Bias-Variance Tradeoff

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Learning Objectives

1. Learn about the intuitions and definitions of bias, variance
2. Understand the concept of bias-variance decomposition
3. Interpret its relationships with model complexity

Graphical Illustration

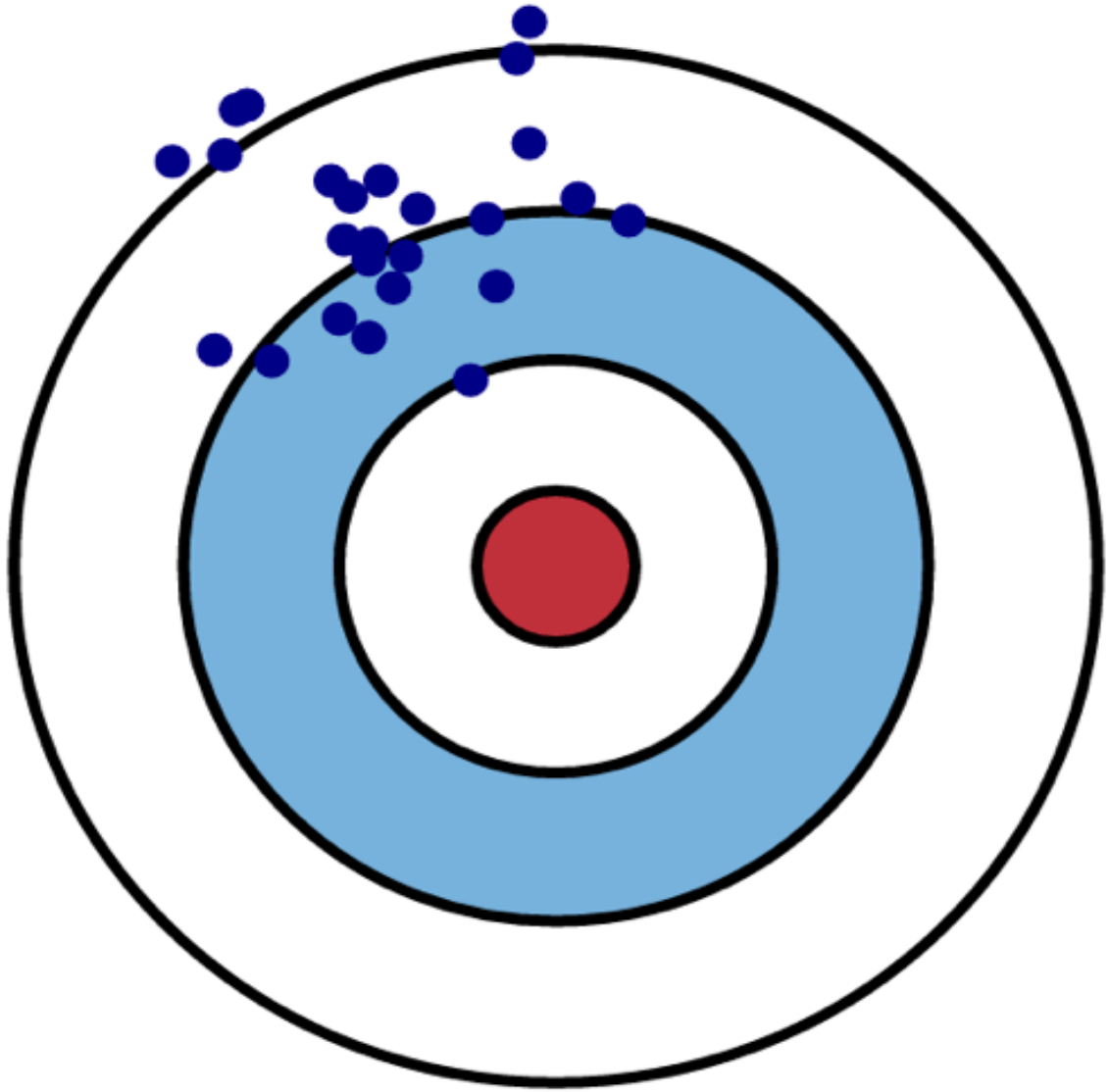


Figure 1 : Taking multiple shots in a dart game

Graphical Illustration

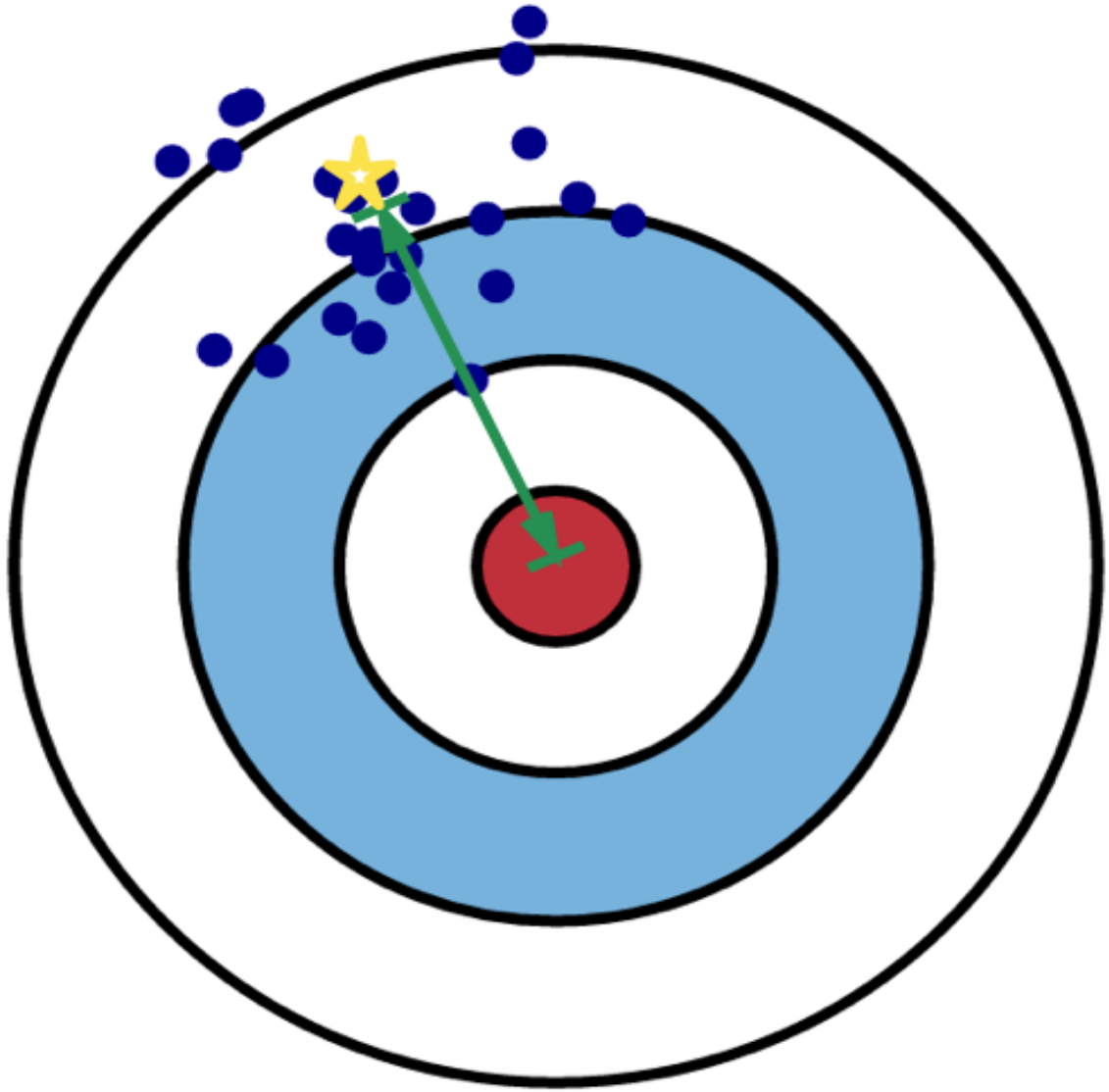


Figure 2 : Taking multiple shots in a dart game

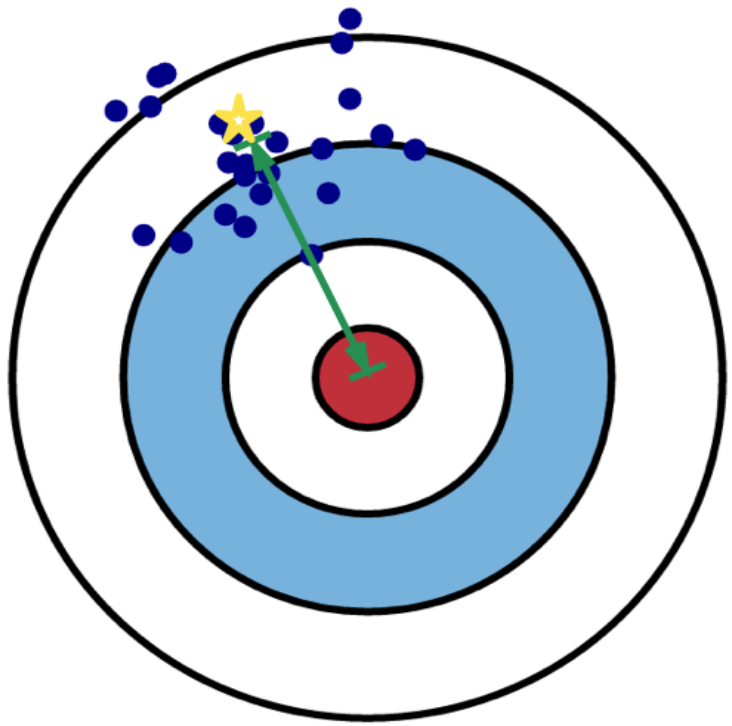
Bias-Variance Decomposition

Bias: difference between **the truth** and **what you expect to learn**

$$\text{Bias} = f(x_0) - \mathbb{E}\hat{f}(x_0) \tag{1}$$

Variance: difference between **what you learn** from a particular dataset and **what you expect to learn**

$$\text{Variance} = \mathbb{E}[\hat{f}(x_0) - \mathbb{E}\hat{f}(x_0)]^2 \tag{2}$$



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Expected prediction error:

$$\begin{aligned}\text{Err}(x_0) &= \underbrace{\mathbb{E}[(Y - \hat{f}(x_0))^2 | X = x_0]}_{\text{Mean Squared Error}} \\ &= \underbrace{[f(x_0) - \mathbb{E}\hat{f}(x_0)]^2}_{\text{Squared Bias}} + \underbrace{\mathbb{E}[\hat{f}(x_0) - \mathbb{E}\hat{f}(x_0)]^2}_{\text{Variance}}\end{aligned} \tag{3}$$

Bias-Variance Tradeoff (1/2): Regression

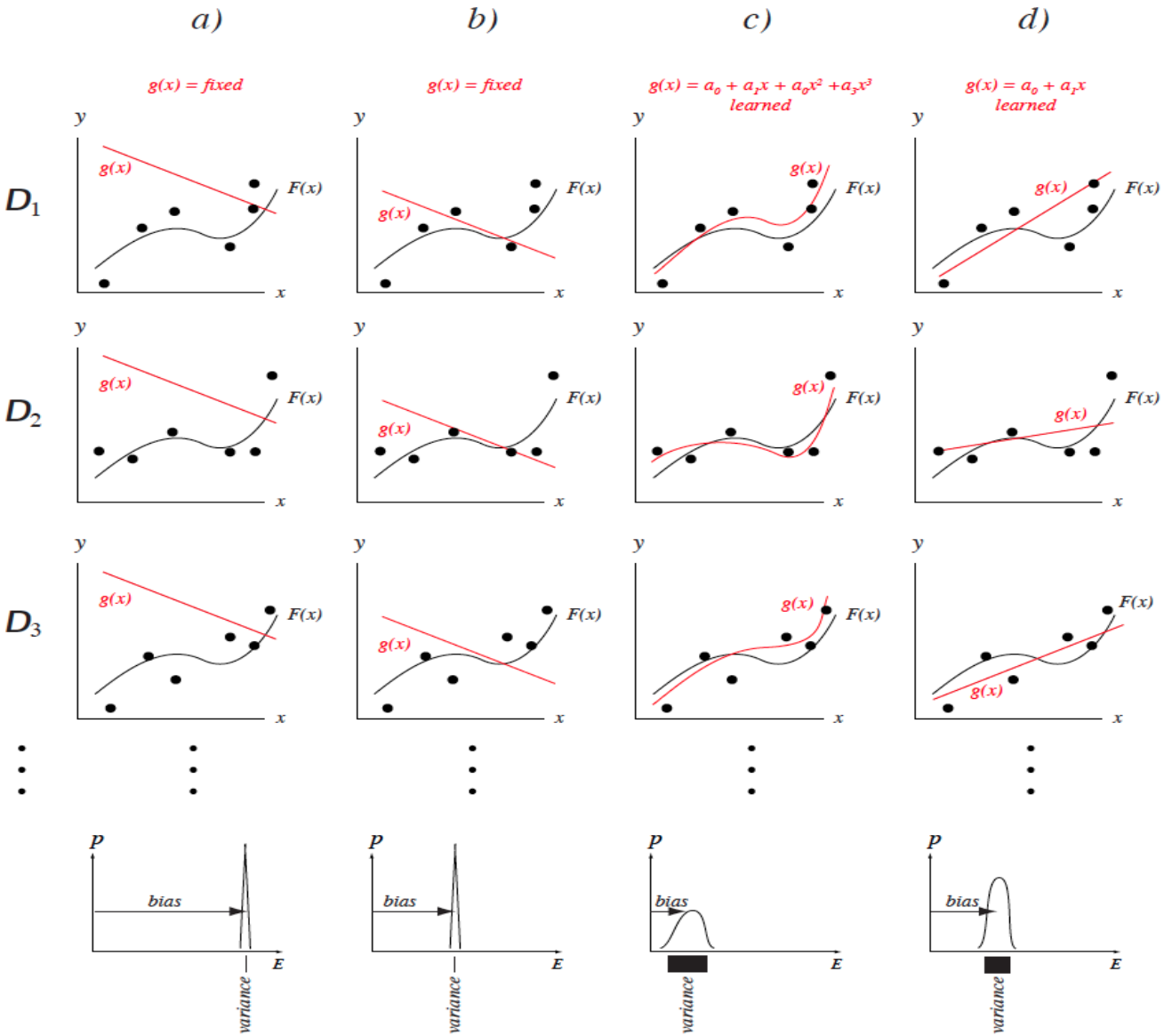

Figure 3 : Illustration of bias-variance tradeoff in regression

Bias-Variance Tradeoff (2/2): Model Complexity

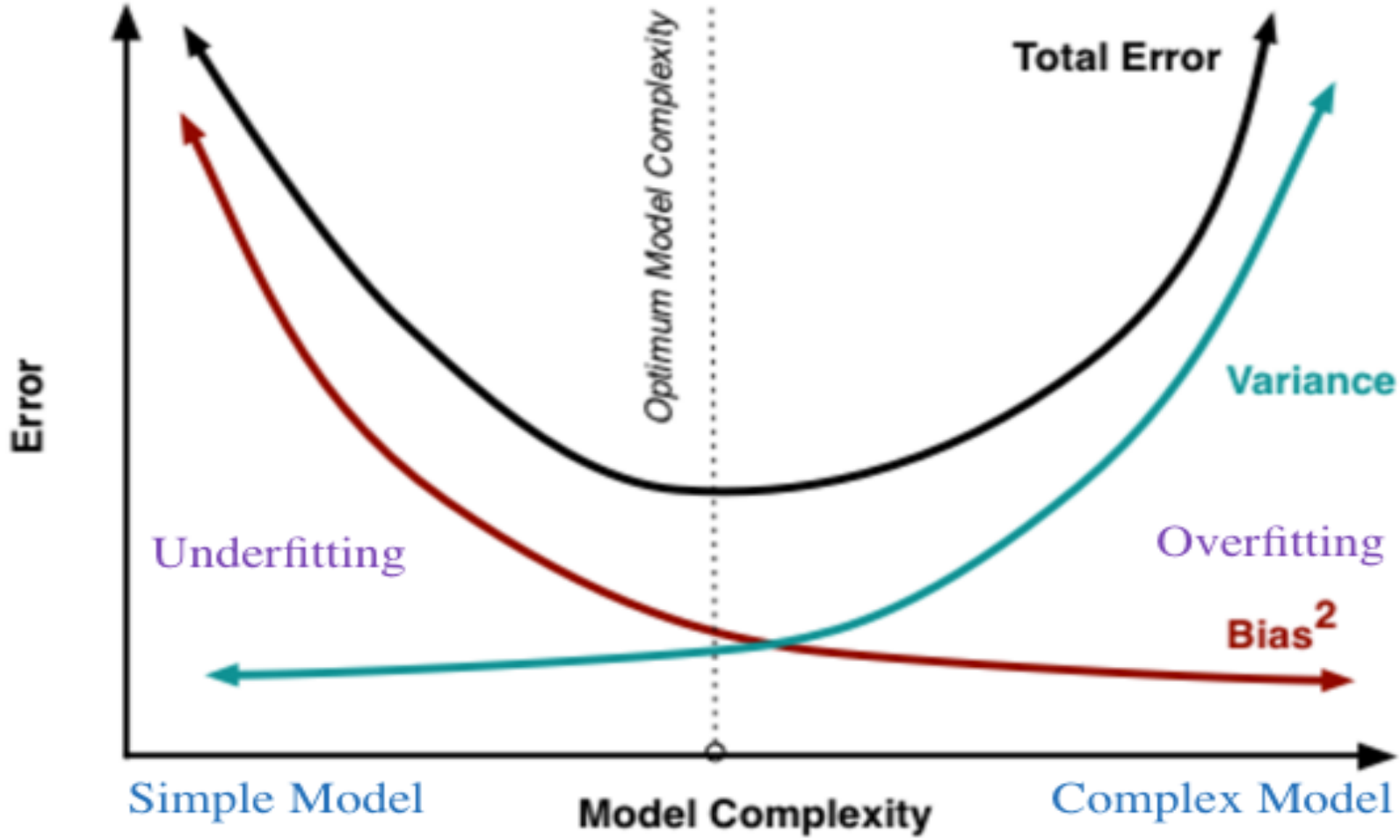

Figure 4 : Bias, variance and total error as a function of model complexity

Summary

- **Bias** is due to **erroneous model assumptions**
- **Variance** is from the **variability** of data gathered and model
- **Prediction error** decomposes into **bias** and **variance**
- **Bias-variance tradeoff** $\Leftrightarrow$ picking the right **model complexity**

More Readings

References:

1. Understanding the Bias-Variance Tradeoff, `http://scott.fortmann-roe.com/docs/BiasVariance.html`
2. Chapter 7: The Elements of Statistical Learning: Data Mining, Inference, and Prediction. (2nd Edition)
3. Chapter 9: Pattern Classification. (2nd edition)

Thank you!

Image credits: Figure 1 & 4 (reference 1), Figure 2 (reference 2), Figure 3 (reference 3).