Let X be a random variable defined by:

$$\mathbb{P}(X = 1) = 3/4$$
$$\mathbb{P}(X = -1) = 1/4$$

What are $\mathbb{E}X$, $\mathbb{E}X^2$, $Var(X)$?

The expected value is just:

$$\begin{aligned}\mathbb{E}X &= \mathbb{P}(X = 1) \cdot 1 + \mathbb{P}(X = -1) \cdot -1 \\ &= 3/4 - 1/4 \\ &= 1/2.\end{aligned}$$

The expected square, also called the second moment, is:

$$\begin{aligned}\mathbb{E}X^2 &= \mathbb{P}(X = 1) \cdot 1^2 + \mathbb{P}(X = -1) \cdot (-1)^2 \\ &= 3/4 + 1/4 \\ &= 1.\end{aligned}$$

Therefore, the variance is:

$$
\begin{aligned}
Var(X) &= \mathbb{E}X^2 - (\mathbb{E}X) \\
&= 1 - (1/2)^2 \\
&= 3/4.
\end{aligned}
$$

Suppose $X \sim Geom(1/3)$, what is the probability that X is greater than or equal to 2?

The probability that X is less than 2 is:

$$\begin{aligned}\mathbb{P}[X \leq 1] &= \mathbb{P}[X = 0] + \mathbb{P}[X = 1] \\ &= (1 - 1/3)^0 \cdot (1/3) + (1 - 1/3)^1 \cdot (1/3) \\ &= (1/3) + (2/9) \\ &= (5/9).\end{aligned}$$

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And so:

$$\begin{aligned}\mathbb{P}[X \geq 2] &= 1 - \mathbb{P}[X \leq 1] \\ &= (4/9).\end{aligned}$$

The Z score for a random variable X is given by:

$$Z = \frac{X - \mathbb{E}X}{\sqrt{Var(X)}}$$

The variable Z always has a variance of 1 and a mean of 0.

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If $X \sim Bin(100, 0.2)$. What is the Z-score of the value 5? How about 20, 21, or 90?

The Z-score for this binomial distribution is:

$$
\begin{aligned}
Z(X) &= \frac{X - np}{\sqrt{np \cdot (1-p)}} \\
&= \frac{X - 20}{\sqrt{20 \cdot 0.8}} \\
&= \frac{X - 20}{4}
\end{aligned}
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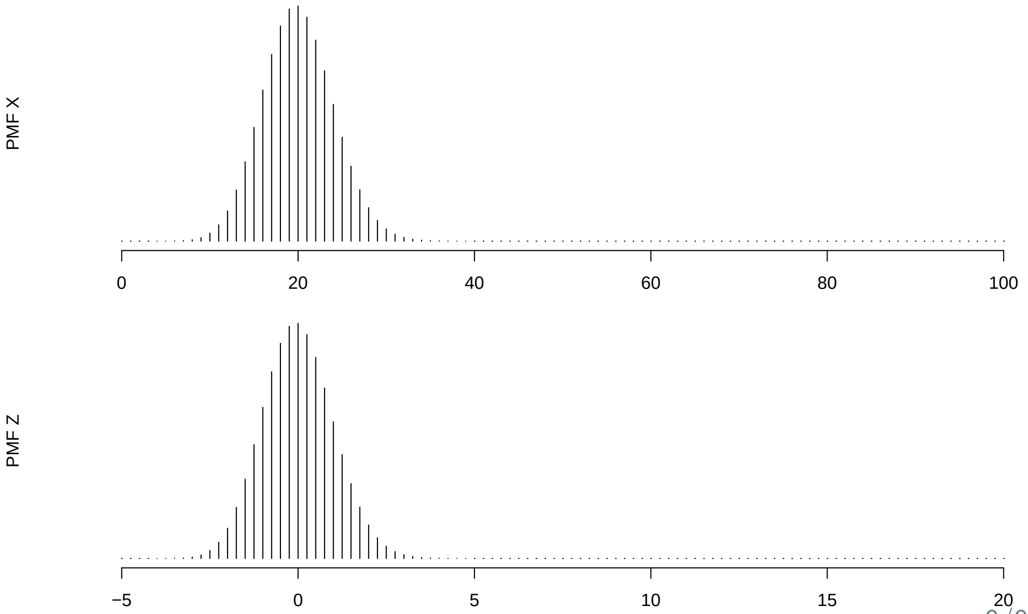
So, the Z scores are:

$$\begin{aligned} Z(5) &= \frac{5 - 20}{4} = -3.75 \\ Z(20) &= \frac{20 - 20}{4} = 0 \\ Z(21) &= \frac{21 - 20}{4} = 0.25 \\ Z(90) &= \frac{90 - 20}{4} = 17.5 \end{aligned}$$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