

4.19 We need to find $P(|X - \mu| \leq 3\sigma)$ for the given distributions of X .

(a) $P(|X - \mu| \leq 3\sigma) = P(|Z| \leq 3) = \Phi(3) - \Phi(-3) = 0.9987 - 0.0013 = \boxed{0.9974}$

(b) For the Uniform(a, b) distribution, $\mu = \frac{a+b}{2}$ and $\sigma = \frac{b-a}{\sqrt{12}}$. Therefore,

$$3\sigma = \frac{\sqrt{3}}{2}(b-a) > \frac{1}{2}(b-a),$$

so the interval $\mu \pm 3\sigma$ covers the entire range $[a, b]$ of X . Hence

$$P(|X - \mu| \leq 3\sigma) \geq P\left(|X - \mu| \leq \frac{1}{2}(b-a)\right) = P(a \leq X \leq b) = \boxed{1}$$