Bootstrap Confidence Intervals

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Overview

- What and why?
- How?
- Always good?

Example: Chick weight

- 578 observations

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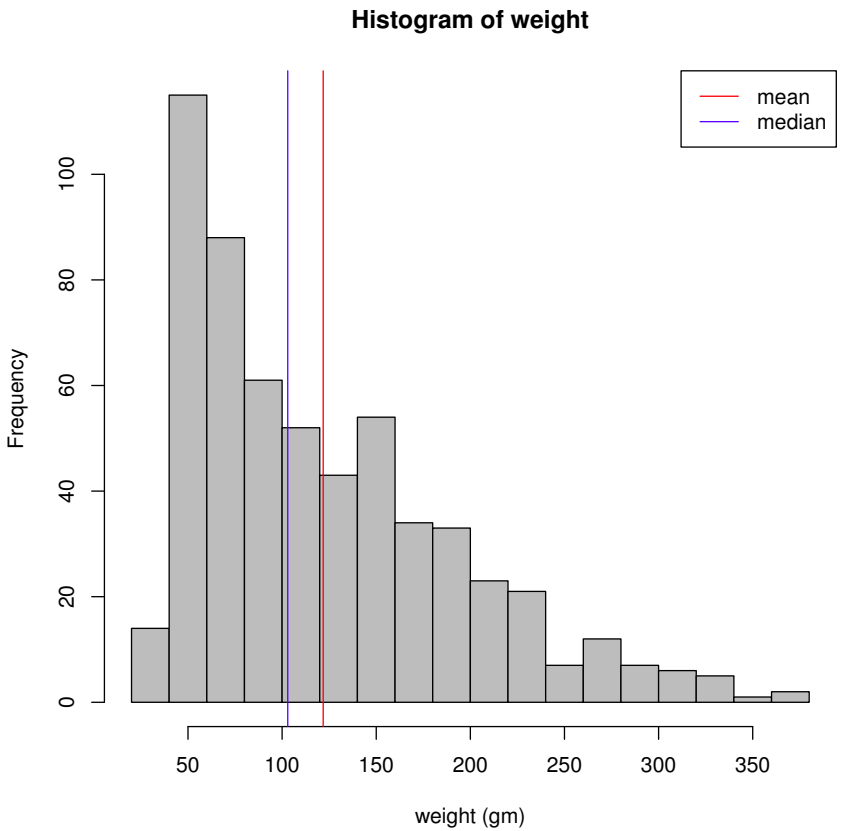
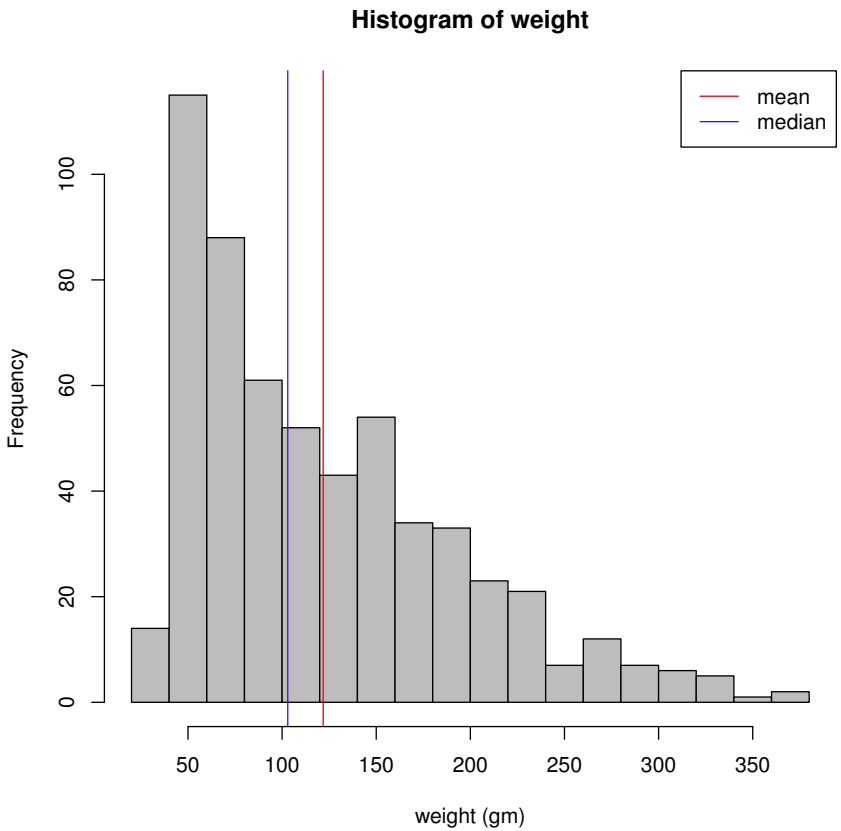


- 578 observations
- Right-skewed

Example: Chick weight



- 578 observations
- Right-skewed
- median=103

Bootstrapping

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- Bootstrap as a metaphor, meaning to better oneself by one's own unaided efforts, was in use in 1922.

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1. Get a bootstrap sample $(x_1^{*(i)}, \ldots, x_n^{*(i)})$ from the original sample;
   - a random sample;
   - same size;
   - sample with replacement.
2. Calculate the bootstrap statistic $T^{*(i)}$ with the bootstrap sample in the first step.

Then we can use the quantiles of these N bootstrap statistics to construct a bootstrap confidence interval.

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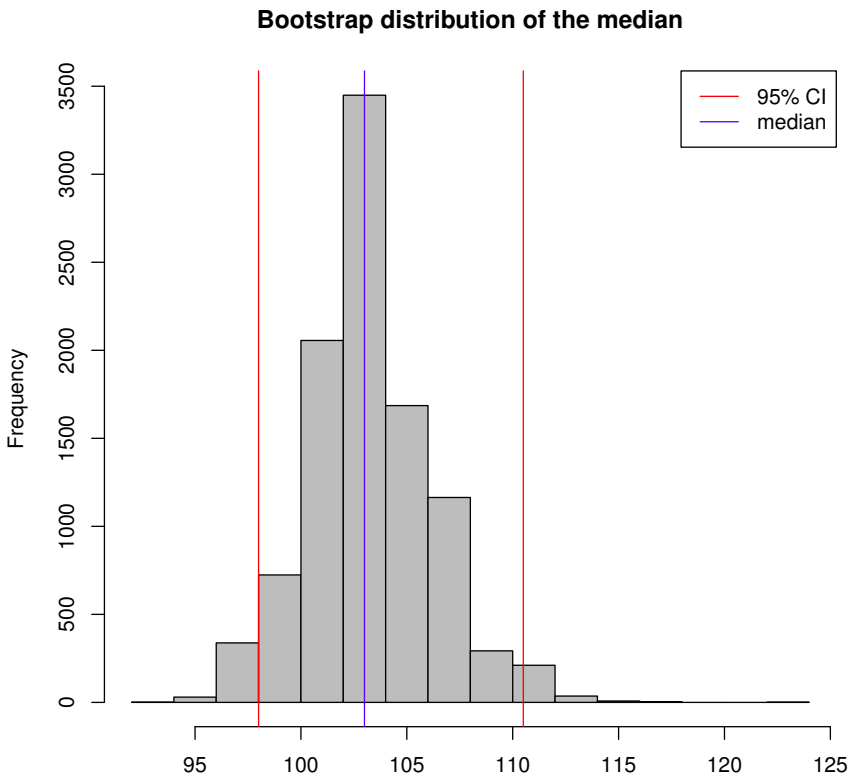

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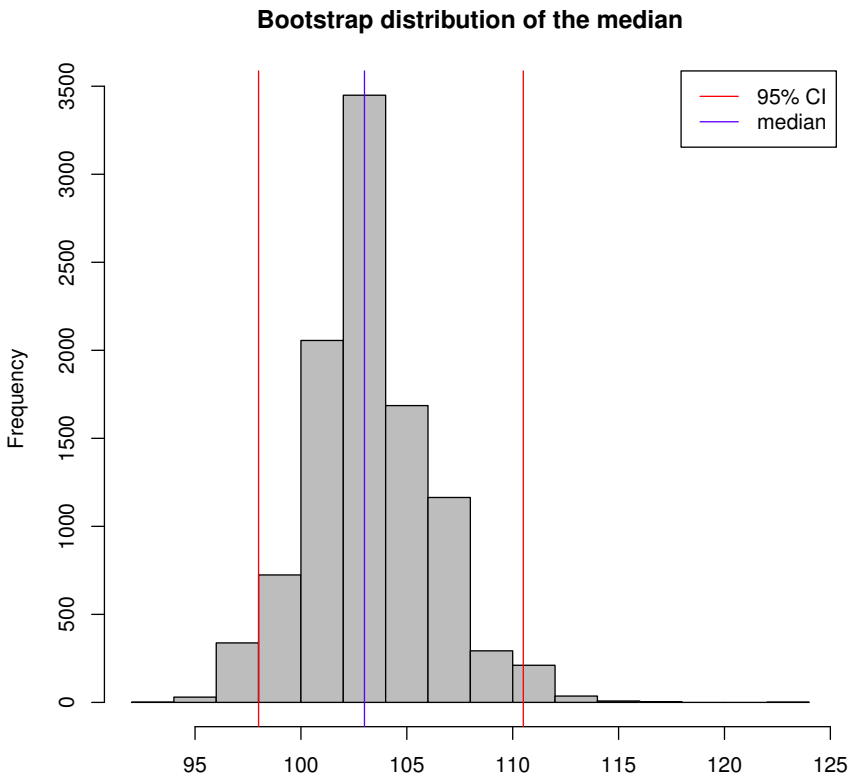

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- bootstrap confidence interval: $(98.0, 110.5)$

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- need a representative sample to start;

In class exercise: Simulation study

- For $n = 100$, generate $x_1, \ldots, x_n \sim N(\mu, 1)$, $\mu = 1$;
- Use the CLT based method to calculate the 95% CI for μ;
- Use the Bootstrap method ($N = 5000$) to construct a 95% CI for μ;
- Compare these two CIs;
- Change the values of n and N, re-do the experiment.