Conditional Probability

Introduction

Sta 771 - Spring 2016

Duke University, Department of Statistical Science

Princeton Lee

Outline

1. Main Topics

2. Example/Definitions

3. Tricks/Shortcuts

4. Review

Ideas for Today:

Today, we are going to discuss, define and learn how to use *Conditional Probability* to solve real world problems. The terms that you should be familiar with at the end of the lesson are:

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Today, we are going to discuss, define and learn how to use *Conditional Probability* to solve real world problems. The terms that you should be familiar with at the end of the lesson are:

- *Marginal Probability*
- *Joint Probability*
- *Conditional Probability*

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Table: Flu Shot Contingency Table

		Vaccinated		
		Yes	No	
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Source: Dr. Roy Benaroch, The Pediatric Insider

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What is the marginal probability of getting a positive flu test?

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What is the joint probability of being vaccinated and getting a positive flu test?

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What is the conditional probability of a person having a positive flu test given that s/he has been vaccinated?

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- Continuing example:

What is the conditional probability of a person having a negative flu test given that s/he has been vaccinated?

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  - Given that a person has been vaccinated, how many different flu test outcomes are there?

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What is the conditional probability of a person having a negative flu test given that s/he has been vaccinated?

- Trick
  - Given that a person has been vaccinated, how many different flu test outcomes are there?
  - If we let $C_1 =$ Positive Flu Test and $C_2 =$ Negative Flu Test. Notice that $P(C_1|A) + P(C_2|A) = 1$

- Assume a standard deck of cards for the following questions. Remember that a standard deck has 4 suits. Each suit has 13 unique cards from Ace to King.
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  - What is the conditional probability of getting a King, given that you drew a red card?

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- Here are some easy and simple tips to help you work faster and more accurately:
  - $P(C_1|A) + P(C_2|A) + \cdots + P(C_n|A) = 1$

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  - If A and B are independent, then

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B)$$

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- Conditional Probability refers to the probability of an event *conditional* on another event happening first.

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- You decide to tell your fortune by drawing two cards from a standard deck of 52 cards. What is the probability of drawing two cards of the same suite in a row? The cards are not replaced in the deck.

- What is the probability that the sum of two die will be greater than 8, given that the first die is 6?

- A new superman MasterCard has been issued to 2000 customers. Of these customers, 1500 hold a Visa card, 500 hold an American Express card and 40 hold a Visa card and an American Express card. Find the probability that a customer chosen at random holds a Visa card, given that the customer holds an American Express card.