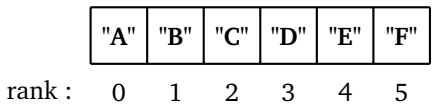
Lists I

A list is a way to group elements into a single object. After defining a list, you can retrieve each item of the list one by one, but also add new ones...

Lesson 1 (List (1))**.**
A ***list*** is a series of elements. This can be a list of integers, for example `[5,-7,12,99]`, or a list of strings, for example `["March","April","May"]` or objects can be of different types `[3.14,"pi",10e-3,"x",True]`.

- **Construction of a list.** A list is defined by elements between square brackets:
  - `mylist1 = [5,4,3,2,1]` a list of 5 integers,
  - `mylist2 = ["Friday","Saturday","Sunday"]` a list of 3 strings,
  - `mylist3 = []` the empty list (very useful to complete it later).
- **Get an item.** To get an item from the list, simply write `mylist[i]` where i is the rank of the desired item.
  Beware! The trap is that you start counting from the rank 0.
  For example after the instruction `mylist = ["A","B","C","D","E","F"]` then
  - `mylist[0]` returns `"A"`
  - `mylist[1]` returns `"B"`
  - `mylist[2]` returns `"C"`
  - `mylist[3]` returns `"D"`
  - `mylist[4]` returns `"E"`
  - `mylist[5]` returns `"F"`



- **Add an element.** To add an item at the end of a list, just use the command `mylist.append(element)`. For example if `primes = [2,3,5,7]` then `primes.append(11)` adds 11 to the list, if you then execute the instruction `primes.append(13)` then the list `primes` is now `[2,3,5,7,11,13]`.
- **Example of construction.** Here is how to build the list that contains the first squares:

```
list_squares = []              # Start from the empty list
for i in range(10):
    list_squares.append(i**2)    # Add squares one by one
```

At the end `list_squares` is:

`[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]`

Lesson 2 (List (2))**.**

- **Length of a list.** The length of a list is the number of elements it contains. The command `len(mylist)` returns the length. The list `[5,4,3,2,1]` is 5 long, the list `["Friday","Saturday","Sunday"]` has length 3, the empty list `[]` has length 0.
- **Browse a list.** Here is the easiest way to scan a list (and here to display each item):

```
for item in mylist:
    print(item)
```

- **Browse a list (bis).** Sometimes you need to know the rank of the elements. Here is another way to do it (which here displays the rank and the element).

```
n = len(mylist)
for i in range(n):
    print(i,mylist[i])
```

- To get a list from `range()` you have to write:

`list(range(n))`

- It's a bad idea to name your list "`list`" because this word is already used by `Python`.

Activity 1 (Simple or compound interests)**.**

Goal: create two lists to compare two types of interests.

1. **Simple interest.** We have a sum of S_0. Each year this investment earns interest based on the initial amount.
   For example, with an initial amount of $S_0 = 1000$ and simple interest of $p = 10\%$. The interest is 100. So after one year, I have a sum of $S_1 = 1100$, after two years $S_2 = 1200\ldots$
   Program a `simple_interest(S0,p,n)` function that returns the list of amounts for the n first years. For example `simple_interest(1000,10,3)` returns `[1000, 1100, 1200, 1300]`.
2. **Compound interest.** A sum of S_0 brings in compound interest. This time the interest is calculated each year on the basis of the sum of the previous year, i.e. according to the formula:
   $$I_{n+1} = S_n \times \frac{p}{100}$$
   Program a function `compound_interest(S0,p,n)` which returns the list of amounts of the n first years. For example `compound_interest(1000,10,3)` returns `[1000, 1100, 1210, 1331]`.
3. I have the choice between a simple interest investment of 10% or a compound interest investment of 7%. What is the most advantageous solution depending on the duration of the placement?

Lesson 3 (List (3))**.**

- **Concatenate two lists.** If you have two lists, you can merge them by the operator "+". For example with `mylist1 = [4,5,6]` and `mylist2 = [7,8,9]`

`mylist1 + mylist2` is `[4,5,6,7,8,9]`.

- **Add an item at the end.** The operator "+" provides another method to add an item to a list:
  `mylist = mylist + [element]`
  For example `[1,2,3,4] + [5]` is `[1,2,3,4,5]`. Attention! The element to be added must be surrounded by square brackets. It is an alternative method to `mylist.append(element)`.
- **Add an element at the beginning.** With :
  `mylist = [element] + mylist`
  the item is added at the beginning of the list. For example `[5] + [1,2,3,4]` is `[5,1,2,3,4]`.
- **Slicing lists.** You can extract a whole part of the list at once: `mylist[a:b]` returns the sublist of items with ranks a to $b-1$.

	"A"	"B"	"C"	"D"	"E"	"F"	"G"
rank :	0	1	2	3	4	5	6

  For example if `mylist = ["A","B","C","D","E","F","G"]` then
  - `mylist[1:4]` returns `["B","C","D"]`
  - `mylist[0:2]` returns `["A","B"]`
  - `mylist[4:7]` returns `["E","F","G"]`

  Once again, it is important to remember that the rank of a list starts at 0 and that the slicing `mylist[a:b]` stops at the rank $b-1$.

Activity 2 (Manipulate lists)**.**

Goal: program small routines that manipulate lists.

1. Program a function `rotate(mylist)` that shifts all the elements of a list by one rank (the last element becoming the first). The function returns a new list.
   For example, `rotate([1,2,3,4])` returns the list `[4,1,2,3]`.
2. Program a function `inverse(mylist)` that inverts the order of the elements in a list.
   For example, `inverse([1,2,3,4])` returns the list `[4,3,2,1]`.
3. Program a function `delete_rank(mylist,rank)` that returns a list of all elements, except the one at the given rank.
   For example, `delete_rank([8,7,6,5,4],2)` returns the list `[8,7,5,4]` (the item 6 that was at rank 2 is deleted).
4. Program a function `delete_element(mylist,element)` returning a list that contains all items except those equal to the specified element.
   For example, `delete_element([8,7,4,6,5,4],4)` returns the list `[8,7,6,5]` (all items equal to 4 have been deleted).

Lesson 4 (Manipulate lists)**.**
You can now use the `Python` functions which do some of these operations.

- **Invert a list.** Here are three methods:
  - `mylist.reverse()` modifies the list in place (i.e. `mylist` is now reversed, the command returns nothing);
  - `list(reversed(mylist))` returns a new list;
  - `mylist[::-1]` returns a new list.

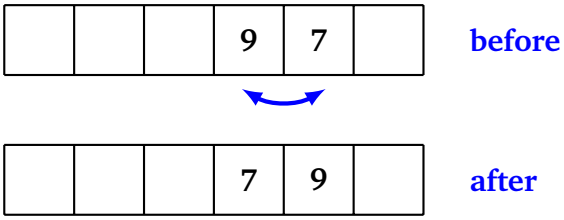
- **Delete an item.** The command `mylist.remove(element)` deletes the first occurrence found (the list is modified). For example with `mylist = [2,5,3,8,5]` the call `mylist.remove(5)` modifies the list which is now `[2,3,8,5]` (the first 5 has disappeared).
- **Delete an element (bis).** The command `del mylist[i]` deletes the element of rank i (the list is modified).

Activity 3 (Bubble sort)**.**

Goal: order a list from the smallest to the largest element.

The bubble sort is a simple way to order a list, here it will be from the smallest to the largest element. The principle is as follows:

- We go through the list from the beginning. As soon as you encounter two consecutive elements in the wrong order, you exchange them.
- At the end of the first pass, the largest element is at the end and it will not move anymore.
- We restart from the beginning (until the penultimate element), this time the last two elements are well placed.
- We continue this way. There is a total of $n-1$ passages if the list is of length n.



Here is the bubble sort algorithm:

Algorithm.

- – Input: a list ℓ of n numbers
  – Output: the ordered list from the smallest to the largest
- For i ranging from $n-1$ to 0:
  For j ranging from 0 to $i-1$:
  If $\ell[j+1] < \ell[j]$ then exchange $\ell[j]$ and $\ell[j+1]$.
- Return the list ℓ.

Program the bubble sort algorithm into a `bubble_sort(mylist)` function that returns the ordered list of elements. For example `bubble_sort([13,11,7,4,6,8,12,6])` returns the list `[4,6,6,7,8,11,12,13]`.

Hints.

- Begin by defining `new_mylist = list(mylist)` and works only with this new list.
- For the index i to run backwards from $n-1$ to 0, you can use the command :
  `for i in range(n-1,-1,-1):`
  Indeed `range(a,b,-1)` corresponds to the decreasing list of integers i satisfying $a \geqslant i > b$ (as usual the right bound is not included).

Lesson 5 (Sorting).
You can now use the `sorted()` function from `Python` which orders lists.

python : sorted()

Use: `sorted(mylist)`
Input: a list
Output: the ordered list of elements

Example: `sorted([13,11,7,4,6,8,12,6])` returns the list `[4,6,6,7,8,11,12,13]`.

Attention! There is also a `mylist.sort()` method that works a little differently. This command returns nothing, but on the other hand the list `mylist` is now ordered. We are talking about a modification *in place*.

Activity 4 (Arithmetic).

Goal: improve some functions of the chapter "Arithmetic – While loop – I".

1. **Prime factors.** Program a function `prime_factors(n)` that returns a list of all the prime factors of an integer $n \geqslant 2$. For example, for $n = 12936$, whose decomposition into prime factors is $n = 2^3 \times 3 \times 7^2 \times 11$, the function returns `[2, 2, 2, 3, 7, 7, 11]`.
   Hints. Consult the chapter "Arithmetic – While loop – I". The core of the algorithm is as follows:

   As long as $d \leqslant n$:
   If d is a divisor of n, then:
   add d to the list,
   n becomes n/d.
   Otherwise increment d by 1.

2. **List of prime numbers.** Write a function `list_primes(n)` that returns the list of all prime numbers less than n. For example `list_primes(100)` returns the list:
   `[2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]`
   To do this, you will program an algorithm that is a simple version of the sieve of Eratosthenes:

   Algorithm.
   - – Input: an integer $n \geqslant 2$.
   - – Ouput: the list of prime numbers $< n$.
   - Initialize `mylist` by a list that contains all integers from 2 to $n-1$.
   - For d ranging from 2 to $n-1$:
     For k in `mylist`:
     If d divides k and $d \neq k$, then remove the element k from `mylist`.
   - Return `mylist`.

   Hints.
   - Start from `mylist = list(range(2,n))`.
   - Use `mylist.remove(k)`.

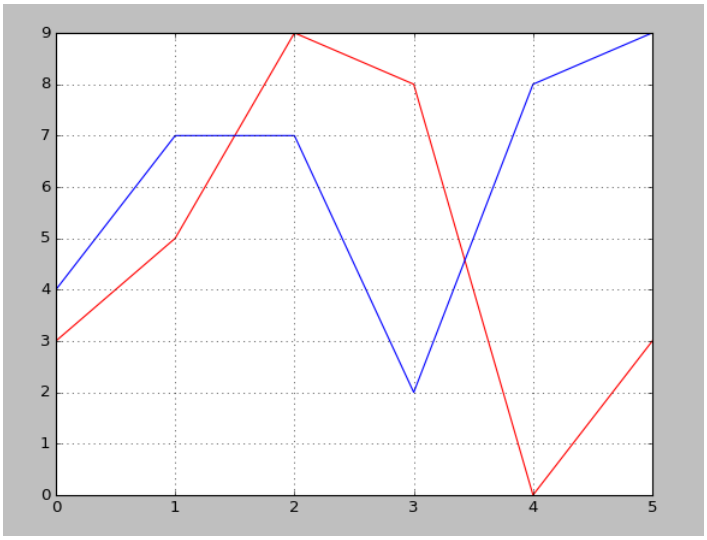
Explanations. Let's see how the algorithm works with $n = 30$.

- At the beginning the list is

  $[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]$

- We start with $d = 2$, we eliminate all the numbers divisible by 2, unless it is the number 2: so we eliminate 4, 6, 8,..., the list is now $[2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29]$.
- We continue with $d = 3$, we eliminate multiples of 3 (except 3), after these operations the list is: $[2, 3, 5, 7, 11, 13, 17, 19, 23, 25, 29]$.
- With $d = 4$, we eliminate multiples of 4 (but there are no more).
- With $d = 5$ we eliminate multiples of 5 (here we just eliminate 25), the list becomes $[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]$.
- We continue (here nothing happens anymore).
- At the end, the list is $[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]$.

Lesson 6 (Plot a list)**.**
With the `matplotlib` module it is very easy to visualize the elements of a list of numbers.




```
import matplotlib.pyplot as plt

mylist1 = [3,5,9,8,0,3]
mylist2 = [4,7,7,2,8,9]

plt.plot(mylist1,color="red")
plt.plot(mylist2,color="blue")
plt.grid()
plt.show()
```

Explanations.

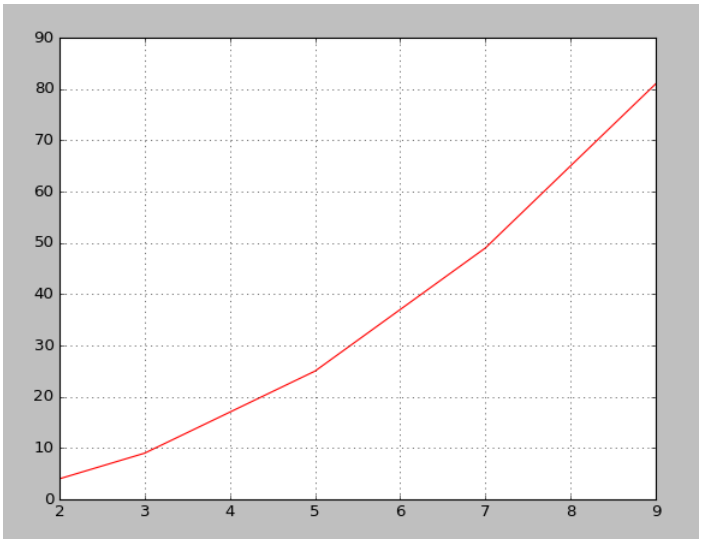
- The module is `matplotlib.pyplot` and is given the new simpler name of `plt`.
- Attention! The `matplotlib` module is not always installed by default with `Python`.
- `plt.plot(mylist)` traces the points of a list (in the form of (i, ℓ_i)) that are linked by segments.
- `plt.grid()` draws a grid.

- `plt.show()` displays everything.

To display points (x_i, y_i) you must provide the list of abscissa then the list of ordinates:

```
plt.plot(mylist_x,mylist_y,color="red")
```

Here is an example of a graph obtained by displaying coordinate points of the type (x, y) with $y = x^2$.

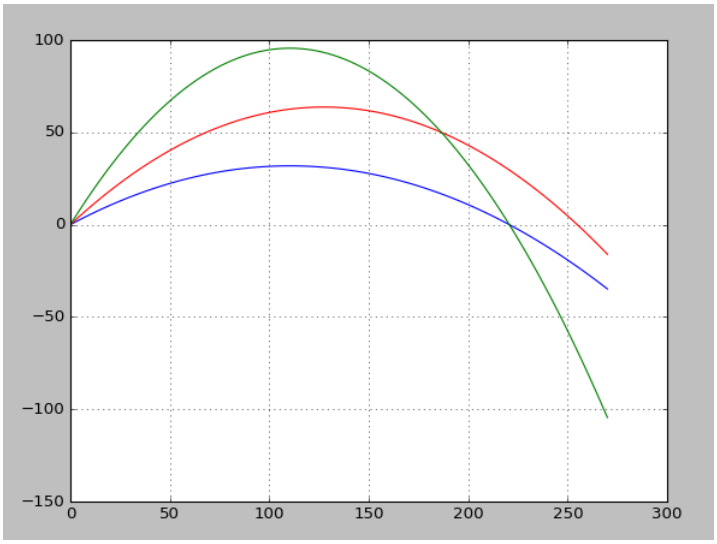



```
import matplotlib.pyplot as plt

mylist_x = [2, 3, 5, 7, 9]
mylist_y = [4, 9, 25, 49, 81]
plt.plot(mylist_x,mylist_y,color="red")
plt.grid()
plt.show()
```

Activity 5 (Ballistics)**.**

Goal: visualize the firing of a cannonball.

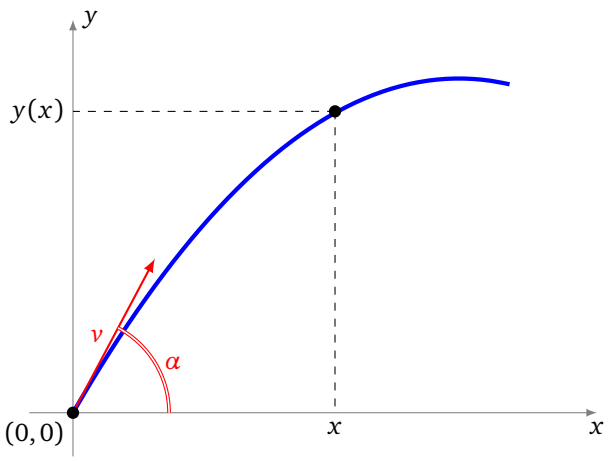


A cannonball has been fired from the point $(0,0)$. The trajectory equation is given by the formula:

$$y(x) = -\frac{1}{2} g \frac{1}{v^2 \cos^2(\alpha)} x^2 + \tan(\alpha) x$$

where

- α is the angle of the shot,
- v is the initial speed,
- g is the gravitational constant: we will take $g = 9.81$.



1. Program a function `parabolic_shot(x,v,alpha)` which returns the value $y(x)$ given by the formula.
   Hint. Be careful with the units for the angle α. If for example you choose that the unit for the angle is degrees, then to apply the formula with `Python` you must first convert the angles to radians :
   $$\alpha_{\text{radian}} = \frac{2\pi}{360} \alpha_{\text{degree}}$$
2. Program a function `list_trajectory(xmax,n,v,alpha)` that calculates the list of ordinates y of the $n+1$ points of the trajectory whose abscissa are regularly spaced between 0 and $x_{\max}$.
   Method. For i ranging from 0 to n:
   - calculate $x_i = i \cdot \frac{x_{\max}}{n}$,
   - calculate $y_i = y(x_i)$ using the trajectory formula,
   - add y_i to the list.
3. For $v = 50$, $x_{\max} = 270$ and $n = 100$, displays different trajectories according to the values of α. What angle α allows to reach the point $(x,0)$ at ground level as far away from the shooting point as possible?