Worksheet 13

1. Let $X \sim Gamma(\alpha, \beta)$ and $Y \sim Bin(n, p)$ be independent random variables. Find the following two quantities:

$$\mathbb{E}\left(2X + 5Y\right) = ?$$
$$Var\left(2X + 5Y\right) = ?$$

2. Let $X_s \sim Geom(\lambda/s)$ and $Y_s = X_s/s$. Find the moment generating function of Y_s.[1]

[1] I am using an s to denote the time variable so you do not confuse it with the t in the moment generating function.

3. Using the notation from the previous question, show that if $X_s \to X$, then $X \sim Exp(\lambda)$ by evaluating the limit of the moment generating function. Hint: Once simplifying, you may need the following limit:

$$\lim_{s \to \infty} s\left[e^{-t/s} - 1\right] = -t.$$

4. Let the arrival of students to my office hours follow a Poisson distribution with an average of 2 students arriving each hour. What is the probability that nobody comes during the first hour?