The "Boxes and Bulbs" Problem

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The problem

The "Boxes and Bulbs" problem¹ is as follows:

Box A has 10 lightbulbs, of which 4 are defective. Box B has 6 lightbulbs, of which 1 is defective. Box C has 8 lightbulbs, of which 3 are defective.

I) If we randomly choose a box, and then randomly choose a lightbulb from that box, what is the probability that we will choose a non-defective bulb? II) If we do choose a nondefective bulb, what is the probability it came from Box C?

¹This problem is taken from *Schaum's Outlines: Probability* (2nd ed., 2000), pages 87-88.

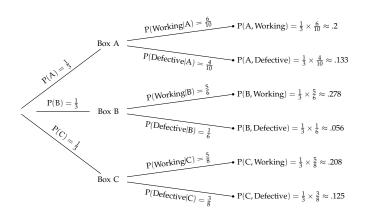
The solution

First, note that the solution requires the calculation of two probabilities:

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P(Bulb = Working) and P(Box = C|Bulb = Working).
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These are the answers to Part I and Part II, respectively. Second, if you correctly work out the first probability, i.e. P(Bulb = Working), then will have all the information necessary to obtain the second probability, i.e. P(Box = C|Bulb = Working).

The generative model



▶ This tree structure gives you all the information you need to answer the questions. Note that on the right are given all the joint probabilities, which can be arranged to a joint probability table as follows:

	Working	Defective
Box A	.2	.133
Box B	.278	.056
Box C	.208	.125

- Now it is simple to see the overall probability of choosing a working bulb. It is probability of choosing Box A and choosing a working bulb or choosing Box B and choosing a working bulb or choosing Box C and choosing a working bulb. This is the sum of the first column, which is $.2 + .278 + .208 \approx .686$.
- ▶ Likewise, to get the probability of choosing Box C *given* that we have chosen a working bulb, we see what proportion of the total probability of choosing a working bulb, i.e. .686 is from when we choose Box C and a working bulb, i.e. .208. In other words, it is $.208/.686 \approx .304$.

Using probability rules

► The first question asks us to calculate P(Bulb = Working). We know that this is equal to

$$\begin{split} P(\text{Working}) &= P(\text{Working}, A) + P(\text{Working}, B) + P(\text{Working}, C), \\ &= .2 + .278 + .208. \\ &\approx .686. \end{split}$$

▶ When asked for P(C|Working) we use the rule of conditional probability being the joint probability divided by the marginal probability, i.e.

$$P(C|Working) = \frac{P(Working, C)}{P(Working)},$$
$$= \frac{.208}{.686},$$
$$\approx .304.$$