Stat 343: Common Discrete Distributions

Bernoulli(p)

X = the result of a single experiment with one of two outcomes ("success", coded as 1, or "failure", coded as 0), where the probability of success is p.

parameters
$$p = \text{probability of success}$$

p.f. $f(x|p) = p^x (1-p)^{(1-x)}$
Mean p
Variance $p(1-p)$

$\mathbf{Binomial}(n, p)$

X = the total number of successes in n independent and identically distributed Bernoulli trials, each with probability of success p.

parameters
$$n = \text{number of trials}, p = \text{probability of success}$$

p.f. $f(x|n,p) = \binom{n}{x} p^x (1-p)^{(n-x)}$
Mean np
Variance $np(1-p)$

$\mathbf{Uniform}(a,b)$

X = an integer between a and b (inclusive), where each integer from a to b is equally likely.

parameters
$$a = \text{lower endpoint}, b = \text{upper endpoint}$$
p.f. $f(x|a,b) = \frac{1}{b-a+1}$
Mean $\frac{a+b}{2}$
Variance $\frac{(b-a)(b-a+2)}{12}$

$\mathbf{Geometric}(p)$

X = the number of failures that occur before the first success in a sequence of independent and identically distributed Bernoulli trials.

parameters
$$p=$$
 probability of success on each trial p.f. $f(x|p)=p(1-p)^x$ Mean $\frac{1-p}{p}$ Variance $\frac{1-p}{p^2}$

Negative Binomial(r, p)

X = the number of failures which occur in a sequence of independent and identically distributed Bernoulli trials before r successes occur.

parameters
$$r=$$
 target number of successes, $p=$ probability of success on each trial p.f. $f(x|r,p)=\binom{r+x-1}{x}p^r(1-p)^x$
Mean $\frac{r(1-p)}{p}$
Variance $\frac{r(1-p)}{p^2}$

${\bf Hypergeometric}(A,B,n)$

X = the number of successes in n draws (without replacement) from a finite population that contains exactly A successes and B failures.

parameters
$$A=$$
 number of successes in the population, $B=$ number of failures in the population, $n=$ sample size
$$\text{p.f.} \quad f(x|A,B,n) = \frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$$
 Mean
$$\text{Variance} \quad \frac{nA}{(A+B)^2} \frac{A+B-n}{A+B-1}$$

Note that for R functions dhyper, phyper, and rhyper, the arguments are m = A, n = B, and k = n.

$\mathbf{Poisson}(\lambda)$

X = the number of events occurring in a fixed interval of time or space if these events occur with a known constant rate and independently of the time since the last event.

$$\begin{array}{ll} \text{parameters} & \lambda = \text{rate parameter} \\ \text{p.f.} & f(x|\lambda) = e^{-\lambda} \frac{\lambda^x}{x!} \\ \text{Mean} & \lambda \\ \text{Variance} & \lambda \end{array}$$