

Today

- Two-Way Between-Subjects Factorial Designs
 - 2 x 2 design
 - concept of interaction
 - model comparison approach
 - controlling type-I error
 - follow-up tests

The 2 x 2 Design

- hypothetical study:
- explore effects of biofeedback and drug therapy on blood pressure
- one approach could be:
 - 1 factor, four groups:
 - (1) biofeedback + drug
 - (2) biofeedback, no drug
 - (3) no biofeedback + drug
 - (4) no biofeedback, no drug

TABLE 7.1 Blood Pressure Data for 2×2 Factorial Design

	Group			
	1: Biofeedback and Drug	2: Biofeedback Alone	3: Drug Alone	4: Neither
	158	188	186	185
	163	183	191	190
	173	198	196	195
	178	178	181	200
	168	193	176	180
Mean	168	188	186	190
s	7.9057	7.9057	7.9057	7.9057

TABLE 7.2 ANOVA for Data in Table 7.1

Source	SS	df	MS	F	p
Between	1540.00	3	513.33	8.21	.002
Within	1000.00	16	62.50		
Total	2540.00	19			

- $[+1 +1 -1 -1]$: effect of biofeedback: $F=8.00$, $p < .05$
- $[+1 -1 +1 -1]$: effect of drug: $F=11.52$, $p < .05$
- our conclusion would be that
 - both drug and biofeedback have an effect

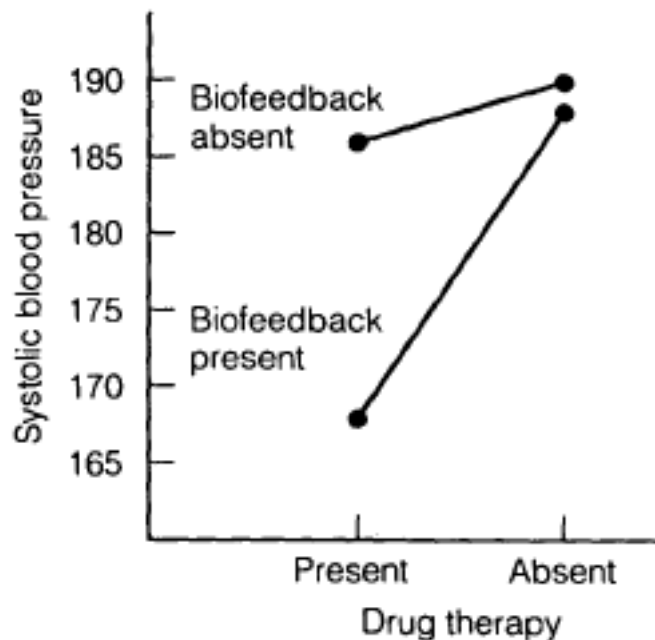
TABLE 7.3 Factorial Arrangement of Means from Table 7.1

		Biofeedback		Average
		<i>Present</i>	<i>Absent</i>	
Drug Therapy	<i>Present</i>	168	186	177
	<i>Absent</i>	188	190	189
	Average	178	188	183

- effect of drug therapy, averaged over levels of biofeedback
 - Present: 177
 - Absent: 189
 - $F=11.52, p < .05$; drug therapy has an effect on blood pressure
- effect of biofeedback, averaged over levels of drug therapy
 - Present: 178
 - Absent: 188
 - $F=8.00, p < .05$; biofeedback has an effect on blood pressure
- Is this an accurate representation of what's going on here?
- no! both main effects are driven by one cell
 - drug therapy + biofeedback

TABLE 7.3 Factorial Arrangement of Means from Table 7.1

		Biofeedback		Average
		Present	Absent	
Drug Therapy	Present	168	186	177
	Absent	188	190	189
	Average	178	188	183

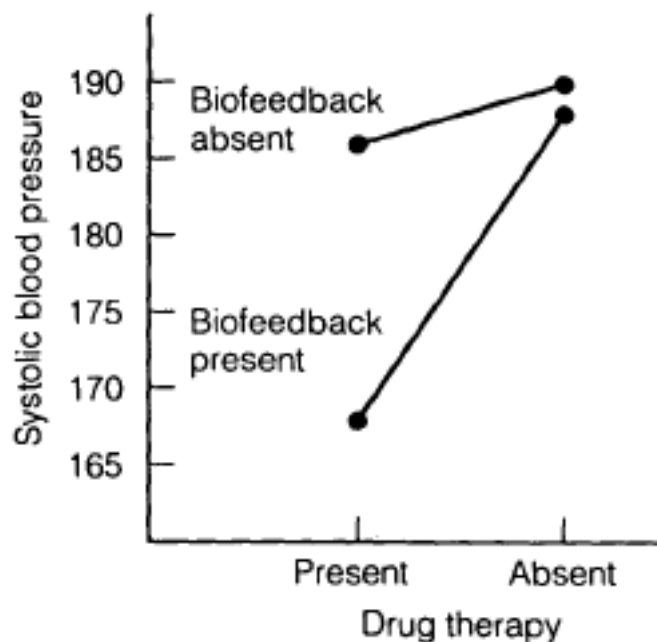


(a)

- there is an **interaction** between drug therapy and biofeedback
 - effect of drug therapy depends on the level of the biofeedback factor
 - effect of biofeedback depends on the level of the drug therapy factor
 - the level of biofeedback modulates the effect of drug therapy
 - the level of drug therapy modulates the effect of biofeedback

TABLE 7.3 Factorial Arrangement of Means from Table 7.1

		Biofeedback		Average
		Present	Absent	
Drug Therapy	Present	168	186	177
	Absent	188	190	189
	Average	178	188	183



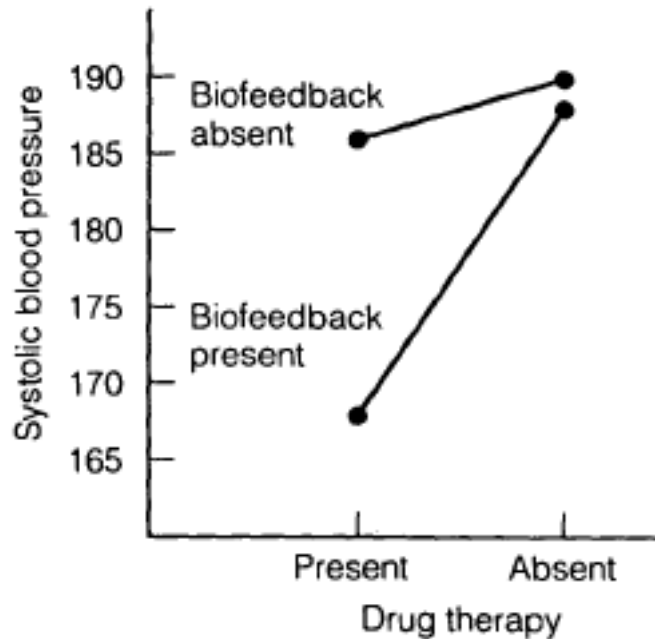
OR

- there is an **interaction** between drug therapy and biofeedback
 - effect of drug therapy depends on the level of the biofeedback factor
 - effect of biofeedback depends on the level of the drug therapy factor
 - the level of biofeedback modulates the effect of drug therapy
 - the level of drug therapy modulates the effect of biofeedback

(a)

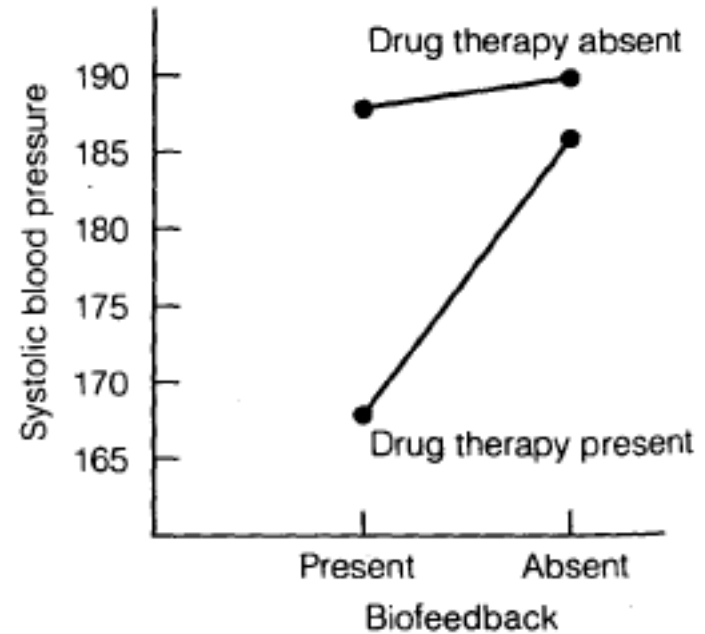
TABLE 7.3 Factorial Arrangement of Means from Table 7.1

		Biofeedback		Average
		Present	Absent	
Drug Therapy	Present	168	186	177
	Absent	188	190	189
	Average	178	188	183



(a)

OR

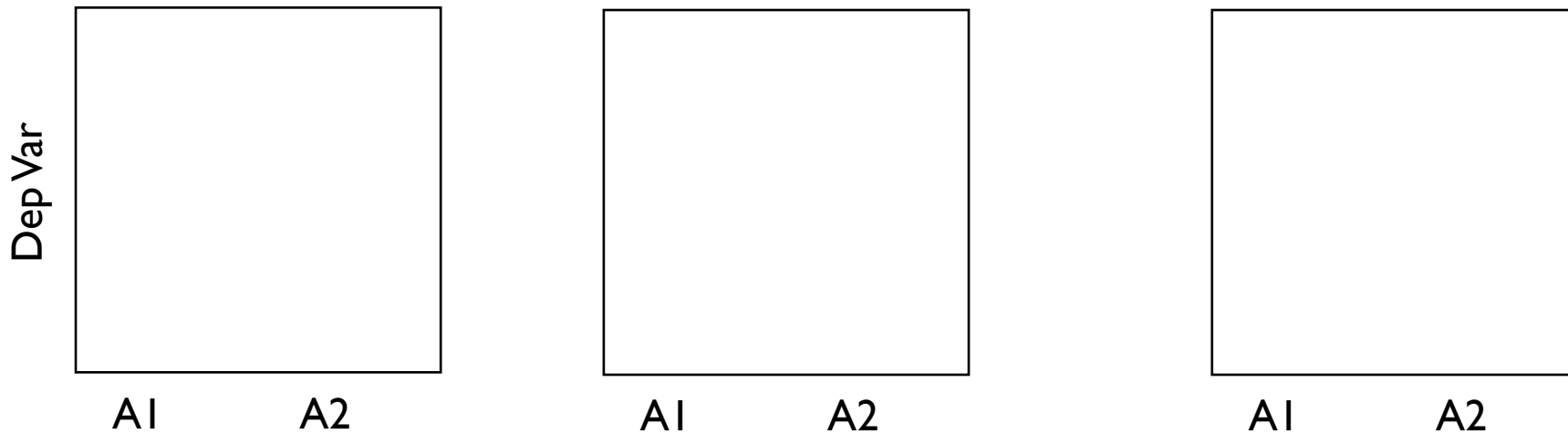


(b)

- there is an **interaction** between drug therapy and biofeedback
 - effect of drug therapy depends on the level of the biofeedback factor
 - effect of biofeedback depends on the level of the drug therapy factor
 - the level of biofeedback modulates the effect of drug therapy
 - the level of drug therapy modulates the effect of biofeedback

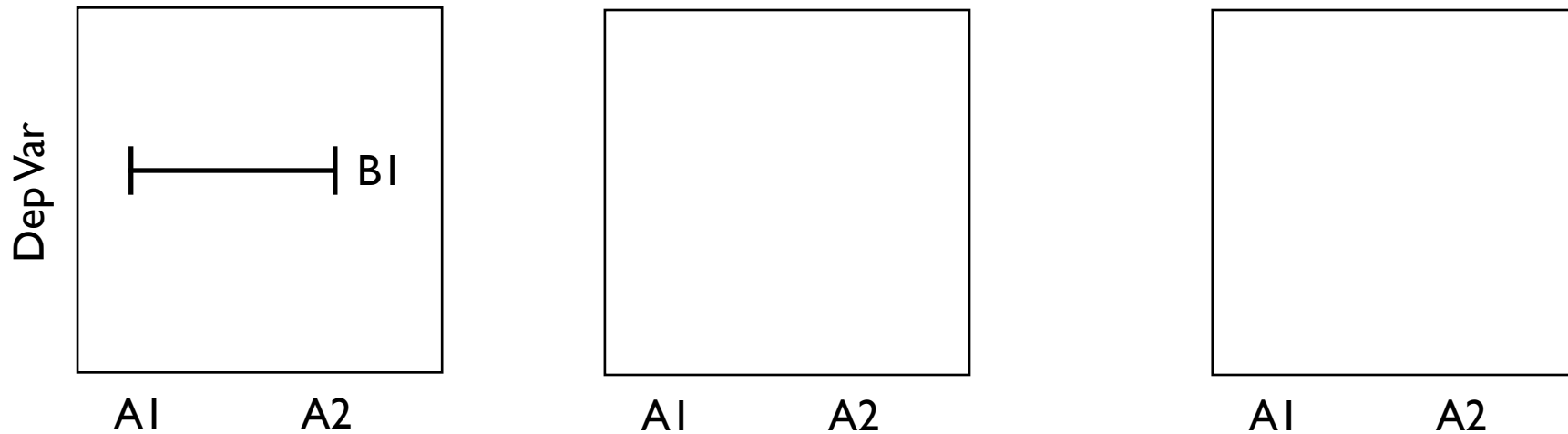
Main Effects

- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



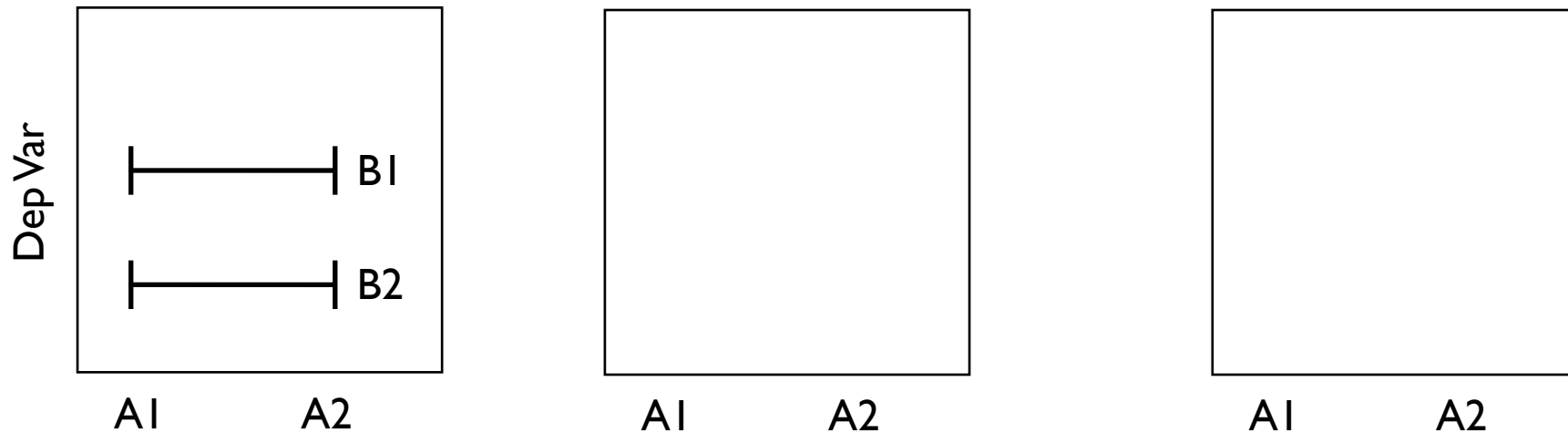
Main Effects

- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



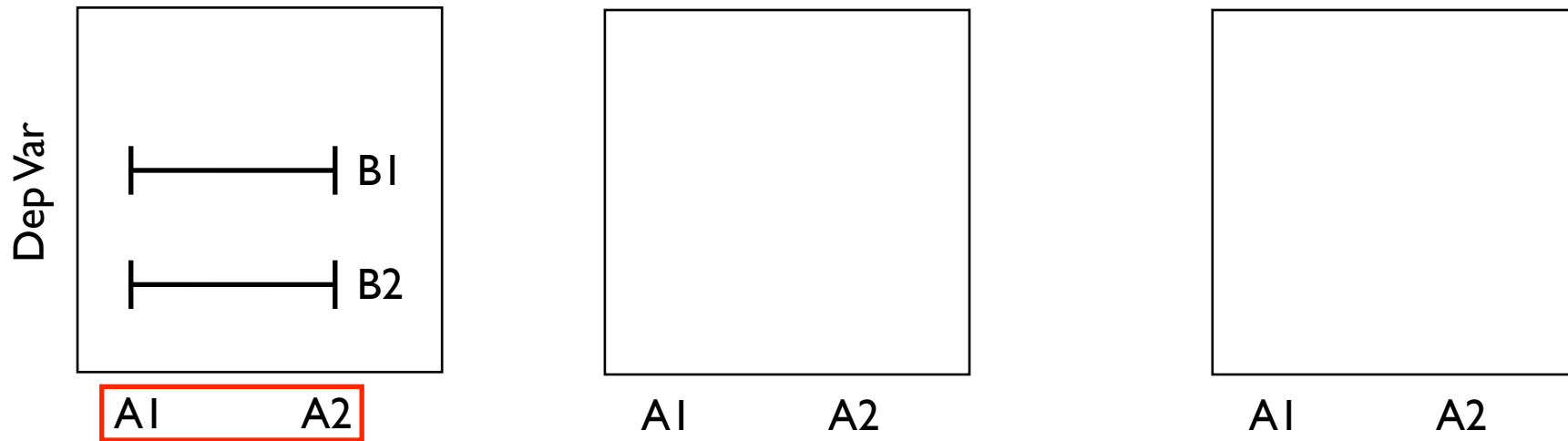
Main Effects

- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



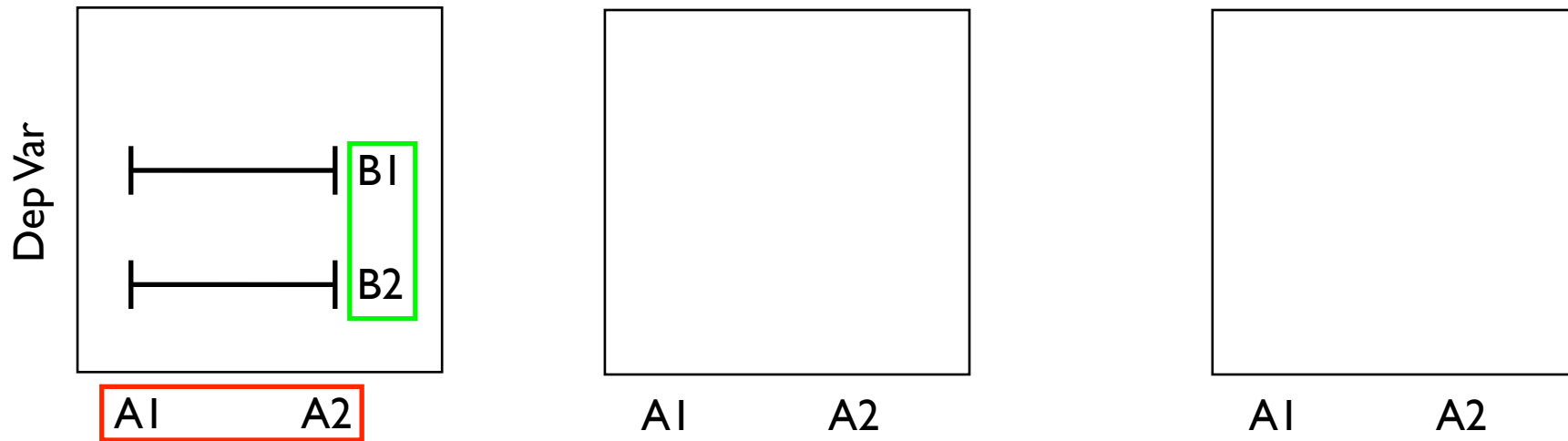
Main Effects

- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



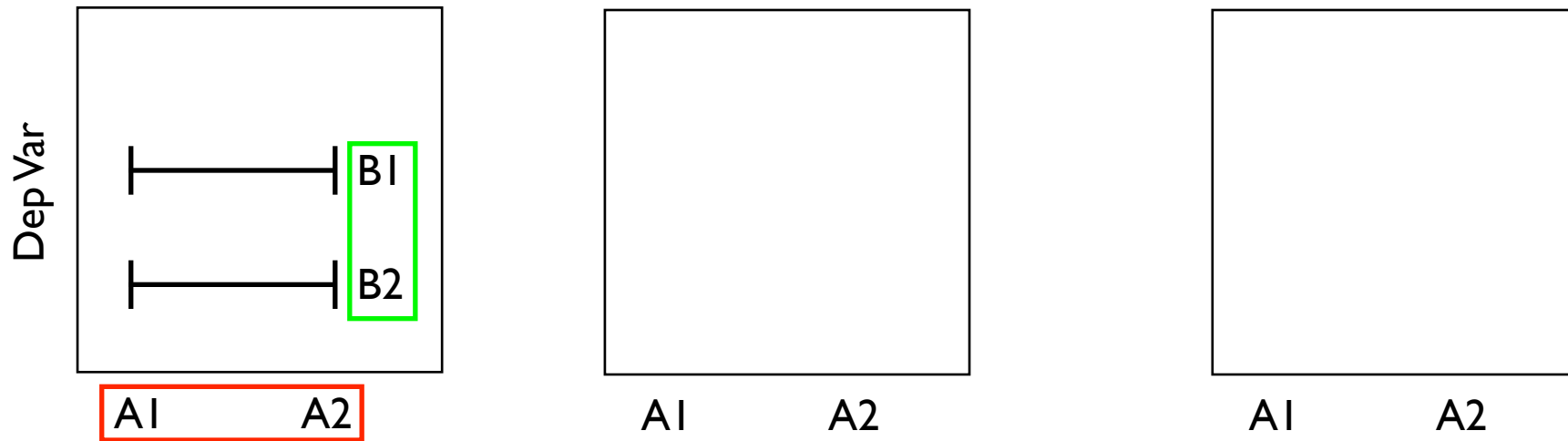
Main Effects

- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main Effects

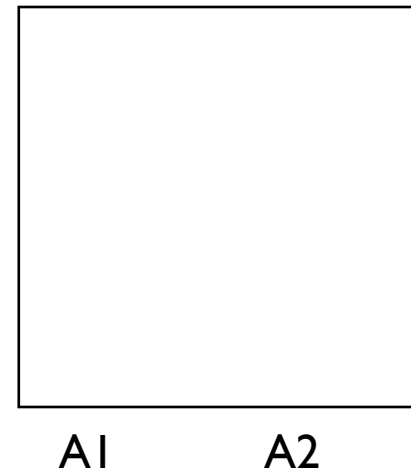
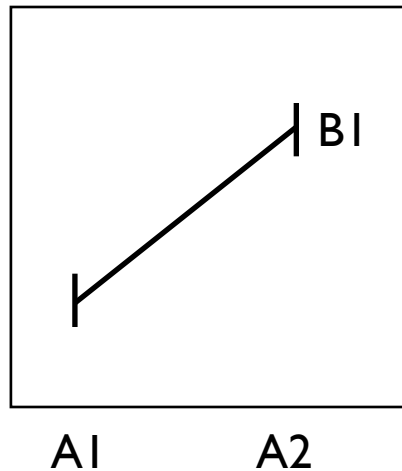
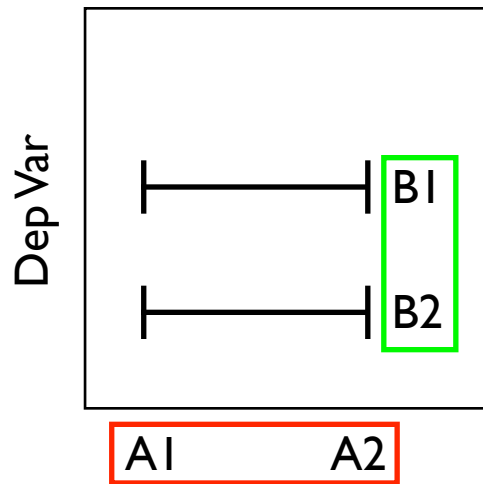
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

Main Effects

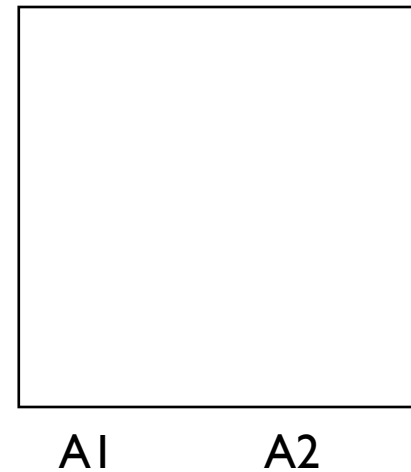
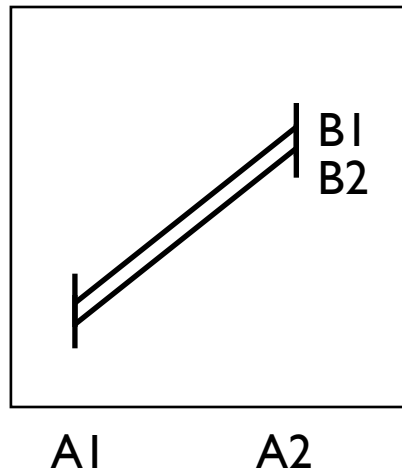
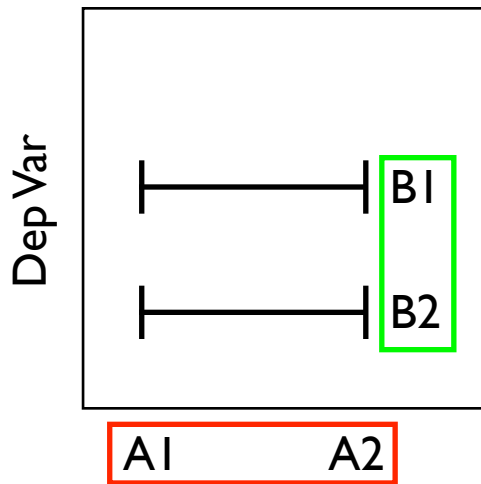
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

Main Effects

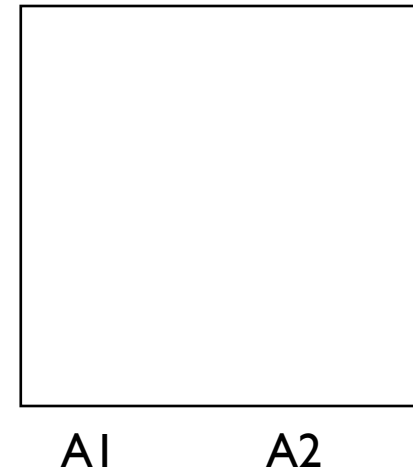
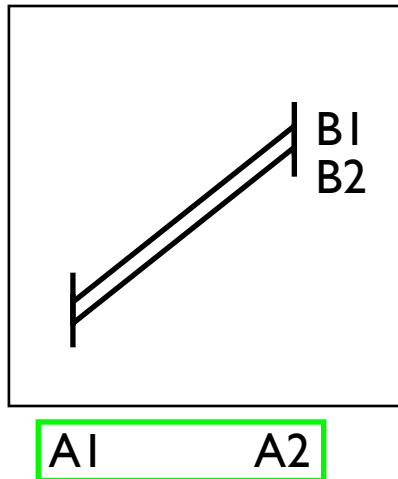
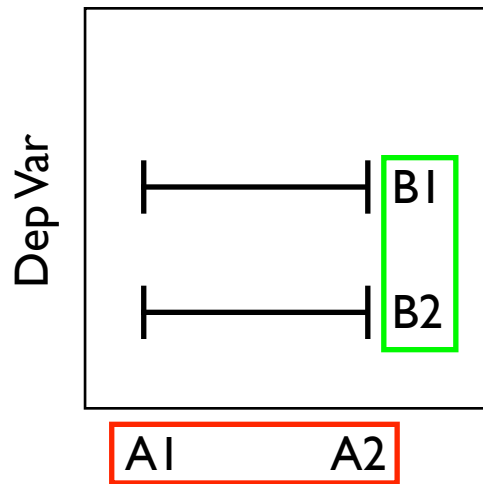
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

Main Effects

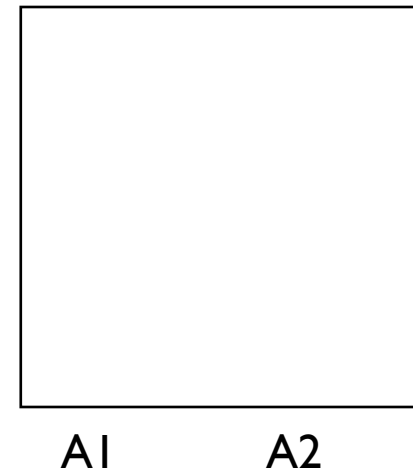
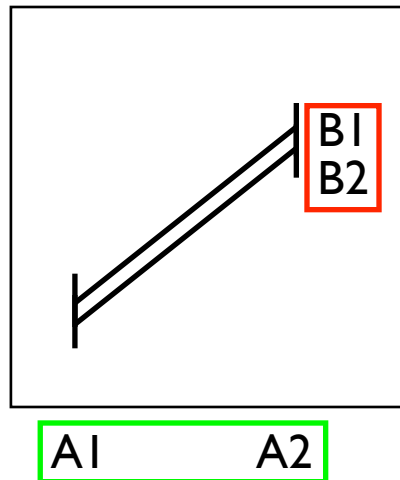
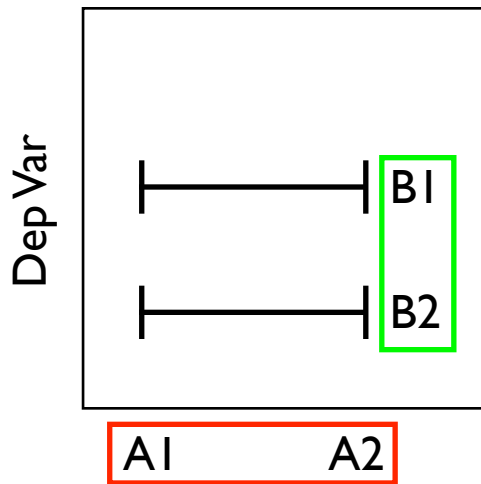
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

Main Effects

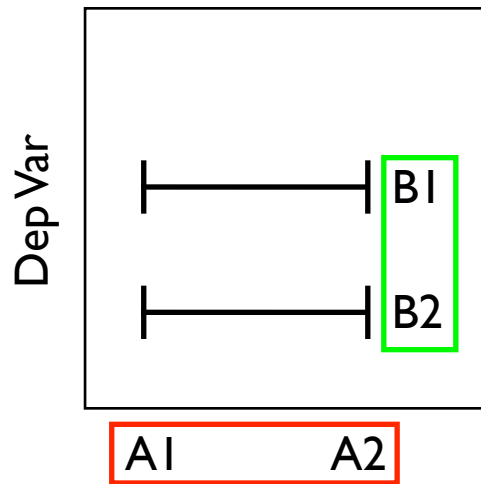
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



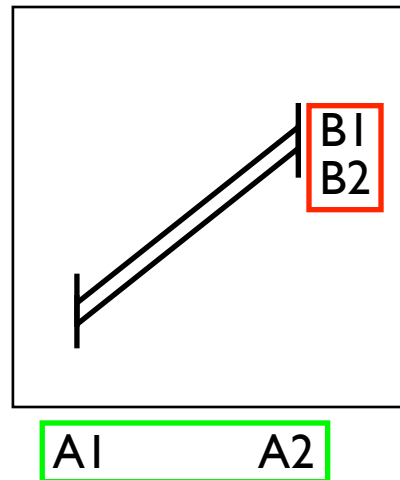
Main effect of B

Main Effects

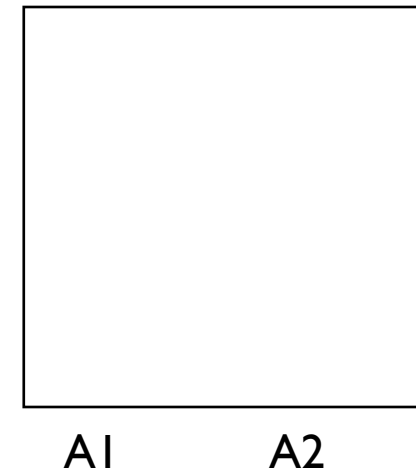
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

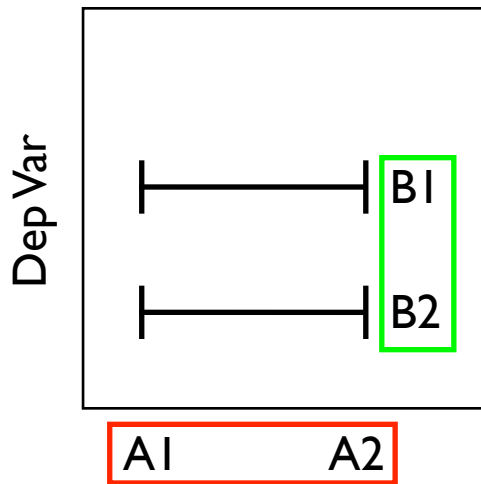


Main effect of A

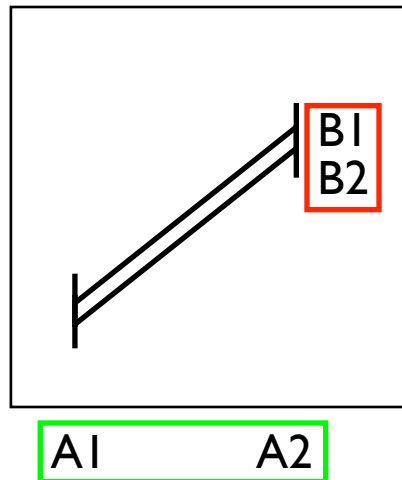


Main Effects

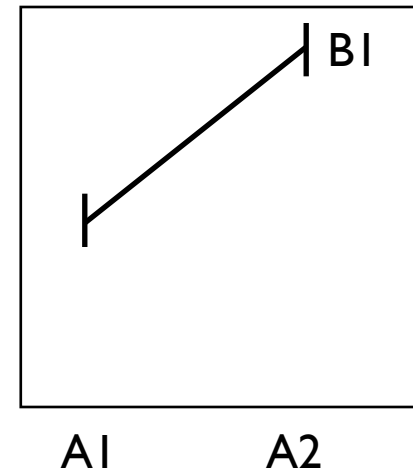
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

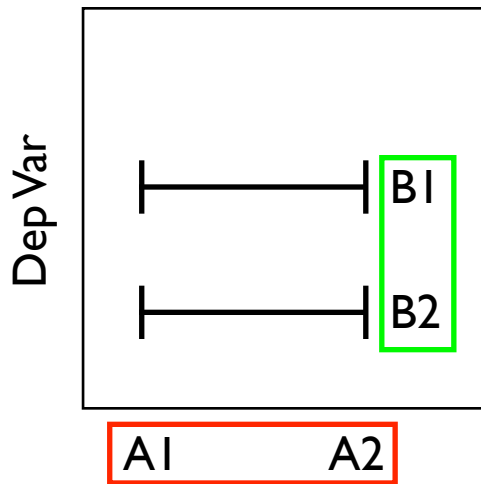


Main effect of A

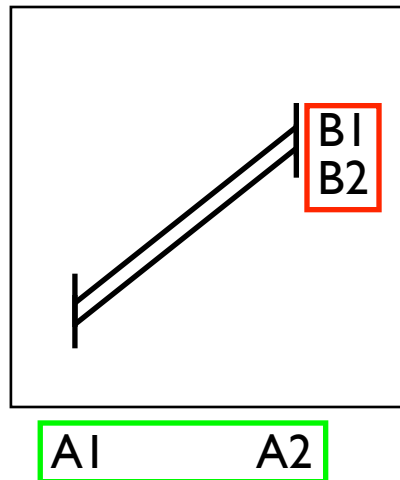


Main Effects

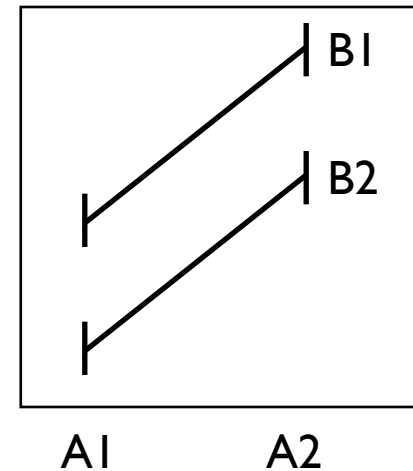
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

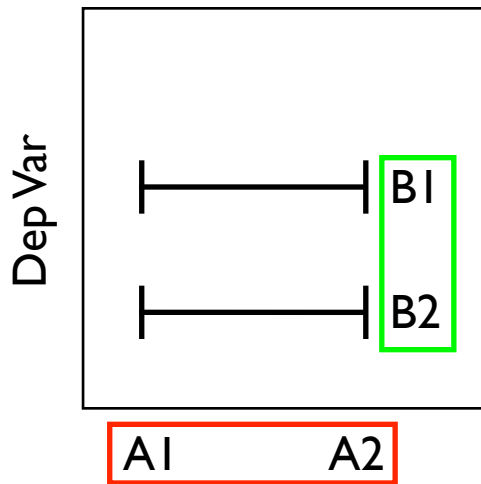


Main effect of A

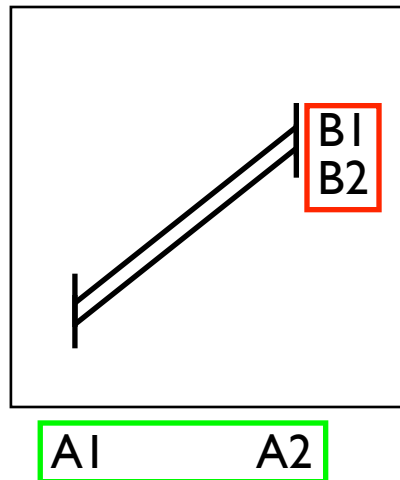


Main Effects

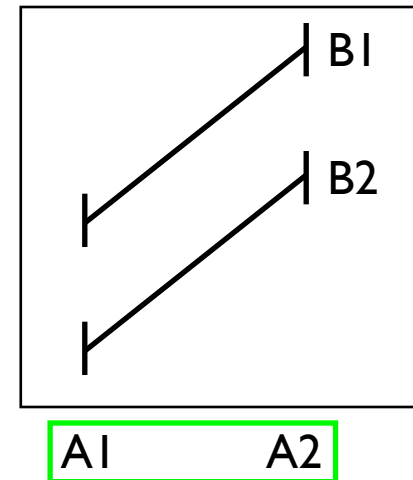
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

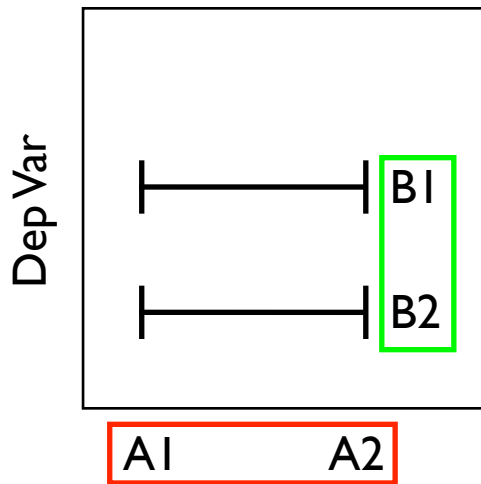


Main effect of A

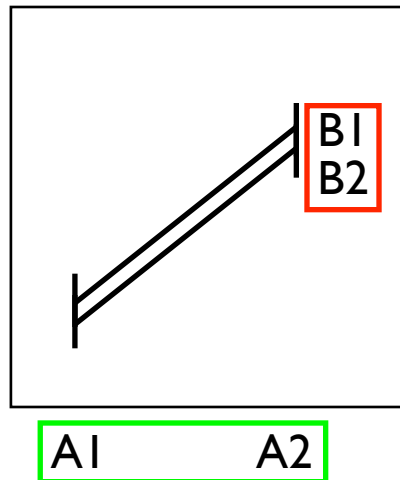


Main Effects

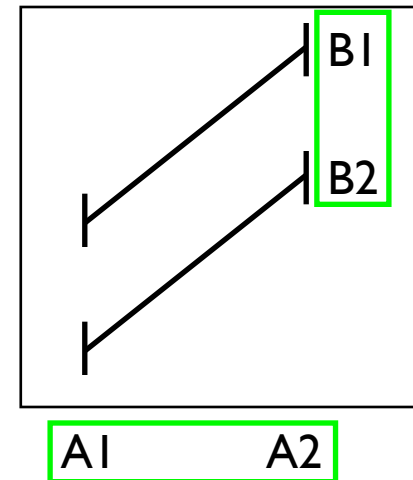
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B

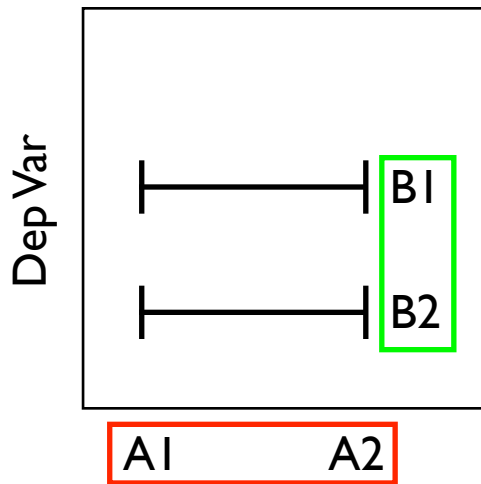


Main effect of A

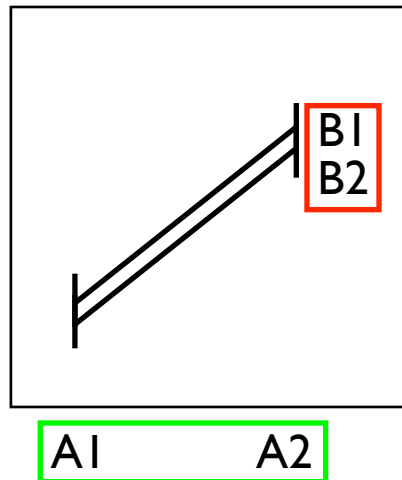


Main Effects

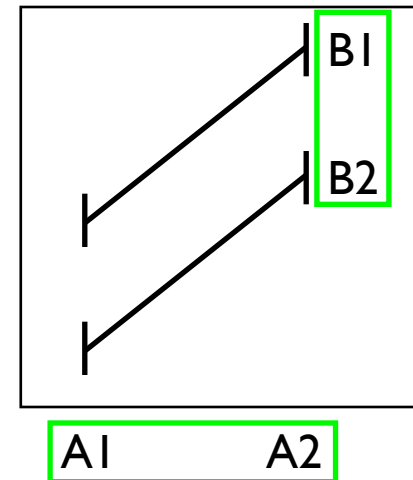
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B



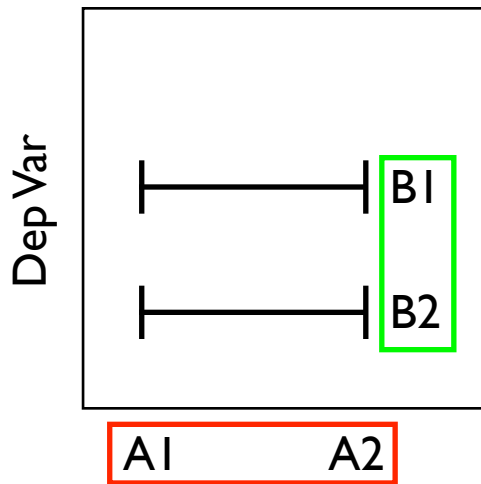
Main effect of A



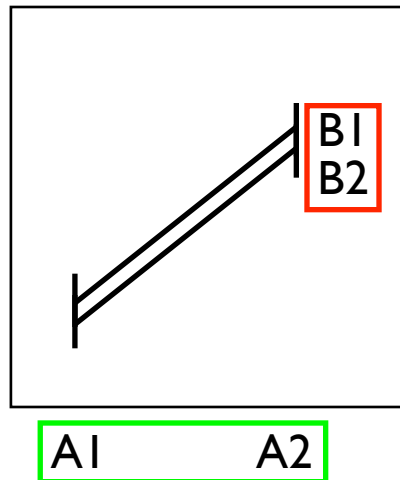
Main effects of A and B

Main Effects

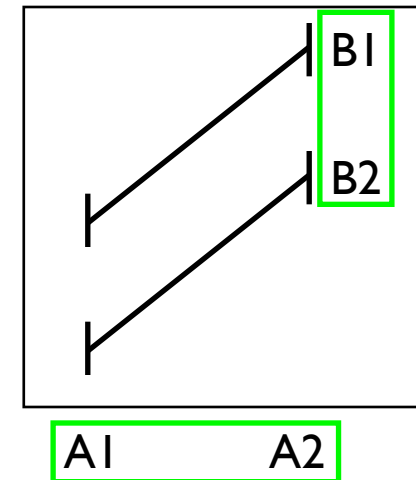
- typical main effects look like this
 - Factor A (A1, A2) and Factor B (B1, B2) fully crossed design



Main effect of B



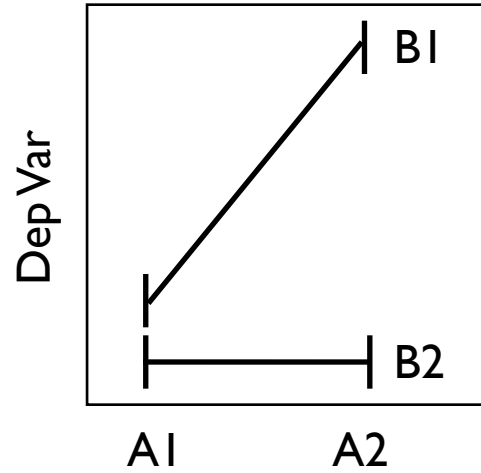
Main effect of A



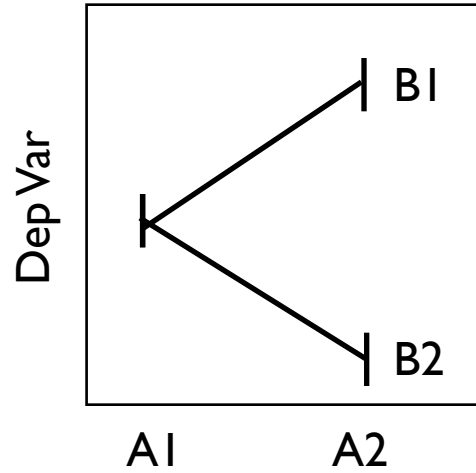
Main effects of A and B

in all 3 cases: **no A x B interaction effect**

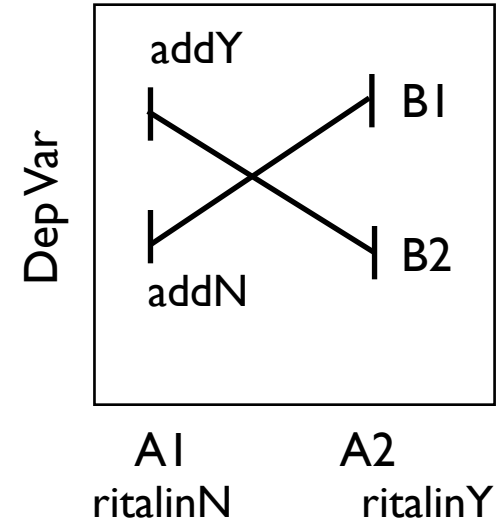
What about these datasets?



A:
B:
AxB:

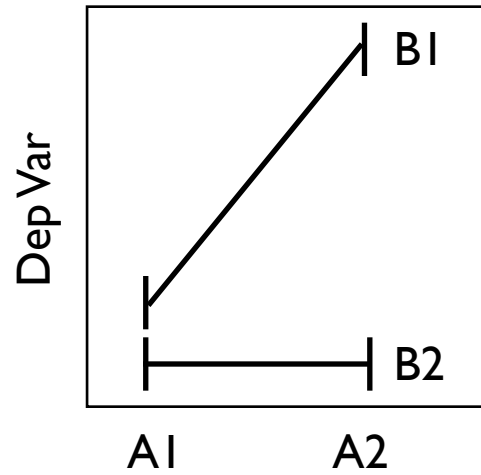


A:
B:
AxB:

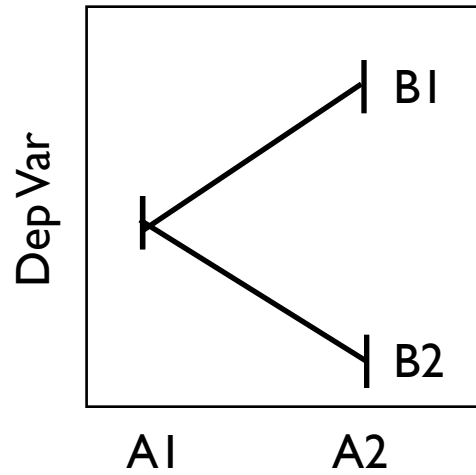


A:
B:
AxB:

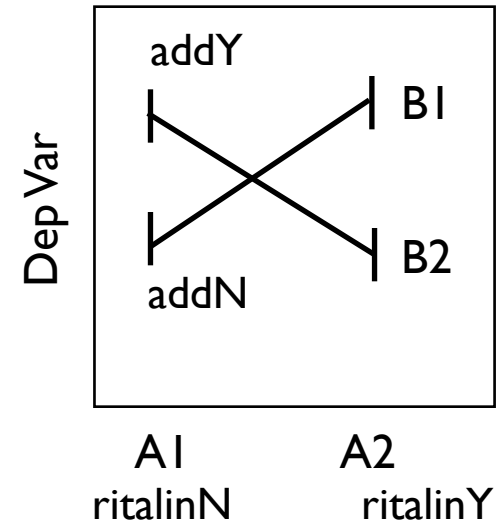
What about these datasets?



A: ●
B:
AxB:

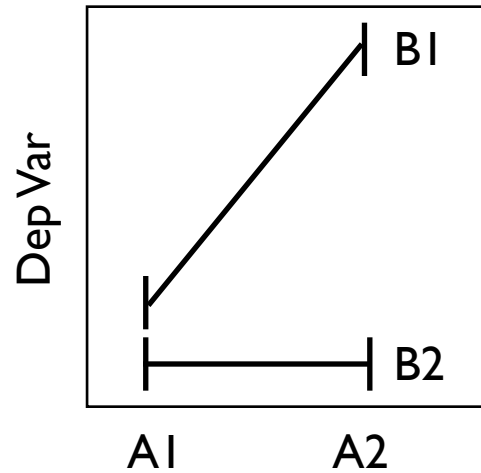


A:
B:
AxB:

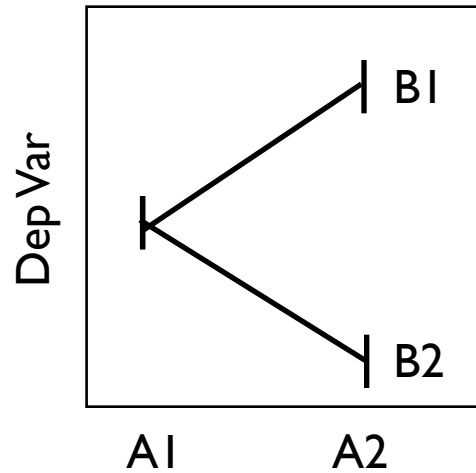


A:
B:
AxB:

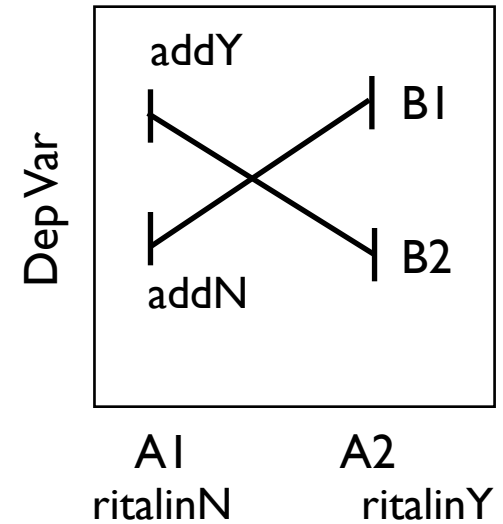
What about these datasets?



A: ●
B: ●
AxB:

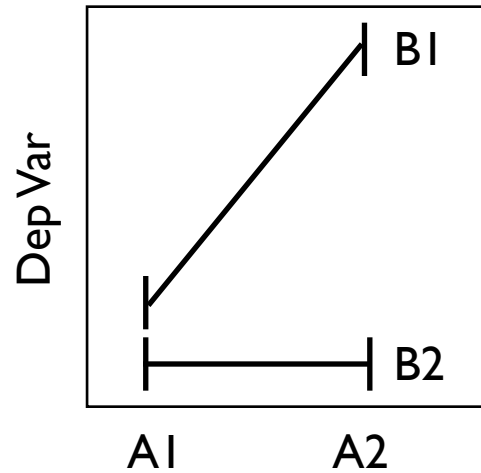


A:
B:
AxB:

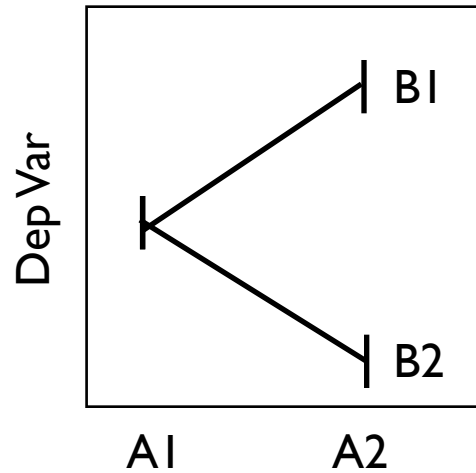


A:
B:
AxB:

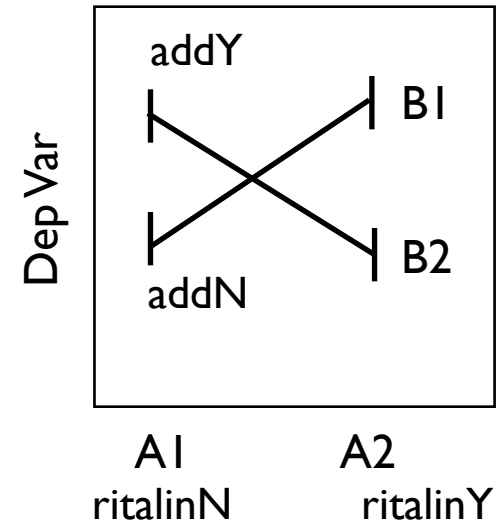
What about these datasets?



A: ●
B: ●
AxB: ●

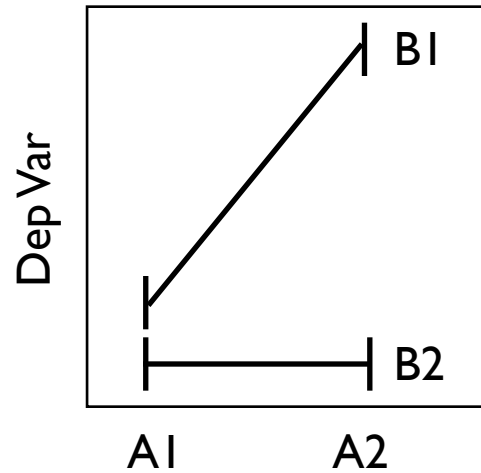


A:
B:
AxB:



A:
B:
AxB:

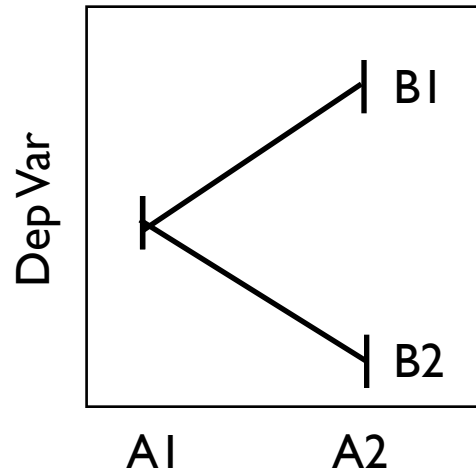
What about these datasets?



A: ●

B: ●

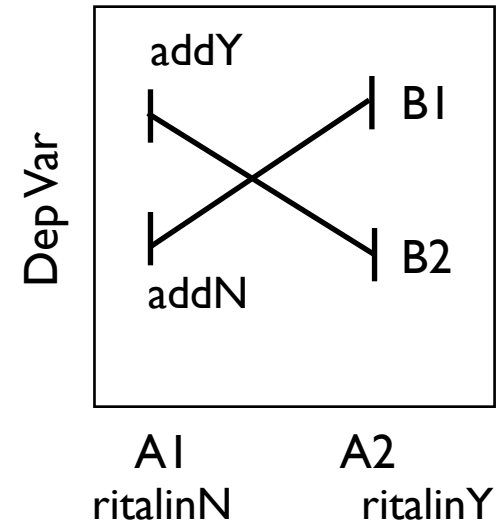
AxB: ●



A: ●

B:

AxB:

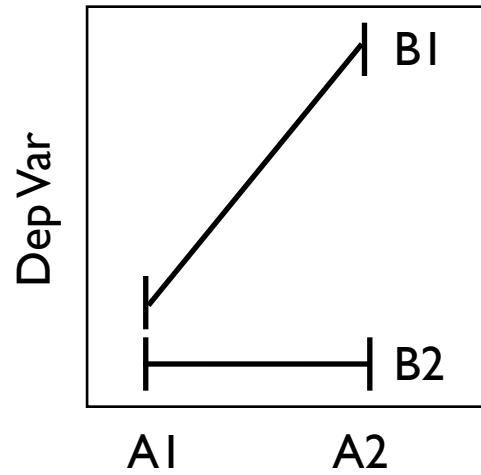


A:

B:

AxB:

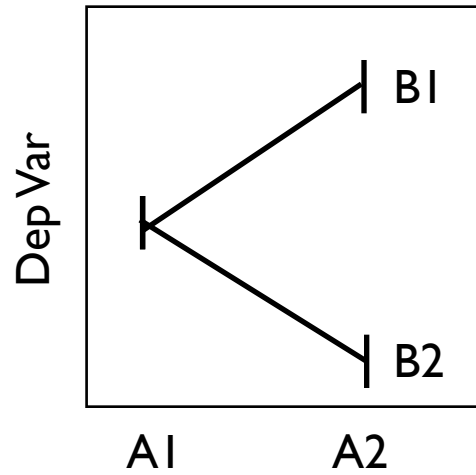
What about these datasets?



A: ●

B: ●

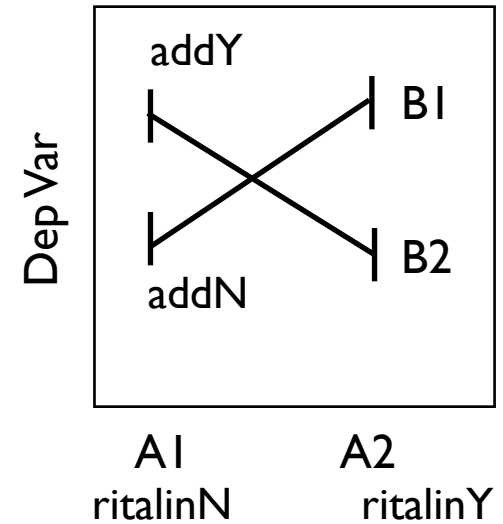
AxB: ●



A: ●

B: ●

AxB:

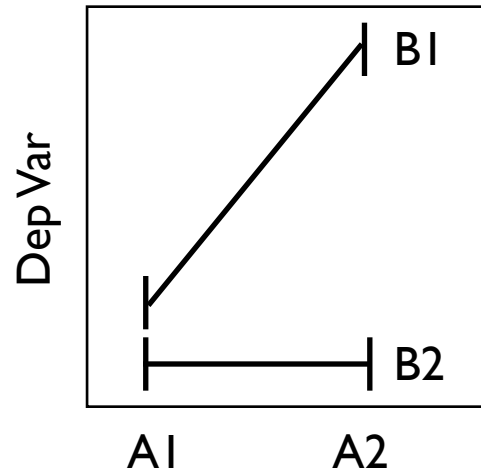


A:

B:

AxB:

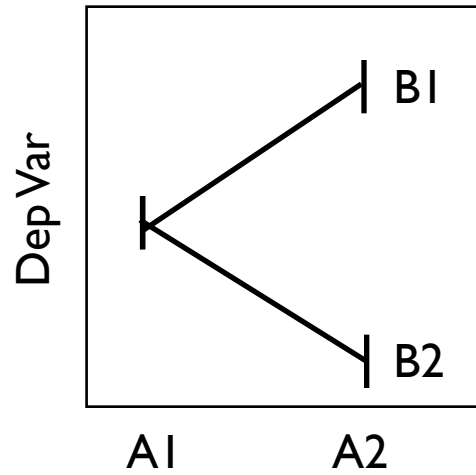
What about these datasets?



A: ●

B: ●

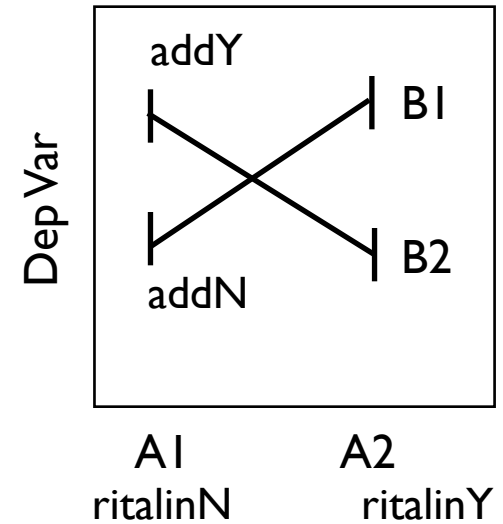
AxB: ●



A: ●

B: ●

AxB: ●

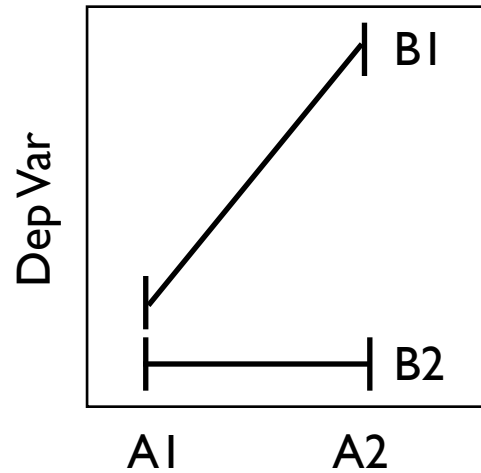


A:

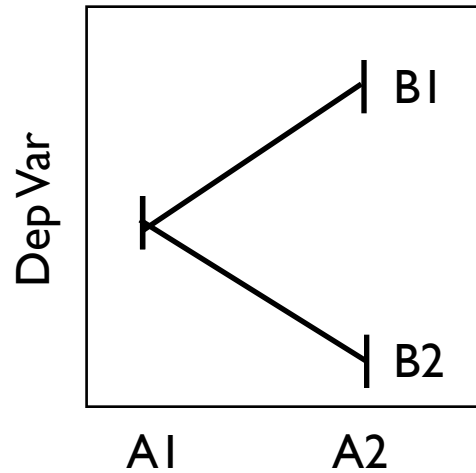
B:

AxB:

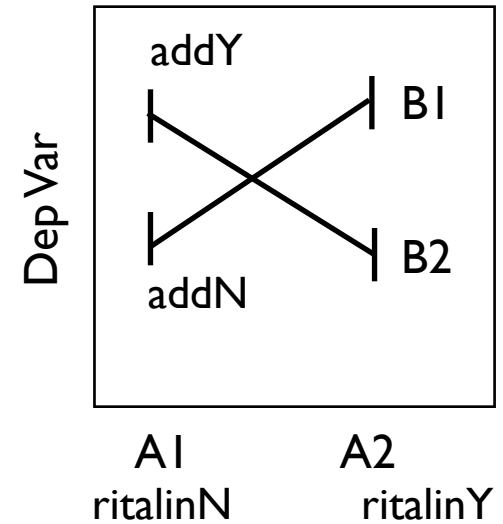
What about these datasets?



A: ●
B: ●
AxB: ●

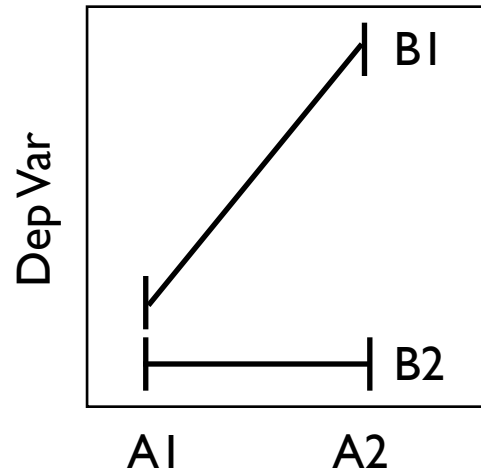


A: ●
B: ●
AxB: ●

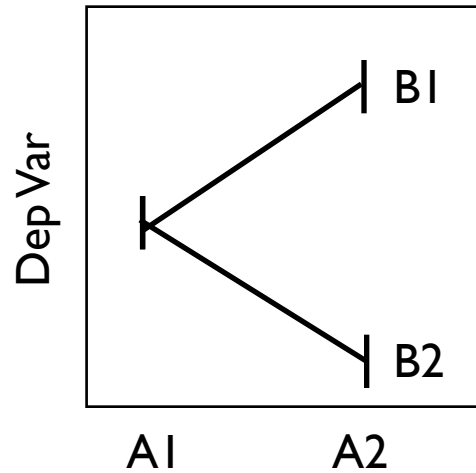


A: ●
B:
AxB:

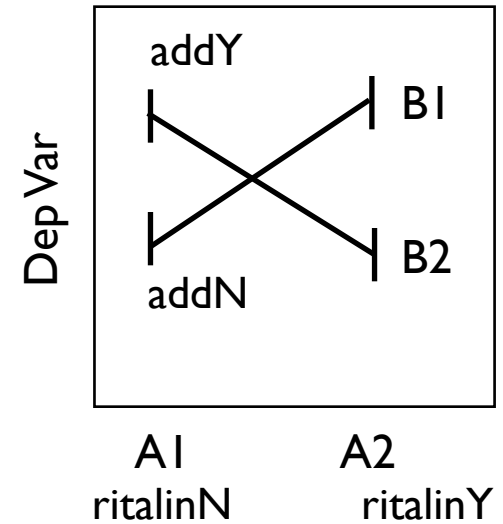
What about these datasets?



A: ●
B: ●
AxB: ●

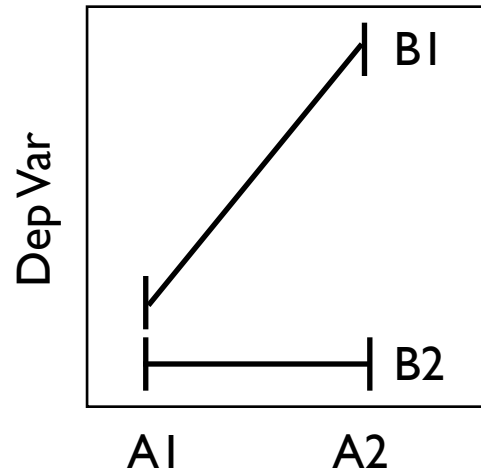


A: ●
B: ●
AxB: ●

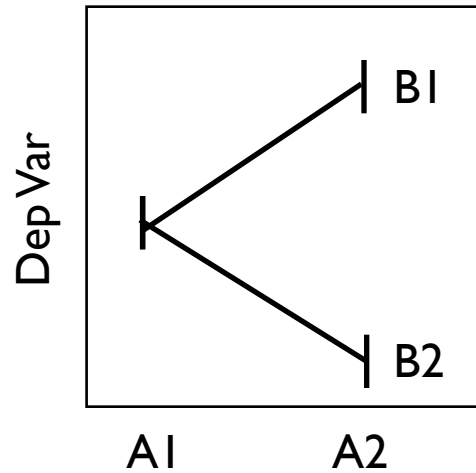


A: ●
B: ●
AxB:

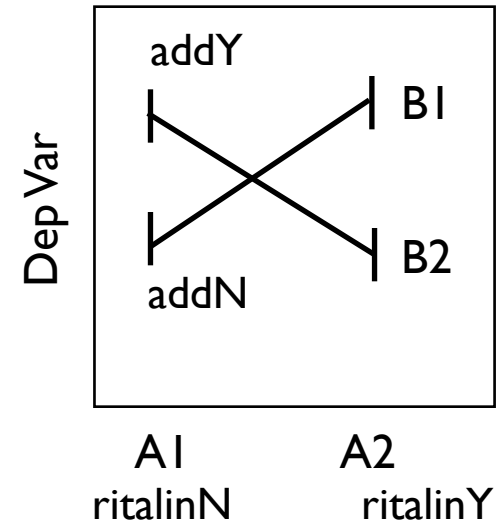
What about these datasets?



A: ●
B: ●
AxB: ●



A: ●
B: ●
AxB: ●



A: ●
B: ●
AxB: ●

rule of thumb

- parallel lines: main effect
- non-parallel lines: interaction effect

Two Factor Design: Model Comparison Approach

- Let's assume two factors
 - Factor A with a levels
 - Factor B with b levels
- Fully crossed design
 - every level of factor A is tested with every level of factor B
 - total # groups (cells) is $a \times b$
- we will see how to formulate in terms of model comparisons:
 - main effect of A
 - main effect of B
 - interaction effect $A \times B$

Our approach will be as before

1. write the equation for the full and restricted models
2. derive the equations for model **error** E_r and E_f
3. derive the expressions for **degrees of freedom** df_R and df_F
4. end up with an equation for the **F ratio**

The Full Model

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

- Y_{ijk} is an individual score in the j th level of factor A and the k th level of factor B (i indexes subjects within each (j,k) cell)
- μ is the overall mean of all cells
- α_j is the effect of the j th level of factor A
- β_k is the effect of the k th level of factor B
- $(\alpha\beta)_{jk}$ is the interaction effect of level j of A and level k of B

Hypothesis testing using Restricted Models

- Two-Factor (A x B) design: 3 null hypotheses to be tested:
 - main effect of A
 - main effect of B
 - interaction effect A x B
- We will formulate a separate restricted model for each hypothesis test
- **each test will involve the same full model**
- we will use the usual **F test**:

$$F = \frac{(E_R - E_F) / (df_R - df_F)}{E_F / df_F}$$

Main effect of A

- **full model:** $Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

- null hypothesis is that A main effect is zero

$$H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_a = 0$$

- restricted model:

$$Y_{ijk} = \mu + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

From Chapter 7:

$$E_F = \sum_{all\ obs} (Y_{ijk} - \bar{Y}_{jk})^2$$

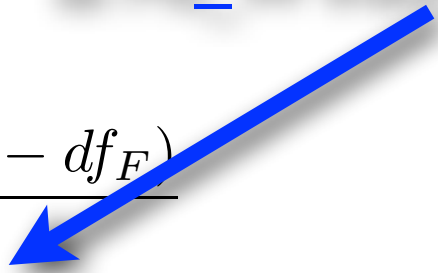
$$E_R - E_F = n \sum_{j=1}^a (\bar{Y}_j - \bar{Y})^2$$

$$df_F = ab(n - 1)$$

$$df_R - df_F = a - 1$$

denominator is always the same
as MS_W from ANOVA table

- so now we can do our F-test!

$$F = \frac{(E_R - E_F)/(df_R - df_F)}{E_F/df_F}$$


Main Effect of B

- full model again is:

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

- restricted model is:

$$Y_{ijk} = \mu + \alpha_j + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

- See Chapter 7 for equations for EF and ER-EF

Interaction effect AB

- full model again is:

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

- restricted model:

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \epsilon_{ijk}$$

Controlling Alpha level

- huh? we are doing three tests here and we are doing nothing about controlling the Type-I error rate. Why not?
- each test is conceptualized as a separate “family” of tests
- each test is addressing an **independent** question
- the approach is to control the **family-wise alpha level** at 0.05
- each **major effect** (A, B, AB) is considered a **family**
- within each family of tests we control alpha at 0.05 level

Controlling Alpha level

- so we are allowing experiment-wise alpha level to exceed 0.05
- we are controlling the family-wise alpha level at 0.05
- does this seem rather arbitrary to you?
- it's not entirely arbitrary BUT
- it's not entirely non-arbitrary either
- statistics is a framework for formulating rational approaches to inferences based on data
- you are responsible for your own convincing arguments

Follow-up Tests

- ok - so we've done F-tests for the main effect A, main effect B, and interaction effect AB; now what?
- investigate the nature of each significant effect
- there is a good rule of thumb for how to proceed:

Follow-up Tests

- **first look at the interaction effect**
- **IF** interaction effect is significant,
 - perform analyses of “simple effects”
 - (i.e. investigate the nature of the interaction)
 - and DON'T bother looking into the main effects (they are not informative anyway)
- **ELSE** if interaction effect is not significant,
 - perform contrasts within each significant main effect to understand the nature of the differences
- so if interaction is significant don't bother looking at the main effects

Follow-up Tests

- **Further Investigations of Main Effects**

- upon finding a significant main effect, the precise effect is not known
- we do not know in what way the different levels of the factor differ
- contrasts are formed and tested in the same way as in a one-way design
- e.g. to test a contrast ψ in the main effect of A (averaged over levels of B):

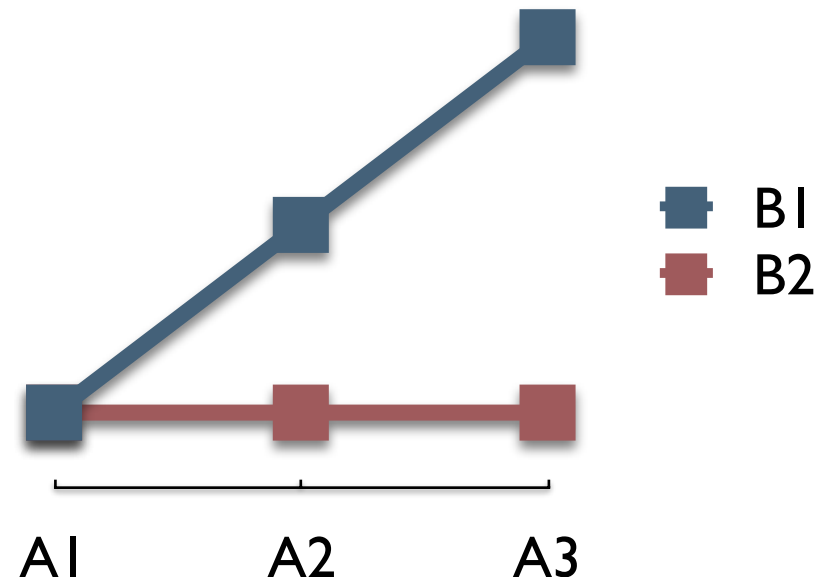
$$SS_{\psi} = nb(\psi)^2 / \sum_{j=1}^a c_j^2 \qquad F = SS_{\psi} / MS_W$$

Follow-up Tests

- critical value of F (F_{crit}) will depend on the same kinds of decisions we discussed in Chapter 5 on multiple-comparison procedures
- possibilities:
 - no correction
 - Bonferroni
 - Tukey
 - Scheffé
- I can tell you about different approaches but ultimately it's up to you to decide how to control family-wise alpha level

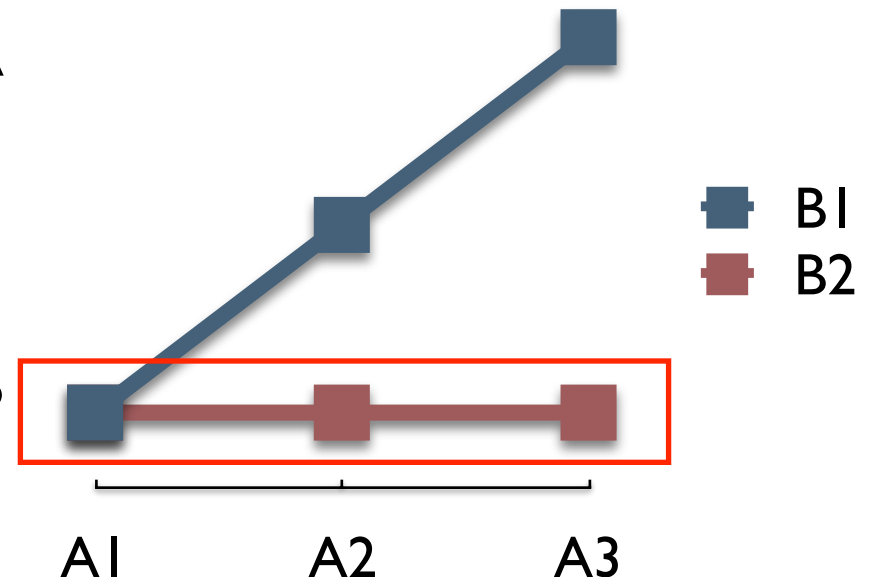
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



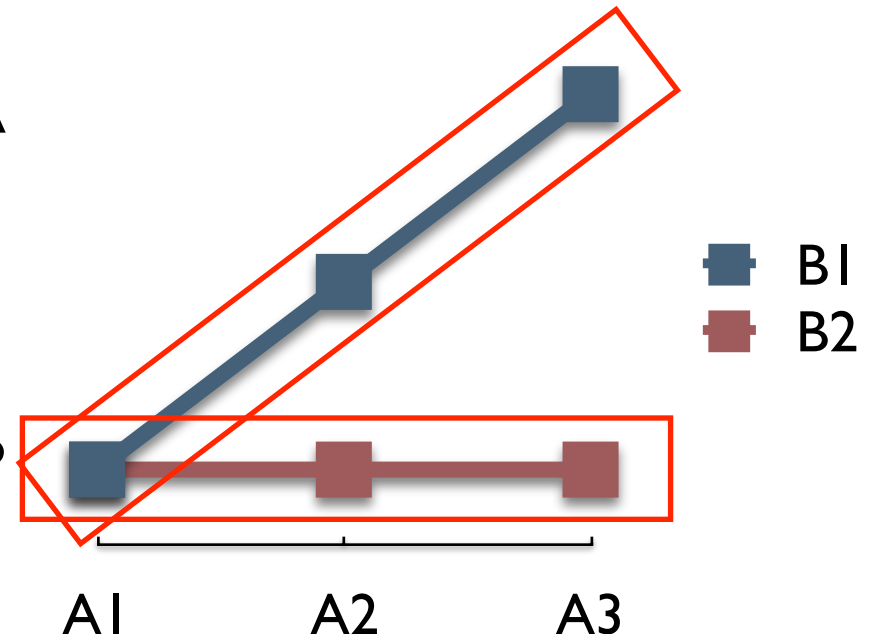
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



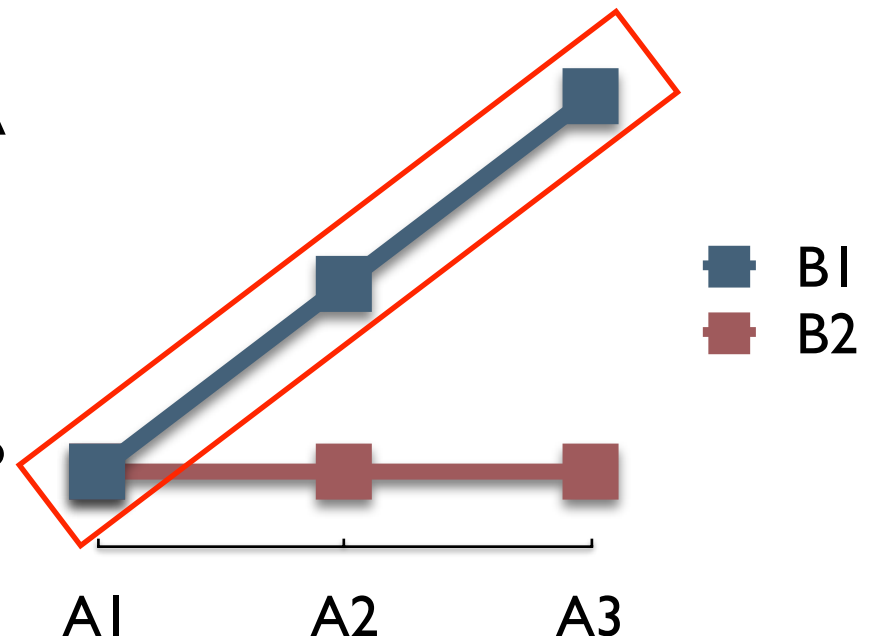
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



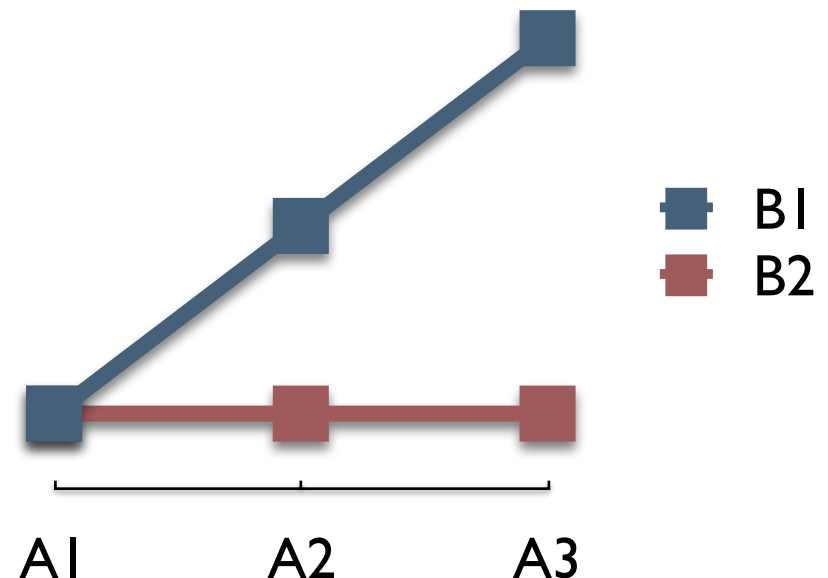
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



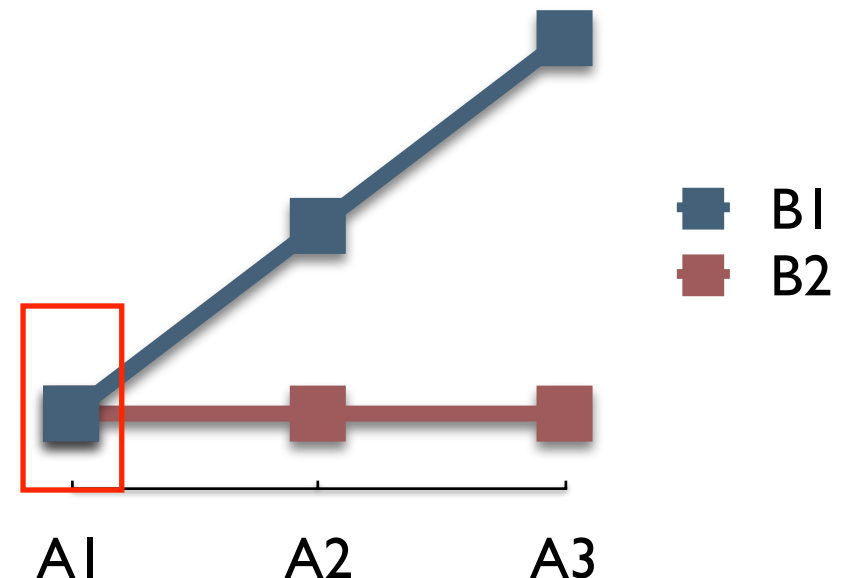
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



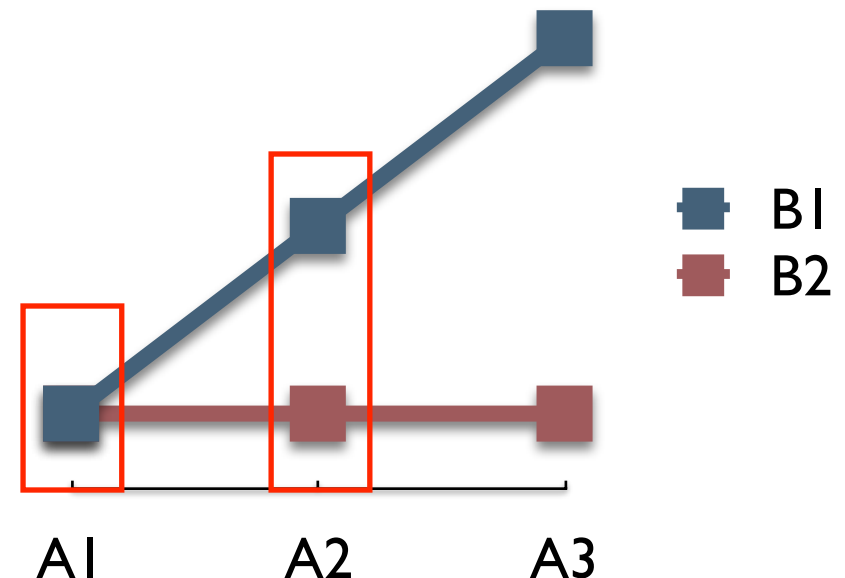
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



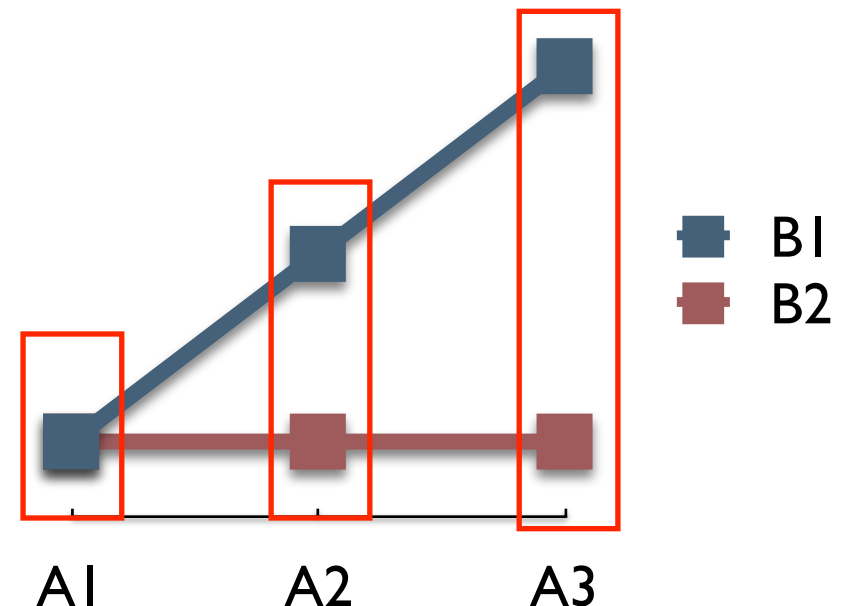
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



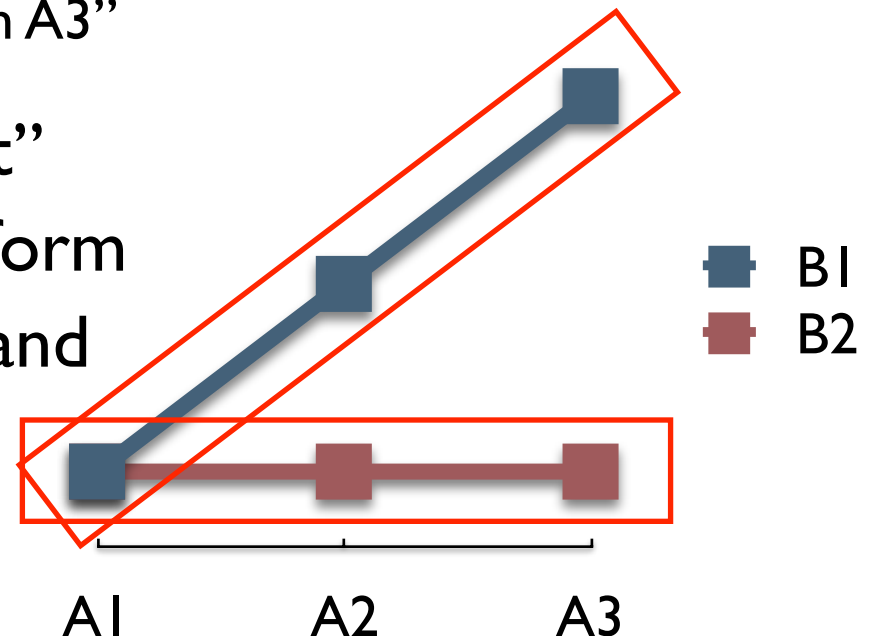
Investigating Interactions: Simple Effects

- like testing contrasts of a main effect, **except** we perform contrasts separately in each level of the other factor
- *like* a mini one-way anova (but **NOT** a one-way anova)
- e.g. two-factors A (3 levels) and B (2 levels)
- let's say we have a significant AB interaction
- test contrasts across levels of A
 - but within each level of B separately
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately



Investigating Interactions: Simple Effects

- test contrasts across levels of A
 - but within each level of B separately
 - called A “within B1” and A “within B2” simple effects
- OR alternatively,
test contrasts across levels of B
 - but within each level of A separately
 - B “within A1”, B “within A2”, B “within A3”
- Upon a significant “simple effect”
we would then proceed to perform
additional contrasts to understand
the nature of the differences



Investigating Interactions

$$F = \frac{SS_{contrast} / df_{contrast}}{MS_W}$$

- we can perform an F test on **any contrast** we want as long as we can compute `SS_contrast` and `df_contrast`
- `MS_W` always comes directly from ANOVA table

$$SS_{\psi} = n(\psi)^2 / \sum_{j=1}^a c_j^2$$

this equation
is your friend

- see Chapter 7 for some numerical examples

Type-I Error Rate

- when you test a bunch of contrasts in order to follow up a significant interaction effect, what should you do to control Type-I error rate?
- one school of thought: nothing! you are only performing the tests if the interaction is significant at 0.05 - so probability that any of the followup tests will be a Type-I error is also 0.05
- M & D don't like this - they say this logic can be flawed if the interaction null hypothesis is “partially” true
- what to do depends on what you constitute as a “family”

Type-I Error Rate

- M & D: suggest we consider all tests regarding differences among levels of a given factor as a separate “family” of tests
- Goal should be to maintain $\alpha = 0.05$ within each family
- they suggest a Bonferroni-like approach
- take # of tests done in each family and divide the alpha level (0.05) by that number
- I suggest: if # comparisons is small (2 or 3) this is ok. If # comparisons is much greater than 2 or 3, use Tukey instead

Statistical Power

- Chapter 7 gives some computational formulas for computing statistical power of
 - main effect of A
 - main effect of B
 - interaction effect AB
- We won't go into it here
- Read it on your own time

Non-orthogonal designs

- orthogonal design = a design with equal number of subjects within each cell
- non-orthogonal design = a design with different numbers of subjects within each cell
- There is controversy about best approach for analysing non-orthogonal designs
- one approach is to compute a new version of n called a “harmonic mean”, sort of like an average # of subjects
- read about it in the Chapter
- my advice: avoid non-orthogonal designs

Advantages of Factorial Designs

Advantages of Factorial Designs

- suppose we are interested in effects of various treatments for hypertension: biofeedback vs drugs X,Y,Z (vs nothing)

Advantages of Factorial Designs

- suppose we are interested in effects of various treatments for hypertension: biofeedback vs drugs X,Y,Z (vs nothing)
- is it better to conduct a 2 x 3 factorial study OR two separate single-factor studies?

Advantages of Factorial Designs

- suppose we are interested in effects of various treatments for hypertension: biofeedback vs drugs X,Y,Z (vs nothing)
- is it better to conduct a 2 x 3 factorial study OR two separate single-factor studies?
- factorial design enables us to test for an interaction

Advantages of Factorial Designs

- suppose we are interested in effects of various treatments for hypertension: biofeedback vs drugs X,Y,Z (vs nothing)
- is it better to conduct a 2 x 3 factorial study OR two separate single-factor studies?
- factorial design enables us to test for an interaction
- factorial design allows for greater generalizability

Advantages of Factorial Designs

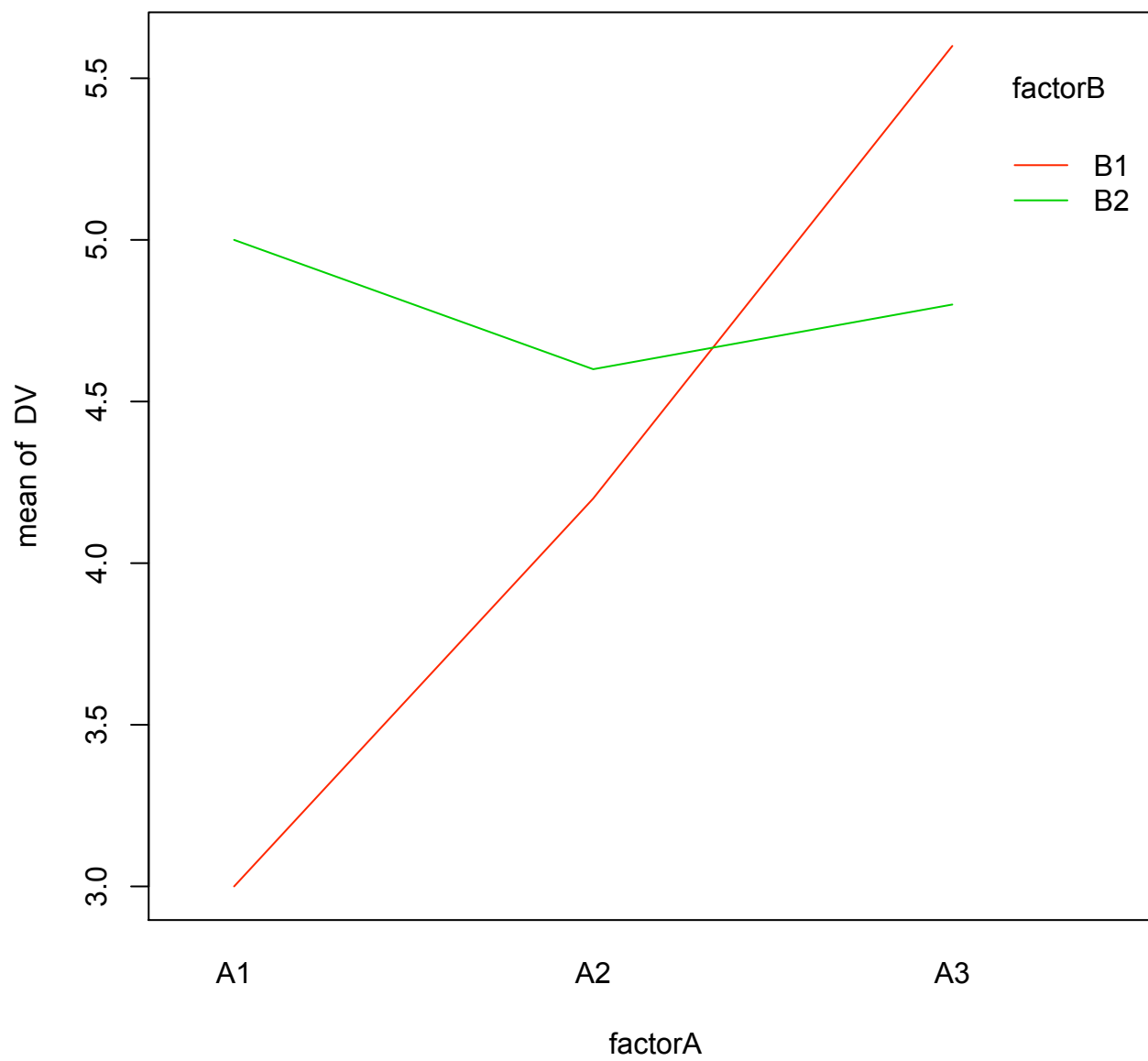
- suppose we are interested in effects of various treatments for hypertension: biofeedback vs drugs X,Y,Z (vs nothing)
- is it better to conduct a 2 x 3 factorial study OR two separate single-factor studies?
- factorial design enables us to test for an interaction
- factorial design allows for greater generalizability
- ★ factorial design can produce the **same statistical power** as 2 single-factor designs using **half as many subjects!**

An example using R

Group	B1	B2
A1	2,3,4,3,3 (3.00)	4,5,6,5,5 (5.00)
A2	3,4,5,4,5 (4.20)	6,5,4,4,4 (4.60)
A3	4,6,5,6,7 (5.6)	5,4,6,5,4 (4.8)

<http://www.gribblelab.org/stats/exercises/twoWay.R>

<http://www.gribblelab.org/stats/data/2waydata.csv>



```
summary(aov(DV~factorA*factorB))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factorA	2	7.4667	3.7333	4.9778	0.015546	*
factorB	1	2.1333	2.1333	2.8444	0.104648	
factorA:factorB	2	9.8667	4.9333	6.5778	0.005275	**
Residuals	24	18.0000	0.7500			

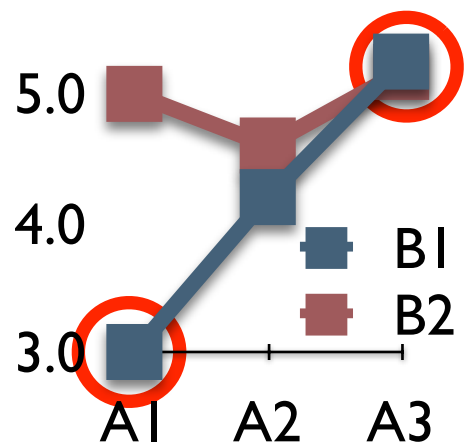
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

- what now? possibilities:
- “simple effects” (mini-anova) of A within B1 & within B2
- simple effects of B within A1, within A2 and within A3
- or just go directly to pairwise contrasts

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factorA	2	7.4667	3.7333	4.9778	0.015546	*
factorB	1	2.1333	2.1333	2.8444	0.104648	
factorA:factorB	2	9.8667	4.9333	6.5778	0.005275	**
Residuals	24	18.0000	0.7500			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

- as a demonstration, let's do the following contrast within B1
 - A1 vs A3
- and the same contrast within B2
 - A1 vs A3
- strategy for controlling Type-I error?
 - how about since we are doing 2 tests we divide each alpha by 2



	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factorA	2	7.4667	3.7333	4.9778	0.015546	*
factorB	1	2.1333	2.1333	2.8444	0.104648	
factorA:factorB	2	9.8667	4.9333	6.5778	0.005275	**
Residuals	24	18.0000	0.7500			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

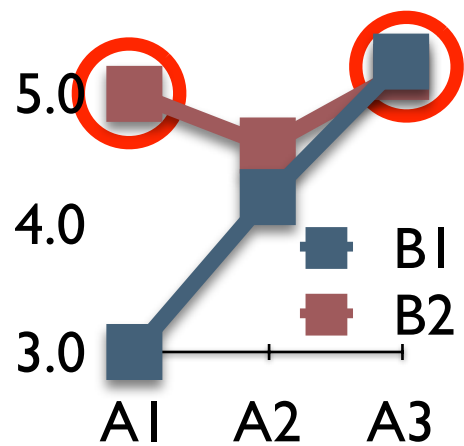
$$\psi = \sum_{j=1}^a c_j \mu_j$$

$$SS_{\psi} = n(\psi)^2 / \sum_{j=1}^a c_j^2$$

$$F = \frac{SS_{contrast} / df_{contrast}}{MS_W}$$

- for A1 vs A3 **within B1**
- $\psi = (+1)(3.00) + (-1)(5.6) = -2.6$
- $SS = 5((-2.6)^2) / ((+1)^2 + (-1)^2) = 33.8 / 2 = 16.9$
- $df_{contrast} = 1$
- $MS_W = 0.75$; $df_{denom} = 24$ (from ANOVA table)
- $F_{obs} = 16.9 / 0.75 = \mathbf{22.53}$
- $pf(22.5333, 1, 24, lower.tail=F) \rightarrow \mathbf{p=0.000079}$

**uncorrected for
Type-I error**



	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factorA	2	7.4667	3.7333	4.9778	0.015546	*
factorB	1	2.1333	2.1333	2.8444	0.104648	
factorA:factorB	2	9.8667	4.9333	6.5778	0.005275	**
Residuals	24	18.0000	0.7500			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$$\psi = \sum_{j=1}^a c_j \mu_j$$

$$SS_{\psi} = n(\psi)^2 / \sum_{j=1}^a c_j^2$$

$$F = \frac{SS_{contrast} / df_{contrast}}{MS_W}$$

- for A1 vs A3 **within B2**
- $\psi = (+1)(5.00) + (-1)(4.8) = 0.2$
- $SS = 5((0.2)^2) / ((+1)^2 + (-1)^2) = 0.2 / 2 = 0.1$
- $df_{contrast} = 1$
- $MS_W = 0.75; df_{denom} = 24$ (from ANOVA table)
- $F_{obs} = 0.1 / 0.75 = \mathbf{0.133}$
- $pf(0.133, 1, 24, lower.tail=F) \rightarrow \mathbf{p=0.719}$

**uncorrected for
Type-I error**

Controlling Alpha Level

- As we saw there are other approaches
- If you are following up tests based on how the data look post-hoc, perhaps you would feel more comfortable using Tukey tests instead
- If you are performing a whole bunch of planned tests then perhaps Bonferroni will actually be too conservative and you might feel better using Scheffé
- Here is the rule to follow:
- you must have some well defined and well understood rationale for how (or if) you control for Type-I error

tukeyHSD(myanova)

Tukey multiple comparisons of means
95% family-wise confidence level

Fit: aov(formula = DV ~ factorA * factorB, data = mydata)

\$factorA

	diff	lwr	upr	p adj
A2-A1	0.4 -0.5671951	1.367195	0.5639204	
A3-A1	1.2 0.2328049	2.167195	0.0131180	
A3-A2	0.8 -0.1671951	1.767195	0.1185021	

\$factorB

	diff	lwr	upr	p adj
B2-B1	0.5333333 -0.1193286	1.185995	0.1046482	

\$`factorA:factorB`

	diff	lwr	upr	p adj
A2:B1-A1:B1	1.2 -0.49352039	2.8935204	0.2782133	
A3:B1-A1:B1	2.6 0.90647961	4.2935204	0.0009965	
A1:B2-A1:B1	2.0 0.30647961	3.6935204	0.0141717	
A2:B2-A1:B1	1.6 -0.09352039	3.2935204	0.0717436	
A3:B2-A1:B1	1.8 0.10647961	3.4935204	0.0326534	
A3:B1-A2:B1	1.4 -0.29352039	3.0935204	0.1475933	
A1:B2-A2:B1	0.8 -0.89352039	2.4935204	0.6911401	
A2:B2-A2:B1	0.4 -1.29352039	2.0935204	0.9761219	
A3:B2-A2:B1	0.6 -1.09352039	2.2935204	0.8783892	
A1:B2-A3:B1	-0.6 -2.29352039	1.0935204	0.8783892	
A2:B2-A3:B1	-1.0 -2.69352039	0.6935204	0.4690617	
A3:B2-A3:B1	-0.8 -2.49352039	0.8935204	0.6911401	
A2:B2-A1:B2	-0.4 -2.09352039	1.2935204	0.9761219	
A3:B2-A1:B2	-0.2 -1.89352039	1.4935204	0.9990353	
A3:B2-A2:B2	0.2 -1.49352039	1.8935204	0.9990353	

3-Factor ANOVA

The 2 x 2 x 2 Design

- same example as last time
- test effects of different therapies for hypertension
- last time: 2 x 2
 - biofeedback (yes/no) x drug therapy (yes/no)
- now add a 3rd factor: diet therapy (yes/no)
- 3 factor design: 2 x 2 x 2
- subjects randomly assigned to one of 8 possible groups

	Diet Absent		Diet Present	
	Biofeedback Present	Biofeedback Absent	Biofeedback Present	Biofeedback Absent
Drug Present	180	205	170	190
Drug Absent	200	210	185	190

The 2 x 2 x 2 Design

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects
 - AB interaction

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects
 - AB interaction
 - AC interaction

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects
 - AB interaction
 - AC interaction
 - BC interaction

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects
 - AB interaction
 - AC interaction
 - BC interaction
- One 3-Way Interaction Effect

The 2 x 2 x 2 Design

- There are **7 effects** in a 3 Factor design:
- Three Main Effects
 - main effect of A
 - main effect of B
 - main effect of C
- Three 2-Way Interaction Effects
 - AB interaction
 - AC interaction
 - BC interaction
- One 3-Way Interaction Effect
 - ABC interaction

Main Effects

- main effect for a factor involves comparing the levels of that factor after averaging over all other factors
- e.g. main effect of Factor A (biofeedback):
 - average over levels of B and C
 - marginal means for Factor A are:
 - Biofeedback Present:** $(180 + 200 + 170 + 185)/4 = 183.75$
 - Biofeedback Absent:** $(205 + 210 + 190 + 190)/4 = 198.75$
- Main effect of B and of C in a similar fashion

		C			
		Diet Absent		Diet Present	
B	A	Biofeedback Present	Biofeedback Absent	Biofeedback Present	Biofeedback Absent
	Drug Present	180	205	170	190
	Drug Absent	200	210	185	190

Two-Way Interactions

Two-Way Interactions

- AB Interaction
 - average over Factor C
 - when averaged over Factor C, the effect of Factor A is different depending on the level of Factor B

Two-Way Interactions

- AB Interaction
 - average over Factor C
 - when averaged over Factor C, the effect of Factor A is different depending on the level of Factor B
- AC Interaction
 - average over Factor B
 - when averaged over Factor B, the effect of Factor A is different depending on the level of Factor C

Two-Way Interactions

- AB Interaction
 - average over Factor C
 - when averaged over Factor C, the effect of Factor A is different depending on the level of Factor B
- AC Interaction
 - average over Factor B
 - when averaged over Factor B, the effect of Factor A is different depending on the level of Factor C
- BC Interaction
 - average over Factor A
 - when averaged over Factor A, the effect of Factor B is different depending on the level of Factor C

Three-Way Interaction

- review: meaning of a two-way interaction (e.g. AB)
 - the Main Effect of A is different depending on the level of B
- meaning of a three-way interaction (e.g. ABC)
 - the AB interaction is different depending on the level of C
 - or
 - the AC interaction is different depending on the level of B
 - or
 - the BC interaction is different depending on the level of A
 - (all are equivalent statements)
 - some may have greater meaning than others in the context of your particular experiment

- I find it easiest to understand three-way interactions by referring to a graphical display of the data
- strategy: plot the two-way interaction multiple times, at each level of the third factor
- e.g. plot the drug x biofeedback interaction (1) for the diet absent level and (2) for the diet present level
- the 2-way drug x biofeedback interaction is different when diet is absent vs when diet is present

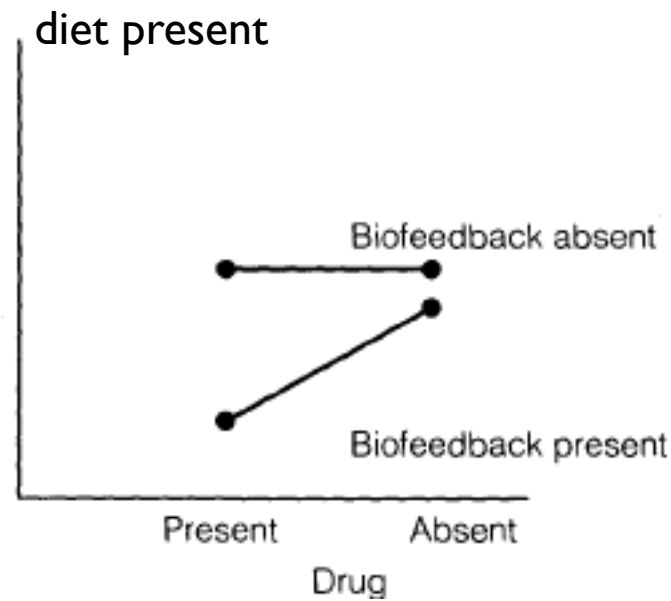
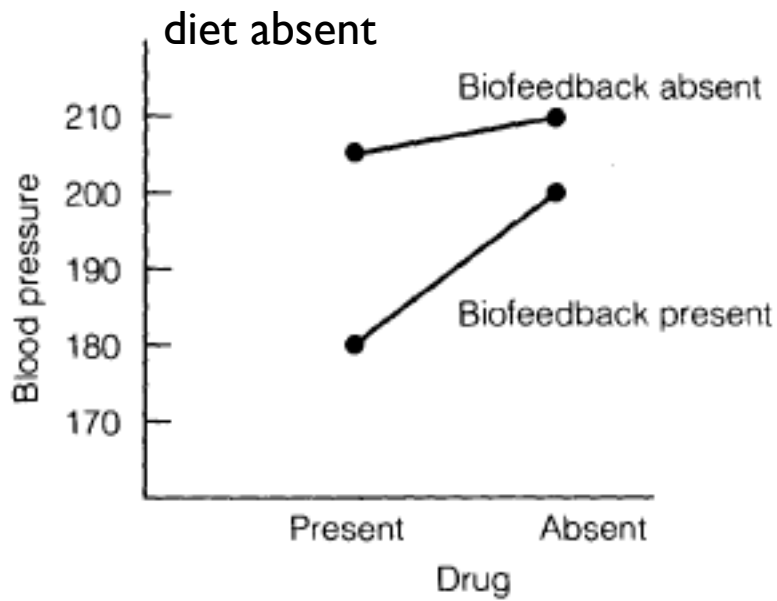


TABLE 8.8 Meaning of Effects in a Three-Way $A \times B \times C$ Design

	Meaning
<i>Main Effects</i>	
A	Comparison of marginal means of A factor, averaging over levels of B and C
B	Comparison of marginal means of B factor, averaging over levels of A and C
C	Comparison of marginal means of C factor, averaging over levels of A and B
<i>Two-Way Interactions</i>	
$A \times B$	Examines whether the A effect is the same at every level of B, averaging over levels of C (equivalently, examines whether the B effect is the same at every level of A, averaging over levels of C)
$A \times C$	Examines whether the A effect is the same at every level of C, averaging over levels of B (equivalently, examines whether the C effect is the same at every level of A, averaging over levels of B)
$B \times C$	Examines whether the B effect is the same at every level of C, averaging over levels of A (equivalently, examines whether the C effect is the same at every level of B, averaging over levels of A)
<i>Three-Way Interaction</i>	
$A \times B \times C$	Examines whether the two-way $A \times B$ interaction is the same at every level of C (equivalently, examines whether the two-way $A \times C$ interaction is the same at every level of B; equivalently, examines whether the two-way $B \times C$ interaction is the same at every level of A)

Model Comparison Approach

- just as before we can write a full model that contains all seven effects
- for each significance test (7 of them) we can write a restricted model in which the effect being tested is absent
- just as before we end up with an F-ratio
- just as before the denominator is equal to the MS_W from the ANOVA table
- **See Chapter 8 M&D for all the details**

Implications of a Three-Way Interaction

- Two-way interactions cannot be interpreted unambiguously
- e.g. there may be a significant two-way interaction between A and B within C1 but not within C2
- **so: do not interpret two-way interactions if the three-way interaction is significant!**
- (just like our previous rule about not interpreting main effects if a two-way interaction is significant)

Implications of a Three-Way Interaction

- Also do not interpret main effects
- effect of one factor depends on the level of BOTH of the other 2 factors
- doesn't make sense to average over levels of the other 2 factors
- in general: rule is: if three-way interaction is significant, do not interpret 2-way interactions OR main effects
- go directly to follow-up tests to understand the nature of the three-way interaction

General Guidelines for Analysing Effects

- a flowchart is shown in chapter
- looks more complicated than it should
- basic idea: start by looking at highest-order effect (3-way interaction)
- if significant, do follow-up tests within each level of a factor
- if not significant, move down to lower-order effects (2-way interactions)
- repeat

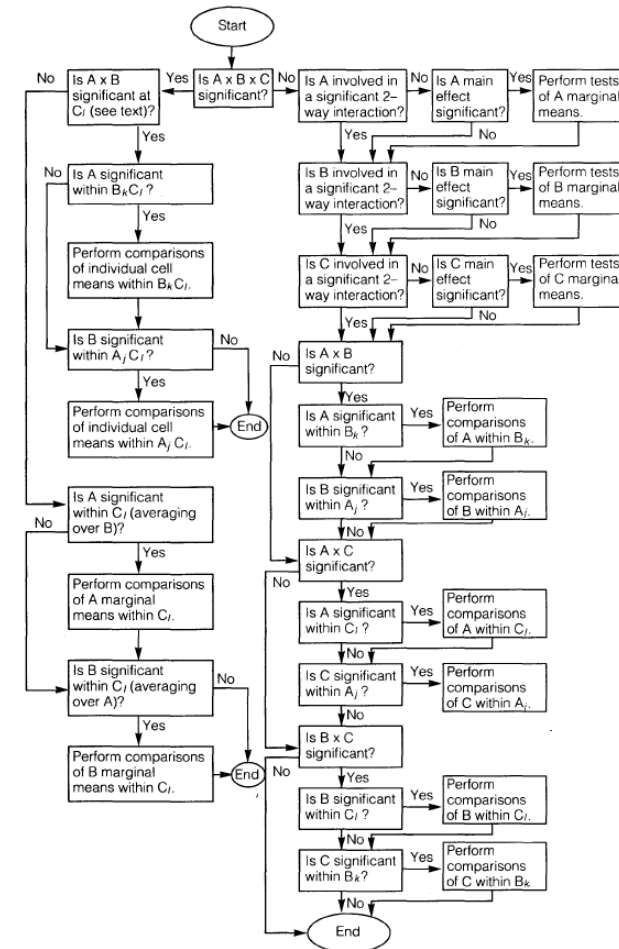


FIGURE 8.2 General guideline for analyzing effects in a three-factor design.

General Guidelines for Analysing Effects

- issues to consider (just as before)
- for follow-up tests, are they planned or post-hoc?
- how are you going to correct (if you do at all) for Type-I error?
- what's the best way of displaying your data graphically?

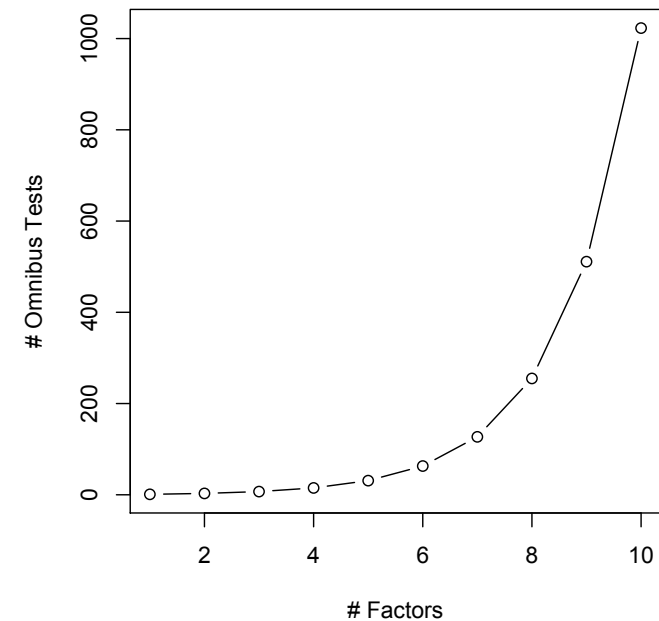
Higher-Order Designs

- 4-Factors (**15** omnibus tests)
 - 4 x main effects: A, B, C, D
 - 6 x 2-way interactions: AB, AC, AD, BC, BD, CD
 - 4 x 3-way interactions: ABC, ABD, ACD, BCD
 - 1 x 4-way interaction: ABCD
- # of groups:
 - e.g. A(2) B(2) C(2) D(2) : $2 \times 2 \times 2 \times 2 = 16$ groups!
 - e.g. A(3) B(3) C(3) D(3) : $3 \times 3 \times 3 \times 3 = 81$ groups!
 - this is ridiculous
- in any case - can you really interpret a 4-way interaction?
- difficult to {visualize, articulate, explain, understand}

Higher-Order Designs

- 4-Factors (**15** omnibus tests)

- 4 x main effects: A, B, C, D
- 6 x 2-way interactions: AB, AC, AD, BC, BD, CD
- 4 x 3-way interactions: ABC, ABD, ACD, BCD
- 1 x 4-way interaction: ABCD



- # of groups:

- e.g. A(2) B(2) C(2) D(2) : $2 \times 2 \times 2 \times 2 = 16$ groups!
- e.g. A(3) B(3) C(3) D(3) : $3 \times 3 \times 3 \times 3 = 81$ groups!
- this is ridiculous

- in any case - can you really interpret a 4-way interaction?
- difficult to {visualize, articulate, explain, understand}