

Please write your first and last name and Section here:

Name \_\_\_\_\_

**Instructions:**

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(a) What is the probability of exactly 1 computer shutdown in a month? (6 points)

**Answer:**

(b) What is the probability of at least 2 computer shutdowns during the next year? (6 points)

**Answer:**

(c) During the next year, what is the probability of at least 3 months (out of 12) with exactly 1 computer shutdown in each? Assume the number of shut downs per month are independent. (6 points)

**Answer:**

2. Suppose  $X$  and  $Y$  are two random variables on the same sample space  $\Omega$  and their joint pmf is given by this table:

		$X$		
		1	2	3
$Y$	2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
	3	$\frac{1}{6}$	0	$\frac{1}{6}$
	4	0	$\frac{1}{3}$	0

- (a) Show that  $X$  and  $Y$  are dependent. (8 points)

**Answer:**

- (b) Give a probability table (like we have for  $X$  and  $Y$ ) for random variables  $U$  and  $V$  that have the same marginal distributions as  $X$  and  $Y$  but are independent. (8 points)

**Answer:**

3. The random number generator on a certain calculator is not will chosen in that values it generates are not adequately described by a distribution uniform on the interval (0,1). Let  $X$  = the next value generated by the calculator's random number generator. Suppose the following probability density function (pdf) is a more appropriate model for  $X$ :

$$f_X(x) = \begin{cases} c(5 - x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show that  $c = \frac{2}{9}$  makes  $f_X(x)$  a valid pdf. (7 points)

**Answer:**

- (b) What is  $E(X)$ ? (8 points)

**Answer:**

- (c) Find  $P(X < .85)$ . Compare this value to what you would get if the calculator truly generated uniform(0,1) values, i.e. give the difference in probabilities. (10 points)

**Answer:**

4. Cows are sent to the slaughterhouse at an average rate of 5 per hour starting at some time  $t = 0$ .

- (a) What is the probability that it takes more than 20 mins for the first cow to be sent? (8 points)

**Answer:**

- (b) You arrive at time  $t = 2$  hours and are told that 11 cows have already been sent. How long should you expect to wait until the next cow enters the slaughterhouse? (8 points)

**Answer:**

- (c) Starting from some time  $t = 0$ , how long do you have to wait to observe the first cow being sent with a probability of 95%? (8points)

**Answer:**

5. The resistance of an assembly of several resistors connected in series is the sum of the resistance of the individual resistors. Suppose that a large lot of resistors has mean resistance  $\mu = 9.91$  ohms and standard deviation of resistances  $\sigma = .08$  ohms. Suppose that 30 resistors are randomly selected from this lot and connected in series. Let  $S$  = resistance of the assembly. Then  $S = X_1 + \dots + X_{30}$ , where  $X_i$  = resistance of the  $i$ th resistor.

- (a) Give the mean and standard deviation of  $S$ . (8 points)

**Answer:**

- (b) Approximate the probability that resistance of the assembly exceeds 298.2 ohms. State any results you use. (9 points)