Lists II

The lists are so useful that you have to know how to handle them in a simple and efficient way. That's the purpose of this chapter!

Lesson 1 (Manipulate lists efficiently)**.**

- **Slicing lists.**
  - You already know `mylist[a:b]` that returns the sublist of elements from the rank a to the rank $b-1$.
  - `mylist[a:]` returns the list of elements from rank a until the end.
  - `mylist[:b]` returns the list of elements from the beginning to the rank $b-1$.
  - `mylist[-1]` returns the last element, `mylist[-2]` returns the penultimate element, ...
  - **Exercise.**

	7	2	4	5	3	10	9	8	3
rank :	0	1	2	3	4	5	6	7	8

    With `mylist = [7,2,4,5,3,10,9,8,3]`, what do the following instructions return?
    - `mylist[3:5]`
    - `mylist[4:]`
    - `mylist[:6]`
    - `mylist[-1]`
- **Find the rank of an element.**
  - `mylist.index(element)` returns the first position at which the item was found. Example: with `mylist = [12, 30, 5, 9, 5, 21]`, `mylist.index(5)` returns 2.
  - If you just want to know if an item belongs to a list, then the statement :

    `element in mylist`

    returns `True` or `False`. Example: with `mylist = [12, 30, 5, 9, 5, 21]`, "`9 in mylist`" is true, while "`8 in mylist`" is false.
- **List comprehension.**
  A set can be defined by listing all its elements, for example $E = \{0, 2, 4, 6, 8, 10\}$. Another way is to say that the elements of the set must verify a certain property. For example, the same set E can be defined by :
  $$E = \{x \in \mathbb{N} \mid x \leqslant 10 \text{ and } x \text{ is even}\}.$$
  With `Python` there is such a way to define lists. It is an extremely powerful and efficient syntax. Let's look at some examples:
  - Let's start from a list, for example `mylist = [1,2,3,4,5,6,7,6,5,4,3,2,1]`.

  - The command `mylist_doubles = [ 2*x for x in mylist ]` returns a list that contains the double of each item of the `mylist` list. So this is the list `[2,4,6,8,...]`.
  - The command `mylist_squares = [ x**2 for x in mylist ]` returns the list of squares of the items in the initial list. So this is the list `[1,4,9,16,...]`.
  - The command `mylist_partial = [x for x in mylist if x > 2]` extracts the list composed only of elements strictly greater than 2. So this is the list `[3,4,5,6,7,6,5,4,3]`.

- **List of lists.**
  A list can contain other lists, for example:
  `mylist = [ ["Harry", "Hermione", "Ron"], [101,103] ]`
  contains two lists. We will be interested in lists that contain lists of integers, which we will call *arrays*. For example:
  `array = [ [2,14,5], [3,5,7], [15,19,4], [8,6,5] ]`
  Then `array[i]` returns the sublist of rank i, while `array[i][j]` returns the integer located at the rank j in the sublist of rank i. For example:
  - `array[0]` returns the list `[2,14,5]`,
  - `array[1]` returns the list `[3,5,7]`,
  - `array[0][0]` returns the integer 2,
  - `array[0][1]` returns the integer 14,
  - `array[2][1]` returns the integer 19.

Activity 1 (Lists comprehension).

Goal: practice list comprehension. In this activity the lists are lists of integers.

1. Program a function `multiplication(mylist,k)` that multiplies each item in the list by k. For example, `multiplication([1,2,3,4,5],2)` returns `[2,4,6,8,10]`.
2. Program a function `power(mylist,k)` that raises each element of the list to the power k. For example, `power([1,2,3,4,5],3)` returns `[1,8,27,64,125]`.
3. Program a function `addition(mylist1,mylist2)` that adds together the elements of two lists of the same length. For example, `addition([1,2,3],[4,5,6])` returns `[5,7,9]`.
   Hint. This one is an example of a task where lists comprehension is not used!
4. Program a function `non_zero(mylist)` that returns a list of all non-zero elements. For example, `non_zero([1,0,2,3,0,4,5,0])` returns `[1,2,3,4,5]`.
5. Program a function `even(mylist)` that returns a list of all even elements. For example, `even([1,0,2,3,0,4,5,0])` returns `[0,2,0,4,0]`.

Activity 2 (Reach a fixed amount).

Goal: try to reach the total of 100 *in a list of numbers.*

We consider a list of n integers between 1 and 99 (included). For example the list of 25 integers:
`[16,2,85,27,9,45,98,73,12,26,46,25,26,49,18,99,10,86,7,42]`
which was obtained at random by the command:
`mylist_20 = [randint(1,99) for i in range(20)]`
We are looking for different ways to find numbers from the list whose sum is exactly 100.

1. Program a function `sum_twoinarow_100(mylist)` that tests if there are two consecutive elements in the list whose sum is exactly 100. The function returns “True” or “False” (but it can also

display numbers and their position for verification). For the example given the function returns `False`.

2. Program a function `sum_two_100(mylist)` that tests if there are two items in the list, located at different positions, whose sum is equal to 100. For the example given the function returns `True` and can display the integers 2 and 98 (at ranks 1 and 6 of the list).
3. Program a function `sum_seq_100(mylist)` that tests if there are consecutive elements in the list whose sum is equal to 100. For the example given the function returns `True` and can display the sequence of integers 25, 26, 49 (at ranks 11, 12 and 13).
4. *(Optional.)* The larger the size of the list, the more likely it is to get the total of 100. For each of the three previous situations, determines from which size n of the list, the probability of obtaining a sum of 100 is greater than 1/2.
   Hints. For each case, you get an estimate of this integer n, by writing a function `proba(n,N)` that performs a large number N of random draws of lists having n items (with for example $N = 10\,000$). The probability is approximated by the number of favorable cases (where the function returns true) divided by the total number of cases (here N).

Activity 3 (Arrays).

Goal: working with lists of lists.

In this activity we work with arrays of size $n \times n$ containing integers. The object `array` is therefore a list of n lists, each having n elements.
For example (with $n = 3$):

```
array = [ [1,2,3], [4,5,6], [7,8,9] ]
```

represents the array:

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}$$

1. Write a function `sum_diagonal(array)` that calculates the sum of the elements located on the main diagonal of an array. The main diagonal of the example given is 1, 5, 9, so the sum is 15.
2. Write a function `sum_antidiagonal(array)` that calculates the sum of the elements located on the other diagonal. The anti-diagonal of the example given is composed of 3, 5, 7, the sum is still 15.
3. Write a function `sum_all(array)` that calculates the total sum of all elements. For the example the total sum is 45.
4. Write a function `print_array(array)` that displays an array properly on the screen. You can use the command:

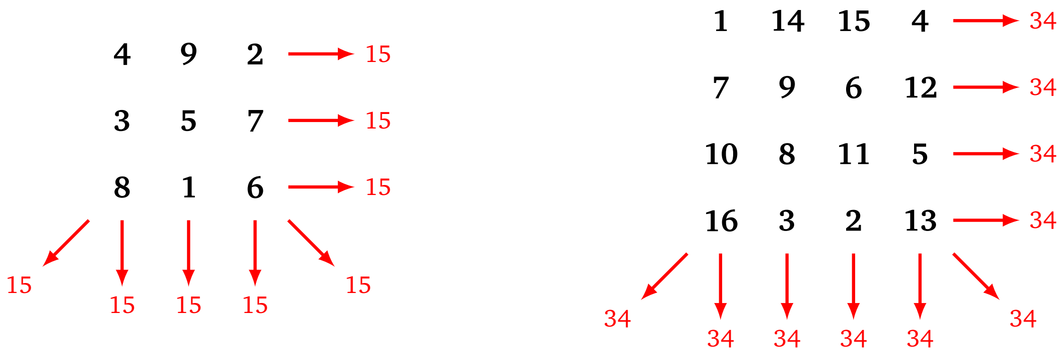
   ```
   print('{:>3d}'.format(array[i][j]), end="")
   ```

   Explanations.
   - The command `print(string,end="")` allows you to display a string of characters without going to the next line.
   - The command `'{:>3d}'.format(k)` displays the integer k on three characters (even if there is only one digit to display).

Activity 4 (Magic Squares).

Goal: build magic squares as big as you want! You must first have done the previous activity.

A ***magic square*** is a square array of size $n \times n$ that contains all integers from 1 to n^2 and satisfies that: the sum of each row, the sum of each column, the sum of the main diagonal and the sum of the anti-diagonal all have the same value.
Here is an example of a magic square with a size of 3×3 and one of size 4×4.



For a magic square of size $n \times n$, the value of the sum is:

$$S_n = \frac{n(n^2+1)}{2}.$$

1. **Examples.** Define an array for each of the above examples 3×3 and 4×4 and display them on the screen (use the previous activity).
2. **To be or not to be.** Define a `is_magic_square(square)` function that tests whether a given array is (or not) a magic square (use the previous activity for diagonals).
3. **Random squares.** *(Optional.)* Randomly generate squares containing integers from 1 to n^2 using a function `random_square(n)`. Experimentally verify that it is rare to obtain a magic square in this way!
   Hints. For a list `mylist`, the command `shuffle(mylist)` (from the `random` module) randomly mixes the list (the list is modified in place).

   The purpose of the remaining questions is to create large magic squares.
4. **Addition.** Define a function `addition_square(square,k)` which adds an integer k to all the elements of the array. With the example of the square 3×3, the command `addition_square(square,-1)` subtracts 1 from all the elements and returns an array that would look like this:

3	8	1
2	4	6
7	0	5

   Hints. To define a new square, start by filling it with 0:
   ```
   new_square = [[0 for j in range(n)] for i in range(n)]
   ```
   then fill it with the correct values by commands of the type:
   ```
   new_square[i][j] = ...
   ```
5. **Multiplication.** Define a function `multiplication_square(square,k)` which multiplies all the elements of the array by k. With the example of the square 3×3, the command `multiplication_square(square,2)` multiplies all the elements by 2 and thus returns an array that would be displayed as follows:

8	18	4
6	10	14
16	2	12

6. **Homothety.** Define a function `homothety_square(square,k)` which enlarges the array by a factor of k as shown in the examples below. Here is an example of the 3×3 square with a homothety ratio of $k = 3$.

4	9	2
3	5	7
8	1	6

$\longrightarrow$

4	4	4	9	9	9	2	2	2
4	4	4	9	9	9	2	2	2
4	4	4	9	9	9	2	2	2
3	3	3	5	5	5	7	7	7
3	3	3	5	5	5	7	7	7
3	3	3	5	5	5	7	7	7
8	8	8	1	1	1	6	6	6
8	8	8	1	1	1	6	6	6
8	8	8	1	1	1	6	6	6

Here is an example of a square 4×4 with a homothety ratio of $k = 2$.

1	14	15	4
7	9	6	12
10	8	11	5
16	3	2	13

$\longrightarrow$

1	1	14	14	15	15	4	4
1	1	14	14	15	15	4	4
7	7	9	9	6	6	12	12
7	7	9	9	6	6	12	12
10	10	8	8	11	11	5	5
10	10	8	8	11	11	5	5
16	16	3	3	2	2	13	13
16	16	3	3	2	2	13	13

7. **Block addition.** Define a `block_addition_square(big_square,small_square)` function that adds a small array of size $n \times n$ to the large array of size $nm \times nm$ per block as shown in the example below with $n = 2$ and $m = 3$ (hence $nm = 6$). The small square 2×2 on the left is added to the large square in the center to give the result on the right. For this addition the large square is divided into 9 blocks, there is a total of 36 additions.

1	2
3	4

4	4	9	9	2	2
4	4	9	9	2	2
3	3	5	5	7	7
3	3	5	5	7	7
8	8	1	1	6	6
8	8	1	1	6	6

$\longrightarrow$

5	6	10	11	3	4
7	8	12	13	5	6
4	5	6	7	8	9
6	7	8	9	10	11
9	10	2	3	7	8
11	12	4	5	9	10

8. **Products of magic squares.** Define a function `product_squares(square1,square2)` which from two magic squares, calculates a large magic square called the product of the two squares. The algorithm is as follows:

Algorithm.

- - Inputs: a magic square C_1 of size $n \times n$ and a magic square C_2 of size $m \times m$.
  - Output: a magic square C of size $(nm) \times (nm)$.

  Set the square C_{3a} by subtracting 1 from all elements of C_2. (Use the command `addition_square(square2,-1)`.)
- Define the square C_{3b} as the homothety of the square C_{3a} of ratio n. (Use the command `homothety(square3a,n)`.)
- Define the square C_{3c} by multiplying all the terms of the square C_{3b} by n^2. (Use the command `multiplication_square(square3b,n**2)`.)
- Define the square C_{3d} by adding the square C_1 to the square C_{3c} per block. (Use the command `block_addition_square(square3c,square1)`.)
- Return the square C_{3d}.

- Implement this algorithm.
- Test it on examples, checking that the square obtained is indeed a magic square.
- Build a magic square of size 36×36.
- Also checks that the order of the product is important: $C_1 \times C_2$ is not the same square as $C_2 \times C_1$.