Discussion 5

1. A sample of size n = 9 from a large population that is thought to be normal has a sample mean of 13.01 and sample standard deviation of 4.85.

   (a) Create a 80% CI for the population mean μ, assuming that σ^2 is unknown.

   (b) If it is now assumed that the variance is known to be $\sigma^2 = 16$, create a 80% CI for the population mean μ. Is it larger or smaller than the one created in (a)?

   (c) What is the minimum sample size n in part (a) (assuming the sample mean and variance don't change) so that a 90% C.I. half-width calculated using the standard normal distribution would be the same as the half-width you calculated in part (b)?

2. An engineer is designing an experiment to estimate the stiffness of beams from a certain factory. A random selection of beams is to be tested by putting a load on each beam and measuring its deflection. She knows that she must have a 90% confidence interval for the population-mean deflection μ where the half-width of the interval (margin of error) is less than 1cm.

   (a) If the population variance were known to be 4.2, what is the minimum sample size in order to attain the required half-width?

   (b) Suppose that the variance is still 4.2, but it was estimated from the sample (which is of the size calculated in part (a)). What is the 90% C.I.'s half-width? Is it acceptable according to the engineer's standard?

   (c) Supposing she has collected a sample sized according to the part (a), where the sample's mean deflection was 7.21 cm and its variance is $s^2 = 3.69$, what is the 90% confidence interval?

   (d) Suppose that the engineer plans to repeat the study described in part (c) 20 times, generating an independent 90% confidence interval each time. How many of those confidence intervals are expected to contain the true value of μ? What is the variance in the number that contain the true value of μ? What is the probability that all of the intervals contain the true μ?