Handout 15: Method of Moments

Today we explore a general procedures for constructing estimators to estimate the parameters of a probability model from observed data.

Method of Moments

When the random sample one draws from a population is large, it is not unreasonable to expect that the characteristics of the population will be fairly well approximated by the characteristics of the sample. This type of intuition gives us some confidence that the sample mean $\bar{X}$ will be a reliable estimator of the population mean μ when the sample size n is large and, similarly, that the sample proportion p will be a reliable estimator of the population proportion p when n is large. Taking advantage of the expected similarity between a population and the sample drawn from it often lies behind the parameter estimation strategy employed. The method of moments is an approach to parameter estimation that is motivated by intuition such as this. It is a method that produces, under mild assumptions, consistent, asymptotically normal estimators of the parameters of interest.

The moments of a probability distribution are, as we have discussed, defined as:

$$\mu_k = \mathbb{E}X^k$$

The sample moments are defined similarly:

$$m_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k.$$

The method of moments estimators simply solve the following system of equations for the first k moments:

$$\begin{aligned} \mu_1(\theta) &= m_1 \\ \mu_2(\theta) &= m_2 \\ &\vdots \\ \mu_k(\theta) &= m_k \end{aligned}$$

Where k is set to the number of parameters in the model. They are often easy to calculate both theoretically and numerically.