

## 1 EKF - Tracking

A logistic growth model with rate  $r$  and carrying capacity  $k$  can be written as

$$\frac{dp}{dt} = rp \left(1 - \frac{p}{k}\right)$$

with initial guess  $p_0$ , the logistic growth model can be analytically solved as.

$$p = \frac{kp_0 \exp(rt)}{k + p_0(\exp(rt) - 1)}$$

A population data is modeled based on the analytical solution of logistic growth model with additive random process error

$$p_t = \frac{kp_{t-1} \exp(r\Delta t)}{k + p_{t-1}(\exp(r\Delta t) - 1)} + v$$

An extended Kalman Filter is used to track the population given that the variance of process and observation error is known. state space is assumed as  $x = [r \ p]^T$  and observation model is given as

$$z = [0 \ 1][r \ p]^T + w$$

In the given code the sample data is synthetically generated and tracking algorithm is implemented.

### 1.1 Exercise 1

In the sample code only population in the state space is assumed to have additive process error. Modify the code such that rate of population growth also has additive process error.

### 1.2 Exercise 2

Implement Kalman filter tracking algorithm for the same logistic population growth by using Euler's explicit time stepping scheme.

$$p_t = p_{t-1} + \Delta t \left( rp_{t-1} \left(1 - \frac{p_{t-1}}{k}\right) + v \right)$$

## 2 EKF - Parameter Estimation

A sample code for estimating parameter  $a$  in the equation

$$y = a^2 x^2 + x + 1$$

is given in the sample code. The data is synthetically generated to observe the error of estimated parameter from the true value of the parameter.

### 2.1 Exercise 3

Estimate the initial velocity ( $v_0$ ) of the projectile for the given data in projectile.txt using initial angle as  $\theta = \pi/4$ . Projectile motion equation is

$$x = v_0 t \cos(\theta)$$

$$y = v_0 t \sin(\theta) - \frac{gt^2}{2}$$