Spring 2016 **Statistics 330** **Exam III**
(100 points)

Please write your first and last name here:

Name ____

Instructions:

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. Suppose that for a particular breed of dog, an offspring of a black dog is black with probability .7 and brown with probability .3. An offspring of a brown dog is black with probability .1 and brown with probability .9.

   (a) Write down the 1-step transition matrix. (6 points)

   (b) Write down the 2-step transition matrix, $P^{(2)}$. (6 points)

   (c) Suppose a dog is brown. What is the probability that its grandchild will be a brown dog? (6 points)

(d) Give the steady state distribution of the dog color by solving $\pi P = \pi$ (6 points)

2. Suppose we have the following 3 by 3 transition matrix for states 1, 2, and 3.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} .4 & .3 & a \\ .5 & b & .5 \\ c & .2 & .6 \end{pmatrix} \end{array}$$

(a) Give the values for a, b and c. (6 points)

(b) The 2-step transition matrix $P^{(2)}$ is:

$$P^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} .37 & .18 & .45 \\ .30 & .25 & .45 \\ .30 & .18 & .52 \end{pmatrix} \end{array}$$

Give the probability of going from state 2 to state 1 in two steps. (4 points)

3. Suppose the mailing list at a website receives, on average, 50 spam messages per day. Assume the arrival of spam messages follows a homogeneous poisson process.

   (a) What is the expected number of spam messages after one week? (6 points)

   (b) What is the expected time between spam messages? (6 points)

   (c) What is the probability that the next spam message occurs within one hour? (6 points)

   (d) What is the expected waiting time until 5 spam messages arrive? (6 points)

4. Customers come to a teller's window according to a Poisson process with a rate of 10 customers every 25 minutes. Service times are Exponential. The average service takes 2 minutes. Using what you know about M/M/1 queuing systems, compute:

   (a) the traffic intensity value, a. (5 points)

   (b) the average number of customers in the system **and** the average number of customers waiting in a line. (7 points)

   (c) the fraction of time when the teller is busy with a customer. (5 points)

   (d) the fraction of time when the teller is busy and at least 2 other customers are waiting in a line. (7 points)

5. A computing server operates as an M/M/1/3 queue, with jobs arriving as a Poisson process at a rate of 3 per day. The service time of a job is exponentially distributed with a mean of 1/6 days.

   (a) Find the (steady state) probability that the server is busy (i.e. the server is not idle). (6 points)

   (b) What is the (steady state) average number of jobs in the server (including jobs in service and in the queue)? (6 points)

   (c) What is the (steady state) rate of arriving jobs being discarded? (6 points)