JOE SEIDEL

STATS 244 HOMEWORK 4

Question 1

Let $Z \sim N(0,1)$, a stand normal distribution and let $X \sim N(\mu\sigma^2)$. Let $\Phi(z)$ be the cdf of Z. Suppose $X \sim N(-4,16)$; find.

It helps to know that

$$P(X < x) = \Phi\Big(\frac{x-\mu}{\sigma}\Big).$$

1. $P(X > 2)$.

$$\begin{aligned} P(X > 2) &= 1 - P(X < 2) \qquad = 1 - \Phi\Big(\frac{2-(-4)}{\sqrt{16}}\Big) \\ &= 1 - .9332 \\ &= .0688 \end{aligned}$$

2. $P(0 < X < 4)$

$$\begin{aligned} P(0 < X < 4) &= P(X < 4) - P(X < 0) \\ &= \Phi\Big(\frac{4-(-4)}{4}\Big) - \Phi\Big(\frac{0-(-4)}{4}\Big) \\ &= .1539 \end{aligned}$$

3. $P(|X+3| \geq 3)$

$$\begin{aligned} P(|X+3| \geq 3) &= P(X \geq 0) + P(X \leq -6) \\ &= 1 - \Phi\Big(\frac{4}{4}\Big) + \Phi\Big(\frac{-2}{4}\Big) \\ &= .1587 + .3085 \\ &= .4672 \end{aligned}$$

4. $P(X \leq 0 \text{ or } X \geq 3)$

$$\begin{aligned} P(X \leq 0 \text{ or } X \geq 3) &= P(X \leq 0) + P(X \geq 3) \\ &= \Phi\Big(\frac{4}{4}\Big) + 1 - \Phi\Big(\frac{7}{4}\Big) \\ &= .8413 + .0003 \\ &= .8416 \end{aligned}$$

Question 2

Based on student A's performace during the first two weeks of a course, the professor has approximately a normal $N(70, 8^2)$ prior distribution about the student's true ability, on a scale of 0 to 100.

Consider the midterm examination as an error-prone measure of the student's true ability, where if the true ability is x, the examination score can be modeled as approximately normally distributed, $N(x, 6^2)$. The student scores 90 on the midterm.

1. What are the posterior expectaion and the probability that the student's true ability is above 85?

   WE HAVE the following information.

   $$f(\theta) \sim N(70, 8^2)$$

   and

   $$f(x \mid \theta) \sim N(x, 6^2)$$

   Using Bayes, $f(\theta \mid X) \propto f(x \mid \theta) f(\theta)$ the posterior distribution is

   $$(\theta \mid X) = \frac{1}{\sqrt{2\pi}B} e^{\frac{(\theta - A)^2}{2B^2}}$$

   Where

   $$A = \frac{6^2(70) + 8^2(90)}{6^2 + 8^2}$$

   and

   $$B^2 \frac{6^2 * 8^2}{6^2 + 8^2}$$