1 EKF - Tracking

A logistic growth model with rate r and carrying capacity k can be written as

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{k}\right)$$

with initial guess p_0, the logistic growth model can be analytically solved as.

$$p = \frac{kp_0 \exp(rt)}{k + p_0(\exp(rt) - 1)}$$

Population data is modeled based on the analytical solution of the logistic growth model with additive random process error

$$p_t = \frac{kp_{t-1} \exp(r\Delta t)}{k + p_{t-1}(\exp(r\Delta t) - 1)} + v$$

An extended Kalman Filter is used to track the population given that the variance of process and observation error is known. The state space is assumed to be $x = [r\, p]^T$ and observation model is given as

$$z = [0\ 1][r\ p]^T + w$$

In the given code the sample data is synthetically generated and tracking algorithm is implemented.

1.1 Exercise 1

In the sample code only the population size is assumed to have additive process error. Modify the code such that rate of population growth also has additive process error.

1.2 Exercise 2

Implement the Kalman filter tracking algorithm for the same logistic population growth by using Euler's explicit time stepping scheme.

$$p_t = p_{t-1} + \Delta t\left(rp_{t-1}\left(1 - \frac{p_{t-1}}{k}\right) + v\right)$$

2 EKF - Parameter Estimation

A sample code for estimating parameter a in the equation $y = a^2x^2 + x + 1$ is given in the sample code. The data is synthetically generated to observe the error of estimated parameter from the true value of the parameter.

2.1 Exercise 3

Estimate the initial velocity (v_0) of the projectile for the given data in `projectile.txt` using the initial angle as $\theta = \pi/4$. The equation of the projectile's motion is

$$x = v_0 t \cos(\theta)$$
$$y = v_0 t \sin(\theta) - \frac{gt^2}{2}$$