If $X \sim Geom(p)$, then the moment generating function is:

$$m_X(t) = \frac{p}{(1-(1-p)e^t)}.$$

What is the expected value of X?

We take the derivative of the moment generating function:

$$\begin{aligned}
\frac{\partial}{\partial t} m_X(t) &= \frac{\partial}{\partial t}\left(\frac{p}{(1-(1-p)e^t)}\right) \\
&= p \cdot (-1) \cdot (1-(1-p)e^t)^{-2} \cdot (p-1) \\
&= p \cdot (1-(1-p)e^t)^{-2} \cdot (1-p)
\end{aligned}$$

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Setting $t = 0$ we get the expected value:

$$\begin{aligned}\mathbb{E}X &= p(1-p) \cdot (p)^2 \\ &= \frac{1-p}{p}.\end{aligned}$$

Let X be a continuous random variable with a probability density function equal to kx^3 for all $x \in (0, 2)$ and some constant k. What does the value of k need to be?

We need the integral to be equal to one, so:

$$\begin{aligned} k^{-1} &= \int_0^2 x^3 \, dx \\ &= \left[\frac{x^4}{4} \right]_{x=0}^{2} \\ &= 4 \end{aligned}$$

So, $k = 0.25$.

What is the CDF of X?

For $z \in (0, 2)$ we have:

$$\begin{aligned} F(z) &= 0.25 \cdot \int_0^z x^3 \, dx \\ &= 0.25 \cdot \frac{z^4}{4} \\ &= \frac{1}{16} z^4 \end{aligned}$$

With $F(z) = 0$ for $z < 0$ and $F(z) = 1$ for $z > 2$.

What is $\mathbb{E}X$?

The expected value is:

$$\begin{aligned}\mathbb{E}X &= 0.25 \cdot \int_0^\infty x^4 \, dx \\ &= 0.25 \cdot \frac{2^5}{5} \\ &= 1.6.\end{aligned}$$

It should make sense that it is somewhere between 0 and 2 but closer to 2.