Your Turn

We want to use the following distribution for inference, where we know the shape, but not the full distributional form.

$$f(x) = c\frac{\log(x)}{1+x^2}, \quad x \in [1, \pi]$$

What do we need for this to be a valid pdf?

We need $\int_1^\pi f(x)\,dx = \int_1^\pi c \frac{\log(x)}{1+x^2}\,dx = 1$

$$\int_1^\pi \frac{\log(x)}{1+x^2}\,dx = \frac{1}{c}$$

1. estimate c using MC integration.

   using $f \sim \text{Unif}(1, \pi)$, $m = 10000$.

   ⇓

   1. find g (write integral as an expected value)
   2. make plan (write pseudo code)
   3. do it! (write code)

      ↳ easy to also estimate $\text{Var}(\hat{\theta})$.

$f(x) =$ pdf unif $(1,\pi)$

$$\int_1^\pi \frac{\log(x)}{1+x^2}\,dx$$

$$= \int_1^\pi \frac{\log(x)}{1+x^2} \frac{\frac{1}{\pi-1}}{\frac{1}{\pi-1}}\,dx$$

$$= \int_1^\pi \underbrace{\frac{\log(x)}{1+x^2}\cdot(\pi-1)}_{g(x)}\cdot\underbrace{\frac{1}{\pi-1}}_{f(x)}\,dx$$

$$\Rightarrow g(x) = \frac{\log(x)}{1+x^2}\cdot(\pi-1)$$

2. Importance sampling using

   $\phi \sim N(1,1)$

   → 1. write our integral as expected value wrt ϕ.

   → write out integral incorporating importance weights.

   2. make plan (write pseudo code)
   3. do it!

3. $$f(x) = \frac{\frac{\log(x)}{1+x^2}}{\int_1^\pi \frac{\log(x)}{1+x^2}\,dx} \text{ for } x \in [1, \pi].$$

   have $\hat{\theta}$

How to sample?

Accept-reject!

need to choose $e(x)$ envelope!

let $e(x) = d \cdot \underbrace{g(x)}_{}$, $\frac{1}{\pi-1}$ $x \in [1, \pi]$. let $g(x) \sim \text{Unif}[1, \pi]$.

find d based on $\boxed{\text{max } f(x) \text{ between } [1, \pi]}$

need $\underline{e(x) \geq f(x)}$ $\forall x$.

① Find $e(x)$

② get $n=$ 1000 samples from $f(x)$ using Accept - reject method.