1 Choice of Prior

2 Conjugate Prior

Let $f(\theta)$ be a prior distribution for parmaeter θ from a particular distribution, and $L(\theta|x)$ be the likelihood function for the data conditional on parameter θ, with its own particular distribution as the basis of the likelihood. If the resulting posterior distribution $f(\theta|x)$ is of the same family of distributions as the prior distribution, then the prior distribution is a conjugate prior for this likelihood

Examples

3 Non-conjugate prior for Binomial Data

$$f(\pi) = e - e^{\pi} \quad \text{for } \pi \in [0, 1]$$

Is this a valid pdf?

After observing 10 successes in 50 trials, calculate the posterior.

4 The Gamma-Poisson Conjugate Family

4.1 Gamma Prior

Let λ be a random variable which can take any value between 0 and ∞, ie. $\lambda \in [0, \infty)$. Then the variability in λ might be well modeled by a Gamma model with **shape parameter** $\alpha > 0$ and **rate parameter** $\beta > 0$:

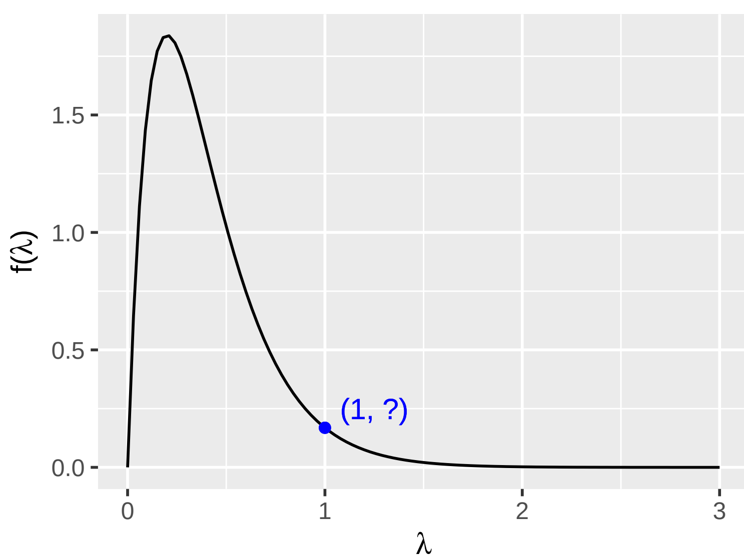
$\lambda \sim \text{Gamma}(\alpha, \beta)$

The Gamma distribution is specified by continuous pdf $f(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \quad \text{for } \lambda \in [0, \infty)$

Exercise

What is the y-coordinate of the blue point in the plot below? In other words what is $f(\lambda)$ if $\lambda = 1$?

```
plot_gamma(shape = 2, rate = 5)
```



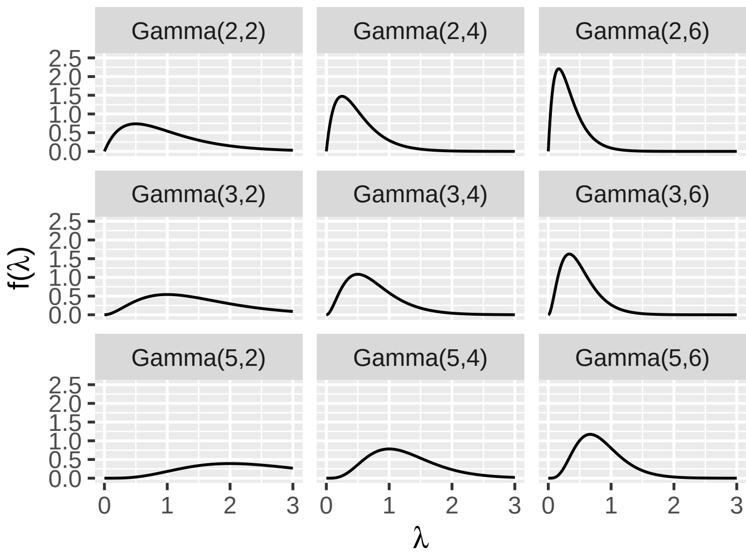
Descriptives of Gamma

$E(\lambda) = \frac{\alpha}{\beta}$

$\text{Mode}(\lambda) = \frac{\alpha-1}{\beta}$ where $\alpha \geq 1$

$\text{Var}(\lambda) = \frac{\alpha}{\beta^2}$

Gamma Tuning



Tuning Gamma example

For our example on spam phone calls, set a prior for λ such that $E(\lambda) = 3$ and λ most likely is between 2 and 4. You can use `plot_gamma()` function to try out different gamma distributions.

4.2 Poisson Likelihood

The Poisson Model Let random variable X be the *number of events* that occur in a fixed amount of time, where λ is the rate at which these events occur. Then the *dependence* of X on λ can be modeled by the Poisson model with **parameter** λ. In mathematical notation:

$X|\lambda \sim \text{Pois}(\lambda)$

Correspondingly, the Poisson model is specified by a conditional pmf:

$f(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x \in \{0, 1, 2, \ldots, n\}$

where $f(x|\lambda)$ sums to one across x:

$\sum_{x=0}^{\infty} f(x|\lambda) = 1$

Likelihood

4.3 Gamma-Poisson Conjugacy

If $f(\lambda) \sim \text{Gamma}(\alpha, \beta)$

and if $x_i \sim iid$ Poissson(λ)for $i \in 1, \ldots, n$

then $f(\lambda|\vec{x}) \sim \text{Gamma}(\alpha + \sum x_i, \beta + n)$.