Spring 2016 **Statistics 330** **Exam I**
(100 points)

Please write your first and last name and Section here:

Name ______________________________

Instructions:

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. Consider two events $A, B \subset \Omega$. If $P(A) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{4}$, can A and B be disjoint? If not, why not? (15 points)

2. A small company has 12 employees. 7 are programmers and 5 are graphic designers. Suppose a team of 4 is going to be selected for a project.

   (a) How many possible teams can be selected? (7 points)

   (b) If a team is selected randomly, what is the probability that there are exactly 2 programmers and 2 graphic designers on the team? (7 points)

   (c) If a team is selected randomly, what is the probability that there is at least 1 graphic designer on the team? (7 points)

3. Suppose that 5% of men and .25% of women are color-blind, and that we are randomly drawing people from a group with equal numbers of males and females.

   (a) What is the probability that a randomly chosen person is both male and color-blind? (10 points)

   (b) Suppose a person chosen at random is color-blind. What is the probability that this person is male? (10 points)

4. Let $c_1, \ldots, c_k$ denote the k components in a serial system. Assume the k components operate independently, and denote $P(c_j \text{ works})$ as p_j. Derive the formula for the reliability of the system in terms of the probabilities p_j. Clearly state which axioms or results you are using at each stage of the development. (15 points)
   Note: Giving only the formula here will result in *no credit*; we want you to show that you understand where the formula comes from and how it is derived from basic facts about probabilities.

5. In a large manufacturing run of PCs, let X be the number of minor flaws in a randomly chosen PC. Suppose the probability mass function for X is given by:

x	0	1	2	3	4
$p(x)$	.45	.35	.10	.05	.05

(a) What are the expectation and variance of X? (10 points)

(b) Suppose a PC is labeled "defective" if there are at least two minor flaws in it. For a randomly chosen PC we can define a random variable Y as:

$$Y = \begin{cases} 1 & X \geq 2 \\ 0 & X < 2 \end{cases}$$

For a randomly chosen PC, what is $P(Y = 1)$? (5 points)

(c) Suppose ten PCs are randomly selected. Let Z = the number of PCs that are "defective". Then $Z = \sum_{i=1}^{10} Y_i$, where each Y_i are independent and identically distributed from part b).

What distribution does Z follow? Give the name and relevant values. (6 points)

(d) What is the probability that in ten randomly chosen PCs, there is more than 1 "defective" PC? (8 points)