Week 9: Hierarchical Models

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Packages

```
library(tidyverse)
library(lme4)
```

Read in data

Household level:

```
d <- read.table(url("http://www.stat.columbia.edu/~gelman/arm/examples/radon/srrs2.dat"), header=T, sep=
```

County level:

```
cty <- read.table(url("http://www.stat.columbia.edu/~gelman/arm/examples/radon/cty.dat"), header = T, se
cty <- cty %>% mutate(fips = 1000 * stfips + ctfips) %>% dplyr::select(fips, Uppm)
```

Join these together, filter to just be Minnesota and tidy up

```
d_mn <- d %>%
  filter(state=="MN") %>%
  mutate(fips = stfips * 1000 + cntyfips) %>%
  dplyr::select(fips, county, floor, activity) %>%
  left_join(cty) %>%
  mutate(log_uran = log(Uppm), log_activity = log(activity)) %>%
  mutate(county = str_trim(county)) %>%
  filter(county!= "") %>%
  filter(!is.na(log_activity), !is.infinite(log_activity))
```

Models using `lm`

- Fit with no pooling
- Fit intercept only
- Fit with floor as covariate

Models using `lmer`

Intercept only

```
mod_hier <- lmer(log_activity ~ (1 | county), data = d_mn)
mod_hier
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: log_activity ~ (1 | county)
##    Data: d_mn
## REML criterion at convergence: 2209.582
## Random effects:
##  Groups   Name        Std.Dev.
##  county   (Intercept) 0.3341
##  Residual             0.7685
## Number of obs: 923, groups:  county, 85
## Fixed Effects:
## (Intercept)
##       1.343
```

With floor

```
mod_hier_floor <- lmer(log_activity ~ floor + (1 | county), data = d_mn)
mod_hier_floor
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: log_activity ~ floor + (1 | county)
##    Data: d_mn
## REML criterion at convergence: 2133.28
## Random effects:
##  Groups   Name        Std.Dev.
##  county   (Intercept) 0.3599
##  Residual             0.7318
## Number of obs: 923, groups:  county, 85
## Fixed Effects:
## (Intercept)        floor
##      1.4783      -0.6273
```

With floor and uranium

```
mod_hier_group <- lmer(log_activity ~ floor + log_uran + (1 | county), data = d_mn)
mod_hier_group
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: log_activity ~ floor + log_uran + (1 | county)
##    Data: d_mn
## REML criterion at convergence: 2095.187
## Random effects:
##  Groups   Name        Std.Dev.
##  county   (Intercept) 0.1878
##  Residual             0.7350
```

```
## Number of obs: 923, groups:  county, 85
## Fixed Effects:
## (Intercept)        floor     log_uran
##      1.4817      -0.5966       0.7547
```

Extracting stuff

Coefficients

```
coef(summary(mod_hier_group))
```

```
##               Estimate Std. Error   t value
## (Intercept)  1.4816705 0.03938412 37.621008
## floor       -0.5965881 0.06705173 -8.897430
## log_uran     0.7547399 0.09618975  7.846364
```

Note difference between `coef` and `ranef`

```
coef(mod_hier_group)
```

```
## $county
##                   (Intercept)      floor  log_uran
## AITKIN               1.450176 -0.5965881 0.7547399
## ANOKA                1.501368 -0.5965881 0.7547399
## BECKER               1.489865 -0.5965881 0.7547399
## BELTRAMI             1.622229 -0.5965881 0.7547399
## BENTON               1.487419 -0.5965881 0.7547399
## BIG STONE            1.438950 -0.5965881 0.7547399
## BLUE EARTH           1.628668 -0.5965881 0.7547399
## BROWN                1.530638 -0.5965881 0.7547399
## CARLTON              1.386773 -0.5965881 0.7547399
## CARVER               1.714548 -0.5965881 0.7547399
## CASS                 1.574838 -0.5965881 0.7547399
## CHIPPEWA             1.490372 -0.5965881 0.7547399
## CHISAGO              1.513207 -0.5965881 0.7547399
## CLAY                 1.582842 -0.5965881 0.7547399
## CLEARWATER           1.448430 -0.5965881 0.7547399
## COOK                 1.431343 -0.5965881 0.7547399
## COTTONWOOD           1.548260 -0.5965881 0.7547399
## CROW WING            1.519865 -0.5965881 0.7547399
## DAKOTA               1.374670 -0.5965881 0.7547399
## DODGE                1.501247 -0.5965881 0.7547399
## DOUGLAS              1.526470 -0.5965881 0.7547399
## FARIBAULT            1.175326 -0.5965881 0.7547399
## FILLMORE             1.428901 -0.5965881 0.7547399
## FREEBORN             1.638321 -0.5965881 0.7547399
## GOODHUE              1.626361 -0.5965881 0.7547399
## HENNEPIN             1.428237 -0.5965881 0.7547399
## HOUSTON              1.445169 -0.5965881 0.7547399
## HUBBARD              1.478691 -0.5965881 0.7547399
## ISANTI               1.504899 -0.5965881 0.7547399
```

```
## ITASCA              1.458571 -0.5965881 0.7547399
## JACKSON             1.556909 -0.5965881 0.7547399
## KANABEC             1.439195 -0.5965881 0.7547399
## KANDIYOHI           1.584684 -0.5965881 0.7547399
## KITTSON             1.486916 -0.5965881 0.7547399
## KOOCHICHING         1.460416 -0.5965881 0.7547399
## LAC QUI PARLE       1.726179 -0.5965881 0.7547399
## LAKE                1.270165 -0.5965881 0.7547399
## LAKE OF THE WOODS   1.655349 -0.5965881 0.7547399
## LE SUEUR            1.505017 -0.5965881 0.7547399
## LINCOLN             1.578466 -0.5965881 0.7547399
## LYON                1.540715 -0.5965881 0.7547399
## MAHNOMEN            1.467351 -0.5965881 0.7547399
## MARSHALL            1.496965 -0.5965881 0.7547399
## MARTIN              1.298596 -0.5965881 0.7547399
## MCLEOD              1.445012 -0.5965881 0.7547399
## MEEKER              1.411486 -0.5965881 0.7547399
## MILLE LACS          1.423776 -0.5965881 0.7547399
## MORRISON            1.379149 -0.5965881 0.7547399
## MOWER               1.483706 -0.5965881 0.7547399
## MURRAY              1.525224 -0.5965881 0.7547399
## NICOLLET            1.584430 -0.5965881 0.7547399
## NOBLES              1.504606 -0.5965881 0.7547399
## NORMAN              1.404291 -0.5965881 0.7547399
## OLMSTED             1.256795 -0.5965881 0.7547399
## OTTER TAIL          1.565523 -0.5965881 0.7547399
## PENNINGTON          1.414943 -0.5965881 0.7547399
## PINE                1.308005 -0.5965881 0.7547399
## PIPESTONE           1.478987 -0.5965881 0.7547399
## POLK                1.476967 -0.5965881 0.7547399
## POPE                1.433791 -0.5965881 0.7547399
## RAMSEY              1.468791 -0.5965881 0.7547399
## REDWOOD             1.530917 -0.5965881 0.7547399
## RENVILLE            1.458814 -0.5965881 0.7547399
## RICE                1.569091 -0.5965881 0.7547399
## ROCK                1.414546 -0.5965881 0.7547399
## ROSEAU              1.612189 -0.5965881 0.7547399
## SCOTT               1.589880 -0.5965881 0.7547399
## SHERBURNE           1.524279 -0.5965881 0.7547399
## SIBLEY              1.395047 -0.5965881 0.7547399
## ST LOUIS            1.240613 -0.5965881 0.7547399
## STEARNS             1.421202 -0.5965881 0.7547399
## STEELE              1.440031 -0.5965881 0.7547399
## STEVENS             1.476446 -0.5965881 0.7547399
## SWIFT               1.329828 -0.5965881 0.7547399
## TODD                1.520344 -0.5965881 0.7547399
## TRAVERSE            1.506571 -0.5965881 0.7547399
## WABASHA             1.547246 -0.5965881 0.7547399
## WADENA              1.546073 -0.5965881 0.7547399
## WASECA              1.262529 -0.5965881 0.7547399
## WASHINGTON          1.440924 -0.5965881 0.7547399
## WATONWAN            1.646621 -0.5965881 0.7547399
## WILKIN              1.516623 -0.5965881 0.7547399
## WINONA              1.372873 -0.5965881 0.7547399
```

```
## WRIGHT               1.583644 -0.5965881 0.7547399
## YELLOW MEDICINE      1.416600 -0.5965881 0.7547399
##
## attr(,"class")
## [1] "coef.mer"
```

```
ranef(mod_hier_group)
```

```
## $county
##                    (Intercept)
## AITKIN            -0.031494298
## ANOKA              0.019697692
## BECKER             0.008194784
## BELTRAMI           0.140558937
## BENTON             0.005748843
## BIG STONE         -0.042720790
## BLUE EARTH         0.146997353
## BROWN              0.048967245
## CARLTON           -0.094897178
## CARVER             0.232877850
## CASS               0.093167824
## CHIPPEWA           0.008701884
## CHISAGO            0.031536892
## CLAY               0.101171057
## CLEARWATER        -0.033240326
## COOK              -0.050327279
## COTTONWOOD         0.066589754
## CROW WING          0.038194440
## DAKOTA            -0.107000484
## DODGE              0.019576281
## DOUGLAS            0.044799624
## FARIBAULT         -0.306344103
## FILLMORE          -0.052769283
## FREEBORN           0.156650565
## GOODHUE            0.144690810
## HENNEPIN          -0.053433824
## HOUSTON           -0.036501665
## HUBBARD           -0.002979588
## ISANTI             0.023228684
## ITASCA            -0.023099524
## JACKSON            0.075238231
## KANABEC           -0.042475703
## KANDIYOHI          0.103013269
## KITTSON            0.005245973
## KOOCHICHING       -0.021254038
## LAC QUI PARLE      0.244508650
## LAKE              -0.211505334
## LAKE OF THE WOODS  0.173678962
## LE SUEUR           0.023346080
## LINCOLN            0.096795050
## LYON               0.059044474
## MAHNOMEN          -0.014319727
## MARSHALL           0.015294138
## MARTIN            -0.183074447
```

```
## MCLEOD            -0.036658151
## MEEKER            -0.070184298
## MILLE LACS        -0.057894692
## MORRISON          -0.102521734
## MOWER              0.002035323
## MURRAY             0.043553614
## NICOLLET           0.102759290
## NOBLES             0.022935726
## NORMAN            -0.077379030
## OLMSTED           -0.224875774
## OTTER TAIL         0.083852459
## PENNINGTON        -0.066727272
## PINE              -0.173665844
## PIPESTONE         -0.002683196
## POLK              -0.004703310
## POPE              -0.047879251
## RAMSEY            -0.012879242
## REDWOOD            0.049246613
## RENVILLE          -0.022856542
## RICE               0.087420952
## ROCK              -0.067124263
## ROSEAU             0.130518271
## SCOTT              0.108209217
## SHERBURNE          0.042608528
## SIBLEY            -0.086623969
## ST LOUIS          -0.241057438
## STEARNS           -0.060468504
## STEELE            -0.041639085
## STEVENS           -0.005224416
## SWIFT             -0.151842775
## TODD               0.038674016
## TRAVERSE           0.024900154
## WABASHA            0.065575587
## WADENA             0.064402202
## WASECA            -0.219141880
## WASHINGTON        -0.040746907
## WATONWAN           0.164950172
## WILKIN             0.034952768
## WINONA            -0.108797774
## WRIGHT             0.101973493
## YELLOW MEDICINE   -0.065070791
##
## with conditional variances for "county"
```

Pull out estimates for alpha's

```
res <- coef(mod_hier_group)[[1]]["(Intercept)"]
```

Pull out standard errors for alpha's and join them to the alphas (horrible)

```
#number of counties
n_counties <- length(unique(d_mn$county))
```

```
ses <- attr(ranef(mod_hier_group)[[1]], "postVar")[,,1:n_counties]
df <- res %>% bind_cols(ses)
colnames(df) <- c("alpha", "se")
df$county <- rownames(df)
rownames(df) <- NULL
```

Pull out estimates for gamma0 and gamma 1

```
gamma0 <- coef(summary(mod_hier_group))[1,1]
gamma1 <- coef(summary(mod_hier_group))[3,1]
```

Plotting stuff

Plot the alphas for each county (county intercepts)

Plot expected log radon versus uranium level

Get uranium level for each county

```
urans <- d_mn %>%
  group_by(county, log_uran) %>%
  slice(1) %>%
  dplyr::select(county, log_uran)
```

Join to alpha estimates

```
df %>%
  left_join(urans) %>%
  filter(county!="") %>%
  mutate(y_hat =  log_uran*gamma1 + alpha)
```

```
##       alpha          se            county     log_uran     y_hat
## 1  1.450176 0.027970182           AITKIN -0.689047595 0.9301245
## 2  1.501368 0.008025134            ANOKA -0.847312860 0.8618674
## 3  1.489865 0.029497489           BECKER -0.113458774 1.4042334
## 4  1.622229 0.024209635         BELTRAMI -0.593352526 1.1744026
## 5  1.487419 0.027970182           BENTON -0.142890481 1.3795742
## 6  1.438950 0.029497489        BIG STONE  0.387056708 1.7310768
## 7  1.628668 0.018428405       BLUE EARTH  0.271613664 1.8336655
## 8  1.530638 0.027970182            BROWN  0.277578705 1.7401374
## 9  1.386773 0.021340445          CARLTON -0.332315488 1.1359615
## 10 1.714548 0.021340445           CARVER  0.095864572 1.7869011
## 11 1.574838 0.026593248             CASS -0.608219807 1.1157906
## 12 1.490372 0.027970182         CHIPPEWA  0.273684563 1.6969330
## 13 1.513207 0.025345523          CHISAGO -0.735320087 0.9582320
## 14 1.582842 0.018428405             CLAY  0.343781175 1.8423069
## 15 1.448430 0.027970182       CLEARWATER -0.059860414 1.4032511
## 16 1.431343 0.031201228             COOK -0.504995983 1.0502026
## 17 1.548260 0.029497489       COTTONWOOD  0.339560321 1.8045399
## 18 1.519865 0.019777809        CROW WING -0.633390700 1.0418197
```

```
## 19 1.374670 0.006897914           DAKOTA -0.024145162 1.3564467
## 20 1.501247 0.029497489            DODGE  0.263855460 1.7003890
## 21 1.526470 0.022218169          DOUGLAS  0.155712317 1.6439924
## 22 1.175326 0.025345523        FARIBAULT  0.295025047 1.3979935
## 23 1.428901 0.031201228          FILLMORE  0.414913663 1.7420531
## 24 1.638321 0.022218169          FREEBORN  0.224206986 1.8075390
## 25 1.626361 0.018428405           GOODHUE  0.196610646 1.7747512
## 26 1.428237 0.004489932          HENNEPIN -0.096520812 1.3553885
## 27 1.445169 0.025345523           HOUSTON  0.503529069 1.8252023
## 28 1.478691 0.026593248           HUBBARD -0.400596977 1.1763444
## 29 1.504899 0.029497489            ISANTI -0.751872233 0.9374312
## 30 1.458571 0.020529434            ITASCA -0.663347631 0.9579160
## 31 1.556909 0.026593248           JACKSON  0.309020285 1.7901386
## 32 1.439195 0.027970182           KANABEC -0.053386009 1.3989022
## 33 1.584684 0.027970182        KANDIYOHI  0.109732943 1.6675036
## 34 1.486916 0.029497489           KITTSON -0.007803367 1.4810269
## 35 1.460416 0.024209635       KOOCHICHING -0.881828921 0.7948650
## 36 1.726179 0.027970182     LAC QUI PARLE  0.311029879 1.9609258
## 37 1.270165 0.022218169              LAKE -0.691596384 0.7481898
## 38 1.655349 0.027970182 LAKE OF THE WOODS -0.681708848 1.1408366
## 39 1.505017 0.026593248          LE SUEUR  0.194447737 1.6517740
## 40 1.578466 0.027970182           LINCOLN  0.444903746 1.9142521
## 41 1.540715 0.023171191              LYON  0.394734406 1.8386367
## 42 1.467351 0.033113843          MAHNOMEN  0.149600343 1.5802601
## 43 1.496965 0.022218169          MARSHALL  0.013764829 1.5073535
## 44 1.298596 0.024209635            MARTIN  0.165861836 1.4237786
## 45 1.445012 0.019777809            MCLEOD  0.140422594 1.5509949
## 46 1.411486 0.026593248            MEEKER  0.023950874 1.4295629
## 47 1.423776 0.031201228        MILLE LACS -0.210059522 1.2652355
## 48 1.379149 0.022218169          MORRISON -0.093226652 1.3087869
## 49 1.483706 0.019079277             MOWER  0.260932471 1.6806419
## 50 1.525224 0.033113843            MURRAY  0.398849943 1.8262520
## 51 1.584430 0.027970182          NICOLLET  0.248046873 1.7716406
## 52 1.504606 0.029497489            NOBLES  0.405451775 1.8106168
## 53 1.404291 0.029497489            NORMAN  0.265221717 1.6044648
## 54 1.256795 0.014099483           OLMSTED  0.243150079 1.4403098
## 55 1.565523 0.023171191        OTTER TAIL -0.204730369 1.4110048
## 56 1.414943 0.029497489        PENNINGTON -0.074027668 1.3590716
## 57 1.308005 0.025345523              PINE -0.163292170 1.1847615
## 58 1.478987 0.027970182         PIPESTONE  0.478604039 1.8402088
## 59 1.476967 0.027970182              POLK  0.266111083 1.6778118
## 60 1.433791 0.031201228              POPE  0.281148274 1.6459850
## 61 1.468791 0.011417466            RAMSEY -0.418053511 1.1532696
## 62 1.530917 0.026593248           REDWOOD  0.366322259 1.8073951
## 63 1.458814 0.029497489          RENVILLE  0.380577977 1.7460513
## 64 1.569091 0.020529434              RICE  0.193146093 1.7148665
## 65 1.414546 0.031201228              ROCK  0.528024865 1.8130676
## 66 1.612189 0.018428405            ROSEAU -0.212045365 1.4521497
## 67 1.589880 0.019079277             SCOTT  0.063115634 1.6375156
## 68 1.524279 0.023171191         SHERBURNE -0.683436482 1.0084622
## 69 1.395047 0.027970182            SIBLEY  0.237212123 1.5740799
## 70 1.240613 0.004113815          ST LOUIS -0.474673717 0.8823579
## 71 1.421202 0.013399990           STEARNS  0.116395407 1.5090502
## 72 1.440031 0.021340445            STEELE  0.269805739 1.6436645
```

```
## 73 1.476446 0.031201228           STEVENS  0.470778329 1.8317612
## 74 1.329828 0.027970182             SWIFT  0.316028976 1.5683474
## 75 1.520344 0.029497489              TODD -0.046840067 1.4849924
## 76 1.506571 0.027970182          TRAVERSE  0.497594477 1.8821250
## 77 1.547246 0.024209635           WABASHA  0.150082416 1.6605192
## 78 1.546073 0.026593248             WADENA -0.672029732 1.0388650
## 79 1.262529 0.027970182             WASECA  0.212414197 1.4228461
## 80 1.440924 0.008810457         WASHINGTON -0.147484283 1.3296113
## 81 1.646621 0.029497489           WATONWAN  0.183237804 1.7849175
## 82 1.516623 0.033113843             WILKIN  0.236036084 1.6947691
## 83 1.372873 0.019079277             WINONA  0.463211867 1.7224772
## 84 1.583644 0.019079277             WRIGHT -0.090024275 1.5156991
## 85 1.416600 0.031201228   YELLOW MEDICINE  0.355286981 1.6847489
```

Questions

Using model with household and county covariates:

- What is expected value of log radon for household in Carlton with basement measure and log uranium = 0?
- What is expected value of log radon for household in Carlton with first floor measure and log uranium = 0?
- What is expected value of log radon for household in Carlton with basement measure and log uranium = 0.3?