Worksheet 10

1. Let X be a random variable defined over the set $[0, b]$ for some $b > 0$ with a density function:

$$f(x) = C \cdot (b - x)$$

For some constant $C > 0$. Find the constant C that makes this a valid density function.

2. What are $\mathbb{E}X$ and $Var(X)$ for X as defined in question 1?

3. Let X be a continuous random variable with density $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and some fixed $\lambda > 0$. This is called the exponential distribution, which we can write $X \sim Exp(\lambda)$. What is the cumulative distribution $F(x)$? Find $\mathbb{P}[x \geq 1]$.

4. Find the MGF of the exponential distribution for $t < \lambda$.

5. If $X \sim Exp(\lambda)$, find $\mathbb{E}X$ and $Var(X)$.