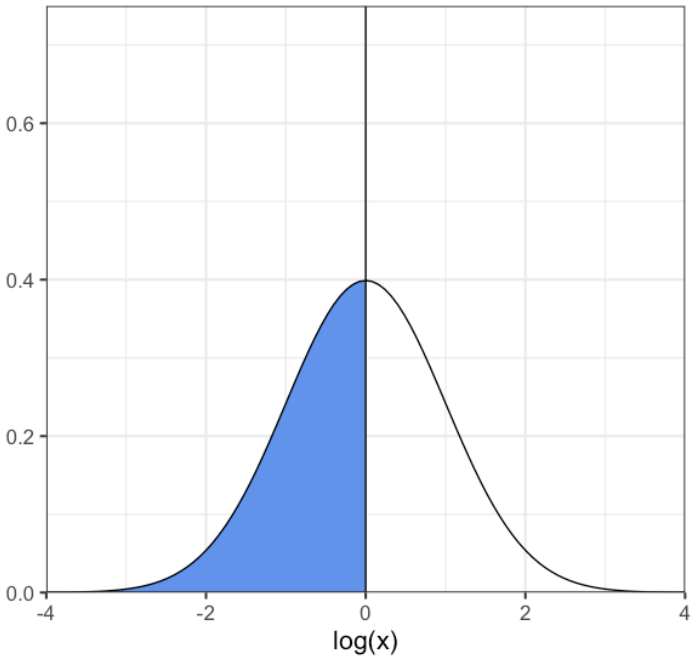
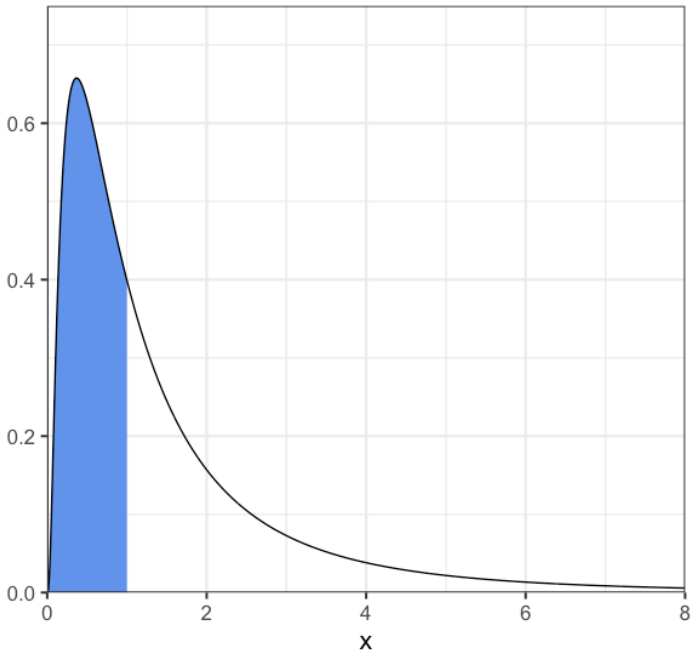
Interpretation of a single mean on the original scale (valid for all transformations, illustrated with log here)

Shaded area is 0.5 in both cases.





On transformed scale, median = mean (ideally, distribution is close to symmetric after transformation)

Example: on log scale, median is 0

Median on original scale is exponential transformation of median on log scale.

Example: on original scale, median is $e^0 = 1$

Interpretation of a difference between means on the original scale (valid for log transformation only!)

$$\begin{aligned}
&\exp\{\textbf{Mean Group 2 on log scale} - \textbf{Mean Group 1 on log scale}\} \\
&= \exp\{\log(\textbf{Median group 2}) - \log(\textbf{Median group 1})\} \\
&= \exp\left\{\log\left(\frac{\textbf{Median group 2}}{\textbf{Median group 1}}\right)\right\} \\
&= \frac{\textbf{Median group 2}}{\textbf{Median group 1}}
\end{aligned}$$

Rearranging, we obtain:

$$\textbf{Median group 2} = \textbf{Median group 1} \times \exp(\textbf{Mean Group 2 on log scale} - \textbf{Mean Group 1 on log scale})$$

Equivalently, …

$$\textbf{Median group 1} = \textbf{Median group 2} \times \exp(\textbf{Mean Group 1 on log scale} - \textbf{Mean Group 2 on log scale})$$