PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 1		Player 2: L (q)	Player 2: R (1-q)
	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

Highlight the best responses in pure strategies.

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 1	Player 2: L (q)	Player 2: R (1-q)
T (p)	1, 1	0, 0
B (1-p)	1, 0	2, 1

For which values of q is Player 1 indifferent?

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1(T)|q] = E[u_1(B)|q]$$
$$=$$

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 2

Player 1		L (q)	R (1-q)
	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

Find q such that Player 1 expects to have equal payoffs from playing T and B:

$$E[u_1(T)|q] = E[u_1(B)|q]$$
$$q = q + 2(1-q) \Leftrightarrow q = 1$$

For which values of p is Player 2 indifferent?

Find p such that Player 2 expect to have equal payoffs from playing L and R:

$$E[u_2(L)|p] = E[u_2(R)|p]$$
$$=$$

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all equilibria (pure and mixed), first analytically and then through plotting the BR functions.

Player 2

Player 1		L (q)	R (1-q)
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$$E[u_1(T)|q] = E[u_1(B)|q]$$

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Find p such that Player 2 expect to have equal payoffs from playing L and R:

$$E[u_2(L)|p] = E[u_2(R)|p]$$

$$p = 1 - p \Leftrightarrow p = \frac{1}{2}$$

and chooses $q = 1$ for $p > 1/2$.

Write up all NE (pure and mixed).

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

Player 1 \ Player 2	L (q)	R (1-q)
T (p)	1, 1	0, 0
B (1-p)	1, 0	2, 1

Player 1 is indifferent for:

$$E[u_1(T)|q] = E[u_1(B)|q]$$
$$q = q + 2(1-q) \Leftrightarrow q = 1$$

Player 2 is indifferent for:

$$E[u_2(L)|p] = E[u_2(R)|p]$$
$$p = 1 - p \Leftrightarrow p = \frac{1}{2}$$

and chooses $q = 1$ for $p > 1/2$.

The pure and mixed NE, (p^*, q^*), are:

$$\left\{(0,0); (1,1); \left(p \in \left[\frac{1}{2}, 1\right), q = 1\right)\right\}$$

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

		Player 2	
		L (q)	R (1-q)
Player 1	T (p)	1, 1	0, 0
	B (1-p)	1, 0	2, 1

$BR_1(q) = \{$

Player 1 is indifferent for:

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The pure and mixed NE, (p^*, q^*), are:

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Write up Player 1's best-response (BR) function, $p^*(q)$

PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

		Player 2	
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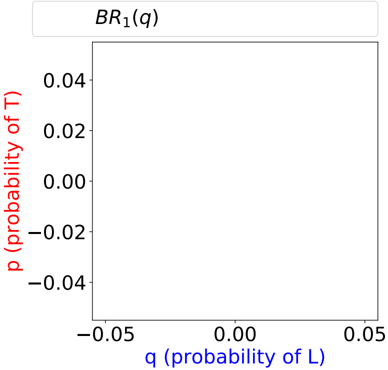
The pure and mixed NE, (p^*, q^*), are:

$$\left\{(0,0); (1,1); \left(p \in \left[\frac{1}{2}, 1\right), q = 1\right)\right\}$$

Plot Player 1's best-response (BR) function, $p^*(q)$

Then through plotting the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 1 \\ p \in [0,1] & \text{if} \quad q = 1 \end{cases}$$



PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

Player 2 (columns) / Player 1 (rows)

	L (q)	R (1-q)
T (p)	1, 1	0, 0
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Player 1 is indifferent for:

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Player 2 is indifferent for:

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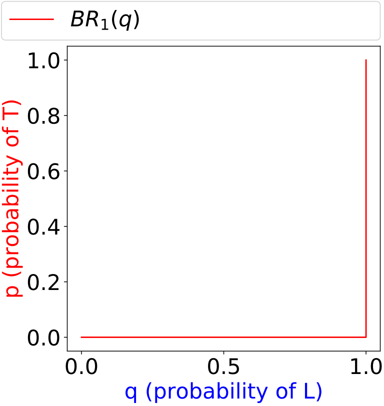
Write up Player 2's BR function, $q^*(p)$

Then through plotting the BR functions:

$$BR_1(q) = \begin{cases} p = 0 & \text{if} \quad q < 1 \\ p \in [0,1] & \text{if} \quad q = 1 \end{cases}$$

$$BR_2(p) = \{$$



PS4, Ex. 1.b (A): MSNE and best-response functions

(b) Find all NE, first analytically:

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Plot Player 2's BR function, $q^*(p)$

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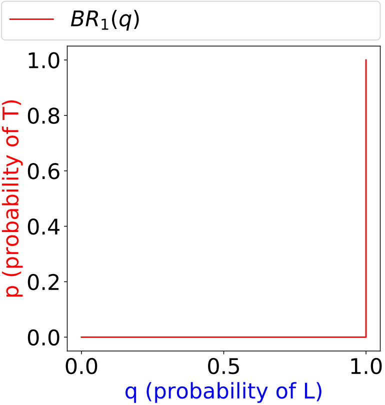

PS4, Ex. 1.b (A): MSNE and best-response functions

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