Worksheet 07

1. Let X be a random variable with the following probability mass function:

$$p_X(x) = \begin{cases} 0.6, & \text{if } x = 1 \\ 0.2, & \text{if } x = 2 \\ 0.2, & \text{if } x = 3 \end{cases}$$

Find $\mathbb{E}X$.

2. Let Y be a random variable with the following probability mass function:

$$p_Y(y) = \begin{cases} (1-p), & \text{if } y = 0 \\ p, & \text{if } y = 1 \end{cases}$$

For some $p \in [0, 1]$. Find (a) $\mathbb{E}Y$ and (b) $Var(Y)$. (c) Sketch a plot of $Var(Y)$ in terms of p. (d) What value of p maximizes the variance?

3. Let X be a random variable defined as follows:

$$p_X(x) = \frac{1}{n}, \quad x \in \{1, 2, \ldots, n\}$$

This is called a *discrete uniform distribution.* Calculate $\mathbb{E}X$ and simplify the result.

4. Let Z be a random variable uniformly distributed over the integers $\{-n, -(n-1), \ldots - 1, 0, 1, \ldots, n\}$. Calculate $\mathbb{E}Z$ using the transformation of variance theorem and your solution to the previous question.

5. Consider flipping a fair coin until it comes up heads. Let X be a random variable equal to the number of flips that are made. Calculate (a) $p_X(1)$, (b) $p_X(2)$, and (c) $p_X(3)$. (d) Write a general formula for $p_X(n)$. (e) Write down the quantity $\mathbb{E}X$. Notice that the summation is very difficult to simplify (you may leave it as is). (f) Write down the quantity $Var(X)$, for X defined as above. It is also very difficult to simplify.