Chapter 11: Multiple Regression, Pairs Plots, and Added Variable Plots

Duncan's Occupational Prestige Data

Intro to data

We have a data set with measurements on 45 different U.S. occupations as of 1950 (descriptions below are quotes from Fox and Weisberg, 2011):

- `type`: Type of occupation. A factor with the following levels: `prof`, professional and managerial; `wc`, white-collar; `bc`, blue-collar.
- `income`: Percentage of occupational incumbents in the 1950 US Census who earned \$3,500 or more per year (about \$36,000 in 2017 US dollars).
- `education`: Percentage of occupational incumbents in 1950 who were high school graduates (which, were we cynical, we would say is roughly equivalent to a PhD in 2017)
- `prestige`: Percentage of respondents in a social survey who rated the occupation as "good" or better in prestige

```
head(Duncan)
```

```
##            type income education prestige occupation
## accountant prof     62        86       82 accountant
## pilot      prof     72        76       83      pilot
## architect  prof     75        92       90  architect
## author     prof     55        90       76     author
## chemist    prof     64        86       90    chemist
## minister   prof     21        84       87   minister
```

References:

- Fox, J. and Weisberg, S. (2011) An R Companion to Applied Regression, Second Edition, Sage.
- Duncan, O. D. (1961) A socioeconomic index for all occupations. In Reiss, A. J., Jr. (Ed.) Occupations and Social Status. Free Press [Table VI-1].

Let's consider a model for occupational prestige as a function of income, education, and type of occupation.

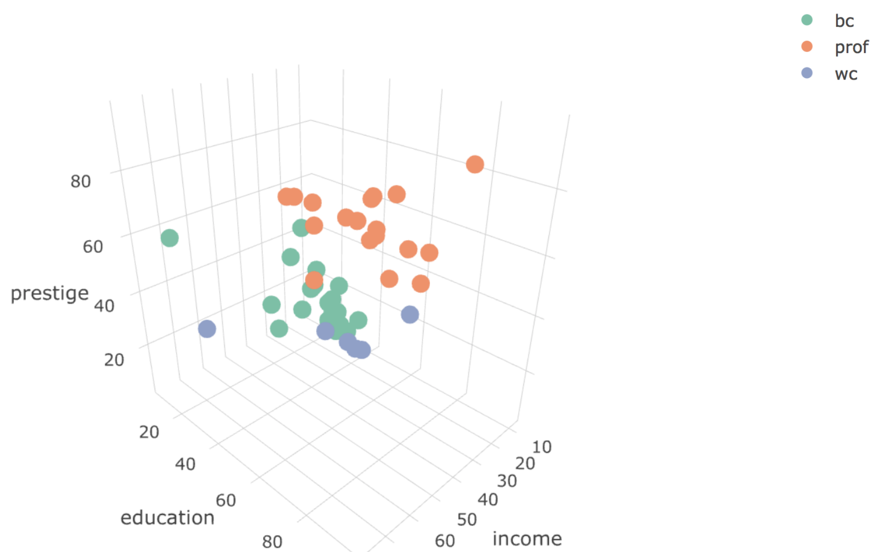
We should always start with plots, but we're really hitting the limits of what's plottable now…

Option 1: plotly

- Formatting very similar to, but not exactly the same as, ggplot2
- **Can't show output in pdf, only for html output or interactive use**
- Can't be used for any more variables than we have in this example.
- If plotly code doesn't give you what you want right away, it can be essentially impossible to fix (not a fully developed and functional package).

```
library(plotly)
plot_ly(Duncan, x = ~income, y = ~education, z = ~prestige, color = ~type) %>%
  add_markers()
```

Here's a screenshot, will demo live:



Option 2: Pairs Plots

```r
library(GGally)
```

```
##
## Attaching package: 'GGally'

## The following object is masked from 'package:dplyr':
##
##     nasa
```

```r
ggpairs(Duncan %>% select(-occupation))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

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```
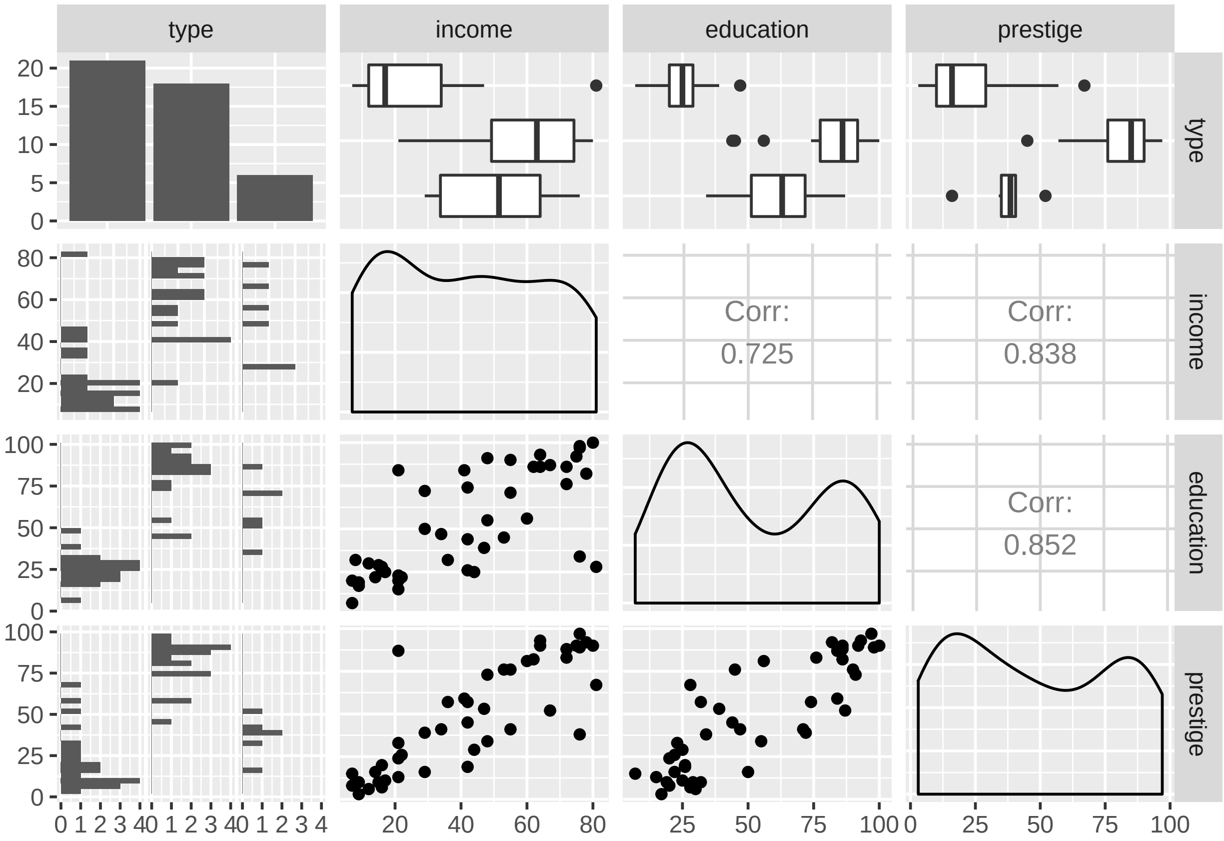



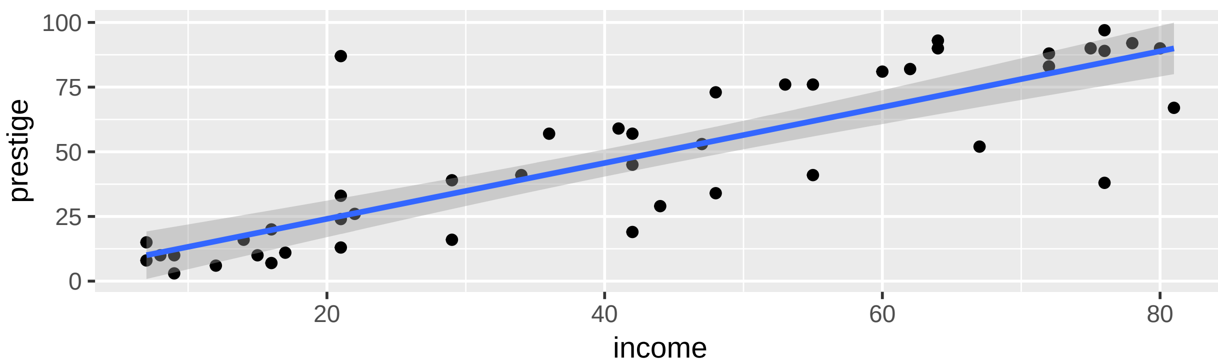
Possible evidence of one outlier? Let's fit some models and come back to that later.

A first model - income only explanatory variable

```
lm_fit_1 <- lm(prestige ~ income, data = Duncan)
summary(lm_fit_1)
```

```
##
## Call:
## lm(formula = prestige ~ income, data = Duncan)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -46.566  -9.421   0.257   9.167  61.855
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.4566     5.1901   0.473    0.638
## income        1.0804     0.1074  10.062 7.14e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.4 on 43 degrees of freedom
## Multiple R-squared:  0.7019, Adjusted R-squared:  0.695
## F-statistic: 101.3 on 1 and 43 DF,  p-value: 7.144e-13
```

```
ggplot(data = Duncan, mapping = aes(x = income, y = prestige)) +
  geom_point() +
  geom_smooth(method = "lm")
```



What is the equation of the estimated line?

What is the interpretation of the coefficient estimate for income?

Second Model: income and education as explanatory variables

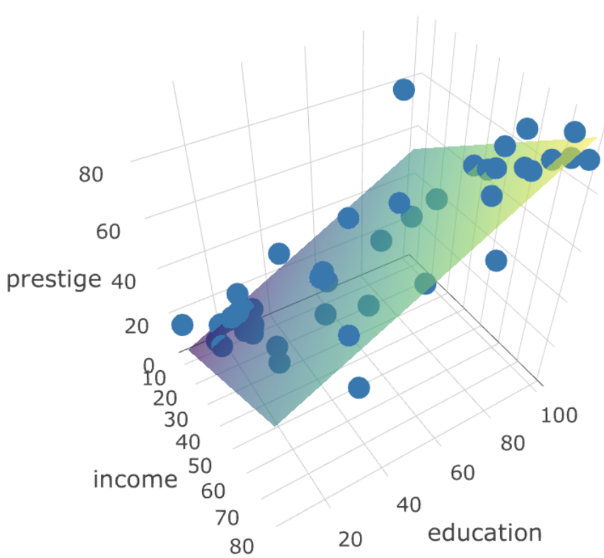
```
lm_fit_2 <- lm(prestige ~ income + education, data = Duncan)
summary(lm_fit_2)
```

```
##
## Call:
## lm(formula = prestige ~ income + education, data = Duncan)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -29.538  -6.417   0.655   6.605  34.641
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6.06466    4.27194  -1.420    0.163
## income       0.59873    0.11967   5.003 1.05e-05 ***
## education    0.54583    0.09825   5.555 1.73e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.37 on 42 degrees of freedom
## Multiple R-squared:  0.8282, Adjusted R-squared:   0.82
## F-statistic: 101.2 on 2 and 42 DF,  p-value: < 2.2e-16
```

What's the estimated equation of this model?

This can be visualized as a plane

Plotly code suppressed because it's awful.

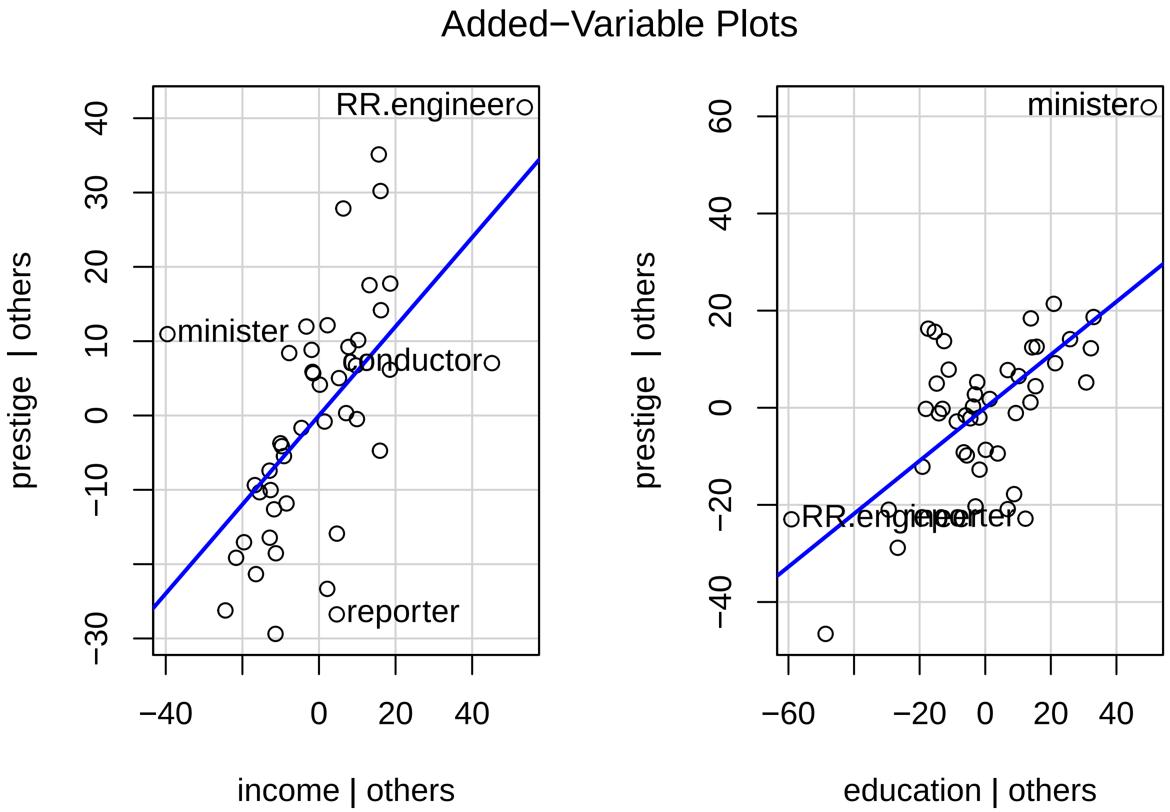


What is the interpretation of the coefficient estimate for income?

Added Variables Plots

- **After accounting for the effects of other variables in the model, what is the relationship between income and prestige?**
- Focus on added variable plot for income: **after accounting for education**,
  - New X variable is *residuals* from a regression of income on education (what's left over in income, after accounting for education level?)
  - New Y variable is *residuals* from a regression of prestige on education (what's left over in prestige, after accounting for education level?)
  - The slope of the line describing the relationship between these residuals is the estimated coefficient for income from the model fit.

```r
library(car)
avPlots(lm_fit_2)
```




```r
fit_income_education <- lm(income ~ education, data = Duncan)
fit_prestige_educaiton <- lm(prestige ~ education, data = Duncan)
av_df <- data.frame(
  income_resids = residuals(fit_income_education),
  prestige_resids = residuals(fit_prestige_educaiton)
)
fit_prestige_income_accounting_education <- lm(prestige_resids ~ income_resids, data = av_df)
```

```r
summary(fit_prestige_income_accounting_education)
```

```
##
## Call:
## lm(formula = prestige_resids ~ income_resids, data = av_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -29.538  -6.417   0.655   6.605  34.641
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -5.954e-16  1.970e+00   0.000        1
## income_resids  5.987e-01  1.183e-01   5.063 8.25e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.21 on 43 degrees of freedom
## Multiple R-squared:  0.3734, Adjusted R-squared:  0.3589
## F-statistic: 25.63 on 1 and 43 DF,  p-value: 8.246e-06
```

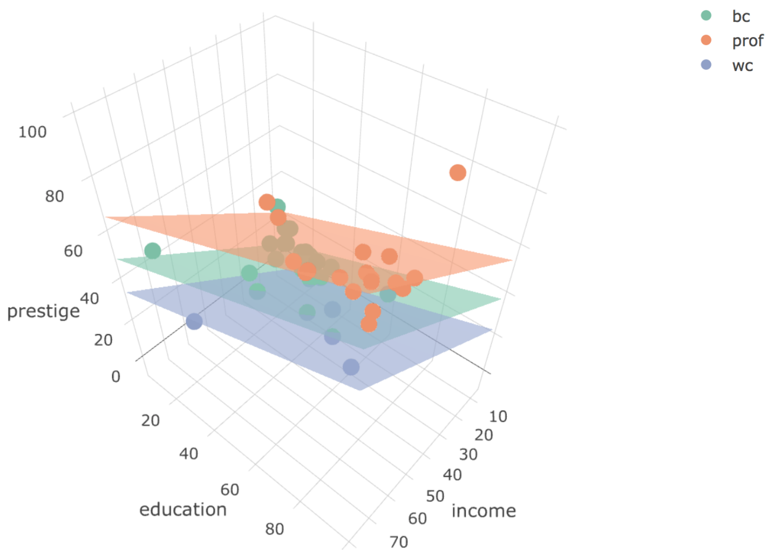
Compare to coefficient from model with income and education as explanatory variables.

Third Model: All 3 explanatory variables!

```
lm_fit_3 <- lm(prestige ~ income + education + type, data = Duncan)
summary(lm_fit_3)
```

```
##
## Call:
## lm(formula = prestige ~ income + education + type, data = Duncan)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.890  -5.740  -1.754   5.442  28.972
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.18503    3.71377  -0.050  0.96051
## income        0.59755    0.08936   6.687 5.12e-08 ***
## education     0.34532    0.11361   3.040  0.00416 **
## typeprof     16.65751    6.99301   2.382  0.02206 *
## typewc      -14.66113    6.10877  -2.400  0.02114 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.744 on 40 degrees of freedom
## Multiple R-squared:  0.9131, Adjusted R-squared:  0.9044
## F-statistic:   105 on 4 and 40 DF,  p-value: < 2.2e-16
```

Plotly code suppressed because it's awful.

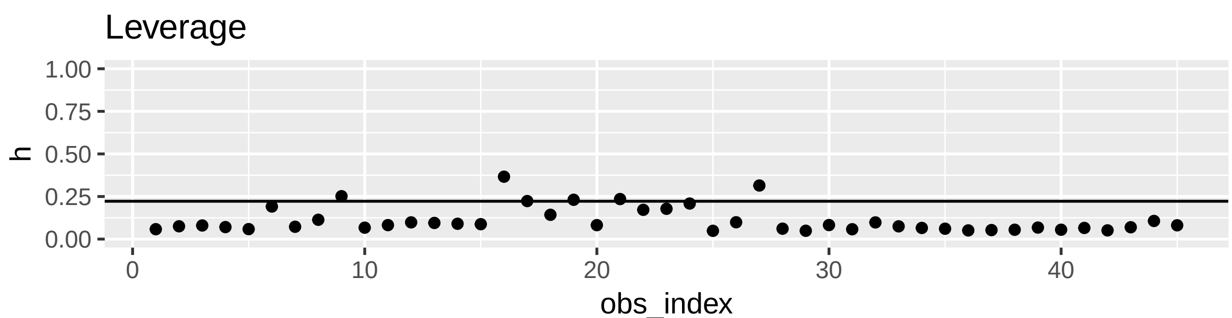


What is the equation of the model fit?

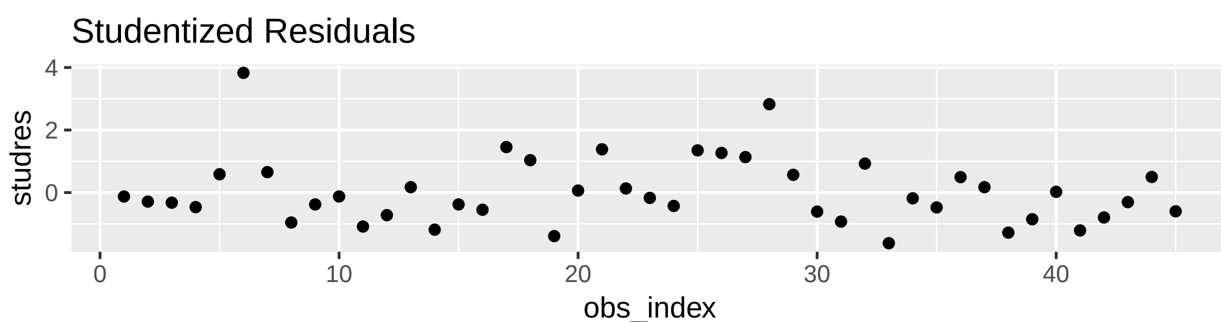
What is the interpretation of the estimated coefficient for income?

Diagnostic Plots

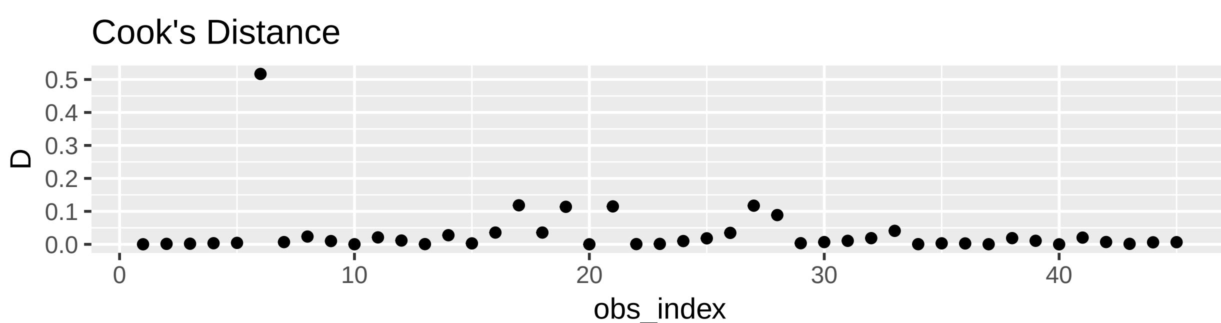
```
Duncan <- Duncan %>%
  mutate(
    obs_index = row_number(),
    h = hatvalues(lm_fit_3),
    studres = rstudent(lm_fit_3),
    D = cooks.distance(lm_fit_3)
  )

ggplot(data = Duncan, mapping = aes(x = obs_index, y = h)) +
  geom_point() +
  geom_hline(yintercept = 2 * 5 / nrow(Duncan)) +
  ylim(0, 1) +
  ggtitle("Leverage")
```




```
ggplot(data = Duncan, mapping = aes(x = obs_index, y = studres)) +
  geom_point() +
  ggtitle("Studentized Residuals")
```




```
ggplot(data = Duncan, mapping = aes(x = obs_index, y = D)) +
  geom_point() +
  ggtitle("Cook's Distance")
```

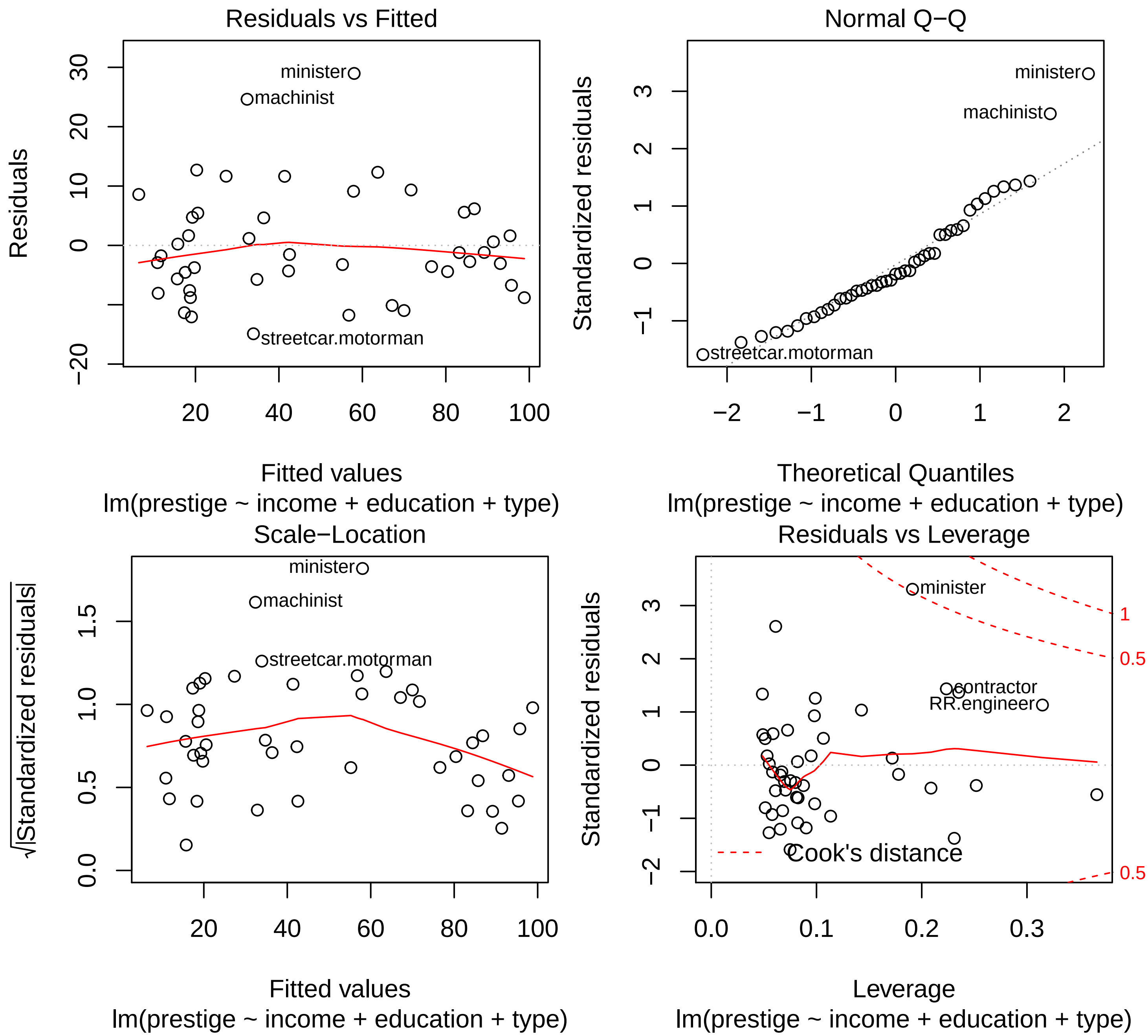



```
obs_to_investigate <- c(6, 16, 27)
```

```
Duncan[obs_to_investigate, ]
```

```
##    type income education prestige  occupation obs_index         h
## 6  prof     21        84       87    minister         6 0.1912053
## 16   wc     76        34       38   conductor        16 0.3663519
## 27   bc     81        28       67 RR.engineer        27 0.3146829
##       studres          D
## 6   3.8293960 0.51680533
## 16 -0.5505711 0.03567303
## 27  1.1339763 0.11725367
```

```
plot(lm_fit_3)
```



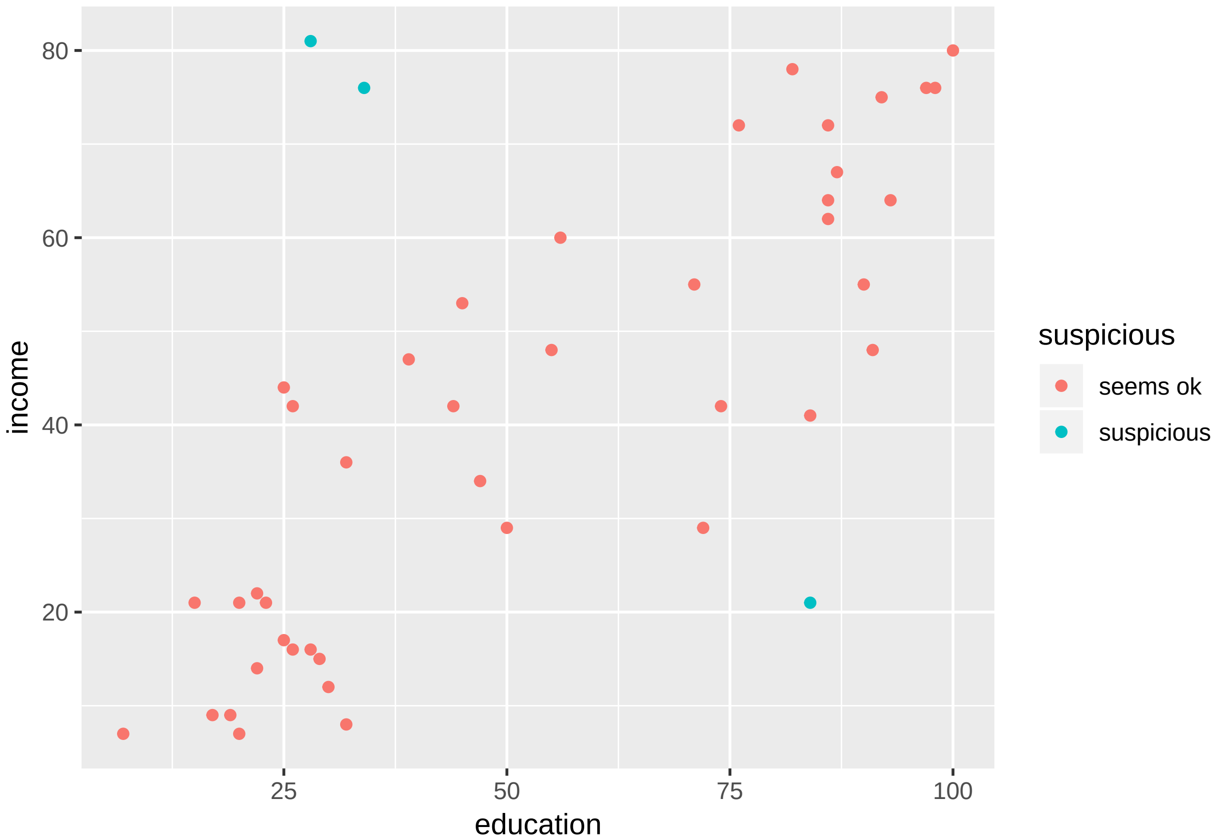
```r
Duncan_minus_suspicious <- Duncan[-obs_to_investigate, ]
lm_fit_without_suspicious <- lm(prestige ~ income + education + type, data = Duncan_minus_suspicious)
summary(lm_fit_without_suspicious)
```

```
##
## Call:
## lm(formula = prestige ~ income + education + type, data = Duncan_minus_suspicious)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.0415  -5.3802  -0.6189   5.0992  23.2906
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.1053     3.2745  -0.338   0.7376
## income        0.7733     0.1171   6.607 9.53e-08 ***
## education     0.2180     0.1174   1.857   0.0714 .
## typeprof     15.2512     6.4123   2.378   0.0227 *
## typewc      -12.3622     5.9478  -2.078   0.0447 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.432 on 37 degrees of freedom
## Multiple R-squared:  0.9368, Adjusted R-squared:   0.93
## F-statistic: 137.1 on 4 and 37 DF,  p-value: < 2.2e-16
```

```r
Duncan_minus_minister <- Duncan[-6, ]
lm_fit_without_minister <- lm(prestige ~ income + education + type, data = Duncan_minus_minister)
summary(lm_fit_without_minister)
```

```
##
## Call:
## lm(formula = prestige ~ income + education + type, data = Duncan_minus_minister)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.0521  -6.4105  -0.7819   4.6552  23.5212
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.62984    3.22841  -0.505  0.61651
## income        0.71813    0.08332   8.619 1.44e-10 ***
## education     0.28924    0.09917   2.917  0.00584 **
## typeprof     13.43111    6.09592   2.203  0.03355 *
## typewc      -15.87744    5.28357  -3.005  0.00462 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.413 on 39 degrees of freedom
## Multiple R-squared:  0.9344, Adjusted R-squared:  0.9277
## F-statistic:   139 on 4 and 39 DF,  p-value: < 2.2e-16
```

```r
Duncan <- Duncan %>%
  mutate(
    suspicious = ifelse(row_number() %in% obs_to_investigate, "suspicious", "seems ok")
  )

ggplot(data = Duncan, mapping = aes(x = education, y = income, color = suspicious)) +
  geom_point()
```



What do we say?