Table of Distributions and Estimators

The following table provides the notation, density, mean, variance, and moment generating function of the eight main distributions we have studied.

Distribution	Notation	pmf/pdf	Mean	Variance	MGF
Bernoulli	$Bernoulli(p)$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$(1-p+pe^t)$
Binomial	$Bin(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^t)^n$
Geometric	$Geom(p)$	$(1-p)^{x-1}p$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$
Poisson	$Poisson(\lambda)$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^t-1)}$
Exponential	$Exp(\lambda)$	$\lambda e^{-\lambda x}$	λ^{-1}	λ^{-2}	$\frac{\lambda}{\lambda-t}$
Gamma	$Gamma(\alpha,\beta)$	$\frac{1}{\Gamma(\alpha)\beta^\alpha}\cdot x^{\alpha-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Beta	$Beta(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Normal	$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	μ	σ^2	$\exp\{\mu t+\frac{1}{2}\sigma^2t^2\}$

Below, are the MLE, MME and Bayesian estimators for several of these families of distributions. Some cases are missing when there is either no closed form solution or only a complex formula that we did not cover.

Distribution	MME	MLE	Prior	Posterior
Bernoulli	$\widehat{p}=\bar{X}$	$\widehat{p}=\bar{X}$	$p\sim Beta(\alpha,\beta)$	$p\|X\sim Beta(\alpha+\sum_i X_i,\beta+\sum_i(1-X_i))$
Geometric	$\widehat{p}=\bar{X}^{-1}$	$\widehat{p}=\bar{X}^{-1}$	$p\sim Beta(\alpha,\beta)$	$p\|X\sim Beta(\alpha+n,\beta+\sum_i X_i$
Poisson	$\widehat{\lambda}=\bar{X}$	$\widehat{\lambda}=\bar{X}$	$\lambda\sim Gamma(\alpha,\beta)$	$\lambda\|X\sim Gamma(\alpha+\sum_i X_i,\beta/(n\beta+1))$
Uniform	$\widehat{a}=\bar{X}-\sqrt{3}s$ $\widehat{b}=\bar{X}+\sqrt{3}s$	$\widehat{a}=\min_i X_i$ $\widehat{b}=\max_i X_i$		
Gamma	$\widehat{\alpha}=\bar{X}^2/s^2$ $\widehat{\beta}=s^2/\bar{X}$			
Normal	$\widehat{\mu}=\bar{X}$ $\widehat{\sigma^2}=s^2$	$\widehat{\mu}=\bar{X}$ $\widehat{\sigma^2}=\frac{1}{n}(X_i-\bar{X})^2$		