Midterm

Bayes' Rule for Events

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}$

where, by the Law of Total Probability,

$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$

Posterior Model

$f(\pi|x) = \frac{f(\pi)L(\pi|x)}{f(x)} \propto f(\pi)L(\pi|x)$

Beta Model

$\pi \sim \text{Beta}(\alpha, \beta)$

The Beta model is specified by continuous pdf

$f(\pi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1} \quad \text{for } \pi \in [0,1], \alpha > 0, \text{and } \beta > 0$

where $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$ and $\Gamma(z+1) = z\Gamma(z)$. Fun fact: when z is a positive integer, then $\Gamma(z)$ simplifies to $\Gamma(z) = (z-1)!$.

Beta Descriptives

$E(\pi) = \frac{\alpha}{\alpha+\beta}$

$\text{Mode}(\pi) = \frac{\alpha-1}{\alpha+\beta-2}$

$\text{Var}(\pi) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

The Beta-Binomial Model

Let $\pi \sim \text{Beta}(\alpha, \beta)$ and $X|n \sim \text{Bin}(n, \pi)$ then

$\pi|(X = x) \sim \text{Beta}(\alpha + x, \beta + n - x)$