Worksheet 02

1. Assume for the sake of simplicity that the probability that someone is born in a particular month is 1/12. What is the probability that 6 randomly selected people do not share a birth month? In this case, try to simplify the result to a decimal version.

2. Three students get on a bus to downtown at the same time. The bus makes three stops once it arrives in downtown Richmond. If each student randomly decides which stop to disembark, what is the probability that everyone gets off at the same stop?

3. Assume for the sake of simplicity that the probability that someone has a particular birthday is 1/365. What is the probability that 28 randomly choosen people share a birthday?

4. Write the formula for the log-probability that k randomly selected elements from a set of n items (with replacement) have no 'matches'. In writing your result, use the *log-factorial* function $lf(n) = \log(n!)$. Why might taking logs be useful?

5. Queen Elizabeth I of England (1533− 1603) was fond of picnics. It is said that William Shakespeare attended several and that Sir Walter Raleigh attended most of them. An Elizabethan picnic consisted of the Queen and seven guests. The eight of them would sit on the ground in a circle, each buoyed by an overstuffed goose down pillow. The picnics were legendary for the sparkling conversation, but they were also famous as a culinary experience. Because the Queen always served herself first and then passed food to the right, sitting at the Queen's right was considered the greatest honor, and sitting as close to her right as possible was considered desirable. The phrases "right-hand man" and "left-handed compliment" might have originated at these picnics. How many different seating orders were possible at these picnics? Should Ben Jonson have been miffed because he never got closer than fourth place at the three picnics he attended?

6. Kindergarten teacher Monica Speller gave each of her students a box of twenty crayons on the first day of class. Miss Speller (an unfortunate moniker for a teacher) has told her students that she would like them to draw a new picture every day using exactly three different colors. She also wants the three colors chosen to be a different combination each day. Given that her kindergarten class will meet 150 times during the course of the year, is she asking the impossible? Anyone who knows a five-year-old will know that she is! But suppose she is willing to replace crayons as needed, and will personally keep track of the color combinations used so far. Are there at least 150 combinations possible?

7. Let's take a look at the lottery (in an imaginary principality referred to as "the State"). After the public has selected its numbers, the State, using a time-honored procedure involving air-blown ping pong balls, will identify the six numbers against which a player's chosen numbers must be matched. The State's numbers may be thought of as having been chosen at random from the integers 1, 2,...,52,53. The number of possible sixsomes that can be chosen is

$$\binom{53}{6} = \frac{53!}{6!47!} = 22,957,480.$$

You win five dollars when exactly three of your choices match up with the State's. What is the probability that you get exactly 3 numbers to match?

8. Back to poker, what is the probability that a randomly dealt hand of 5 cards yields a full house? That is, three cards of the same type and two cards of a different type.

9. Now, what is the probability of getting a three of a kind in poker, but not a full house or a four of a kind? That is, you have three cards of equal types by the other two are of different types.

10. Suppose that two evenly matched teams (say team A and team B) make it to the baseball World Series. The series ends as soon as one of the teams has won four games. Thus, it can end as early as the 4th game (a "sweep") or as late as the 7th game, with one team winning its fourth game compared to the other team's three wins. What's the probability that the series ends in 4 games, 5 games, 6 games, 7 games?