Simple Linear Regression

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Data Example

Name	Midterm	Final
Alice	84	90
Bob	71	68
Carol	92	89
Dick	86	93
Emily	65	77
Frank	75	?
⋮	⋮	⋮

- Frank had an emergency and could not take the final.
- Unfortunately, there is no time for a make-up exam.
- How to predict his score on final exam using the midterm grades?

- Expected score on final $= \alpha + \beta \times$ midterm
- α, β are coefficients to be determined

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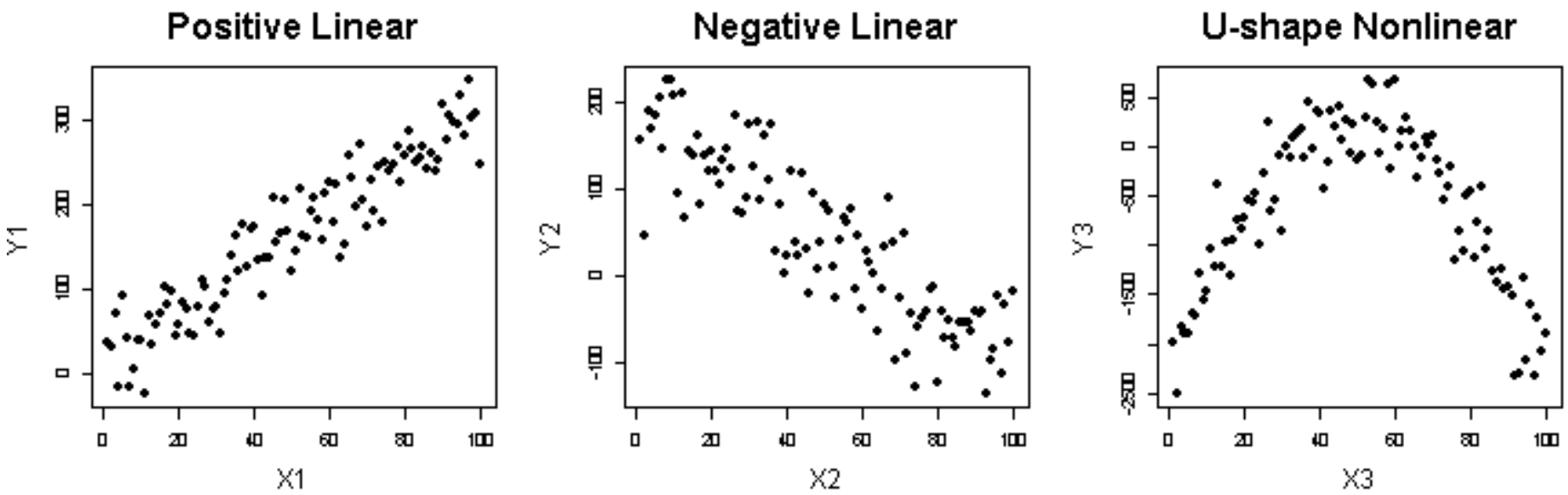
Simple Linear Regression

- **Linear** regression: $Y = \alpha + \beta X + \epsilon$, with $\epsilon \sim N(0, \sigma^2)$
- Predict Y (response variable) from X (explanatory variable)
- Error term ϵ is independent of X, Y, and the coefficients α, β
- Which trend do the midterm-final data look like?



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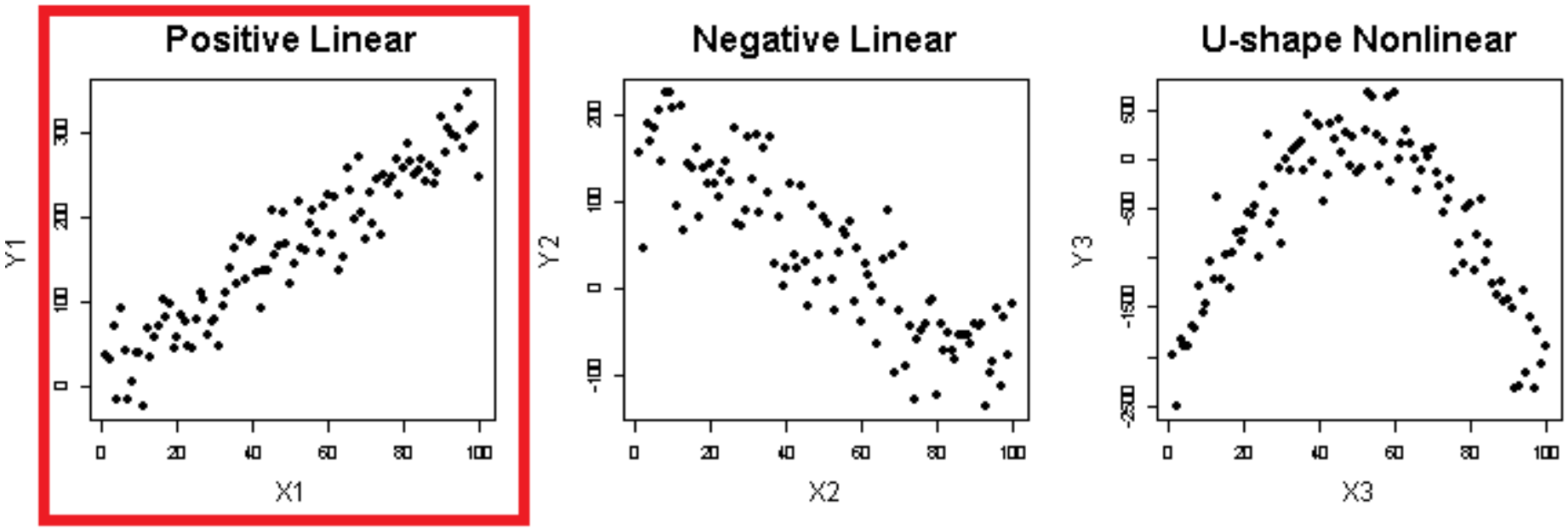

Applications

- $Y = \alpha + \beta X + \epsilon$, with $\epsilon \sim N(0, \sigma^2)$
- Use the R function `lm` to determine the coefficients
- In the midterm-final dataset, $\alpha = 22.13$ and $\beta = 0.770$
- Frank's expected final score $= 22.13 + 0.770 \times 75 = 79.88 \approx 80$

- For every point increase in the midterm score, the final score **is expected to** increase by $\beta = 0.770$ points
- $\alpha = 22.13$ serves as an intercept – the value of Y when $X = 0$
- α may or may not have a meaning

- Do you think it is possible to score 0 points on the midterm exam?

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