Matrix ~~and vector~~ multiplication:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} (a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}) & (a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32}) \\ (a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31}) & (a_{21}b_{22} + a_{22}b_{22} + a_{23}b_{32}) \end{bmatrix}$$

~~Simple Linear Regression in Matrix Notation~~:

$$\cancel{\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}}$$

$$\cancel{\underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X =}$$

Simple Linear Regression for obs. i:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\varepsilon_i \sim \text{Normal}(0, \sigma^2)$$

Simple linear regression in matrix notation:

$$\left.\begin{aligned} y_1 &= \beta_0 + \beta_1 x_1 + \varepsilon_1 \\ y_2 &= \beta_0 + \beta_1 x_2 + \varepsilon_2 \\ &\vdots \\ y_n &= \beta_0 + \beta_1 x_n + \varepsilon_n \end{aligned}\right\} \longrightarrow \underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}, \text{ where}$$

$$\underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \quad \underset{\sim}{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \quad \underset{\sim}{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Note:

$$X\underset{\sim}{\beta} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1 \\ \beta_0 + \beta_1 x_2 \\ \vdots \\ \beta_0 + \beta_1 x_n \end{bmatrix}$$

$$X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon} = \begin{bmatrix} \beta_0 + \beta_1 x_1 + \varepsilon_1 \\ \beta_0 + \beta_1 x_2 + \varepsilon_2 \\ \vdots \\ \beta_0 + \beta_1 x_n + \varepsilon_n \end{bmatrix}$$

so $y = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$ means

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1 + \varepsilon_1 \\ \beta_0 + \beta_1 x_2 + \varepsilon_2 \\ \vdots \\ \beta_0 + \beta_1 x_n + \varepsilon_n \end{bmatrix}$$

Another version of matrix notation (probability form):

$$\underset{\sim}{y} \sim \text{Normal}\left(X\underset{\sim}{\beta},\ \sigma^2\right)$$

- emphasizes that $\underset{\sim}{y}$ is random: relationship between $\underset{\sim}{y}$ and $\underset{\sim}{x}$ is not deterministic!