A 1985 survey found that 29.8% of respondents were self-reported smokers. In this survey, 53% of the responses came from female, and only 14.5% were females who also smoked.

What is the chance that a randomly selected respondent is a male who smokes?

Let's define the events S (smokers), N (non-smokers), F (females), and M (males).

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$$\mathbb{P}(S \cap M).$$

To find this probability, let's write down one formula that we know from the probability axioms:

$$\mathbb{P}(S) = \mathbb{P}(S \cap M) + \mathbb{P}(S \cap F).$$

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Make sure you understand where this comes from! We know two of the quantities and can use basic arithmetic to solve for the third:

$$\begin{aligned} 0.298 &= \mathbb{P}(S \cap M) + 0.145 \\ \mathbb{P}(S \cap M) &= 0.153. \end{aligned}$$

And so the answer if 15.4%.

What is the chance that a randomly selected respondent is a male who does not smoke?

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We can use the same logic to deduce that:

$$\mathbb{P}(M) = \mathbb{P}(M \cap S) + \mathbb{P}(M \cap N).$$

Filling in the known quantities yields the result:

$$\begin{aligned} 0.470 &= 0.153 + \mathbb{P}(M \cap N) \\ \mathbb{P}(M \cap N) &= 0.317. \end{aligned}$$

And so the answer if 31.7%.

What is the chance that a randomly selected respondent is a female, a smoker, or both?

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Here we use the addition rule:

$$\begin{aligned}\mathbb{P}(S \cup F) &= \mathbb{P}(S) + \mathbb{P}(F) - \mathbb{P}(F \cap S) \\ &= 0.298 + 0.530 - 0.145 \\ &= 0.683\end{aligned}$$

And so the answer if 68.3%.

A useful way to think of these types of problems is as a table. Here is the table as we started:

	Smoker	Non-smoker	Total
Female	0.145	-	0.530
Male	-	-	-
Total	0.298	-	-

And here it is all filled in (using just basic arithmetic combined with the probability axioms):

	Smoker	Non-smoker	Total
Female	0.145	0.385	0.530
Male	0.153	0.317	0.470
Total	0.298	0.702	1.000