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5

The Theory of Signal Detection

The discovery that expectancy and payoff have such a dramatic influence upon detection behavior was incorporated into a new theoretical conception of the detection situation. Tanner and Swets (1954) proposed that statistical decision theory and certain ideas about electronic signal-detecting devices might be used to build a model closely approximating how people actually behave in detection situations. The model is called the *theory of signal detection* (TSD) and is described in detail by Green and Swets (1966). Fundamental to TSD is the concept of noise.

DISTRIBUTIONS OF NOISE AND SIGNAL PLUS NOISE

Signals (stimuli) are always detected—whether by electronic devices or by humans—against a background of activity. The level of this background activity, called *noise*, is assumed to vary randomly and may be either external to the detecting device or caused by the device itself (e.g., physiological noise caused by spontaneous activity of the nervous system). In the detection situation, the observer must therefore first make an *observation* (x) and then make a decision about the observation. On each trial, the observer must decide whether x is due to a signal added to the noise background or to the noise alone. When a weak signal is applied, the decision becomes difficult, and errors are frequent. One factor contributing to the difficulty of the problem is the random variation of background noise. On some trials, the noise level may be so high as to be mistaken for a signal, and on other trials it may be so low that the addition of a weak signal is mistaken for noise. This state of affairs can

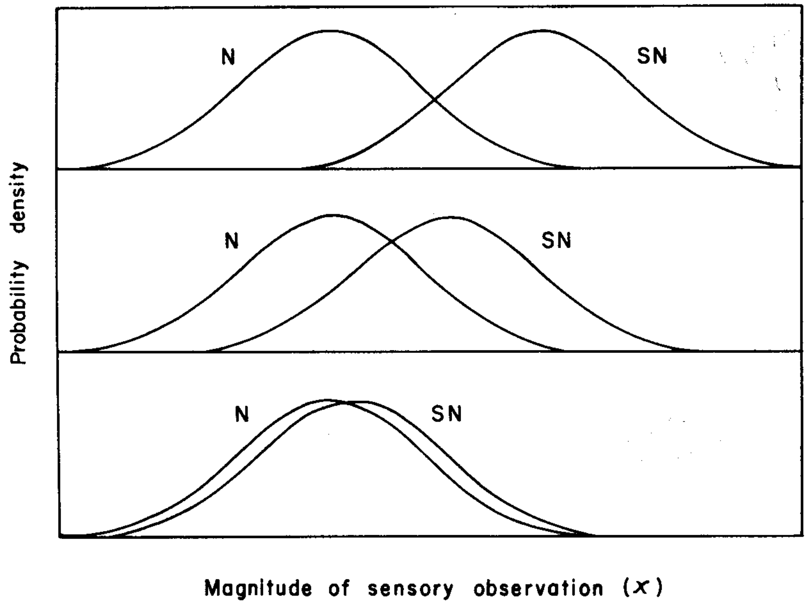

FIG. 5.1. Theoretical frequency distributions of noise and signal noise for three different values of signal strength.

be represented graphically by two probability distributions describing the random variation of noise (N) and the signal plus noise (SN) (Figure 5.1). Since the signal is added to the noise, the average sensory observation magnitude will always be greater for the *signal-plus-noise distribution*, SN, than for the *noise distribution*, N. However, the difference between the means becomes smaller and smaller as the signal strength is decreased, until the distributions are essentially the same. It is when the two distributions greatly overlap that decision making becomes difficult.

THE LIKELIHOOD RATIO

On any given trial the observer makes a sensory observation x which consists of a random sample from one or the other of the two distributions. Based on its sensory magnitude, the observer decides whether x was sampled from the N or the SN distribution. The ordinate of N gives the *probability density*,[1] or likelihood, of x occurring when only noise is pre-

[1]The term *probability density* is used because x is continuous rather than discrete. If x had a limited number of discrete values, each could be described as having a particular probability of occurrence. In either case, the ordinate gives the relative likelihood of a particular value of x.

sented. Similarly, the ordinate of SN gives the likelihood of x occurring when a signal is presented. Each value of x can now be expressed in terms of these two likelihoods or probability densities. For each value of x there exists a particular *likelihood ratio, $l(x)$*, defined as

$$l(x) = \frac{\text{ordinate of SN}}{\text{ordinate of N}}. \tag{5.1}$$

The likelihood ratio provides the observer with a basis for making a decision, since it expresses the likelihood of x in the SN situation relative to the likelihood of x in the N situation. Even though x may vary on several dimensions (e.g., hue, saturation, brightness, shape), each x can be located on a single dimension of likelihood ratio, since for each x there exists a single ordinate value of N and a single ordinate value of SN. Thus, the observer's final decision of whether x is due to N or SN can be based on a single quantity, the likelihood ratio.

Figure 5.2 illustrates how the value of the likelihood ratio, $l(x)$, changes as the value of the sensory observation, x, changes. At a point on the sensory observation dimension where the noise and signal-plus-noise distributions cross, the ordinate values for the two distributions are the same, resulting in a likelihood ratio of 1.0. This likelihood ratio of 1.0 indicates that a sensory observation, x, located at this point on the sensory observation dimension, is equally likely to result from noise as it is from signal plus noise. When the value of x is higher than this point, the likelihood ratio is greater than 1.0; and when the value of x is below this point, the likelihood ratio is less than 1.0. A likelihood ratio greater than 1.0 indicates that it is more likely that the sensory observation resulted from signal plus noise than from noise alone. A likelihood ratio less than 1.0 indicates that it is more likely that the sensory observation resulted from noise alone than from signal plus noise.

THE OBSERVER'S CRITERION

One of the assumptions of TSD is that an observer establishes a particular value of $l(x)$ as a cutoff point, or criterion (β), and that the decision will be determined by whether a particular observation, x, is above or below the criterion. Proponents of the theory assume that the observer operates by a *decision rule*: when $l(x)$ is equal to or greater than β, the observer chooses SN, and when $l(x)$ is below β, the observer chooses N. If the observer properly sets the criterion, performance will be optimal in a long series of observations. For example, in many situations it would be best for the observer to set a criterion so that the value of β was equal to 1.0. Since sensory observations having likelihood ratios greater than 1.0 are

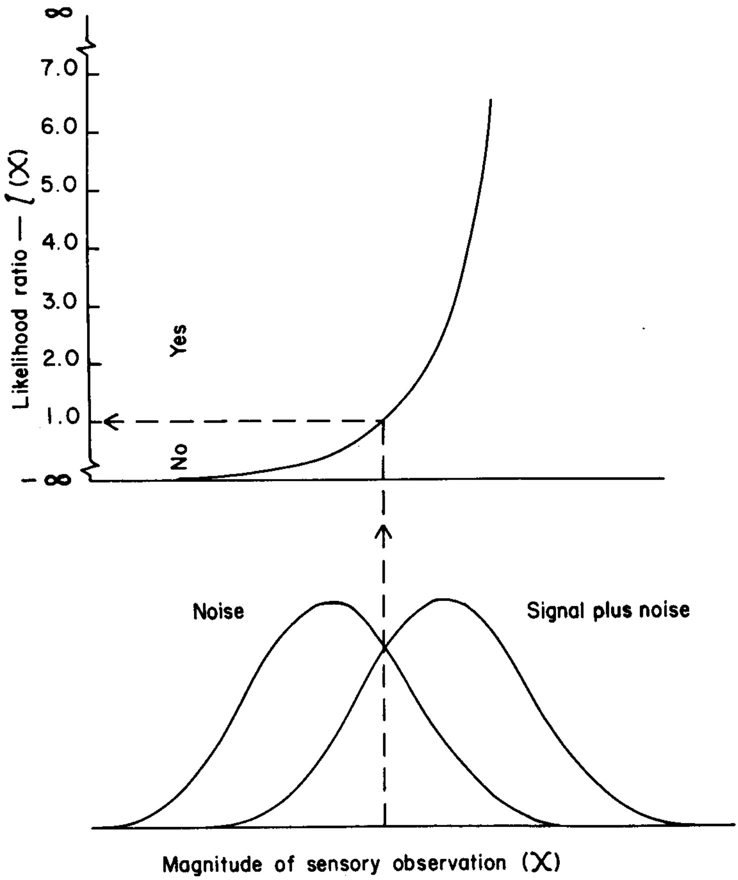

FIG. 5.2. Change of likelihood ratio as a function of the value of x.

more likely to have resulted from signal plus noise than from noise alone, the observer should always choose SN when $l(x)$ is greater than 1.0. When $l(x)$ is less than 1.0, the observer should always choose N, because sensory observations having likelihood ratios less than 1.0 are more likely to be the result of noise than signal plus noise. Figure 5.2 illustrates how an observer should report "yes, a signal was presented" when the likelihood ratio is greater than 1.0, and report "no, a signal was not presented" when the likelihood ratio is less than 1.0.

Figure 5.3 illustrates a case in which the observer has set a criterion at a point on the sensory observation dimension where β is equal to 1.0 (the ordinates of the N and SN distributions are equal). When sensory observations are above the criterion, the observer reports "yes, there is a signal." When the sensory observation is below the criterion, the observer reports

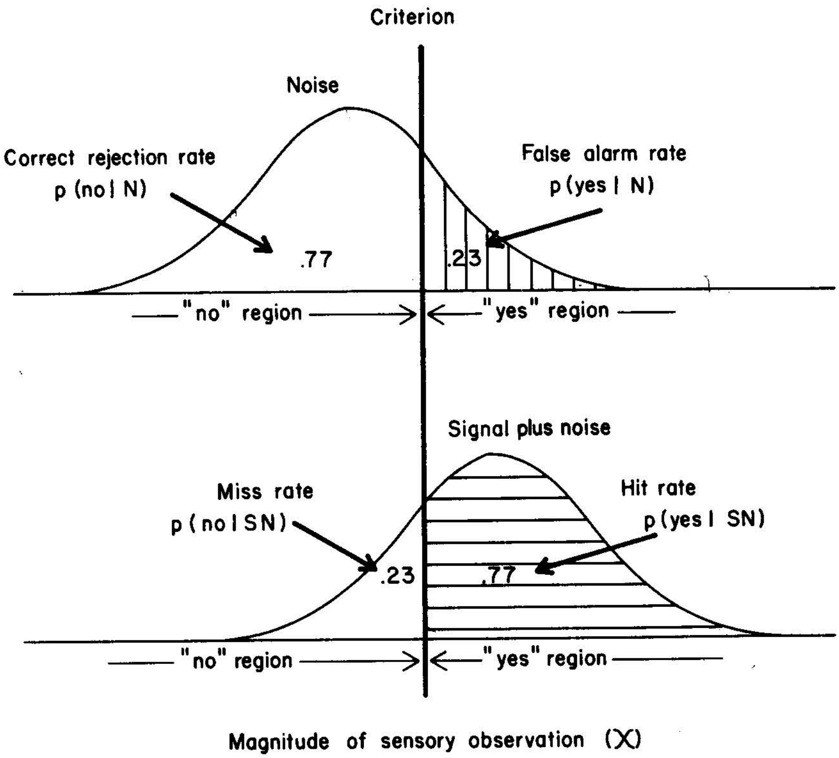

FIG. 5.3. Theoretical frequency distributions of noise and signal plus noise. The location of the observer's criterion determines whether a particular sensory observation, x, results in a no or a yes response.

"no, there is no signal." Sometimes, the observer will be correct and report yes when the signal is presented (hit) and no when only noise is present (correct rejection). Sometimes, however, the observer will be incorrect and report yes when only noise is present (false alarm) and no when the signal is presented (miss).

In a typical signal detection experiment, there are many trials in which the signal is presented, and there are many trials in which no signal is presented. The order of presentation of SN and N trials is random. The intensity of the signal is usually the same on each SN trial. On each trial, the observer must report whether or not a signal was presented. The hit rate is the proportion of SN trials in which the observer reported the signal, and the false alarm rate is the proportion of N trials in which the observer reported a signal. If such an experiment were conducted with the observer whose N and SN distributions and criterion are depicted in Figure 5.3, it would be possible to predict the hit and false alarm rates.

The predicted proportion of hits, the hit rate [p(yes | SN)], is the proportion of the SN distribution above the criterion. The hit rate in this example is .77. The predicted proportion of false alarms, the false alarm rate [p(yes | N)], is the proportion of the N distribution above the criterion. In this example, this rate is .23. For this observer, the location of the criterion on the sensory observation dimension, and the location of the N and SN distributions, were known in advance; therefore, it was possible to predict the hit and false alarm rates for an experiment. More typically, the reverse is done; that is, the location on the sensory observation dimension of the criterion, and N and SN distributions, are derived from the observer's experimentally determined hit and false alarm rates. It is here that the power of TSD lies. If the theory is correct, it becomes possible, through analysis of experimentally obtainable hit and false alarm rates, to specify quantitatively the processes going on inside the observer that are the bases of sensory decision making.

Swets et al. (1961) consider the detection situation to be analogous to a game of chance in which three dice are thrown. Two of the dice are ordinary, but the third is a special die with three spots on each of three sides and no spots on the other three sides. When the dice are thrown, the player is told only the total number of spots on all three dice. This information is analogous to the information given for each observation in a detection situation. On the basis of the total number of spots showing, the player must decide whether the unusual die showed a zero or a three. Similarly, in the detection situation the observer must decide whether an observation was a product of noise alone or of signal plus noise. To come out ahead in the long run, the player of the dice game would compute the probability of occurrence of each of the possible totals (2 to 12) when the unusual die shows zero, and likewise, the probabilities of each of the totals (5 to 15) when the unusual die shows three. The results can be plotted as two probability distributions and should be thought of as the analogs of the noise and signal-plus-noise distributions (Figure 5.4). Furthermore, as in the detection situation, a criterion can be set so that if the total number of spots were greater than some number the player would say "three," and if the total were less than the number he would say "zero." In our example, where the probabilities of a three and a zero are both .50 and the costs and values are the same for the various decision outcomes, the optimal criterion is the point where the two curves cross. In a detection situation, where the stimulus probability is .50 and the costs and values are equal for the various decision outcomes, the optimal criterion is also the point on the observation magnitude dimension where the two distributions cross.

It can be demonstrated mathematically that in the dice game the optimal cutoff point changes when the conditions of the game are changed.

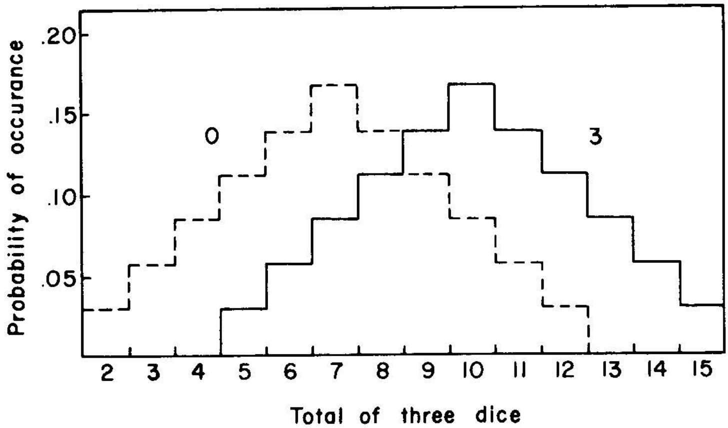

FIG. 5.4. Probability distributions for the dice game. (From Swets, Tanner, & Birdsall, 1961. Copyright 1961 by the American Psychological Association. Reprinted by permission.)

For example, if the unusual die were changed to one having three spots on five of the six sides and no spots on only one side, the probability of obtaining three spots would be .83 instead of .50, and the optimal criterion would be lowered. A good player would lower the cutoff point for saying "three." Likewise, the optimal criterion would be raised if only one side of the die had three spots and the other five sides had none. In the detection situation, changing the stimulus probability is assumed to have a similar effect on the observer's criterion. In order to perform optimally in a situation where stimulus occurrence is highly probable, an observer will report a signal after a less intense sensory observation than when the stimulus is relatively improbable.

The location of the optimal cutoff point in the dice game is also influenced by changes in the payoff conditions. For example, if the reward is great for correctly saying "three" and the punishment slight for saying "three" when "zero" is correct, the optimal criterion will, of course, be relatively low for saying "three." Rewards and punishments in the detection situation are assumed to have a similar effect on the observer's criterion. A radar scope observer, for instance, maintains a low criterion for reporting signals because of the extreme importance of detecting enemy aircraft and the possible disastrous consequences of failing to do so.

The criterion set in the dice game will also be influenced by the degree of overlap of the two distributions. If the number of spots on the unusual die is made zero or four instead of zero or three, the distributions will overlap less because the distribution, when four is correct, will be shifted further up the scale, and the game becomes easier. Since the point where the two distributions cross is also shifted up the scale, the optimal criterion will be higher. This manipulation in the dice game can be translated into an increase in stimulus intensity in the detection situation. When the signal-plus-noise distribution is shifted to a higher point on the observation magnitude dimension, detection becomes easier, and the optimal criterion is higher.

In summary, the detection of energy changes in our environment involves, according to TSD, the establishing of a decision rule in the same way as the efficient playing of a game of chance does. The decision rule is the setting of a criterion determining which hypothesis about a given piece of information will be accepted and which rejected. High criteria are used to minimize false alarms and maximize correct rejections, whereas low criteria are used to minimize misses and maximize hits. The location of optimal criterion is a function of (a) the probabilities of the N and SN presentations, and (b) the costs and values for the various decision outcomes.

In the detection situation, where the costs and values of the various decision outcomes and the probability of signal presentation are precisely known, the optimum value of the criterion, β_{opt}, can be calculated by

$$\beta_{opt} = \frac{p(\mathrm{N})}{p(\mathrm{SN})} \times \frac{\text{value(correct rejection)} - \text{cost(false alarm)}}{\text{value(hit)} - \text{cost(miss)}}. \tag{5.2}$$

β_{opt} is the value of the likelihood ratio, $l(x)$, which, when used as the criterion, will result in the largest possible winnings in the long run; p(N) is the probability of a noise trial; p(SN) is the probability of a signal-plus-noise trial; value is the amount given to the observer for each correct decision, and cost is the amount taken away from the observer for each incorrect observation. Costs are entered into the equation as negative numbers. For example, if a nickel were taken away for each false alarm, the cost of a false alarm would be –5.

When the value of β, as calculated from the judgments of observers, is compared with β_{opt}, it is generally found that observers do fairly well at optimizing their winnings. An exception to this rule occurs, however, when β_{opt} is very small or very large, in which case β will not be as extreme as β_{opt}, and the observer will fail to optimize his winnings. Observers tend not to set extremely low or extremely high criteria, even in situations where these strategies would lead to optimal performance.

An interesting early application of setting the optimal criterion is seen in Pascal's famous "wager." In 1670, Blaise Pascal, a French mathematician, claimed that to believe in God was rational. He noted that there are two possibilities, existence of God or nonexistence of God, and two possible responses, belief in God or disbelief in God. Pascal argued that, even if the probability of God's existence is extremely small, the gain (value) of asserting His existence and the cost of denying it make belief in God the rational choice. In TSD terms, the decision criterion should be set infinitely low because the value of a hit is infinitely high as is the cost of a miss, and at the same time there is no cost to a false alarm and no value to a correct rejection. Thus, "If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is" (Pensée No. 233, Pascal, 1958).

THEORETICAL SIGNIFICANCE OF THE RECEIVER-OPERATING CHARACTERISTIC CURVE

One of the main sources of evidence supporting TSD comes from the experimental manipulation of variables resulting in data plotted as ROC curves. In fact, shapes of ROC curves for various stimulus intensities can be generated from the theory and checked against empirical data. It should be recalled that the high threshold theory was rejected because of its failure to predict empirical ROC curves.

The manner in which ROC curves are predicted from TSD is illustrated in Figures 5.5, 5.6, and 5.7. The ROC curve in Figure 5.5 represents a situation where the signal strength is sufficient to result in only a slight overlap of the N and SN probability distributions. The vertical lines represent the locations of the criterion that might be associated with specific conditions of stimulus probability and payoff. Each point on an ROC curve, according to the theory, is determined by the location of the observer's criterion on the x dimension. If an observation is to the right of the criterion, the observer will say yes. The proportion of the area under the curve to the right of the criterion gives the proportion of yes decisions. Therefore, the hit rate [$p(\text{yes} \mid \text{SN})$] and the false alarm rate [$p(\text{yes} \mid \text{N})$] can be determined by finding the areas under the SN and N distribution curves, respectively, which are located to the right of the criterion. As the criterion is changed from high to low, the false alarm rate and the hit rate increase and, when plotted, form an ROC curve. The illustration in Figure 5.6 shows the ROC curve predicted from TSD when the signal strength is so weak as to result in considerable overlap of the N and SN distributions. Figure 5.7 is the predicted ROC curve when there is no separation between the N and SN distributions. In this case, the signal is too weak to have an effect on the nervous system. Comparing the ROC curves of Figures 5.5, 5.6, and 5.7 shows that, as the separation between N and SN distributions increases, the predicted ROC curve rises more rapidly and departs from the positive diagonal of the graph by a greater amount. In all three cases, it is seen that, as the criterion is lowered, the predicted point on the ROC curve becomes higher.

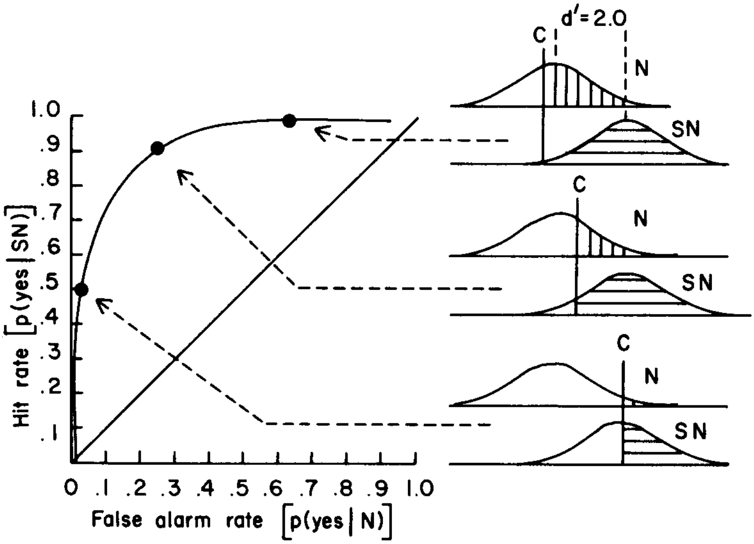

FIG. 5.5. Relation between the ROC curve and the theoretical noise and signal-plus-noise distributions. Variation in the observer's criterion results in different points along the ROC curve. The hit rate is equal to the proportion of the area of the signal-plus-noise distribution that is above criterion. The false alarm rate is equal to the proportion of the area of the noise distribution that is above criterion.

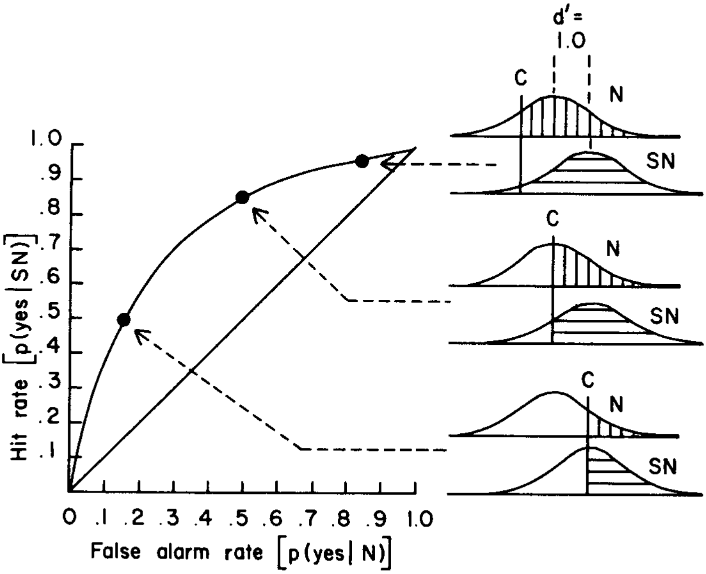

FIG. 5.6. Same as Figure 5.5 but the noise and signal-plus-noise distributions are closer together.

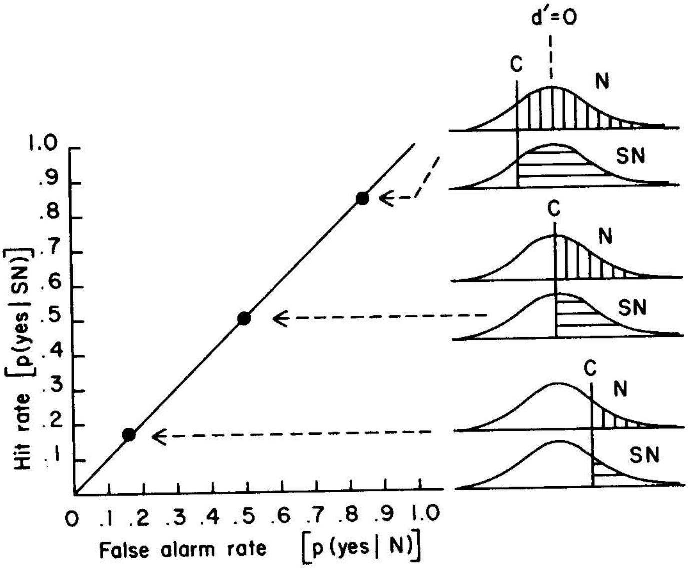

FIG. 5.7. Same as Figures 5.5 and 5.6 but the locations of the noise and signal-plus-noise distributions are identical.

The shape of the ROC curve is a function of the way the areas under the N and SN distributions above the criterion change when the location of the criterion is changed. When N and SN distributions are identical, as in Figure 5.7, lowering the criterion increases the area above criterion by exactly the same amount in the N and SN distributions, and the resulting ROC curve is linear with a slope of 1.0. When the two distributions are separated, as in Figures 5.5 and 5.6, lowering the criterion from high to moderate changes the area above criterion more rapidly for the SN than for the N distribution, and consequently, the lower section of the ROC curve rises at a rapid rate with slopes greater than 1.0. Lowering the criterion from moderate to low, on the other hand, changes the area above the criterion less rapidly for the SN than for the N distribution, and although the ROC curve continues to rise, it does so at a diminished rate with slopes less than 1.0.

It is important to understand that the ROC curves presented in Figures 5.5, 5.6, and 5.7 are predicted from TSD. Testing the validity of TSD requires that ROC curves determined in experiments in which observers detect signals be compared with the ROC curves predicted from the theory. To obtain an ROC curve, the observer is presented with a random series of trials that either contain or do not contain a signal. On each trial, the observer must report the presence or absence of the signal. At various stages of the experiment, the observer is induced to change the location of the criterion. This criterion change can be accomplished in many ways, including changing the proportion of trials containing a signal, or changing the payoff for correct responses and the punishment for errors. Hit and false alarm rates are recorded for each criterion set by the observer. The ROC curve is obtained when the hit rates are plotted as a function of the false alarm rates. Thus, each data point on an ROC curve represents the hit and false alarm rates obtained for a single criterion of the observer. Many experiments designed in this manner have been conducted, and, generally, the shapes of the ROC curves obtained in the laboratory are in close agreement with those predicted from TSD. Because TSD has been supported so well by experimental research, most psychophysicists accept the theory, and many have used it to solve the old problem of obtaining pure measurements of an observer's sensitivity to stimuli uninfluenced by the location of the criterion.

MEASURING SENSITIVITY

In the classical psychophysical experiment, the results expressed as thresholds were a function of both stimulus detectability and the location of the observer's criterion. Thus, as a measure of sensitivity to stimuli, the threshold may be hopelessly contaminated by changes in the observer's criterion. Such contamination can lead to faulty conclusions about the results of an experiment. For example, there have been cases in which investigators incorrectly attributed large changes in thresholds to changes in the sensitivity of sensory processes. In fact, as revealed by subsequent experimentation, only the criterion had changed. As seen in Figure 4.16, psychometric functions obtained by the method of constant stimuli can be substantially influenced by changing the proportion of trials on which the signal was presented. When the proportion of signal trials was increased, the proportion of "yes" responses increased at all intensities of the signal. The increase in the proportion of "yes" responses can be attributed to a lowering of the observer's decision criterion. One consequence of this increase in the proportion of "yes" responses is a shift in the position of the psychometric function such that the estimated *absolute threshold,* defined as the intensity of a signal that is detectable 50% of the time, is substantially lowered. The threshold, estimated from the psychometric function, becomes a biased estimate of sensitivity insofar as performance in the task is influenced by the location of the observer's criterion, as well as by sensitivity level. Figure 5.8 illustrates the possible confounding effects of criterion location on threshold estimation. The psychometric functions on the top of the figure illustrate a case in which observer A and observer B, having the same sensitivity and decision

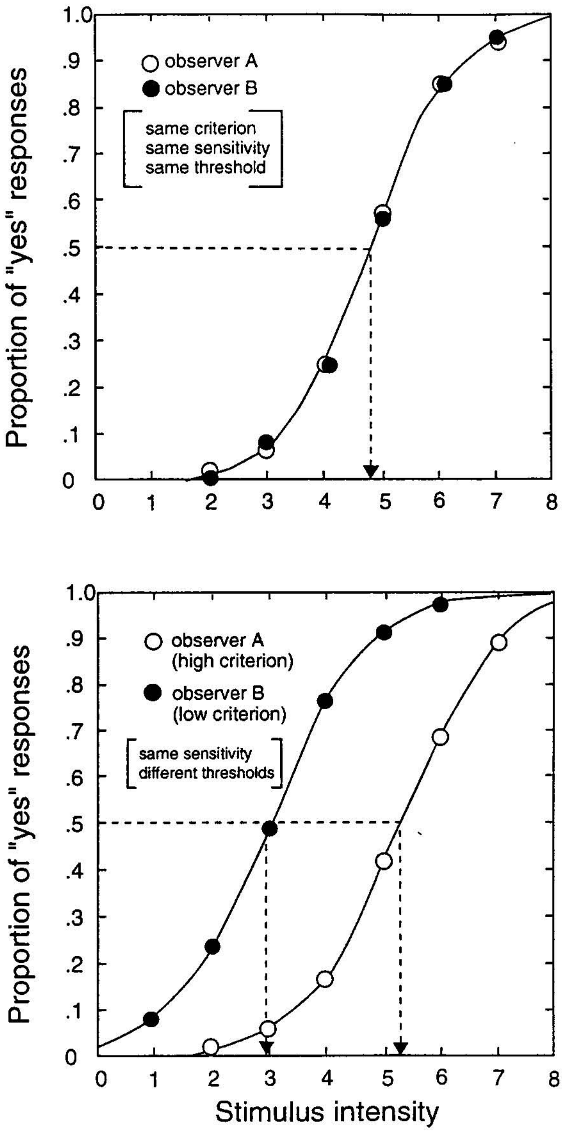

FIG. 5.8. Psychometric functions of observers with the same criterion and sensitivity (top) and psychometric functions of observers with different criteria and the same sensitivity (bottom).

criterion, have the same estimated threshold. In the bottom of the figure, the estimated threshold of observer B is lower than that of observer A because, although the observers have the same sensitivity, observer B has a lower criterion than does observer A.

TSD and its associated methodology afford a means of independently measuring the observer's sensitivity and criterion location. The theory proposes that d', a measure of detectability, is equal to the difference between the means of the SN and N distributions ($M_{SN} - M_N$) divided by the standard deviation of the N distribution (σ_N):

$$d' = \frac{M_{SN} - M_N}{\sigma_N}. \tag{5.3}$$

Because the location of the SN distribution in relation to that of the N distribution is entirely a function of stimulus intensity and properties of the sensory system, d' is a pure index of stimulus detectability that is uncontaminated by the location of the observer's criterion.

But how can this theoretical concept of signal detectability be measured? Since different d' values predict different ROC curves, the value of d' in a particular situation can be ascertained by determining on which member of the family of ROC curves an observer's response probabilities fall. A family of ROC curves corresponding to d' values ranging from 0 to 3.0 is seen in Figure 5.9. Because only a limited number of curves are usually presented in such a graph, it is best to use them when only approximate values of d' are needed. Fortunately, simple methods are available for the determination of exact values of d'.

The value of d' can be quickly computed from the experimentally determined values of the false alarm and hit rates. The proportion of false alarms, when subtracted from 1.0 and converted to a z score through Table A of the appendix, gives Z_N, the location of the criterion on the abscissa of the noise distribution. These operations are summarized below and are illustrated in Figure 5.10.

$$1.0 - p(\text{false alarms}) \rightarrow Z_N \tag{5.4}$$

The location of the criterion on the abscissa of the signal-plus-noise distribution, Z_{SN}, is found by subtracting the hit rate from 1.0 and converting this p value to a z score. These operations are summarized below and are illustrated in Figure 5.10.

$$1.0 - p(\text{hit}) \rightarrow Z_{SN} \tag{5.5}$$

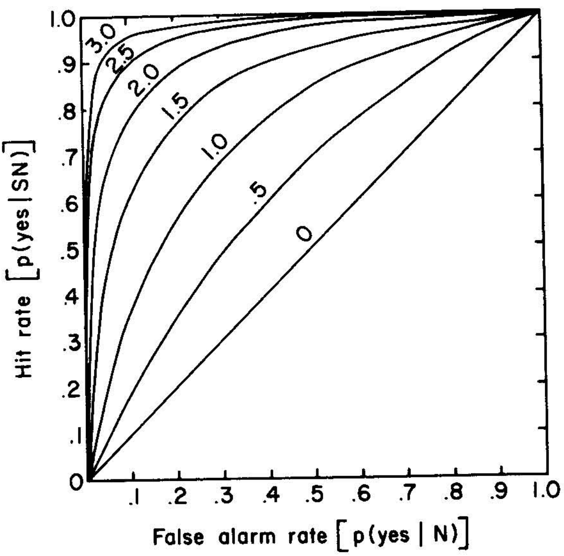

FIG. 5.9. A family of ROC curves corresponding to d' values ranging from 0 to 3.0.

To obtain d', Z_{SN} is subtracted[2] from Z_N.

$$d' = Z_N - Z_{SN} \tag{5.6}$$

In the example in Figure 5.10, 1.0 minus the false alarm rate of .02 (i.e., .98) gives the proportion of the area under the noise distribution below the criterion. Converting .98 to a z score yields a Z_N value of 2.05, which represents the location of the criterion on the abscissa of the noise distribution. The hit rate of .35 subtracted from 1.0 (i.e., .65) gives the proportion of the area under the signal-plus-noise distribution below the criterion. When .65 is converted to a z score, Z_{SN} is found to be .39. This value represents the location of the criterion on the abscissa of the signal-plus-noise distribution. To find d', the Z_{SN} value of .39 is subtracted from the Z_N value of 2.05 to yield a d' value of 1.66. This value of 1.66 is the number of z-score units between the mean of the noise distribution and the mean of the signal-plus-noise distribution.

[2]An alternative, and frequently used method for calculating d', is to use the formula $d' = Z\text{ (hit)} - Z\text{ (false alarm)}$. Although the same result is obtained with this method and the one specified above, I prefer teaching the concept of d' in terms of Z_N and Z_{SN} rather than Z (false alarm) and Z (hit), because Z_{SN} and Z_N specify the location of the observer's criterion in terms of z-score units on the abscissa of the SN and N distributions, respectively (see Fig. 5.10).

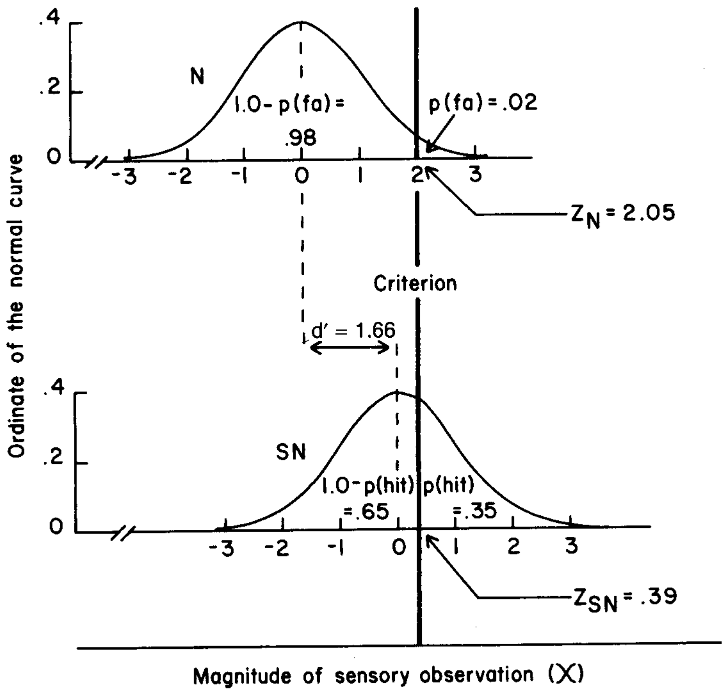

FIG. 5.10. Distributions of noise and signal plus noise expressed in z scores. The location of the criterion on the noise distribution is found by subtracting the false alarm rate from 1.0 and converting this value to a z score. The location of the criterion on the signal-plus-noise distribution is found by subtracting the hit rate from 1.0 and converting this value to a z score. The value of d', a measure of the observer's sensitivity to the signal, is found by subtracting Z_{SN} from Z_N ($d' = Z_N - Z_{SN}$).

The d' values for each data point on the ROC curves of Figures 5.5, 5.6, and 5.7 can be determined by the method just described. When these calculations are performed, we find that d' will be 2.0 for each point on the ROC curve in Figure 5.5, 1.0 for each point on the curve in Figure 5.6, and .00 for each point on the curve in Figure 5.7. It should be clear that, for a particular separation of the noise and signal-plus-noise distributions, the value of d' will remain constant for all possible criterion positions. Thus, an ROC curve is a description of performance changes that are accounted for by a constant d' and a continuously variable criterion.

It has been experimentally demonstrated for both visual stimuli (Swets, Tanner, & Birdsall, 1955) and auditory stimuli (Tanner, Swets, & Green, 1956) that d', as a measure of sensitivity, is not contaminated by the effects of variables which shift an observer's response criterion. Furthermore, d' values, unlike the different threshold values obtained through the use of the various classical psychophysical methods, remain relatively invariant when measured by different experimental procedures (Swets, 1959; Tanner & Swets, 1954).

Once the correct ROC curve has been determined, the location of the observer's criterion, β, can be determined by observing exactly where on the ROC curve the point is located. If the point is near the bottom of the ROC curve where the slope is great, the criterion is high; if the point is near the top of the curve where the slope is slight, the criterion is low. The exact value of β is equal to the slope of the ROC curve at a particular point.

To reiterate, β is a value of the likelihood ratio. It is the ratio of the ordinate of the SN distribution at the criterion to the ordinate of the N distribution at the criterion, as follows:

$$\beta = \frac{\text{ordinate of SN distribution at criterion}}{\text{ordinate of N distribution at criterion}}. \tag{5.7}$$

Ordinate values (O) of the normal distribution curve for various p values and z-score values are found in Table A in the appendix. Figure 5.2 illustrates that moving the criterion to the right increases the value of β, and moving it to the left decreases β. A low value of β represents a lax criterion where the observer will be liberal about reporting signals, while a high value of β represents a strict criterion where the observer will be conservative about reporting signals.

The value of β can be calculated from a pair of hit and false alarm rates. The ordinate of the N distribution at criterion can be estimated as the ordinate value given in Table A that corresponds to 1.0 minus the false alarm rate. Likewise, the ordinate of the SN distribution at criterion is obtained by converting 1.0 minus the hit rate into the ordinate value on the normal distribution curve. For example, ordinate values for a false alarm rate of .20 and a hit rate of .85 are .2801 and .2333, respectively:

$$\beta = \frac{.233}{.280} = .83.$$

The techniques of computing d' and β equip investigators who wish to study the effects of a particular variable with a means of testing whether the effects of that variable are on detectability or on the location of the criterion. They have only to observe whether systematic changes in the variable result in different points along a single ROC curve or points located on different ROC curves. The values of d' and β can also be calculated for various experimental conditions. In some experiments, manipulation of an independent variable has led to changes in both β and d'.

RESPONSE BIAS

Response bias is a tendency of the observer, determined by factors other than signal intensity, to favor one response over another. The sensitivity measure, d', depends on stimulus parameters but remains constant when the situation dictates that one response should be made more frequently than another. Thus, d' is independent of response bias. Up to this point we have described how response bias can be measured through the calculation of β. Calculating β from the hit and false alarm rates, although providing a useful description of the location of the observer's criterion in terms of a likelihood ratio, is not the only feasible way to describe response bias. An alternative index of response bias is C (for criterion), which is defined as

$$C = 0.5\ [Z_{SN} + Z_N]. \tag{5.8}$$

When the false alarm rate and the miss rate are equal, the hit rate and correct rejection rate will also be equal and the value of C will be zero. For example, as illustrated in Figure 5.11, when both the false alarm and miss rate is 0.1 and the hit rate and correct ejection rate is .9, the value of Z_N is $(1.0 - 0.1) \rightarrow Z_N = 1.28$, while the value of Z_{SN} was $(1.0 - 0.9) \rightarrow Z_{SN} = -1.28$. Thus, the value of C is $.5(-1.28 + 1.28) = 0.0$. It should also be kept in mind that C is zero at the point where the N and SN distributions cross. This point, illustrated in Figure 5.11, is regarded as the point at which the observer's responses are neither biased toward "yes" responses nor toward "no" responses. Because the area of the SN distribution above the criterion is exactly equal to the area of the N distribution below the criterion the hit and correct rejection rates are exactly equal. Likewise, because the area of the SN distribution below the criterion is equal to the area of the N distribution above the criterion the miss and false alarm rates are equal. Indicative of the absence of response bias in this situation in which the value of C is zero, the total number of "yes" responses (hits plus false alarms) will be the same as the total number of "no" responses (misses plus correct rejections). Negative values of C, on the other hand, would reflect a bias toward frequent "yes" responses and positive values of C would reflect a bias toward frequent "no" responses. It should be recognized that C, because it is computed from Z_N and Z_{SN}, is the number of standard deviation units (z-score units) that the criterion is above or below the zero bias point where the N and SN distributions cross. A convenient feature of expressing the criterion in this way is that both the sensitivity measure, d', and the criterion are expressed in the common units of z scores of the two distributions. In the present example, we can say that the means of the N and SN distributions are separated

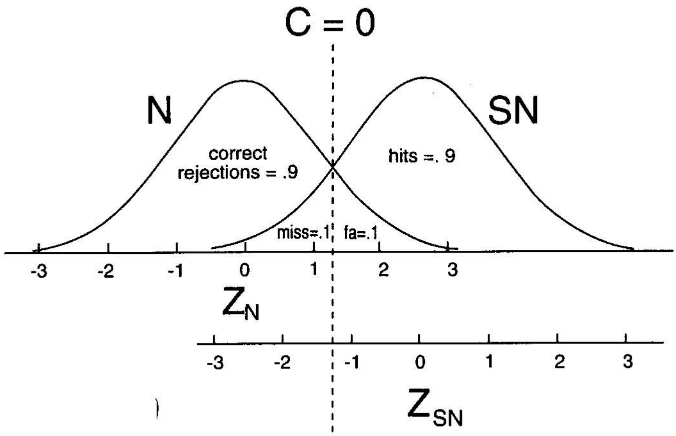

FIG. 5.11. The determination of C from Z_N and Z_{SN}. In this example, C = 0, the point where the N and SN distributions cross.

by 2.56 *z*-score units ($d' = Z_N - Z_{SN} = 1.28 - -1.28 = 2.56$), while the location of the criterion in *z*-score units from the point at which the N and SN distributions cross is 0.0 ($C = .5\ [Z_{SN} + Z_N = .5[-1.28 + 1.28] = 0$).

In addition to β and C, a third measure of response bias is C′. While β is the value of the likelihood ratio at the criterion, and C is the distance of the criterion in *z*-score units from the crossing point of the N and SN distributions, C′ represents the value of C as a proportion of sensitivity distance (*d′*). The value of C′ can be calculated as

$$C' = C/d' = .5[Z_{SN} + Z_N]\ /\ [Z_N - Z_{SN}]. \tag{5.9}$$

The answer to the question of which index of response bias is better depends on the degree to which each of the three measures, β, C, and C′, are independent of changes in sensitivity. It has been demonstrated that neither β or C′ is statistically independent of *d′*, whereas C is (Macmillan & Creelman, 1991). Furthermore, the range of C does not depend on *d′*, whereas the range of β and C′ does (Banks, 1970; Ingham, 1970). When *d′* is large, the range of C′ is small while the range of β is large. When *d′* is small, the opposite is true. In situations where both the sensitivity and criterion of the observer change as experimental conditions change, C is the only measure of response bias that can be interpreted without knowledge of *d′* (Snodgrass & Corwin, 1988; Macmillan & Creelman, 1990; Macmillan & Creelman, 1991).

The three measures of response bias are simply related. If *d′* and one of the bias measures is known, it is easy to calculate the other two. The value of C′ is obtained by dividing C by *d′*, while multiplying C by *d′*

yields log β. Taking the antilog of log β where log β is the natural rather than common log of β yields β. It is important to carefully consider the results obtained with different bias measures because, in some circumstances, they may each lead to different conclusions about the experimental results.

PROBLEMS

5.1. Assume that the variance of the N and SN distributions are equal. Plot ROC curves on proportion coordinates for a d' of .8 and for a d' of 1.6. Plot at least 5 points on each curve. Use the equation $d' = Z_N - Z_{SN}$ to solve the problem.

5.2. Assuming that N and SN distributions are normal with equal variance, calculate the values d', β, C, and C' for the hit and false alarm rates found in the table below.

Observer	False Alarm Rate	Hit Rate
1	.18	.84
2	.38	.96
3	.42	.69
4	.30	.48
5	.70	.98

5.3. An observer performed a signal detection task in which he was paid \$2.00 for each correct response and was charged \$1.00 for each error. Calculate β_{opt} when $p(S)$ is .30 and when $p(S)$ is .70.

5.4. An observer performed a signal detection task in which she was paid \$2.00 for each hit, and \$1.00 for each correct reject. Each miss and each false alarm cost her \$1.50 and 50 cents respectively. The value of p(s) was .50. Calculate the value of β_{opt}.