Worksheet 14

1. Let $X \sim Bin(n, p)$ and consider the estimator $\widehat{p} = X/n$. Find the variance, bias, and MSE of $\widehat{p}$.

2. Let $X \sim Bin(n, p)$ and consider the estimator $\widehat{p} = 1/2$. Find the variance, bias, and MSE of $\widehat{p}$.

3. Sketch a graph showing for which p the estimator from question 2 is better than the one from question 1 (in terms of MSE)? Note: You do not need to formally work out what the bounds are; it is doably but messy algebra.

4. Let $X \sim Bin(n, p)$. Consider:

$$\widehat{p} = \frac{x + a}{n + 2a}$$

Notice that this is like assuming that you have observered a 1's and a 0's before seeing any data. Find the Bias, Variance, and MSE of $\widehat{p}$. How does this compare to the result from question 1?

5. Let $X \sim Bin(2, p)$ and define $\theta = p^2$. Let $\widehat{\theta}$ be equal to $(X/2)^2$. Is $\widehat{\theta}$ unbiased?

6. Let $X \sim Gamma(\alpha, 1)$. Let $\widehat{\alpha}$ be equal to $n^{-1} \sum_i X_i$. Find the variance, bias, and MSE of $\widehat{\alpha}$.