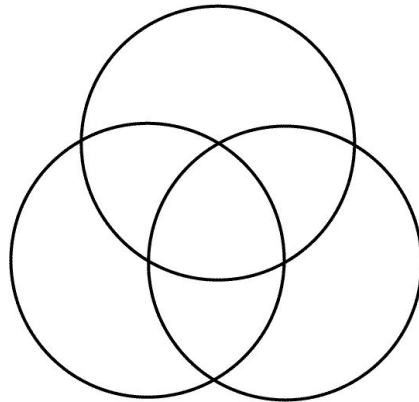


Worksheet 03

1. Let A , B , and C be three events in a random experiment with sample space S . Write expressions for each of the following sets in terms of the set operations “union,” “intersection,” “complement,” and “difference”:

- (a) only A occurs
- (b) A and B occur but C does not occur
- (c) exactly one of the events occurs
- (d) at least one of the events occurs
- (e) at most one of the events occurs
- (f) exactly two of the events occur
- (g) at least two of the events occur
- (h) at most two of the events occur
- (i) all three events occur
- (j) none of the events occur

2. Let A , B , and C be three overlapping sets. That is, their Venn diagram looks like this:



Consider the set $A \cup B \cup C$. Note that this union can alternatively be viewed as the union of seven disjoint sets. Label these seven sets using the set notation we have learned. Hint: For example, the middle area will be $A \cap B \cap C$.

3. Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the sets A , B , and C are given by $A = \{2, 4, 6, 8, 10\}$, $B = \{2, 5, 6, 7, 10\}$, and $C = \{1, 6, 9\}$. Identify each of the following sets:

- (a) $A \cup B$,
- (b) $A \cap B$,
- (c) $A - B$
- (d) $A \cup B^c$,
- (e) $A \cap B \cap C$,
- (f) $B \cap (A \cup C)^c$
- (g) $(A \cap C) \cup (B \cap C)$,
- (h) $(A - C) \cup (C - A)$,
- (i) $A^c \cap B \cap C^c$.

4. Show using Venn diagrams that the set $(A \cap B) - C$ is equal to the set $(A - C) \cap (B - C)$.

5. For arbitrary sets A and B, give a set theoretic proof that $A \cap B^c$ is equal to $A - B$.

6. For arbitrary sets A and B, prove that $A \cup B = A \cup (B - A)$.

7. Specify the sample space for the experiment consisting of three consecutive tosses of a fair coin. Using that model, compute the probability that you (a) obtain exactly one head, (b) obtain more heads than tails, (c) obtain the same outcome each time.

8. (Review) Certain members of the eight-member city council are feuding at present and absolutely refuse to work together on council projects. Specifically, Mr. T refuses to work with Ms. B, and Mr. U refuses to work with Dr. P. How many three-person committees can be formed (to serve as the city's Public Relations Task Force) that involve only council members willing to work together amicably?

9. (Review) Morse code is made up of dots and dashes. A given sequence of dots and dashes stands for a letter. For example, $- \cdot -$ might be one letter, and $\cdot \cdot - \cdot$ might be another. Suppose we are not interested in our own alphabet, but in a more general alphabet with more letters, and suppose we use a Morse code with at least one, and at most n , dots and dashes. How many different letters could be represented by such a code?