

PS6, Ex. 7: To keep or split (imperfect information)

Consider the following 2×2 game where payoffs are monetary:

	L	R
T	3, 3	0, 4
B	4, 0	1, 1

Before this game is played, Player 1 can choose whether, after the game is played, players should keep their own payoffs or split the aggregate payoff evenly between them. Player 2 observes this choice.

- (a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.
- (b) Find the subgame perfect Nash Equilibria (SPNE).
- (c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) and find the subgame perfect Nash Equilibria (SPNE).

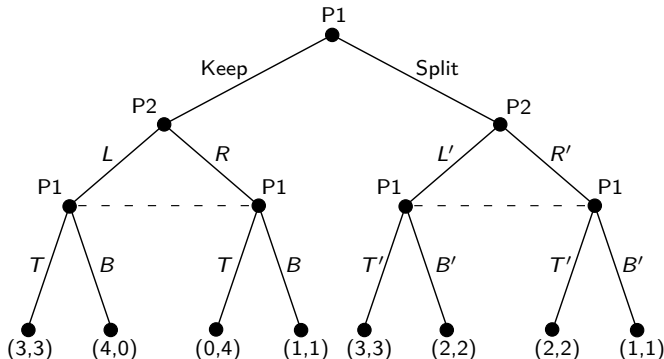
PS6, Ex. 7.a: To keep or split (imperfect information)

(a) Write down the game tree of this two-stage game: be careful to represent the simultaneous-move game in the second stage using information sets.

1st stage: Player 1 chooses Keep or Split. Player 2 observes the choice.

2nd stage: Player 2 chooses L or R (L' or R'). The action is private information.

3rd stage: Player 1 chooses T or B (T' or B') without knowing what Player 2 did.

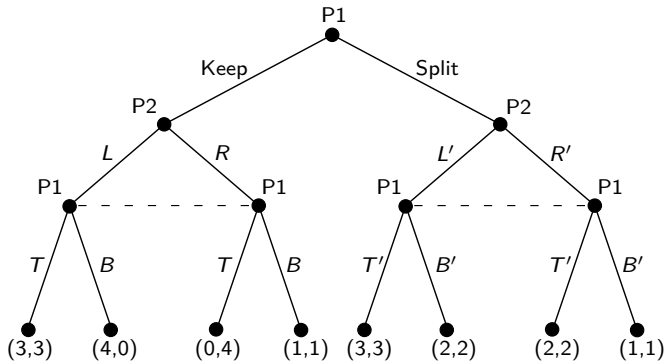


The order of stage 2 and 3 is arbitrary, but the 2nd stage must be private information.

(b) **Find the subgame perfect Nash Equilibria (SPNE).**

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).

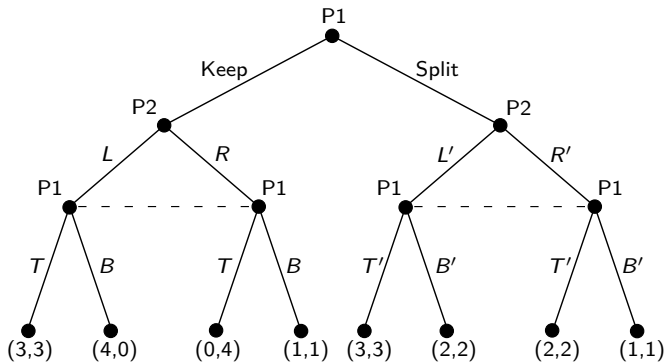


Player 1

	Keep	Split
T	3, 3	2, 2
B	4, 0	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).



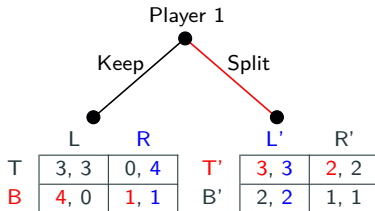
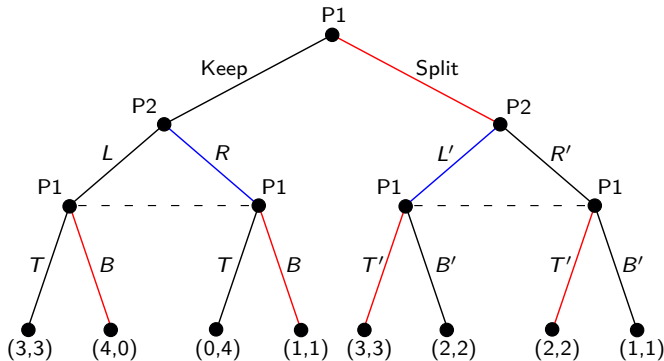
Player 1

	Keep	Split
L	3, 3	0, 4
R	4, 0	1, 1

	L'	R'
T'	3, 3	2, 2
B'	2, 2	1, 1

PS6, Ex. 7.b: To keep or split (imperfect information)

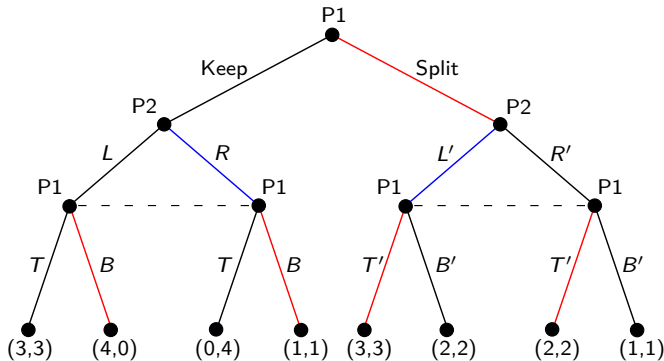
(b) Find the subgame perfect Nash Equilibria (SPNE).



Write up the full strategy profiles for the subgame perfect Nash Equilibria (SPNE).

PS6, Ex. 7.b: To keep or split (imperfect information)

(b) Find the subgame perfect Nash Equilibria (SPNE).



Player 1

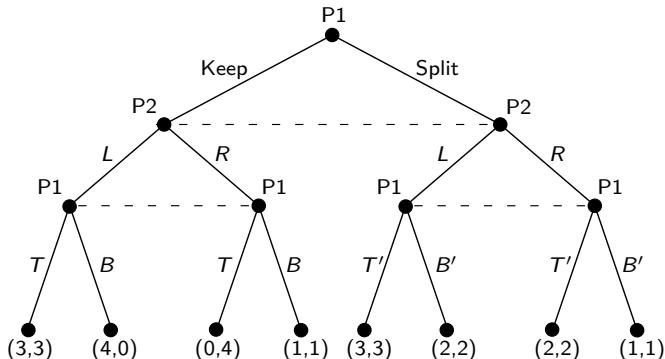
	L	R	L'	R'
T	3, 3	0, 4	3, 3	2, 2
B	4, 0	1, 1	2, 2	1, 1

SPNE = $\{(Split, B, T'), (R, L')\}$ with outcome (3,3).

(c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. **Draw the game tree (again using information sets)** and find the subgame perfect Nash Equilibria (SPNE).

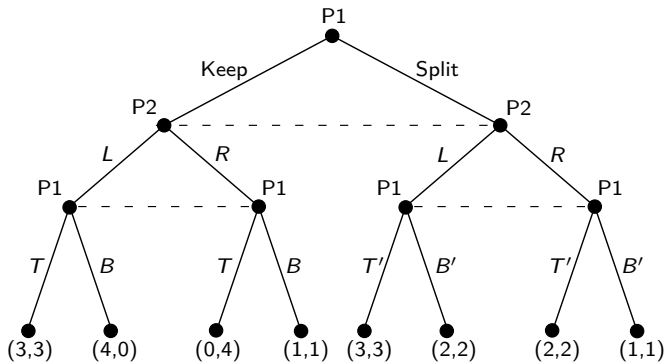
PS6, Ex. 7.c: To keep or split (imperfect information)

- (c) Now suppose that Player 2 cannot observe Player 1's choice in the first stage. Draw the game tree (again using information sets) **and find the subgame perfect Nash Equilibria (SPNE).**



PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the subgame perfect Nash Equilibria (SPNE).



With 2nd and 3rd stage in normal form (Player 1 knows her own action in 1st stage):

Player 1

Keep Split

L R L R

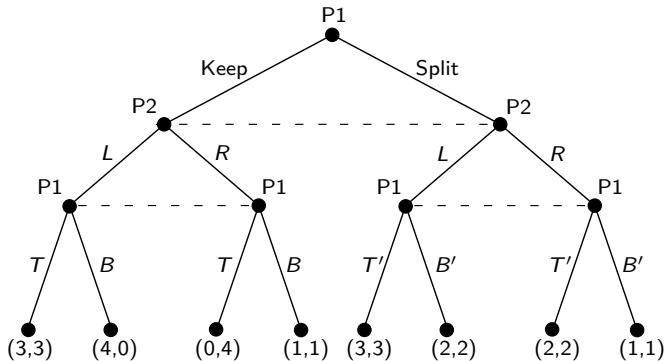
T	3, 3	0, 4
B	4, 0	1, 1

T'	3, 3	2, 2
B'	2, 2	1, 1

How many subgames are there?

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the subgame perfect Nash Equilibria (SPNE).



With 2nd and 3rd stage in normal form:

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graph TD
    P1((Player 1)) -- Keep --> P2L(( ))
    P1 -- Split --> P2R(( ))
    P2L -.- P2R
    P2L -- L --> P1L["3, 3"]
    P2L -- R --> P1R["0, 4"]
    P2R -- L --> P2L1["2, 2"]
    P2R -- R --> P2R1["1, 1"]
  
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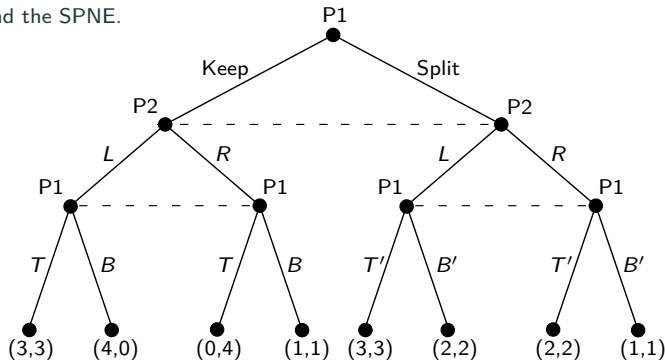
	L	R		L	R
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

There is only one subgame; the full game itself.

Write up the game in normal form and solve it.

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the SPNE.



With 2nd and 3rd stage in normal form:

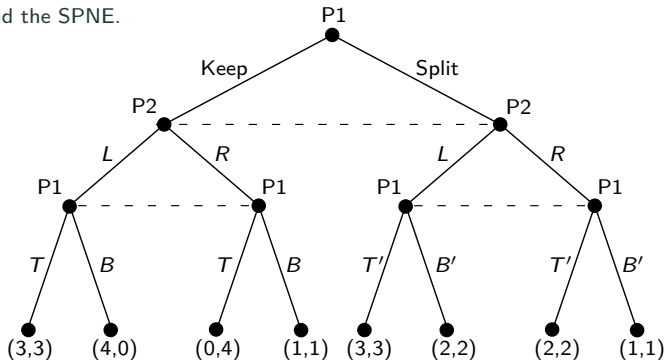
		Player 1			
		Keep	Split		
	L	R	L	R	
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

Full game:

	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) Find the SPNE.



With 2nd and 3rd stage in normal form:

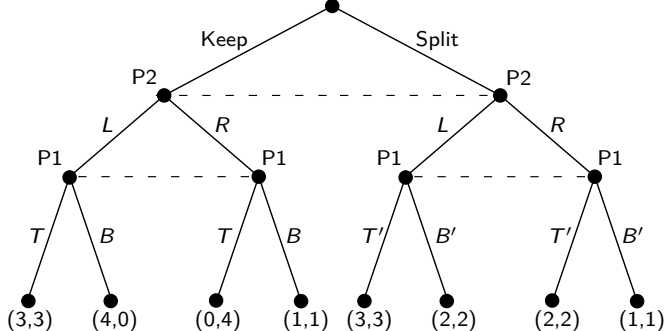
		Player 1			
		Keep	Split		
	L	R	L	R	
T	3, 3	0, 4	T'	3, 3	2, 2
B	4, 0	1, 1	B'	2, 2	1, 1

Full game:

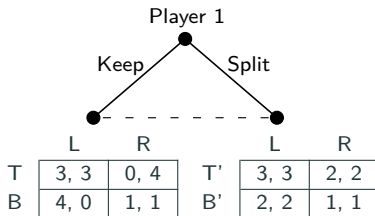
	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1

PS6, Ex. 7.c: To keep or split (imperfect information)

(c) No SPNE exists (in pure strategies) P1



2nd and 3rd stage in normal form:



Full game:

	L	R
Keep, T, T'	3, 3	0, 4
Keep, T, B'	3, 3	0, 4
Keep, B, T'	4, 0	1, 1
Keep, B, B'	4, 0	1, 1
Split, T, T'	3, 3	2, 2
Split, B, T'	3, 3	2, 2
Split, T, B'	2, 2	1, 1
Split, B, B'	2, 2	1, 1