Assume that $X_1, \ldots, X_4 \sim_{i.i.d.} N(\mu, \sigma^2)$**. Find a 99% confidence interval for** μ **if we observe** 1**,** 2**,** 3**, and** 4**.**

First, we need to get the mean of the data points. This is given by:

$$\begin{aligned}\bar{X} &= \frac{1}{n} \cdot \sum_i X_i \\ &= \frac{1}{4} \cdot (1 + 2 + 3 + 4) \\ &= 2.5\end{aligned}$$

Next, we need to compute an estimate of the value σ^2:

$$\begin{aligned}
s^2 &= \frac{1}{n-1} \sum_i (X_i - \bar{X})^2 \\
&= \frac{1}{3} \cdot \left((1-2.5)^2 + (2-2.5)^2 + (3-2.5)^2 + (4-2.5)^2\right) \\
&= \frac{5}{3} \\
&\approx 1.667.
\end{aligned}$$

Finally, we need to figure out $z_{\alpha/2}$ for $\alpha = 0.01$. That is, we need to find a value z such that:

$$\mathbb{P}\left[Z \geq z\right] = 0.01/2 = 0.005.$$

Or, equivalently:

$$\mathbb{P}\left[Z \leq z\right] = 1 - 0.005 = 0.995.$$

Reading off of the table this is 2.57.

Once we have these quantities, we simply plug them into the confidence equation:

$$
\begin{aligned}
&\bar{X} \pm \frac{s}{\sqrt{n}} \cdot z_{\alpha/2} \\
&2.5 \pm \frac{\sqrt{1.667}}{\sqrt{4}} \cdot 2.57 \\
&2.5 \pm 1.659
\end{aligned}
$$

Which we could re-write as:

$$[0.841, 4.160]\,.$$