Fall 2014 **Statistics 330** **Exam I**
(100 points)

Please write your first and last name here:

Name ____

Instructions:

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. Baguette Me Not sells sandwiches with three different types of toppings: meats, cheeses, and condiments. For meats, the shop sells pork, turkey, and beef. For cheeses, the shop sells american, cheddar, provolone, and swiss. For condiments, the shop has ketchup, mustard, relish, hummus, and mayonnaise. Below assume that sandwich toppings are chosen randomly and independently of each other.

   (a) The shop allows one choice of meat, one choice of cheese, and one condiment. How many different sandwich combinations are there? (5 points)

   Answer: There are 3 choices of meats, 4 choices of cheeses, and 5 choices for condiments. Therefore there are $3 \times 4 \times 5 = 60$ different types of sandwiches.

   (b) What is the probability the sandwich will have pork? (7 points)

   Answer: The previous question provides $|\Omega| = 60$, i.e. the total number of sandwiches possible. Define A to be the event that the sandwich has pork. Then there are $|A| = 1 \times 4 \times 5 = 20$ different sandwiches that have pork. So $P(A) = |A|/|\Omega| = 20/60 = 1/3$.

   (c) What is the probability the sandwich will have provolone or swiss and ketchup or mustard? (8 points)

   Answer: Again, $|\Omega| = 60$. Define B to be the event that the sandwich has provolone or swiss and ketchup or mustard. Then $|B| = 3 \times 2 \times 2 = 12$ and thus $P(B) = |B|/|\Omega| = 12/60 = 1/5$.

2. Olympics

   (a) Twelve athletes compete in an archery event at the Olympics. How many ways are there to award the Gold, Silver, and Bronze medals to these athletes? (7 points)

   Answer: This is an ordered sample without replacement. There are

   $$P(12, 3) = \frac{12!}{(12-3)!} = 12 \times 11 \times 10 = 1320$$

   ways.

   (b) How many ways are there to award 3 medals if we do not care about the color of the medal? (7 points)

   Answer: This is an unordered sample without replacement. There are

   $$\binom{12}{3} = \frac{12!}{3!(12-3)!} = 220$$

   ways.

   (c) If we know the three individuals who got a medal, how many ways are there to distribute the Gold, Silver, and Bronze to these three individuals? (6 points)

   Answer: This is an ordered sample without replacement. There are

   $$P(3, 3) = \frac{3!}{(3-3)!} = 3! = 6$$

   ways.

3. You have torn a tendon and are facing surgery to repair it. The surgeon explains the risks to you: site infection occurs in 2% of such operations, the repair fails in 10%, and both infection and failure occur together in 1%.

   (a) What is the probability that a repair succeeds and there is no site infection? (12 points)

   Answer: Let I be the event of infection and R be the event of failed repair.

$$\begin{aligned} P(\bar{R} \cap \bar{I}) &= 1 - P(R \cup I) \\ &= 1 - (P(R) + P(I) - P(R \cap I)) \\ &= 1 - (0.1 + 0.02 - 0.01) \\ &= 0.89 \end{aligned}$$

   (b) If the repair succeeds, what is the probability that there is no site infection? (8 points)

   Answer:

$$P(\bar{I}|\bar{R}) = \frac{P(\bar{I} \cap \bar{R})}{P(\bar{R})} = \frac{0.89}{0.9} = 0.9889$$

4. At the Large Hadron Collider scientists are hunting for the Higgs Boson. The scientists have developed an alarm that rings when a Higgs Boson enters the collider during a particle collision. However, the alarm is not perfect. The alarm will turn on 90% of the time when a Higgs Boson enters the collider. Unfortunately, there is also a 1% probability the alarm will turn on even when no Higgs Boson enters the collider. Assume that the probability a Higgs Boson enters the device is 0.05.

   (a) What is the probability the alarm turns on during one particle collision? (12 points)

   Answer: Let A be the event that the Higgs Boson enters the detector and B be the event that the alarm turns on. From the question, we have $P(B|A) = 0.9$, $P(B|\bar{A}) = 0.01$ and $P(A) = 0.05$.

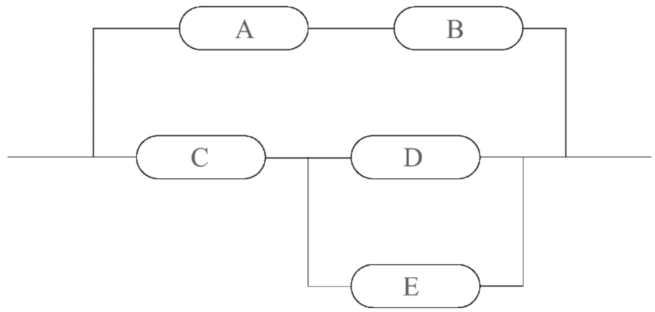
$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|\bar{A})P(\bar{A}) \\ &= (0.9)(0.05) + (0.01)(0.95) \\ &= 0.0545 \end{aligned}$$

   (b) If the alarm turned on, what is the probability that a Higgs Boson has entered the collider? (8 points)

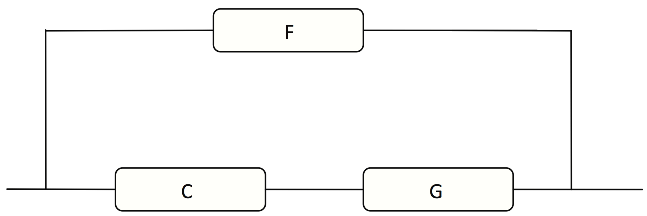
   Answer:

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{(0.9)(0.05)}{0.0545} \\ &= 0.8257 \end{aligned}$$

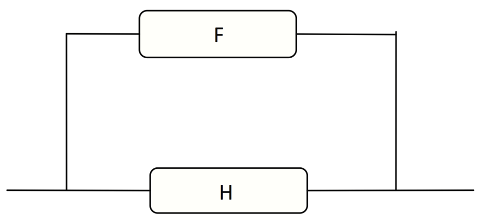
5. In the following system, the reliability of component A, B, C, D and E are 0.7, 0.8, 0.7, 0.8 and 0.9 respectively. Compute the system's reliability. (20 points)



Answer: Conbine A and B to form F, which has reliability $(0.7)(0.8) = 0.56$. Combine D and E to form G, which has reliability $1 - (1 - 0.8)(1 - 0.9) = 0.98$.



Now, further combine C and G to form H, which has reliability $(0.7)(0.98) = 0.686$.



The reliability of the whole system is $1 - (1 - 0.56)(1 - 0.686) = 0.86184$.