The Binomial Distribution

An Introduction

Matt Johnson

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- Flip a fair coin 10 times. How many come up heads?
- Roll two dice 5 times. How many times will they add up to 7?

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Multiple choice question. Four possible answers (A-D).

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$$P(\text{correct}) = 0.25$$
$$P(\text{incorrect}) = 0.75$$

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$$P(2R) = 3 \cdot 0.25^2 \cdot 0.75 = 3/64 \approx 4.7\%$$

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$$P(k \text{ correct}) = \binom{n}{k} p^k (1-p)^{n-k}$$

The Binomial Distribution

$X \sim Bin(n, p)$. *For* $k \in 0 : n$,

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

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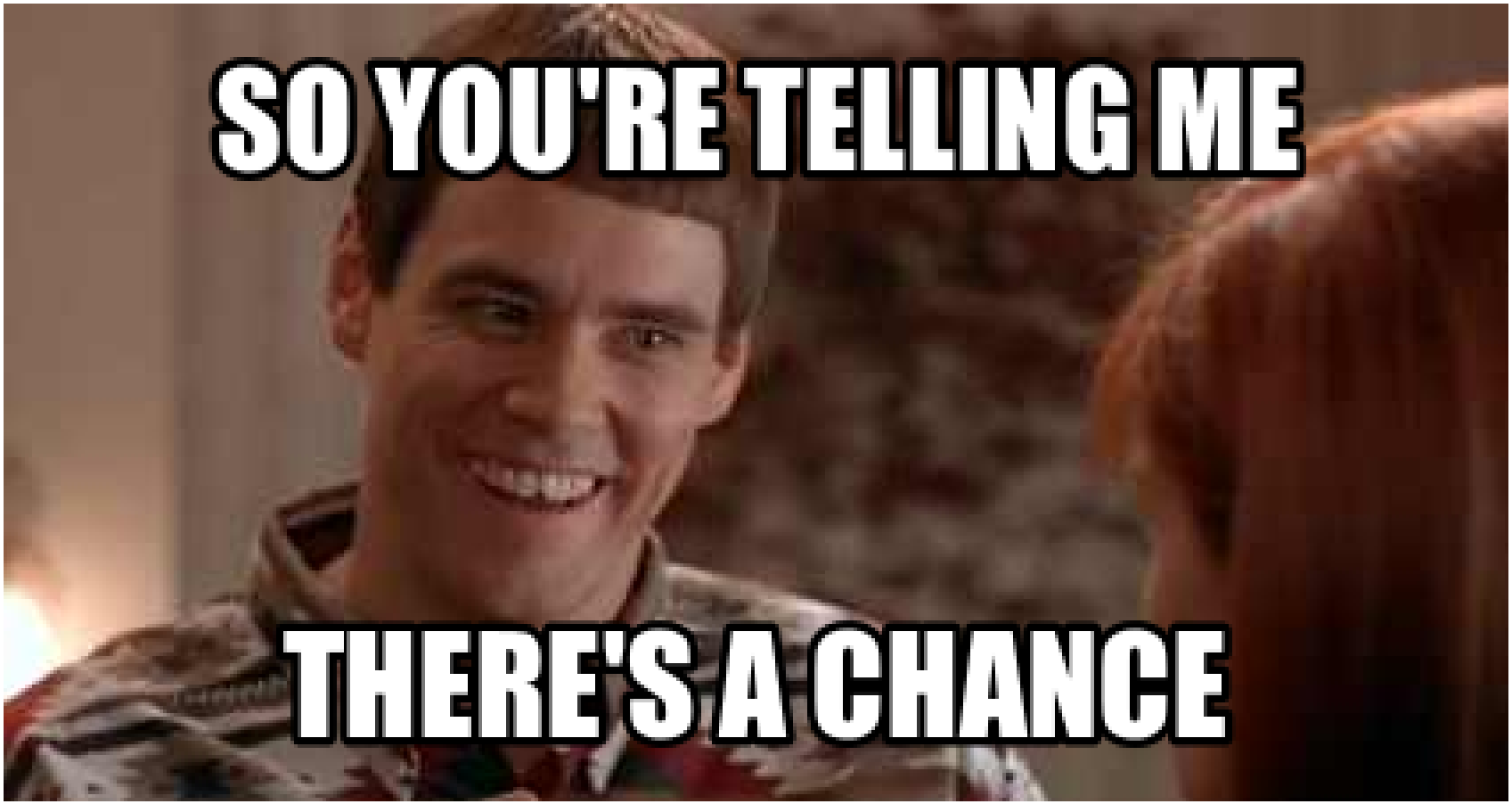
$$P(k \geq 7) \approx 1.97\%$$



Questions?