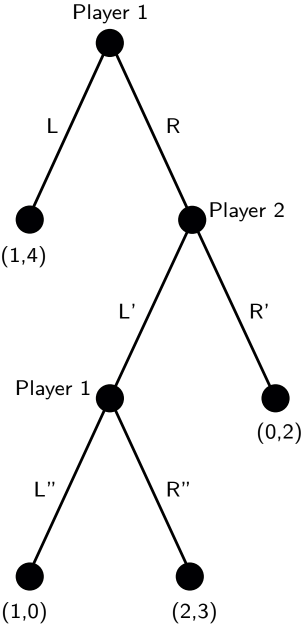
## PS7, Ex. 10:

# PS7, Ex. 10:

# PS7, Ex. 10.a:

# Code examples

# Code examples




Matrix, no player names:

| | L (q) | R (1-q) |
|---|---|---|
| T (p) | | |
| B (1-p) | | |

Matrix, no colors:

| | | Player 2 | |
|---|---|---|---|
| | | L (q) | R (1-q) |
| Player 1 | T (p) | | |
| | B (1-p) | | |

Matrix, with colors:

| | | Player 2 | |
|---|---|---|---|
| | | L (q) | R (1-q) |
| Player 1 | T (p) | 1, 1 | |
| | B (1-p) | | |