Worksheet 11

1. Let $X \sim Gamma(\alpha, \beta)$. Find $\mathbb{E}X^{1/2}$.

2. Let $X \sim N(\mu, \sigma^2)$. Show that $\mathbb{E}(X)$ is equal to μ using the moment generating function.

3. Let $X \sim N(\mu, \sigma^2)$. Show that $Var(X)$ is equal to σ^2 using the moment generating function.

4. Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ be independent random variables. Let $W = X+Y$. Show that W, as defined above, is a normally distributed random variable. Find its mean and variance.

5. Let $X \sim Gamma(\alpha_1, \beta)$ and $Y \sim Gamma(\alpha_2, \beta)$ for independent X and Y. Define $W = X+Y$. Find the distribution of W using moment generating functions.