Let X be a random variable with probability mass function p given by:

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What is the expected value of $Y = X^2$?

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We will do this both directly and with the Transformation of Expected Value theorem.

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$$\begin{aligned} q(1) &= 1/2 \\ q(4) &= 1/4 \\ q(9) &= 1/4 \end{aligned}$$

The expected value is:

$$\begin{aligned} \mathbb{E}\, Y &= \sum_i q(i) \cdot i \\ &= q(1) \cdot 1 + q(4) \cdot 4 + q(9) \cdot 9 \\ &= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 9 \\ &= 3.75 \end{aligned}$$

We can get to the same result via the transformation of expected value, using $g(x) = x^2$:

$$\begin{aligned}
\mathbb{E}\, Y &= \sum_i p(i) \cdot g(i) \\
&= \sum_i p(i) \cdot i^2 \\
&= p(1) \cdot 1 + p(2) \cdot 2^2 + p(3) \cdot 3^2 \\
&= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 9 \\
&= 3.75
\end{aligned}$$

It should make a bit more sense now why the theorem holds.