Stat238: Problem Set 4
Due Wednesday Nov. 9

October 27, 2016

Comments:

- Please note my comments in the syllabus about academic integrity. You can work together on the problems, but your final writeup must be your own and you should wrestle with the problems on your own first.
- It's due at the start of class on paper. The syllabus discusses the penalty for turning it in late.
- Please give the names of anyone you worked with on the problem set on what you hand in.
- Please include your code in your solution. For extensive code that doesn't naturally fit within your response to a given problem, please include the code in an Appendix.

Note that in *skewN.R*, I provide NIMBLE code for using the skew-normal distribution needed for problems 1 and 2. The standard parameterization (e.g., found on Wikipedia) of

$$p(y) = \frac{2}{\omega}\phi\left(\frac{y-\xi}{\omega}\right)\Phi\left(\alpha\frac{y-\xi}{\omega}\right)$$

(where $\phi()$ and $\Phi()$ are the density and CDF functions for the standard normal distribution) has terrible statistical properties because it produces a very strangely-shaped likelihood surface. Instead, the code in the file parameterizes with (μ, σ, γ_1) where μ can be interpreted as the mean, σ as the standard deviation, and γ_1 as a skewness parameter, with $-1 \leq \gamma_1 \leq 1$. When $\gamma_1 = 0$, then $y \sim N(\mu, \sigma^2)$. $\gamma_1 > 0$ indicates positive skewness.

Problems

1. Apply model assessment and model expansion strategies to assess the distributional assumption of normality for the random effects in the beta-blockers study of BDA chapter 5, by comparison with a skew-normal distribution. You should be able to evaluate the posterior for the parameter(s) in the expanded model to assess whether the simpler model is reasonable. Note that for this problem and the next one, that for the base model, one can integrate over $\{\theta_j\}$ and μ analytically, it's probably most straightforward to just use MCMC on the base model without marginalization, since once one expands the model, one can no longer easily do the integrals.
2. Note that the $p(y)$ calculations will be the focus of Lab 9 on November 4, so you should have time to work on this problem then. Apply model selection strategies to the same question of the distribution of the random effects in the beta-blockers study. Compare the normal distribution with the skew-normal distribution. Note that getting reliable numerical estimates may be difficult, and it's fine if that conclusion is part of your answer.

(a) Compute $p(y)$ for the models using at least two of the sampling approaches discussed in class (one approach uses samples from the prior and the two other approaches use posterior samples). Can you get a robust estimate of $p(y)$ or is it difficult?

(b) For at least one of the models and at least one hyperparameter, assess how sensitive $p(y)$ is to the prior distributions for the hyperparameters.

(c) If you're able to get a reliable estimate of the Bayes factor, which model do you prefer?

(d) Compare the models using WAIC. Is the result consistent with part (c)?

3. In class we discussed the INLA approach, which involves the following approximation to the marginal posterior:

$$\tilde{p}(\theta|y) \propto \left. \frac{p(u, \theta, y)}{\tilde{p}_G(u|\theta, y)} \right|_{u=\hat{u}(\theta)}$$

where $\tilde{p}_G$ is a normal approximation to $p(u|\theta, y)$, u is a vector of random effects and θ a vector of hyperparameters. In class, I claimed that this approximation was a Laplace approximation. Consider the Laplace approximation for:

$$p(\theta|y) = \int p(\theta, u|y) du$$

and show that the two representations are equivalent.

4. (Extra credit): BDA problem 13.10 (variational Bayes for probit regression)