**Fall 2014** **Statistics 330** **Exam II**
**(100 points)**

Please write your first and last name here:

**Name** _____

**Instructions:**

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. Suppose $X$ is a discrete random variable with probability distribution given in below table. Find the variance of $X$.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $p(x)$ | 0.25 | 0.1 | 0.3 | 0.1 | $a$ |

(a) What is $a$? (5 points)

(b) What are the expectation and variance of $X$? (10 points)

(c) What is $P(X \leq 0)$? (5 points)

2. You are given a binomial random distribution with 9 trails and probability of success on a single trial being 0.45.

   (a) What is the probability of getting more than 2 successes in 9 trails? (Keep at least 4 decimal places in your calculations.) (10 points)

   (b) What is the expectation and variance of this distribution? (5 points)

   (c) Instead of 9 trials, we repeat the experiment until the first success. Let $W$ be the number of trails. For instance, if the first success happened in the second trial, $W = 2$. What are the expectation and variance of $W$? (5 points)

3. Consider the following joint distribution for two random variables $X$ and $Y$:

| | $Y$ | | | |
|---|---|---|---|---|
| $X$ | 0 | 1 | 2 | 3 |
| 0 | 0.2 | 0.1 | 0.1 | 0 |
| 1 | 0.3 | 0.1 | 0 | 0.2 |

(a) Find the marginal distributions for $X$ and $Y$ and the $E[X]$ and $E[Y]$. (8 points)

(b) Find $Var[X]$, $Var[Y]$, and $\text{Cov}(X, Y)$. (10 points)

(c) Are $X$ and $Y$ independent? Why or why not? (2 points)

4. In an industrial setting, a hard drive has about a 15% chance of failing in a given year. Let $X$ be the failure time in years for an individual hard drive. If we assume $X$ has an exponential distribution, then $X \sim \text{Exp}(\lambda = 0.16)$.

   (a) What is the expected failure time for an individual hard drive? (5 points)

   (b) What is the probability that a hard drive lasts longer than 2 years? (5 points)

   (c) If a hard drive has already lasted 2 years, what is the probability it will last another year? (5 points)

   (d) Given the 15% chance of first year failure, derive $\lambda = 0.16$. (5 points)

5. A continuous random variable X has the probability density function (pdf)

$$f_X(x) = \begin{cases} cx & \text{if } 2 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

(If you cannot do parts a or b, you should still be able to do parts c and d.)

(a) Show that $c = 0.4$ makes $f_X(x)$ a valid pdf. (5 points)

(b) Show that $E[X] = 2.533$. (5 points)

(c) It turns out that, $Var[X] = 4$. Suppose $X_1, \ldots, X_{81}$ are iid from $f_X(x)$ and let $\overline{X} = \frac{1}{81}\sum_{i=1}^{81} x_i$. What is the approximate distribution of $\overline{X}$? Give the name of the distribution and the value(s) of parameter(s). (5 points)

(d) Approximate $P(\overline{X} < 2)$. (5 points)