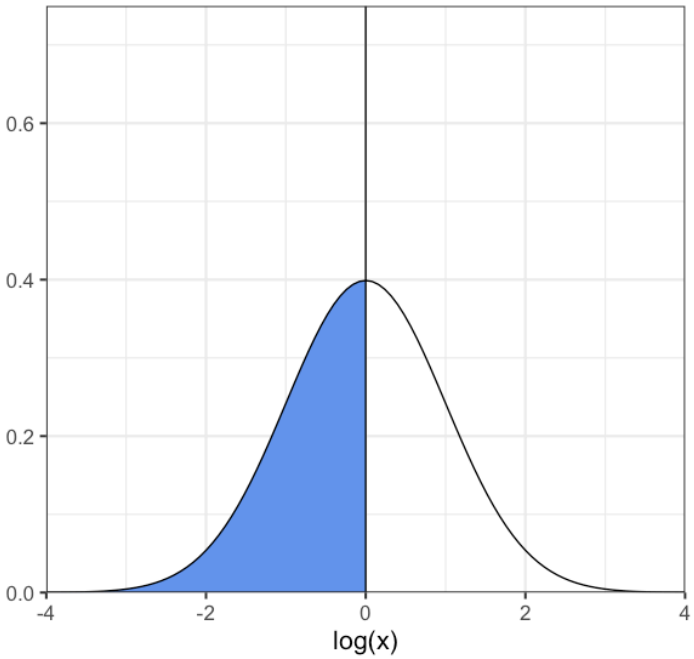
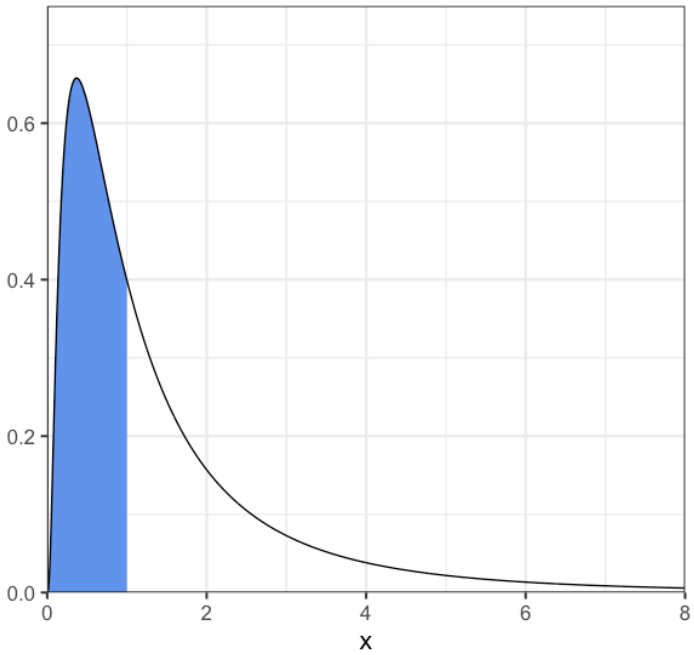
Interpretation of a single mean on the original scale (valid for all transformations, illustrated with log here)

Shaded area is 0.5 in both cases.





On transformed scale, median = mean (ideally, distribution is close to symmetric after transformation)

On log scale: median is 0

Median on original scale is exponential transformation of median on log scale.

On original scale: median is $e^0 = 1$

Interpretation of a difference between means on the original scale (valid for log transformation only!)

$$\begin{aligned}\exp(\mu_2 - \mu_1) &= \exp\{\log(Median(Y_2)) - \log(Median(Y_1))\} \\ &= \exp\left\{\log\left(\frac{Median(Y_2)}{Median(Y_1)}\right)\right\} \\ &= \frac{Median(Y_2)}{Median(Y_1)}\end{aligned}$$

Rearranging, we obtain:

$$Median(Y_2) = Median(Y_1) \times \exp(\mu_2 - \mu_1)$$

Equivalently, …

$$Median(Y_1) = Median(Y_2) \times \exp(\mu_1 - \mu_2)$$