Math329: Exam 03 (Out of Class)

NAME:

Please answer the following questions and bring your solution to the in-class exam. You may consult any class notes you would like, but may not search the internet or discuss the question with anyone else. You may typeset your solution in LaTeX, but this is not required. These will count for 40% of the exam.

All of these questions concern a collection of n random variables Y_i. The Y_i's are defined in terms of random variables $e_i \sim_{i.i.d.} N(0,1)$ according to the following relationship:

$$Y_i = X_i\beta + e_i$$

Where β is a fixed unknown constant and the X_i's are fixed (but possibly different) and known values. The goal is to estimate β after observing n pairs of data (X_i, Y_i).

1. Write down a formula for the joint density of the Y_i's. You should simplify so that your answer involves a summation rather than a product.

2. Find the MLE estimator for β.

3. Find the distribution of $\widehat{\beta}_{MLE}$. Calculate the bias, variance, and MSE of $\widehat{\beta}_{MLE}$.[1]

[1] Hint: It should be a normal with a certain mean and variance, but you must explain why it is normal.

4. You observe the following pairs of data (x is first, y is second):

$$(1,0)\,(2,2)\,(5,3)\,(6,5)\,(7,8)$$

Compute $\widehat{\beta}_{MLE}$. Note: A graph of these looks like this: