MATH 60604A Exercice 3

3.1 We consider three nested regression models for modelling the number of car accidents according to region (region). The variables risk class (risk) has 3 categories and the number of years of driving experience (exp) is broken down into 4 categories.

Model	variables	<i>p</i> + 1	$\ell(\widehat{m{eta}})$	AIC	BIC
M_1	risk	3	-244.566	495.132	510.362
M_2	risk+region	*	-151.620	*	*
M_3	risk + region + exp	10	-139.734	299.468	350.235

Table 1: Goodness-of-fit measures for three nested regression models with the number of parameters in each model (p+1), the value of the log-likelihood function evaluated at the maximum likelihood estimate $(\ell(\widehat{\pmb{\beta}}))$ and information criteria.

What is the difference between AIC and BIC of Model M₂ (in absolute value)?

3.2 A random variable X follows a geometric distribution with parameter p if its probability mass function is

$$P(X = x) = (1 - p)^{x-1}p, \qquad x = 1, 2, ...$$

- (a) Write the likelihood and the log-likelihood of the *n* sample.
- (b) Derive the maximum likelihood estimator for the parameter p.
- (c) Compute the observed information matrix.
- (d) Suppose we have a sample of 15 observations, {5,6,3,7,1,2,11,8,7,34,1,7,10,1,0}, whose sum is 103. Compute the maximum likelihood estimate and its approximate standard error.
- (e) Compute the likelihood ratio and the Wald test statistics. Perform a test at level 5% for \mathcal{H}_0 : $p_0 = 0.1$ against the two-sided alternative \mathcal{H}_a : $p_0 \neq 0.1$.
- 3.3 We consider the failure time of an engine based on its level of corrosion w. Failure time T is modelled with an exponential distribution with density $f(t) = \lambda \exp(-\lambda t)$, but whose rate parameter $\lambda = aw^b$; if b = 0, the failure time is a^{-1} . Assume we have an n sample of independent observations with w_i assumed known. [Coles (2001)]
 - (a) Write down the log-likelihood of the model.
 - (b) Derive the observed and the Fisher information matrices.
 - (c) Show that the profile log-likelihood for *b* is

$$\ell_{\mathsf{p}}(b) = n \ln(\widehat{a}_b) + b \sum_{i=1}^{n} \ln(w_i) - \widehat{a}_b \sum_{i=1}^{n} w_i^b t_i,$$

and give an explicit formula for the partial maximum likelihood estimator \hat{a}_b .

page 1 of 1 Compiled 04/10/2020 at 22:45