Worksheet: 2016-01-09 (Solutions)

1. Write down all of the possible outcomes of flipping a coin three times. Use H for heads and T for tails. What is the probability that all of the flips have the same result (in other words, 3 heads or 3 tails)?

The possible options are: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }. There are four such outcomes. The probability that all are the same is given by: $2/8 = 1/4 = 0.25$.

2. A standard deck of cards has 52 cards with 4 suits. In poker, 5 cards are dealt to each player. A flush occurs when all of a player's cards are of the same suit. What is the probability that a player will be dealt a flush? Note: you can leave this unsimplified.

Using the basic rule of counting there are 52 possible outcomes of for the first cards, as every card could potentially be part of a flush. One decided though, the second card has to be of the same suit. As we already have on card from this suit there are $13 - 1 = 12$ possible options for the second card that preverse the straight. Similarly, there are 11 possible third cards, 10 possible fourth cards, and 9 possible fifth cards. So, there are $52 \cdot 12 \cdot 11 \cdot 10 \cdot 9$ ways of being dealt a flush.

How many total ways are there to be dealt cards? This is just $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$. Then the probability of a flush is simply:

$$\begin{aligned}
\mathbb{P}\,(\text{flush}) &= \frac{52 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \\
&= \frac{12 \cdot 11 \cdot 10 \cdot 9}{51 \cdot 50 \cdot 49 \cdot 48}
\end{aligned}$$

If we plug this in, the result is 0.001981, so only about two hands in one-thousand will be a flush.

3. Re-write your previous solution using only factorials.

We can re-write this as follows:

$$\begin{aligned}
\mathbb{P}\,(\text{flush}) &= \frac{12 \cdot 11 \cdot 10 \cdot 9}{51 \cdot 50 \cdot 49 \cdot 48} \\
&= \frac{12!}{8!} \cdot \frac{47!}{51!}.
\end{aligned}$$

4. Using the 26 letters in the latin alphabet, how many two-letter 'words' can you construct? These do not need to be actual words; just count the unique combinations.

Using the basic rule of counting, this is simply $26 \cdot 26 = 26^2$.

5. How many combinations are there if every word needs a vowel (a,e,i,o,u)? How many combinations are there if we also allow words that end in y (like 'my')?

There are two ways a word can be valid: either the first letter is a vowel or the second letter is a vowel. If we count these seperately and add, this will double count those words with two vowels. There are two ways of dealing with this. In both, we start by calculating how many ways a word can start with a vowel. This is equal to $5 \cdot 26$. We can now simply count those words that end in a vowel but do not start with one; this is just $21 \cdot 5$. This gives a total of:

$$\#\{\text{num. words}\} = 5 \cdot 26 + 21 \cdot 5 = 5 \cdot (47) = 235.$$

Similarly, we can double count those words that have two vowels and then subtract them off at the end:

$$\#\{\text{num. words}\} = 5 \cdot 26 + 5 \cdot 26 - 5 \cdot 5 = 5 \cdot (47) = 235.$$

These, thankfully, yield the same result.

6. How many 'words' are there if we have three letters, without any vowel restrictions? How about four letters? What about words of length k?

Three letter words will have 26^3 and four letter words will have 26^4 options. In generaly, with k length words, we have 26^k options.