

# Bootstrap Confidence Intervals

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February 23, 2016

# Overview

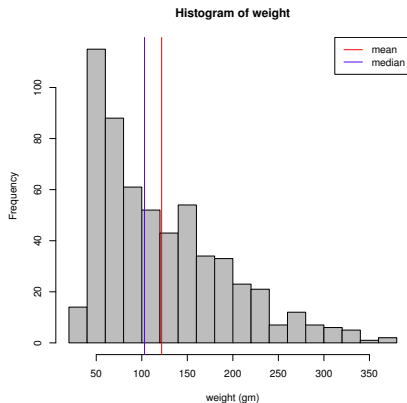
- ▶ What and why?
- ▶ How?
- ▶ Always good?

## Example: Chick weight



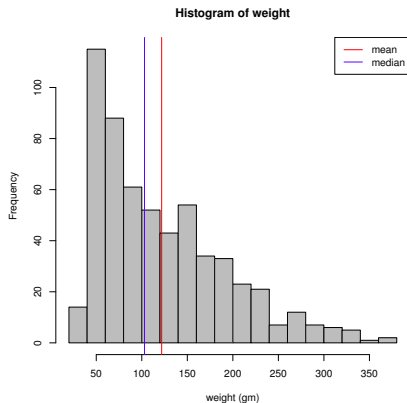
- ▶ 578 observations

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- ▶ Right-skewed

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- ▶ median=103

# Bootstrapping



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- ▶ Bootstrap as a metaphor, meaning to better oneself by one's own unaided efforts, was in use in 1922.

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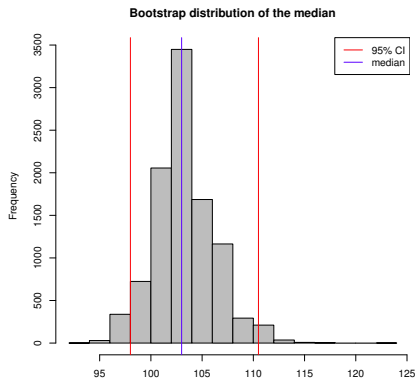
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1. Get a bootstrap sample  $(x_1^{*(i)}, \dots, x_n^{*(i)})$  from the original sample;
  - ▶ a random sample;
  - ▶ same size;
  - ▶ **sample with replacement.**
2. Calculate the bootstrap statistic  $T^{*(i)}$  with the bootstrap sample in the first step.

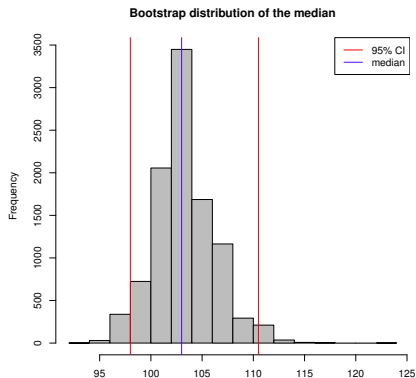
Then we can use the quantiles of these  $N$  bootstrap statistics to construct a bootstrap confidence interval.

# Example: Chick weight



- ▶ central 95% of the bootstrap distribution

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- ▶ central 95% of the bootstrap distribution
- ▶ bootstrap confidence interval: (98.0, 110.5)

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- ▶ your statistic  $T$  is not too weird;



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- ▶ your statistic  $T$  is not too weird;
- ▶ need a representative sample to start;