Statistical Inference: power

Errors in testing

What can happen:

Truth	Decision: Do not reject	Decision: Reject null
Null true	Correct	Type I error
Null false	Type II error	Correct

Tension between truth and decision about truth (imperfect).

- Prob. of type I error denoted α. Usually fix α, eg. $\alpha = 0.05$.
- Prob. of type II error denoted β. Determined by the planned experiment. Low β good.
- Prob. of not making type II error called **power** ($= 1 - \beta$). *High* power good.

Power

- Suppose $H_0 : \theta = 10$, $Ha : \theta \neq 10$ for some parameter θ.
- Suppose H_0 wrong. What does that say about θ?
- Not much. Could have $\theta = 11$ or $\theta = 8$ or $\theta = 496$. In each case, H_0 wrong.
- How likely a type II error is depends on what θ is:
  - If $\theta = 496$, should be able to reject $H_0 : \theta = 10$ even for small sample, so β should be small (power large).
  - If $\theta = 11$, might have hard time rejecting H_0 even with large sample, so β would be larger (power smaller).
- Power depends on true parameter value, and on sample size.
- So we play "what if": "if θ were 11 (or 8 or 496), what would power be?".

Figuring out power

- Time to figure out power is before you collect any data, as part of planning process.
- Need to have idea of what kind of departure from null hypothesis of interest to you, eg. average improvement of 5 points on reading test scores. (Subject-matter decision, not statistical one.)
- Then, either:
  - "I have this big a sample and this big a departure I want to detect. What is my power for detecting it?"
  - "I want to detect this big a departure with this much power. How big a sample size do I need?"

How to understand/estimate power?

- Suppose we test $H_0 : \mu = 10$ against $H_a : \mu \neq 10$, where μ is population mean.
- Suppose in actual fact, $\mu = 8$, so H_0 is wrong. We want to reject it. How likely is that to happen?
- Need population SD (take $\sigma = 4$) and sample size (take $n = 15$). In practice, get σ from pilot/previous study, and take the n we plan to use.
- Idea: draw a random sample from the true distribution, test whether its mean is 10 or not.
- Repeat previous step "many" times.
- "Simulation".

Making it go

- Random sample of 15 normal observations with mean 8 and SD 4:

```
x = rnorm(15, 8, 4)
x
```

```
##  [1] 14.487469  5.014611  6.924277  5.201860
##  [5]  8.852952 10.835874  3.686684 11.165242
##  [9]  8.016188 12.383518  1.378099  3.172503
## [13] 13.074996 11.353573  5.015575
```

- Test whether `x` from population with mean 10 or not (over):

...continued

```r
t.test(x, mu = 10)
```

```
##
##  One Sample t-test
##
## data:  x
## t = -1.8767, df = 14, p-value = 0.08157
## alternative hypothesis: true mean is not equal to 10
## 95 percent confidence interval:
##   5.794735 10.280387
## sample estimates:
## mean of x
##  8.037561
```

- Fail to reject the mean being 10 (a Type II error).

or get just P-value

```r
t.test(x, mu = 10)$p.value
```

```
## [1] 0.0815652
```

Run this lots of times

- Two steps:
  - Generate a bunch of random samples
  - extract the P-value for the t-test from each
- without a loop!
- Use `rerun` to generate the random samples
- Use `map` to run the test on each random sample
- Use `map_dbl` to pull out the P-value for each test
- Count up how many of the P-values are 0.05 or less.

In code

```
rerun(10000, rnorm(15, 8, 4)) %>%
  map( ~ t.test(., mu = 10)) %>%
  map_dbl("p.value") ->
  pvals
tibble(pvals) %>% count(pvals <= 0.05)
```

pvals <= 0.05	n
FALSE	5547
TRUE	4453

We correctly rejected 422 times out of 1000, so the estimated power is 0.422.

Calculating power

- Simulation approach very flexible: will work for any test. But answer different each time because of randomness.
- In some cases, for example 1-sample and 2-sample t-tests, power can be calculated.
- `power.t.test`. delta difference between null and true mean:

```
power.t.test(n = 15, delta = 10-8, sd = 4, type = "one.sample")
```

```
##
##      One-sample t test power calculation
##
##               n = 15
##           delta = 2
##              sd = 4
##       sig.level = 0.05
##           power = 0.4378466
##     alternative = two.sided
```

Comparison of results

Method	Power
Simulation	0.422
`power.t.test`	0.4378

- Simulation power is similar to calculated power; to get more accurate value, repeat more times (eg. 10,000 instead of 1,000), which takes longer.
- CI for power based on simulation approx. 0.42 ± 0.03.
- With this small a sample size, the power is not great. With a bigger sample, the sample mean should be closer to 8 most of the time, so would reject $H_0 : \mu = 10$ more often.

Calculating required sample size

- Often, when planning a study, we do not have a particular sample size in mind. Rather, we want to know how big a sample to take. This can be done by asking how big a sample is needed to achieve a certain power.
- The simulation approach does not work naturally with this, since you have to supply a sample size.
- For the power-calculation method, you supply a value for the power, but leave the sample size missing.
- Re-use the same problem: $H_0 : \mu = 10$ against 2-sided alternative, true $\mu = 8$, $\sigma = 4$, but now aim for power 0.80.

Using power.t.test

- No `n=`, replaced by a `power=`:

```r
power.t.test(power=0.80, delta=10-8, sd=4, type="one.sample")
```

```
##
##      One-sample t test power calculation
##
##               n = 33.3672
##           delta = 2
##              sd = 4
##       sig.level = 0.05
##           power = 0.8
##     alternative = two.sided
```

- Sample size must be a whole number, so round up to 34 (to get at least as much power as you want).

Power curves

- Rather than calculating power for one sample size, or sample size for one power, might want a picture of relationship between sample size and power.
- Or, likewise, picture of relationship between difference between true and null-hypothesis means and power.
- Called power curve.
- Build and plot it yourself.

Building it

- If you feed power.t.test a collection ("vector") of values, it will do calculation for each one.
- Do power for variety of sample sizes, from 10 to 100 in steps of 10:

```
ns=seq(10,100,10)
ns
```

```
##  [1]  10  20  30  40  50  60  70  80  90 100
```

- Calculate powers:

```
ans=power.t.test(n=ns, delta=10-8, sd=4, type="one.sample")
ans$power
```

```
##  [1] 0.2928286 0.5644829 0.7539627 0.8693979
##  [5] 0.9338976 0.9677886 0.9847848 0.9929987
##  [9] 0.9968496 0.9986097
```

Building a plot

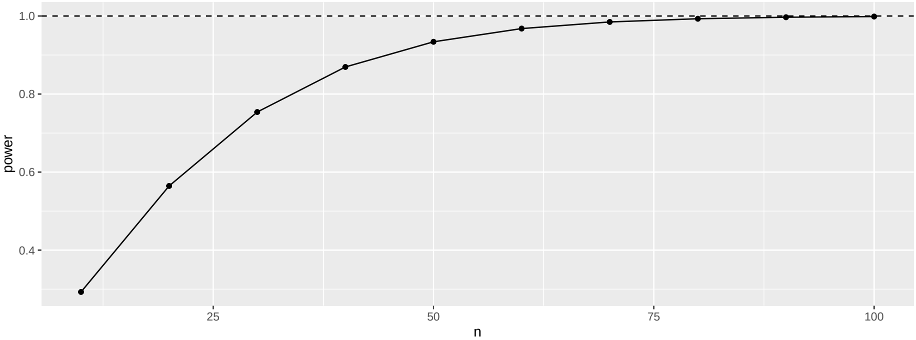
- Make a data frame out of the values to plot:

```
d=tibble(n=ns, power=ans$power)
d
```

n	power
10	0.2928286
20	0.5644829
30	0.7539627
40	0.8693979
50	0.9338976
60	0.9677886
70	0.9847848
80	0.9929987
90	0.9968496
100	0.9986097

The power curve

```
g
```



Another way to do it:

```
tibble(n=ns) %>%
  mutate(power_output=map(n, ~power.t.test(n=., delta=10-8, sd
  mutate(power=map_dbl(power_output, "power")) %>%
```

Power curves for means

- Can also investigate power as it depends on what the true mean is (the farther from null mean 10, the higher the power will be).
- Investigate for two different sample sizes, 15 and 30.
- First make all combos of mean and sample size:

```
means=seq(6,10,0.5)
means
```

```
## [1]  6.0  6.5  7.0  7.5  8.0  8.5  9.0  9.5 10.0
```

```
ns=c(15,30)
ns
```

```
## [1] 15 30
```

```
combos=crossing(mean=means, n=ns)
```

The combos

```
combos
```

mean	n
6.0	15
6.0	30
6.5	15
6.5	30
7.0	15
7.0	30
7.5	15
7.5	30
8.0	15
8.0	30
8.5	15
8.5	30
9.0	15

Calculate and plot

- Calculate the powers, carefully:

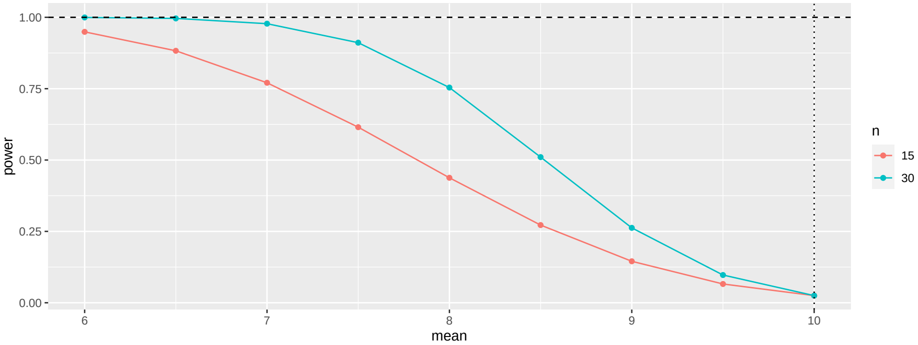
```
ans=with(combos, power.t.test(n=n, delta=mean-10, sd=4,
                              type="one.sample"))
ans
```

```
##
##      One-sample t test power calculation
##
##               n = 15, 30, 15, 30, 15, 30, 15, 30, 15, 30, 1
##           delta = 4.0, 4.0, 3.5, 3.5, 3.0, 3.0, 2.5, 2.5, 2
##              sd = 4
##       sig.level = 0.05
##           power = 0.94908647, 0.99956360, 0.88277128, 0.996
##     alternative = two.sided
```

```
names(ans)
```

The power curves

```
g
```



Comments

- When `mean=10`, that is, the true mean equals the null mean, H_0 is actually true, and the probability of rejecting it then is $\alpha = 0.05$.
- As the null gets more wrong (mean decreases), it becomes easier to correctly reject it.
- The blue power curve is above the red one for any mean $<$ 10, meaning that no matter how wrong H_0 is, you always have a greater chance of correctly rejecting it with a larger sample size.
- Previously, we had $H_0 : \mu = 10$ and a true $\mu = 8$, so a mean of 8 produces power 0.42 and 0.80 as shown on the graph.
- With $n = 30$, a true mean that is less than about 7 is almost certain to be correctly rejected. (With $n = 15$, the true mean needs to be less than 6.)

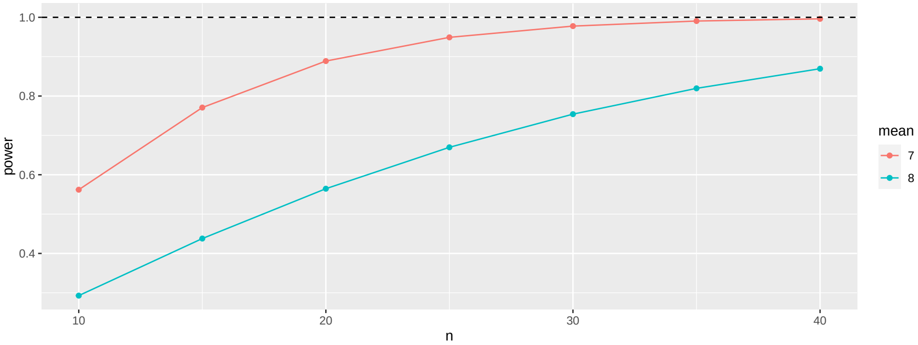
Power by sample size for means 7 and 8

Similar procedure to before:

```
means=c(7, 8)
ns=seq(10, 40, 5)
combos=crossing(mean=means, n=ns)
ans=with(combos, power.t.test(n=n, delta=10-mean, sd=4,
                              type="one.sample"))
d=tibble(mean=factor(combos$mean), n=combos$n,
         power=ans$power)
g=ggplot(d, aes(x=n, y=power, colour=mean)) +
  geom_point() + geom_line() +
  geom_hline(yintercept=1,linetype="dashed")
```

The power curves

```
g
```



Two-sample power

- For kids learning to read, had sample sizes of 22 (approx) in each group
- and these group SDs:

```
kids
```

group	score
t	24
t	61
t	59
t	46
t	43
t	44
t	52
t	43
t	58
t	67
t	62

Setting up

- suppose a 5-point improvement in reading score was considered important (on this scale)
- in a 2-sample test, null (difference of) mean is zero, so `delta` is true difference in means
- what is power for these sample sizes, and what sample size would be needed to get power up to 0.80?
- SD in both groups has to be same in `power.t.test`, so take as 14.

Calculating power for sample size 22 (per group)

```r
power.t.test(n=22, delta=5, sd=14, type="two.sample",
             alternative="one.sided")
```

```
##
##      Two-sample t test power calculation
##
##               n = 22
##           delta = 5
##              sd = 14
##       sig.level = 0.05
##           power = 0.3158199
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

sample size for power 0.8

```
power.t.test(power=0.80, delta=5, sd=14, type="two.sample",
             alternative="one.sided")
```

```
##
##      Two-sample t test power calculation
##
##               n = 97.62598
##           delta = 5
##              sd = 14
##       sig.level = 0.05
##           power = 0.8
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

Comments

- The power for the sample sizes we have is very small (to detect a 5-point increase).
- To get power 0.80, we need 98 kids in *each* group!