Introducing Dynamic Modeing in R (dynr)

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Today's Tutorial at a Glance: What to Expect from It?

- I will provide a brief coverage of the following topics:
  1. What this package does
  2. Coding examples in dynr - growth curve model, vector autoregressive model, other demos

- What this tutorial isn't:
  - A comprehensive workshop on dynr, dynamical systems models, or state-space models

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- What this tutorial isn't:
  - A comprehensive workshop on dynr, dynamical systems models, or state-space models
- BUT I will cover the essential survival basics to (hopefully) interest you enough to get started on these.

How can I get started?

- *dynr* can be downloaded from CRAN - check out the installation instructions for users under "Vignettes"
- Check out the file: dynrInstallation/InstructionsAccessdynrServer.docx on the R bootcamp web page
- Other *dynr* demos and tutorials are available on the QuantDev website → Resources → dynr-package-linear-and-nonlinear-dynamic-modeling-r and (coming soon!) www.dynr.ssri.psu.edu

Links among the Different Modeling Approaches

	Discrete-time dynamical systems	Continuous-time dynamical systems
Deterministic	* Ordinary difference eqn models * Latent difference score models (McArdle & Hamagami, 2001) * Equally spaced growth curve models * Chaotic models of population density (e.g., logistic growth model)	* Ordinary differential eqn model * Differential structural eqn model (Boker, Neale, & Rausch, 2008) * Damped/coupled oscillators model (Boker & Graham, 1998) * Continuous-time growth curve model * Chaotic models (e.g., Lorenz eqs)
Stochastic	* State-space models * Time series models * Dynamic factor analysis models (Molenaar, 1985; Nesselroade et al., 2001) Nesselroade et al., 2001) * Longitudinal mediation models (Cole & Maxwell, 2003) * Nonlinear state-space models	* Exact discrete time models * Ornstein Uhlenbeck model Oravecz et al. 2009 * Stochastic damped oscillator model (Oud, 2007) * Stochastic catastrophe models (Cobb & Zacks, 1985) * Nonlinear stochastic differential eqs

Note. The examples listed are non-exhaustive.

Why dynr?

Why make a new package for this?

- OpenMx (Neale et al., 2015): linear only
- ctsem (Driver, Oud, & Voelkle, 2017): linear, continuous time only
- MATLAB (single subject)
- MKFM6 (Dolan, 2002): (linear only)
- dlm (Petris, 2010; Petris, Petrone, & Campagnoli, 2009): (slow)
- SsfPack (Koopman, Shephard, & Doornik, 1999): no regime-switching
- MPlus 8 (Asparouhov, Hamaker, & Muthn, 2018): linear, Bayesian only
- Structural equation modeling (SEM) programs such as Lavaan: targeted estimation functions for intensive longitudinal data (large T, $n \geq 1$; can handle longitudinal panel data (small T, large n) too

dynr preparation

- Gather data with `dynr.data()`
- Prepare *recipes* with
  - `prep.measurement()`
  - `prep.*Dynamics()`
  - `prep.initial()`
  - `prep.noise()`
  - `prep.regimes()` (optional)
- *Mix* recipes and data into a model with `dynr.model()`
- *Cook* model with `dynr.cook()`
- *Serve* results with
  - `summary()`
  - `plot()`
  - `dynr.ggplot()`
  - `plotFormula()`
  - `printex()`

What Are Dynamical Systems?

- **System** - a set of interrelated variables
- **Dynamical System**:
- A system in which the present state of the system depends on the previous states of the system.
- Scheinerman (1996). Invitation to dynamical systems, p. 1:

> "A dynamical system is a function with an attitude."
>
> "A dynamical system is doing the same thing over and over again."
>
> "A dynamical system is ~~always~~ mostly knowing what you are going to do next."
>
> "The difficulty is that virtually anything that evolves over time can be thought of as a dynamical system."

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Discrete-Time Dynamical Systems Models

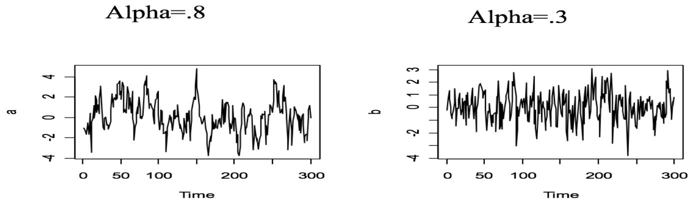
We express the regularity of the dynamics in terms of one-step-ahead difference equations of the form:

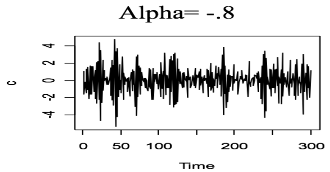
$$\text{Processes}_{i,t} = \text{Processes}_{i,t-1} + \text{Change functions } (.) \\ + \text{Random noise} \qquad (1)$$

i indexes person; t is a discrete-valued time index; "(.)" denotes all elements that affect the change processes of interest (e.g., the processes of interest at some previous time points, covariates, parameters)

- In discrete-time dynamical systems models, time is represented as integers.
- Data are usually equally spaced.
- $\text{Processes}_{i,t}$ can be a vector of observed or latent processes
- The change functions may be linear or nonlinear
- The change functions may be person-specific

Autoregressive Model of Order (AR(1) Model) $NE_{it} = \alpha NE_{i,t-1} + process\ noise_{it}$






```
a =
arima.sim(list(order=c(1,0,0),
ar=.8),n=300)
```

Higher positive AR(1) weight in NE has been likened to **higher inertia**, sluggishness, or "getting stuck" in extreme affective states (Hamaker, Asparouhov, Brose, Schmiedek, & Muthén, 2017, under review; Koval, Kuppens, Allen, & Sheeber, 2012; Kuppens, Allen, & Sheeber, 2010; Suls, Green, & Hillis, 1998).

State-Space Models

- State-space models can be regarded as a modeling framework that composed of:
  1. a measurement model linking some observed (manifest) variables to some unobserved (latent) variables, and
  2. a dynamic model describing the evolution of the latent variables over time.
- The dynamic model in a state-space model takes on the form of the one-step-ahead equation shown in (1).
- Virtually all linear, discrete-time dynamic models can be expressed in state-space form.

ASIDE I: Tutorials on Essential Matrix Operations to Understand a Model Presented in Matrix Form

- Overall Review and Practice
  https://www.khanacademy.org/math/precalculus/precalc-matrices
- Basic Concept (4mins)
  https://www.opened.com/video/introduction-to-the-matrix-khan-academy/181810
- Basic Operations (5mins)
  https://www.opened.com/video/matrices-basic-matrix-operations-add-subtract-multiply-by/116163
- Multiplication (10mins)
  https://www.opened.com/video/linear-algebra-matrix-multiplication/336137
- Matrix equation (10mins)
  https://www.opened.com/video/ex-1-solve-a-system-of-two-equations-using-a-matrix-equation/891337

Linear State-Space Models: Measurement Model

$$\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}. \tag{2}$$

- $\boldsymbol{y}_{it} = p_k \times 1$ vector of manifest variables
- $\boldsymbol{\eta}_{it} = w_k \times 1$ vector of latent variables
- $\boldsymbol{\Lambda}_k = p_k \times w_k$ factor loading matrix,
- $\boldsymbol{\tau}_k = p_k \times 1$ vector of intercepts, and
- $\boldsymbol{\epsilon}_{it} = p_k \times 1$ vector of measurement errors
- The subscript k is used to indicate elements that may be regime-specific
- Vectors of fixed exogenous variables may be added to the measurement model (not shown here), but available in the R package *dynr*, which will be introduced in a separate tutorial.

Linear State-Space Models: Dynamic Model

$$\eta_{it} = \nu_k + B_k \eta_{i,t-1} + \zeta_{it}, \qquad (3)$$

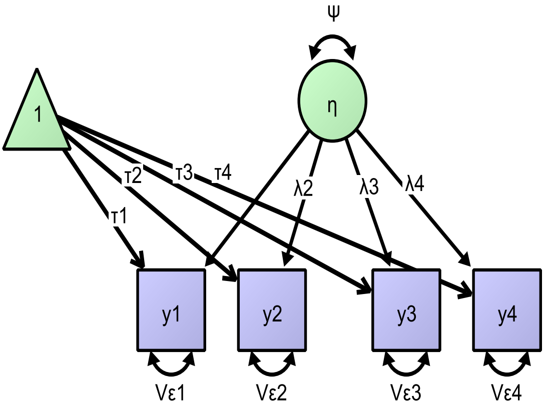
- $\nu_k = w_k \times 1$ vector of intercepts
- $B_k = w_k \times w_k$ regression matrix relating latent variables to each other
- $\zeta_{it} = w_k \times 1$ vector of residuals or dynamic noise
- Vectors of fixed exogenous variables may be added to the dynamic model (not shown here), but available in the R package *dynr*.

Linear State-Space Models: Noise and Initial Condition Structures

$$
\begin{array}{rcll}
\boldsymbol{\eta}_{i1|0} & \sim & N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0) & \text{Initial condition} \\
\boldsymbol{\epsilon}_{it} & \sim & N(\mathbf{0}, \boldsymbol{\Theta}_k) & \text{Measurement errors} \\
\boldsymbol{\zeta}_{it} & \sim & N(\mathbf{0}, \boldsymbol{\Psi}_k) & \text{Dynamic noise/uncertainties}
\end{array}
$$

ASIDE II: Introduction to Path Diagram Notations

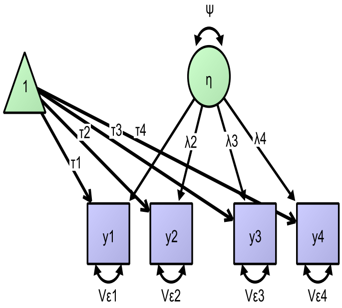
What do these squares, circles, triangle and paths mean?



Examples

- Confirmatory factor analysis (CFA) model
- Linear growth curve model
- Hands-on session: try accessing dynr on the TLT server, Linying Ji and Yanling Li will go over vector autoregressive (VAR) example

Confirmatory Factor Analysis Model



Measurement model

$$\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$$

$$\begin{bmatrix} y_{1it} \\ y_{2it} \\ y_{3it} \\ y_{4it} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} \begin{bmatrix} \eta_{it} \end{bmatrix} + \begin{bmatrix} \epsilon_{1it} \\ \epsilon_{2it} \\ \epsilon_{3it} \\ \epsilon_{4it} \end{bmatrix}$$

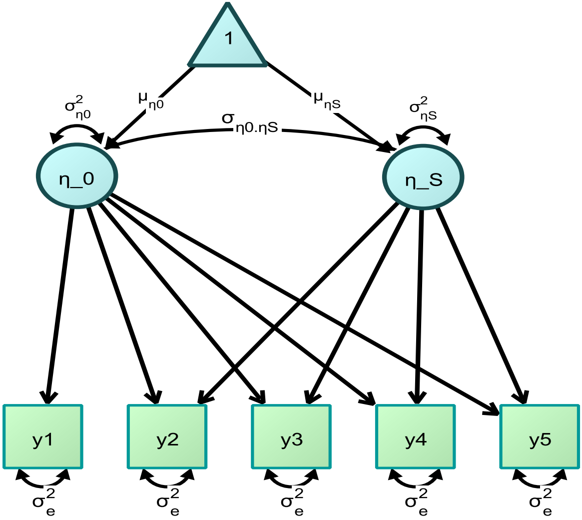
Dynamic model

$$\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$$

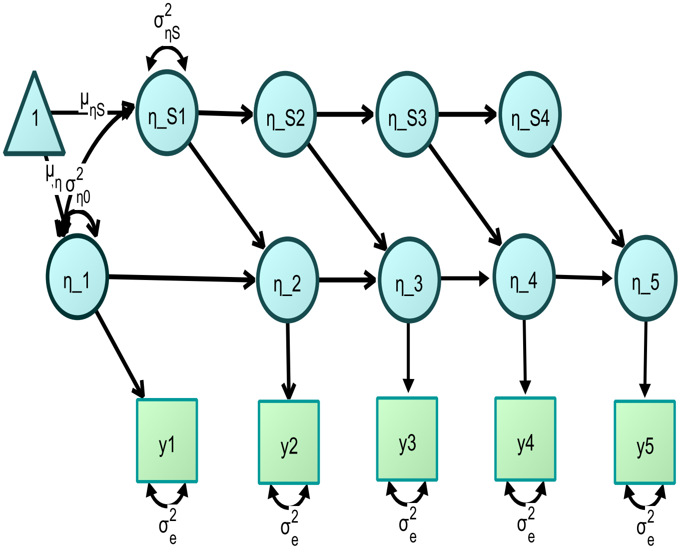
$$\begin{bmatrix} \eta_{it} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \eta_{i,t-1} \end{bmatrix} + \begin{bmatrix} \zeta_{it} \end{bmatrix}$$

Check out CFA.r

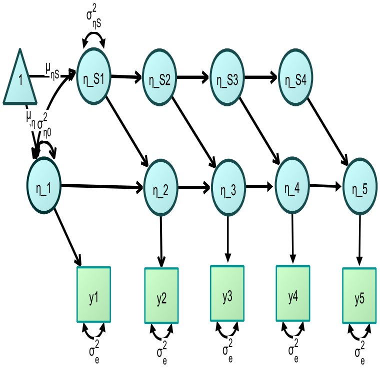
Growth Curve Model as a SEM



Growth Curve Model in One Possible State-Space Form



(1) Growth Curve Model: State-Space Measurement Model



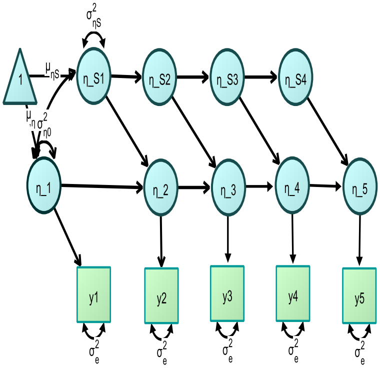
Measurement model (deterministic portion)

$$E(\boldsymbol{y}_{it}|\boldsymbol{\eta}_{it}) = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it}$$

$$y_{it} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{it} \\ \eta_{Sit} \end{bmatrix}$$

Specified via *prep.measurement* in *dynr*

(2) Growth Curve Model: State-Space Dynamic Model



Dynamic model (deterministic)

$$E(\boldsymbol{\eta}_{it}|\boldsymbol{\eta}_{i,t-1}) = \boldsymbol{\nu}_k + \boldsymbol{B}_k\boldsymbol{\eta}_{i,t-1}$$

In matrix form: prep.matrixDynamics()

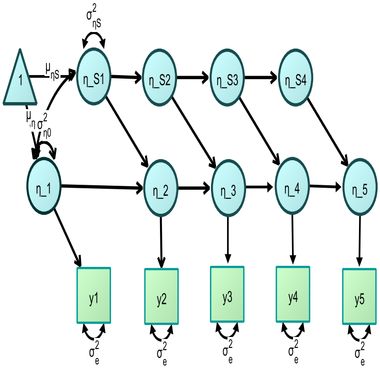
$$\begin{bmatrix} \eta_{it} \\ \eta_{Sit} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_{i,t-1} \\ \eta_{Si,t-1} \end{bmatrix}$$

As formulas: prep.formulaDynamics()

$$\eta_{it} \sim \eta_{i,t-1} + \eta_{Si,t-1}$$

$$\eta_{Sit} \sim \eta_{Si,t-1}$$

(3) Growth Curve Model: State-Space Initial Condition Model



Initial condition (IC)

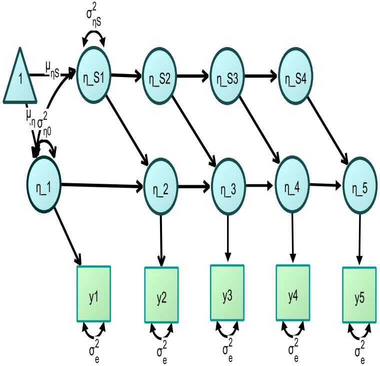
$\boldsymbol{\eta}_{i1|0} \sim N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$

Specified via *prep.initial* in *dynr*):

$$\begin{bmatrix} \eta_{i1} \\ \eta_{Si1} \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_{\eta 0} \\ \mu_{\eta S} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\eta 0} & \\ \sigma_{\eta 0.\eta S} & \sigma^2_{\eta S} \end{bmatrix} \right)$$

Fixed effects of intercept and slope IC means; var-cov parameters for between-person random effects in IC covariance matrix, Σ_0

(4) Growth Curve Model: State-Space Measurement/Process Noise Structure



Noise structure

$$\epsilon_{it} \sim N(\mathbf{0}, \boldsymbol{\Theta}_k)$$
$$\zeta_{it} \sim N(\mathbf{0}, \boldsymbol{\Psi}_k)$$

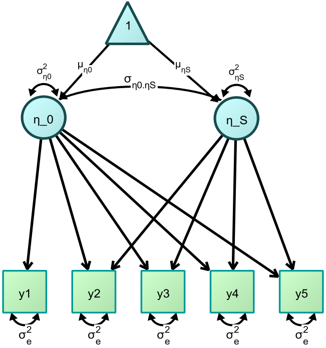
Specified via *prep.noise* in *dynr*):

- $\boldsymbol{\Psi}_k$ is a matrix of zeros
- $\boldsymbol{\Theta}_k$ consists of σ_e^2

No stochastic process noises in standard growth curve models

Check out GCM_demo.html

Growth Curve Model in Another State-Space Form



Measurement model

$$\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$$

$$y_{it} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 1 & Time_{it} \end{bmatrix} \begin{bmatrix} \eta_{it} \\ \eta_{Sit} \end{bmatrix} + \epsilon_{it}$$

Linear trend as time-varying factor loadings

Dynamic model

$$\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$$

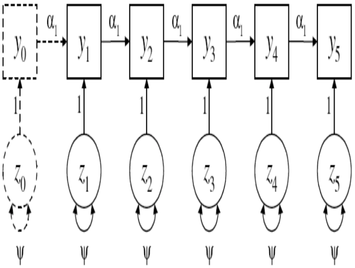
$$\begin{bmatrix} \eta_{it} \\ \eta_{Sit} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_{i,t-1} \\ \eta_{Si,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Examples

Related Variations/Extensions

Autoregressive Model of Order 1 (AR(1) Model)

Figure: From Browne and Nesselroade (2005)



Assumes that all systematic trends have been removed. The process of interest fluctuates around a zero intercept (set point).

Measurement model

- $\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$

$$y_{it} = [0] + [1]\,[\eta_{it}]$$

Dynamic model

- $\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$

$$\eta_{it} = \alpha_1 \eta_{i,t-1} + z_{it}$$

Autoregressive Model of Order 1 (AR(1) Model) with Individual Differences in Set Point

Measurement model

$$\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$$

Within-person centering

$$y_{it} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \eta_{it} \\ \text{SetP}_{0it} \end{bmatrix}$$

- SetPt_{0i} is the person-specific latent mean or set-point (a latent variable)

Dynamic model

$$\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$$

$$\begin{aligned} \eta_{it} &= \alpha_1 \eta_{i,t-1} + z_{it} \\ \text{SetP}_{0it} &= \text{SetP}_{0i,t-1} \end{aligned}$$

Initial condition (IC)

$$\eta_{i1|0} \sim N(\mu_0, \Sigma_0)$$

Between-person centering

- Person-specific trait variables may be included here
- These variables may be (grand-mean) centered to ease interpretation

$$\begin{bmatrix} \eta_{i1} \\ \text{SetP}_{0i1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ B_0 + B_1 \text{trait}_i \end{bmatrix}, \begin{bmatrix} \sigma^2_{\eta 0} & \\ \sigma_{\eta 0.\mu_0} & \sigma^2_{\mu 0} \end{bmatrix} \right)$$

Autoregressive Model of Order 2 (AR(2) Model)

Measurement model

- $\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$

$$y_{it} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{it} \\ \eta_{i,t-1} \end{bmatrix}$$

Dynamic model

- $\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$

$$\begin{aligned} \eta_{it} &= \alpha_1 \eta_{i,t-1} + \alpha_2 \eta_{i,t-2} + z_{it} \\ \eta_{i,t-1} &= \eta_{i,t-1} \end{aligned}$$

$$\begin{bmatrix} \eta_{it} \\ \eta_{i,t-1} \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{i,t-1} \\ \eta_{i,t-2} \end{bmatrix} + \begin{bmatrix} z_{1,it} \\ 0 \end{bmatrix}$$

Vector Autoregressive Model of Order 1 (VAR(1) Model)

Measurement model

$$\boldsymbol{y}_{it} = \boldsymbol{\tau}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \boldsymbol{\epsilon}_{it}$$

$$\begin{bmatrix} y_{1,it} \\ y_{2,it} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_{1,it} \\ \eta_{2,it} \end{bmatrix}$$

Dynamic model

$$\boldsymbol{\eta}_{it} = \boldsymbol{\nu}_k + \boldsymbol{B}_k \boldsymbol{\eta}_{i,t-1} + \boldsymbol{\zeta}_{it}$$

$$\begin{aligned} \eta_{it} &= \alpha_1 \eta_{i,t-1} + \alpha_2 \eta_{i,t-2} + z_{it} \\ \eta_{i,t-1} &= \eta_{i,t-1} \end{aligned}$$

$$\begin{bmatrix} \eta_{1,it} \\ \eta_{2it} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \eta_{1,i,t-1} \\ \eta_{2,i,t-1} \end{bmatrix} + \begin{bmatrix} z_{1,it} \\ z_{2,it} \end{bmatrix}$$

Check out VAR_demo.html

Hands-on Practice: Let's Start Using dynr!

- Connect to the Rstudio server by clicking on this link and log in using your Access ID
  https://lxclusterapps.tlt.psu.edu:8787/
  – To use this server from off-campus, check out:
  InstructionsAccessdynrServer.docx
- Choose R version. At the top-right corner, click the button of R-X.X.X, and choose R-3.5.3.
- Test dynr by typing at the console:
  demo('LinearSDE', package='dynr')
  – If there are error messages complaining regarding not enough space, go to https://www.work.psu.edu and increase the space limit by using the sidebar on the left.

Today's Tutorial at a Glance: What Have We Learned?

- We provided a brief overviewon:
  1. What are discrete-time dynamical systems?
  2. Linkage between state-space models and discrete-time dynamical systems models
  3. Quick introduction to (or refresher of) basic matrix algebra operations so you can understand equations in matrix form.
  4. Examples of state-space models
  5. Coding examples in dynr
  6. Other *dynr* demos and tutorials are available on the QuantDev website → Resources → dynr-package-linear-and-nonlinear-dynamic-modeling-r and (coming soon!) www.dynr.ssri.psu.edu
  7. Again, installation instructions can be found here: dynrInstallation/InstructionsAccessdynrServer.docx on the R bootcamp web page

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