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From which we can reason that:

$$\begin{aligned} \mathbb{P}(S_1 \cap S_2) &= \frac{13}{52} \cdot \frac{12}{51} \\ &\approx 0.0588. \end{aligned}$$

This is actualy how many of you were trying to do probabilities in the first week!

Suppose that 5% of the population have a certain disease, and that the test administered for detecting the disease is reasonably accurate. Specifically, suppose that a subject who has the disease will test positive with probability .98. As is the case with most diagnostic procedures, however, there is a chance that some people without the disease will also test positive. Let's assume that the probability of a false positive in this instance is .1. When a randomly chosen subject is tested, the full testing procedure may be represented as the two events: "D" stands for disease and "$+$" stands for a positive test result.

What is the probability that you have the disease given that you tested positive for it?

The multiplication rule gives us the answer:

$$\mathbb{P}(D|+) = \frac{\mathbb{P}(D \cap +)}{\mathbb{P}(+)}$$
$$= \frac{0.05 * 0.98}{0.05 * 0.98 + 0.95 * 0.10} \approx 0.340.$$

Which you might find surprisingly low.

Consider flipping three fair, independent coins. Let A be the event that exactly one of the first two flips is heads and let B be the event that exactly one of the final two flips is heads. Are A and B independent events?

You may have a knee-jerk reaction that these are not independent, but let's apply the definition. There are 8 possible outcomes, all equally likely: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT. Given this, the following probabilities are the result of simple counting:

$$\begin{aligned}
\mathbb{P}(A \cap B) &= \frac{2}{8} = \frac{1}{4} \\
\mathbb{P}(A) &= \frac{4}{8} = \frac{1}{2} \\
\mathbb{P}(B) &= \frac{4}{8} = \frac{1}{2}
\end{aligned}$$

We then see that:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

And therefore these events are independent.

What is the probability of A conditioned on B? We see that:

$$\begin{aligned}\mathbb{P}(A|B) &= \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \\ &= \frac{1/4}{1/2} \\ &= \frac{1}{2} \\ &= \mathbb{P}(A).\end{aligned}$$

Because these events are independent, we could have jumped to the last line, but it is nice to see that the derivation carries out.

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