

Let $X_i \sim_{i.i.d.} \text{Bin}(n, p)$. Find the MMEs for n and p .

We solve this system of equations:

$$\bar{X} = \mathbb{E}X \tag{1}$$

$$s^2 = \text{Var}(X) \tag{2}$$

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Which simplifies to:

$$\bar{X} = np \tag{3}$$

$$s^2 = np(1 - p) \tag{4}$$

To solve this, set $n = \bar{X}/p$ and plug into the variance equation, solving for p :

$$s^2 = \bar{X}(1 - p) \quad (5)$$

$$s^2 - \bar{X} = \bar{X}p \quad (6)$$

$$\hat{p} = \frac{\bar{X} - s^2}{\bar{X}}. \quad (7)$$

And then:

$$\hat{n} = \frac{\bar{X}}{p} \quad (8)$$

Code to test this:

```
n <- 15  
p <- 0.3  
x <- rbinom(100, prob = p, size = n)  
xbar <- mean(x)  
s2 <- mean((x - xbar)^2)  
  
p_hat <- (xbar - s2) / xbar  
n_hat <- xbar / p_hat  
  
p_hat  
n_hat
```