



**Forward Propagation:** Start with  $x^{(i)<1>}$  and  $a^{(i)<0>[1]}$ , work forwards in time

$$a^{(i)<t>[1]} = g^{[1]}(z^{(i)<t>[1]})$$

$$z^{(i)<t>[1]} = b^{[1]} + W_a^{[1]}a^{(i)<t-1>[1]} + W_x^{[1]}x^{(i)<t-1>}$$

**Backward Propagation:** Start with  $J^{(i)<T_x^{(i)}>}(b, w)$ , work backwards in time

For time  $t$ ,  $J^{(i)}(b, w) = \sum_{t=1}^{T_x^{(i)}} J^{(i)<t>}(b, w)$  depends on  $a^{(i)<t>[1]}$  through  $a^{(i)<t>[2]}$  and  $a^{(i)<t+1>[1]}$   
same time, next layer up next time, same layer

$$\text{Therefore, } \frac{\partial J^{(i)}}{\partial a^{(i)<t>[1]}} = \frac{\partial J^{(i)}}{\partial a^{(i)<t>[2]}} \frac{\partial a^{(i)<t>[2]}}{\partial a^{(i)<t>[1]}} + \frac{\partial J^{(i)}}{\partial a^{(i)<t+1>[1]}} \frac{\partial a^{(i)<t+1>[1]}}{\partial a^{(i)<t>[1]}}$$

$J^{(i)}(b, w)$  depends on  $W_x^{[1]}$  through  $a^{(i)<1>[1]}, \dots, a^{(i)<T_x^{(i)}>[1]}$

$$\text{Therefore, } \frac{\partial J^{(i)}}{\partial W_x^{[1]}} = \sum_{t=1}^{T_x^{(1)}} \frac{\partial J^{(i)}}{\partial a^{(i)<t>[1]}} \frac{\partial a^{(i)<t>[1]}}{\partial W_x^{[1]}} \quad (\text{a similar idea holds for } b^{[1]} \text{ and } W_a^{[1]})$$