PS5, Ex. 4: The Mutated Seabass (backwards induction)

Consider a game where two evil organizations, rather prosaically named A and B, are battling for world domination. The battle takes the form of a three-stage game. Organization A is on the verge of acquiring a new powerful weapon, the *mutated seabass*. In stage 1 of the game, they decide whether to acquire the weapon or not. Their choice is observed by organization B. In stage 2, organization B decides whether to attack organization A. If an attack occurs, the game stops. If no attack occurs, it moves to stage 3, where organization A decides whether or not to attack organization B. The payoffs are as follows. If no-one attacks the other, the payoffs to both organizations are 0. If B attacks A, then the payoffs to both organizations are .1. The same if A attacks B, without having acquired the seabass weapon. If, on the other hand, A acquires the weapon, the payoffs from A attacking B are 2 to A and -2 to B.

(a) Draw the game tree that corresponds to the game. What are the strategies of the players?

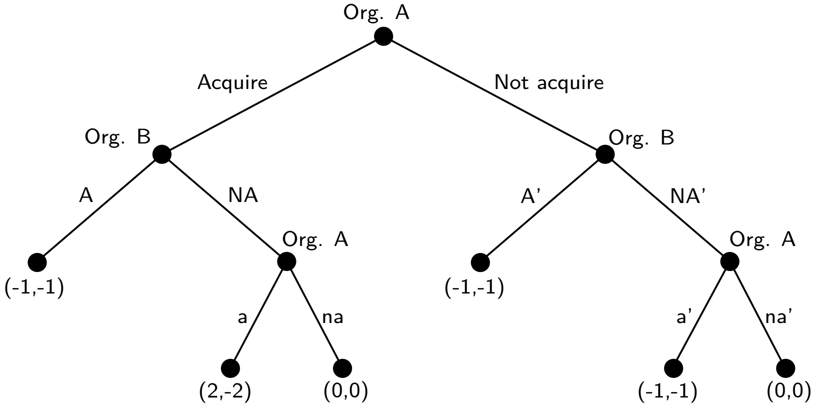
(b) What is the backwards induction outcome?

(c) What is the intuition for the outcome? What role do you think it plays that B observes if A acquires the weapon or not?

PS5, Ex. 4.a: The Mutated Seabass (backwards induction)

(a) Draw the game tree that corresponds to the game. What are the strategies of the players?

$$S_A = \{(Acquire, a, a'), (Acquire, a, na'), (Acquire, na, a'), (Acquire, na, na'),$$
$$(Not\ acquire, a, a'), (Not\ acquire, a, na'), (Not\ acquire, na, a'), (Not\ acquire, na, na')\}$$
$$S_B = \{(A, A'), (A, NA'), (NA, A'), (NA, NA')\}$$



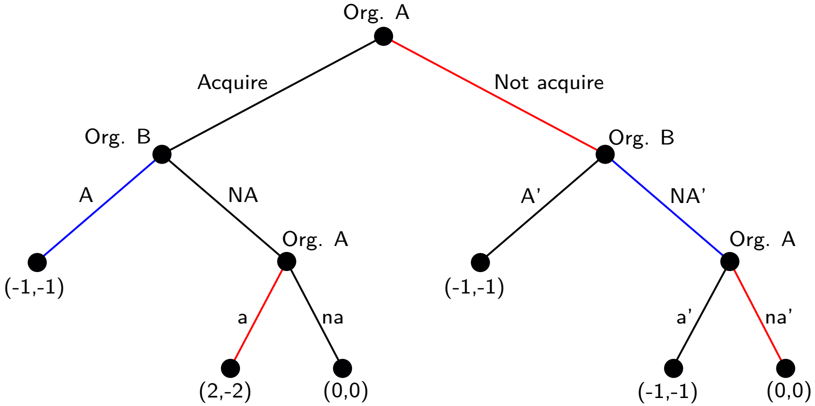
PS5, Ex. 4.b: The Mutated Seabass (backwards induction)

(b) What is the backwards induction outcome?

3^{rd} stage: Org. A will choose to attack if having acquired the weapon and not attack if not having acquired the weapon.

2^{nd} stage: Org. B will choose to attack if Org. A has acquired the weapon and not attack if they have not acquired the weapon.

1^{st} stage: Org. A will choose to not acquire the weapon in order to signal peaceful intentions to Org. B, i.e. giving the payoffs $(0, 0)$.



$SPNE = \{S_A, S_B\} = \{(Not\ acquire, a, na'), (A, NA')\}$

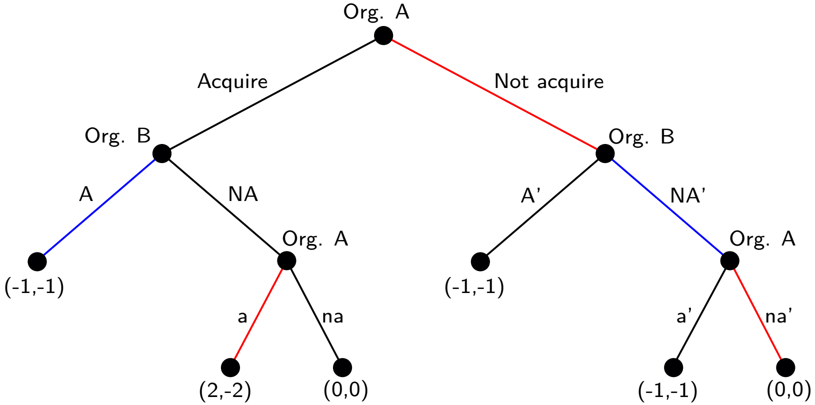
PS5, Ex. 4.c: The Mutated Seabass (backwards induction)

(c) What is the intuition for the outcome?

3$^{\text{rd}}$ stage: Org. A does only benefit from attacking if having acquired the weapon.

2$^{\text{nd}}$ stage: Org. B will only choose to attack if Org. A has acquired the weapon.

1$^{\text{st}}$ stage: Not acquiring the weapon is a credible signal that Org. A will not attack.



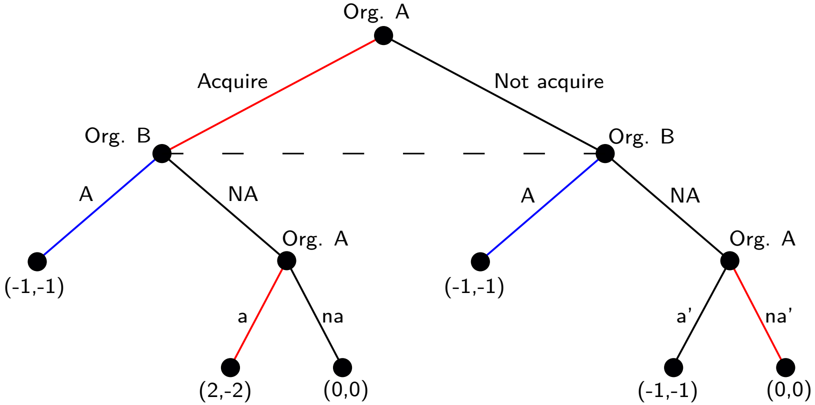
What role do you think it plays that B observes if A acquires the weapon or not?
I.e. what is the outcome if Organization A cannot send a signal in the 1$^{\text{st}}$ stage?

(c) What role do you think it plays that B observes if A acquires the weapon or not? I.e. what is the outcome if Organization A cannot send a signal in the 1st stage?

3rd stage: [unchanged] Org. A will choose to attack if having acquired the weapon and not attack if not having acquired the weapon.

2nd stage: Knowing that Org. A will attack if having acquired the weapon, Org. B chooses to attack first, giving the payoffs $(-1,-1)$ regardless of stage one.

1st stage: Org. A cannot affect the outcome, but acquires it in case Org. B deviates.



$SPNE = \{S_A, S_B\} = \{(Acquire, a, na'), A\}$

(c) Fool-proof alternative solution method:

3rd stage: [unchanged] Org. A will choose to attack if having acquired the weapon and not attack if not having acquired the weapon.

Reduce: Substitute in Org A's best actions and outcomes from the 3rd stage to get the reduced form game which is a static game.

Bi-matrix: Write up the normal form of the reduced game and highlight best responses:

Org. A \ Org. B	A	NA
$(Acquire, a, na')$	-1, -1	2, -2
$(Not\ acquire, a, na')$	-1, -1	0, 0

The strategy $(Not\ acquire, a, na')$ is weakly dominated by $(Acquire, a, na')$ and the unique Subgame Perfect NE is crystal clear:

$$SPNE = \{S_A, S_B\} = \{(Acquire, a, na'), A\}$$