SDS 383D Ex 04:
Hierarchical Models

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Math Tests

The data set in "mathtest.csv" shows the scores on a standardized math test from a sample of 10th-grade students at 100 different U.S. urban high schools, all having enrollment of at least 400 10th-grade students. (A lot of educational research involves "survey tests" of this sort, with tests administered to all students being the rare exception.)

Let θ_i be the underlying mean test score for school i, and let y_{ij} be the score for the jth student in school i. Starting with the "mathtest.R" script, you'll notice that the extreme school-level averages $\bar{y}_i$ (both high and low) tend to be at schools where fewer students were sampled.

Part 1

Briefly explain why this would be.

The extreme school-level averages occur in the schools with smaller sample sizes because we do not do a very good job of estimating the mean when sample size is small. These schools do not have min and max observation values that are more extreme than the other schools; they just have fewer observations to balance out the calculation of the mean. The smaller the sample size, the more influential an extreme observation is over the group mean.

Part 2

Fit a normal hierarchical model to these data via Gibbs sampling:

$$\begin{aligned} y_{ij} &\sim \mathrm{N}(\theta_i, \sigma^2) \\ \theta_i &\sim \mathrm{N}(\mu, \tau^2\sigma^2) \end{aligned}$$

Decide upon sensible priors for the unknown model parameters (μ, σ^2, τ^2).
The model is as follows.

$$\begin{aligned} (y_ij|\theta_i, \sigma^2) &\sim N(\theta_i, \sigma^2) \\ (\theta_i|\mu, \sigma^2, \tau^2) &\sim N(\mu, \sigma^2\tau^2) \\ \mu &\sim I_{\mathbb{R}}(\mu), \text{ a flat prior on the real line} \\ \tau^2 &\sim I_{\mathbb{R}^+}(\tau^2), \text{ a flat prior on the positive real line} \\ \sigma^2 &\sim \left(\frac{1}{\sigma^2}\right) I_{\mathbb{R}^+}(\sigma^2), \text{ Jeffreys prior} \end{aligned}$$

where

$$\begin{aligned} i &= 1, ..., p \text{ indexes the } p \text{ groups.} \\ n_i &= \text{ sample size in each group.} \\ j &= 1, ..., n_i \text{ indexes observations in a group.} \\ n &= \text{ total number of observations.} \end{aligned}$$

The likelihood is

$$L(y|\theta_1, \ldots, \theta_p, \sigma^2) \sim \prod_{i=1}^{p}\prod_{j=1}^{n_i}\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}} \exp\left[-\frac{1}{\sigma^2}\left(y_{ij}-\theta_i\right)^2\right] = \left(\sigma^2\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{p}\sum_{j=1}^{n_i}\left(y_{ij}-\theta_i\right)^2\right]$$

The full conditionals are as follows.

$(\theta_i|y,\mu,\sigma^2,\tau^2)$

Note that $\bar{y}_i$ is a sufficient statistic for the $y's$, with $\bar{y}_i \sim N\left(\theta_i, \frac{\sigma^2}{n}\right)$.

$$(\theta_i|y,\mu,\sigma^2,\tau^2) \propto \left(\sigma^2\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2/n}(\bar{y}_i-\theta_i)^2\right] \left(\tau^2\sigma^2\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2\tau^2}(\theta_i-\mu)^2\right]$$

This is the normal-normal model, therefore

$$\begin{aligned}
(\theta_i|y,\mu,\sigma^2,\tau^2) &\sim N(m^*,v^*) \\
&\text{with} \\
v^* &= \left[\frac{n_i}{\sigma^2}+\frac{1}{\sigma^2\tau^2}\right]^{-1} = \left[\frac{n_i\tau^2+1}{\sigma^2\tau^2}\right]^{-1} = \sigma^2\left[\frac{\tau^2}{n_i\tau^2+1}\right] \\
m^* &= v^*\left[\left(\frac{n_i}{\sigma^2}\right)\bar{y}_i + \left(\frac{1}{\sigma^2\tau^2}\right)\mu\right] \\
&= \sigma^2\left[\frac{\tau^2}{n_i\tau^2+1}\right]\left[\left(\frac{n_i}{\sigma^2}\right)\bar{y}_i + \left(\frac{1}{\sigma^2\tau^2}\right)\mu\right] \\
&= \left[\frac{n_i\tau^2}{n_i\tau^2+1}\right]\bar{y}_i + \left[\frac{1}{n_i\tau^2+1}\right]\mu \\
&= w\bar{y}_i + (1-w)\mu
\end{aligned}$$

So full conditional is

$$(\theta_i|y,\mu,\sigma^2,\tau^2) \sim N\left(\left[\frac{n_i\tau^2}{n_i\tau^2+1}\right]\bar{y}_i + \left[\frac{1}{n_i\tau^2+1}\right]\mu, \sigma^2\left[\frac{\tau^2}{n_i\tau^2+1}\right]\right) \tag{1}$$

$(\mu|\theta,y,\sigma^2,\tau^2)$

$$\begin{aligned}
\left(\mu|\theta,y,\sigma^2,\tau^2\right) &\propto \exp\left[-\frac{1}{2\sigma^2\tau^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right]\cdot 1 \\
&= \exp\left[-\frac{1}{2\sigma^2\tau^2}\left\{(\theta_1-\mu)(\theta_1-\mu)+\ldots+(\theta_p-\mu)(\theta_p-\mu)\right\}\right] \\
&= \exp\left[-\frac{1}{2\sigma^2\tau^2}\left\{p\mu^2 - 2\mu\sum_{i=1}^{p}\theta_i + \sum_{i=1}^{p}\theta_i^2\right\}\right] \\
&= \exp\left[-\frac{p}{2\sigma^2\tau^2}\left\{\mu^2 - 2\mu\left(\frac{\sum_{i=1}^{p}\theta_i}{p}\right) + \frac{\sum_{i=1}^{p}\theta_i^2}{p}\right\}\right] \\
&\propto \exp\left[-\frac{p}{2\sigma^2\tau^2}\left\{\mu^2 - 2\mu\bar{\theta}_i\right\}\right]
\end{aligned}$$

We recognize this as a Normal kernel, therefore

$$\left(\mu|\theta,y,\sigma^2,\tau^2\right) \sim N\left(\bar{\theta}_i, \frac{\sigma^2\tau^2}{p}\right) \tag{2}$$

$(\sigma^2|\theta,y,\mu,\tau^2)$

$$(\sigma^2|\theta,y,\mu,\tau^2) \propto \left(\sigma^2\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{p}\sum_{j=1}^{n_i}(y_{ij}-\theta_i)^2\right]\left(\sigma^2\right)^{-\frac{p}{2}} \exp\left[-\frac{1}{2\sigma^2\tau^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right]\left(\frac{1}{\sigma^2}\right)$$
$$= \left(\sigma^2\right)^{-\frac{(n+p)}{2}-1} \exp\left[-\left(\frac{1}{\sigma^2}\right)\cdot\left\{\frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{n_i}(y_{ij}-\theta_i)^2+\frac{1}{2\tau^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right\}\right]$$

We recognize this as an Inverse-Gamma kernel, therefore

$$(\sigma^2|\theta,y,\mu,\tau^2) \sim IG\left(\frac{(n+p)}{2}, \left\{\frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{n_i}(y_{ij}-\theta_i)^2+\frac{1}{2\tau^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right\}\right) \quad (3)$$

$(\tau^2|\theta,y,\mu,\sigma^2)$

$$(\tau^2|\theta,y,\mu,\sigma^2) \propto \left(\tau^2\right)^{-\frac{p}{2}} \exp\left[-\frac{1}{2\sigma^2\tau^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right]\cdot 1$$

We recognize this as an Inverse Gamma kernel, therefore

$$(\tau^2|\theta,y,\mu,\sigma^2) \sim IG\left(\frac{p}{2}-1, \frac{1}{2\sigma^2}\sum_{i=1}^{p}(\theta_i-\mu)^2\right) \quad (4)$$

Part 3

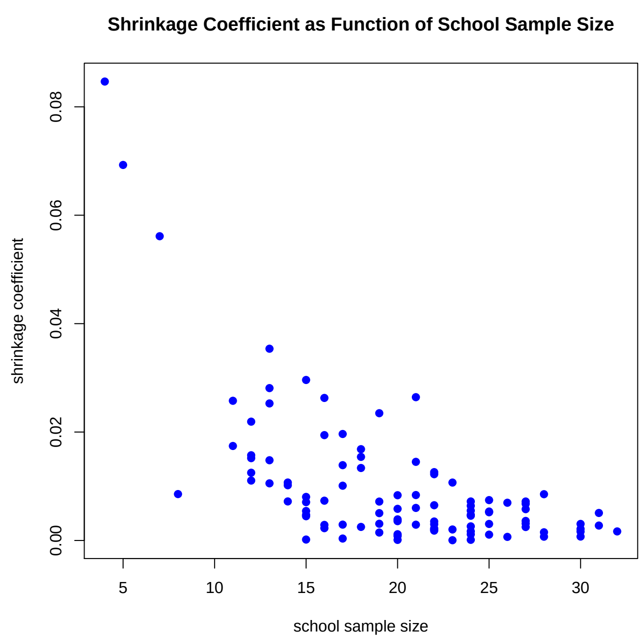


Figure 1: Shrinkage estimator by school sample size

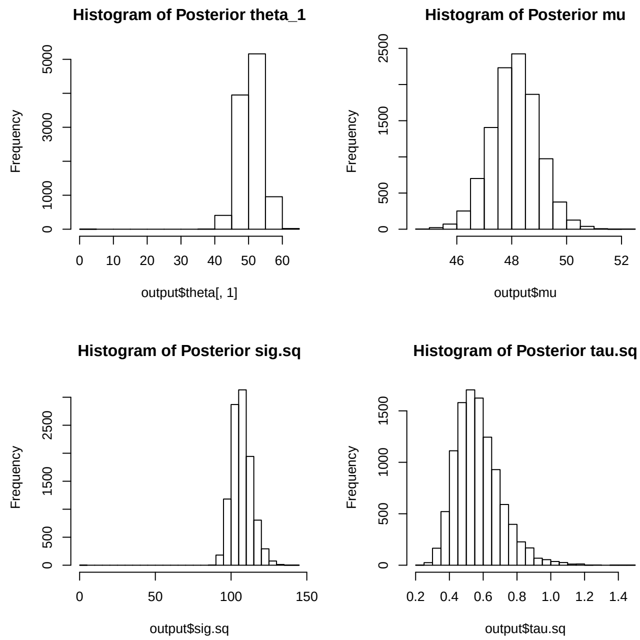


Figure 2: Histograms of Posteriors

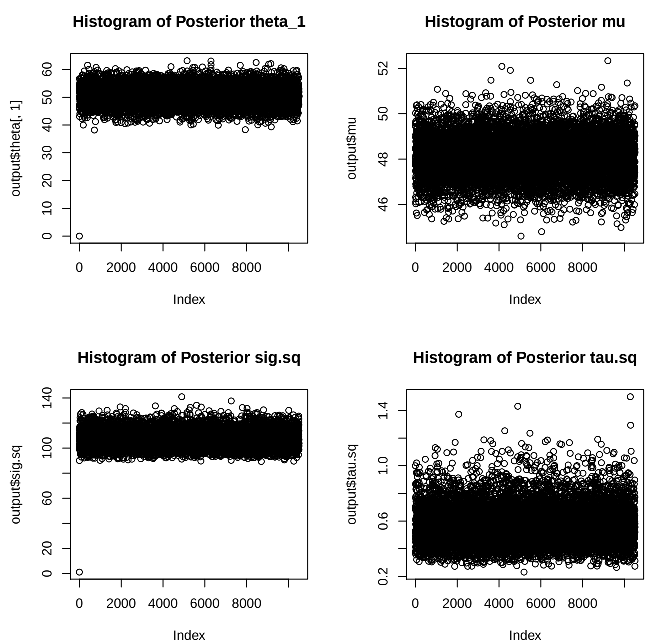


Figure 3: Traces for Gibbs Sampler

Price Elasticity of Demand

Linear Hierarchical Model using Empirical Bayes

Model is specified as

$$
\begin{aligned}
log(Q_{it}) &= log(\alpha_i) + \beta_i log(P_{it}) + \gamma_i x_{it} + \theta_i \left[log(P_{it}) * x_{it}\right] + e_{it} \\
\alpha_i &\sim N(\mu_\alpha, \tau_\alpha^2) \\
\beta_i &\sim N(\mu_\beta, \tau_\beta^2) \\
\gamma_i &\sim N(\mu_\gamma, \tau_\gamma^2) \\
\theta_i &\sim N(\mu_\theta, \tau_\theta^2) \\
e_{it} &\sim N(0, \sigma^2)
\end{aligned}
$$

Where $i = \{1, 2, \ldots, 88\}$ indexes stores, and $t = \{1, 2, \ldots, 68\}$ indexes week (repeated obs on each store).

$$
\begin{aligned}
log(Q_{it}) &= \text{Response; log-volume for store } i \text{ at week } t \\
log(P_{it}) &= \text{Log-price for store } i \text{ at week } t \\
log(\alpha_i) &= \text{Intercept for each store} \\
x_{it} &= \text{Indicator variable for ad display (displayed ad = 1)} \\
log(P_{it}) * x_{it} &= \text{Interaction; shape may change depending on whether ad in store}
\end{aligned}
$$

Variance estimates using lmer to fit the model were as follows.

$$
\begin{aligned}
\hat{\tau}_\alpha^2 &= 5.0478 \\
\hat{\tau}_\beta^2 &= 4.6658 \\
\hat{\tau}_\gamma^2 &= 0.9634 \\
\hat{\tau}_\theta^2 &= 0.7004 \\
\hat{\sigma}^2 &= 0.06733
\end{aligned}
$$

Residual plot does not show evidence of major model mis-fit.

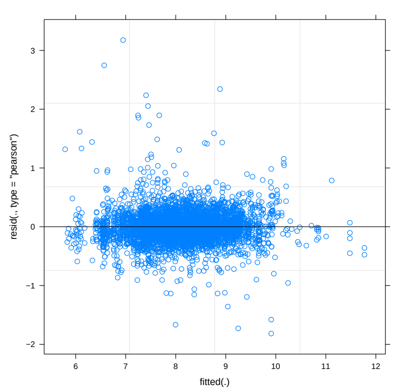


Figure 4: Residual plot for hierarchical model

Model summary:

```
Linear mixed model fit by REML ['lmerMod']
Formula: logQ ~ (logP + disp + disp:logP | store)
   Data: data

REML criterion at convergence: 1811.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-7.0245 -0.4898 -0.0317  0.4348 12.2358

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 store    (Intercept) 5.0478   2.2467
          logP        4.6658   2.1600   -0.94
          disp        0.9634   0.9816    0.47 -0.55
          logP:disp   0.7004   0.8369   -0.32  0.38 -0.97
 Residual             0.0675   0.2598
Number of obs: 5555, groups:  store, 88

Fixed effects:
            Estimate Std. Error t value
(Intercept)  8.18711    0.07794     105
```

Fully Bayesian Hierarchical Linear Model

Model

Define the following variables.

$$y_i = log(Q), \text{ a } (n_s x1) \text{ vector of log-volume observations for each of the } s \text{ stores.}$$
$$X_i = W_i = \begin{bmatrix} | & | & | & | \\ 1 & logP_{it} & ad_{it} & logP_{it}{*}ad_{it} \\ | & | & | & | \end{bmatrix}, \text{ a } (n_s x p) \text{ matrix of covariates for each store.}$$

where

$$\begin{aligned} s &= \text{ Number of stores. Stores are indexed by } i = \{1, 2, \ldots, s\} \\ n_i &= \text{ Number of observations (weeks) within store } i. \end{aligned}$$

We can write the model as follows. This model includes an overall mean of each covariate β_j, plus store-varying offsets.

$$\begin{aligned} y_i &= X_i\beta + W_i b_i + e_i, \text{ with } e_i \sim N(0, \sigma^2 I_{n_i}) \\ \beta &\sim N_p(\mu_\beta, V_\beta) \\ b_i &\sim N_p(0, \Sigma) \\ \sigma^2 &\sim \frac{1}{\sigma^2} \\ \Sigma &\sim IW(d, C) \end{aligned}$$

Likelihood

$$\begin{aligned} y_i &\sim N_{n_i}(X_i\beta + W_i b_i, \sigma^2 I_{n_i}) \\ y_i &\propto \left(\sigma^2\right)^{-\frac{n_i}{2}} \exp\left[-\frac{1}{2\sigma^2}\left(y_i - X_i\beta - W_i b_i\right)^T \left(y_i - X_i\beta - W_i b_i\right)\right] \\ y_1, \ldots, y_s &\propto \left(\sigma^2\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{s}\left(y_i - X_i\beta - W_i b_i\right)^T \left(y_i - X_i\beta - W_i b_i\right)\right] \end{aligned}$$

Full Conditionals

<u>$(b_i | \ldots)$</u>

$$\begin{aligned} (b_i | \ldots) &\propto \exp\left[-\frac{1}{2} b_i^T \Sigma^{-1} b_i\right] \cdot \exp\left[-\frac{1}{2\sigma^2}\left(y_i - X_i\beta - W_i b_i\right)^T \left(y_i - X_i\beta - W_i b_i\right)\right] \\ &\propto \exp\left[-\frac{1}{2} b_i^T \Sigma^{-1} b_i\right] \cdot \exp\left[-\frac{1}{2\sigma^2}\left(b_i^T W_i^T W_i b_i - 2 b_i^T W_i^T y_i - 2 b_i^T W_i^T X_i \beta\right)\right] \\ &\propto \exp\left[-\frac{1}{2} b_i^T \Sigma^{-1} b_i\right] \cdot \exp\left[-\frac{1}{2\sigma^2}\left(b_i^T W_i^T W_i b_i - 2 b_i^T W_i^T \left(y_i - X_i\beta\right)\right)\right] \end{aligned}$$

We recognize this as the multivariate normal kernel.

$$(b_i|\ldots) \sim N\left(m^*, V^*\right), \text{with} \tag{5}$$
$$V^* = \left[\Sigma^{-1} + \frac{1}{\sigma^2} W_i^T W_i\right]^{-1} \tag{6}$$
$$m^* = V^* \left[\frac{1}{\sigma^2} W_i^T \left(y_i - X_i\beta\right)\right] \tag{7}$$

<u>$(\beta|\ldots)$</u>

$$\begin{aligned}
(\beta|\ldots) &\propto \exp\left[-\frac{1}{2}\left(\beta-\mu_\beta\right)^T V_\beta^{-1}\left(\beta-\mu_\beta\right)\right] \cdot \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{s}\left(y_i - X_i\beta - W_i b_i\right)^T\left(y_i - X_i\beta - W_i b_i\right)\right] \\
&\propto \exp\left[-\frac{1}{2}\left(\beta^T V_\beta^{-1}\beta - 2\beta^T V_\beta^{-1}\mu_\beta\right) - \frac{s}{2\sigma^2}\left(\beta^T\left(\sum_{i=1}^{s} X_i^T X_i\right)\beta - 2\beta^T\left(\sum_{i=1}^{s} X_i^T y_i - \sum_{i=1}^{s} X_i^T W_i b_i\right)\right)\right] \\
&= \exp\left[-\frac{1}{2}\left(\beta^T V_\beta^{-1}\beta - 2\beta^T V_\beta^{-1}\mu_\beta\right) - \frac{s}{2\sigma^2}\left(\beta^T\left(\sum_{i=1}^{s} X_i^T X_i\right)\beta - 2\beta^T\left(\sum_{i=1}^{s} X_i^T\left(y_i - W_i b_i\right)\right)\right)\right]
\end{aligned}$$

We recognize this as the univariate normal kernel.

$$(\beta|\ldots) \sim N_p\left(m^*, V^*\right), \text{with} \tag{8}$$
$$V^* = \left[V_\beta^{-1} + \left(\frac{1}{\sigma^2}\right)\sum_{i=1}^{s} X_i^T X_i\right]^{-1} \tag{9}$$
$$m^* = V^* \left[V_\beta^{-1}\mu_\beta + \left(\frac{1}{\sigma^2}\right)\sum_{i=1}^{s} X_i^T\left(y_i - W_i b_i\right)\right] \tag{10}$$

<u>$(\sigma^2|\ldots)$</u>

$$(\sigma^2|\ldots) \propto \left(\frac{1}{\sigma^2}\right)\left(\sigma^2\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}\underbrace{\sum_{i=1}^{s}\left(y_i - X_i\beta - W_i b_i\right)^T\left(y_i - X_i\beta - W_i b_i\right)}_{RSS_{\sigma^2}}\right]$$

We recognize this as the inverse gamma kernel.

$$(\sigma^2|\ldots) \sim IG\left(\frac{n}{2}, \frac{RSS_{\sigma^2}}{2}\right) \tag{11}$$

<u>$(\Sigma|\ldots)$</u>

$$\begin{aligned}
(\sigma^2|\ldots) &\propto |\Sigma|^{-\left(\frac{d+p+1}{2}\right)} \exp\left[-\frac{1}{2} tr\left(C\Sigma^{-1}\right)\right] \cdot |\Sigma|^{-\left(\frac{s}{2}\right)} \exp\left[-\frac{1}{2}\sum_{i=1}^{s} b_i^T \Sigma^{-1} b_i\right] \\
&= |\Sigma|^{-\left(\frac{d+s+p+1}{2}\right)} \exp\left[-\frac{1}{2} tr\left(C\Sigma^{-1}\right) - \frac{1}{2} tr\left(\sum_{i=1}^{s} b_i b_i^T \Sigma^{-1}\right)\right]
\end{aligned}$$

We recognize this as the Inverse Wishart kernel.

$$(\sigma^2|\ldots) \sim IW\left(d+s, C + \sum_{i=1}^{s} b_i b_i^T\right) \tag{12}$$

The demand curves for the 88 stores are as follows. Stores are ordered in decreasing order by average price. Red represents weeks where an ad is displayed; blue represents no ad.

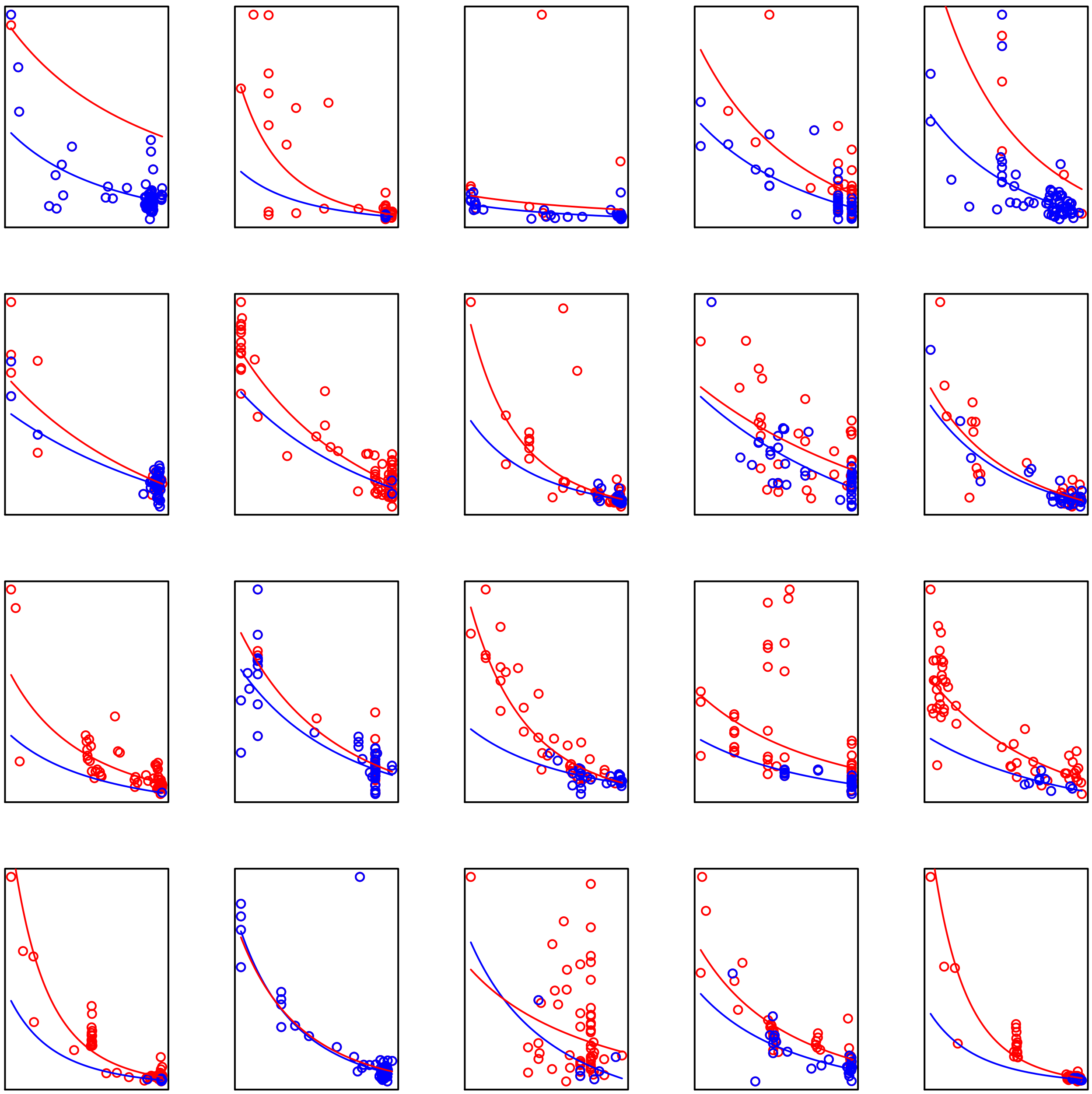


Figure 5: Demand Curves for 88 Stores

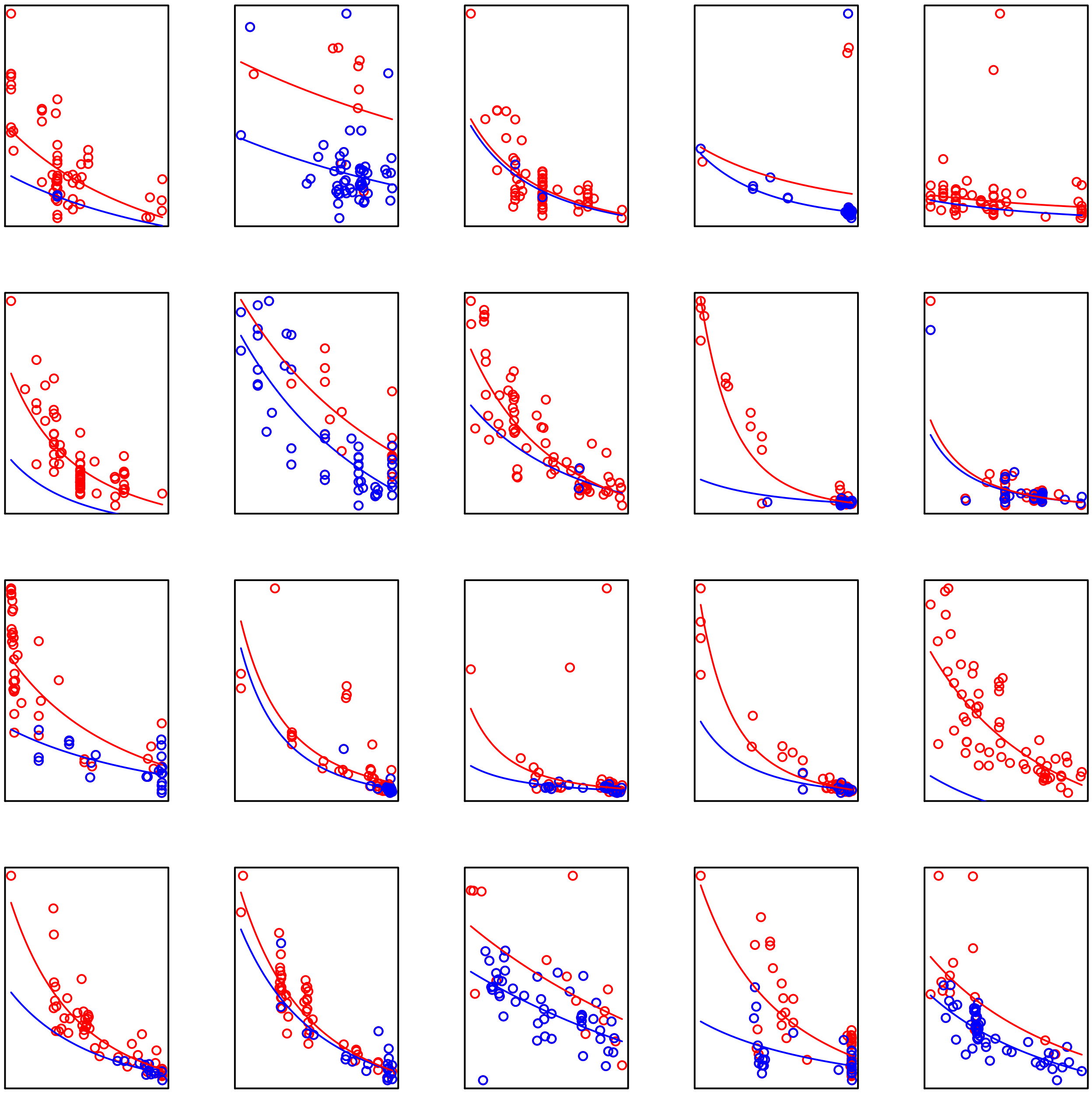


Figure 6: Demand Curves for 88 Stores

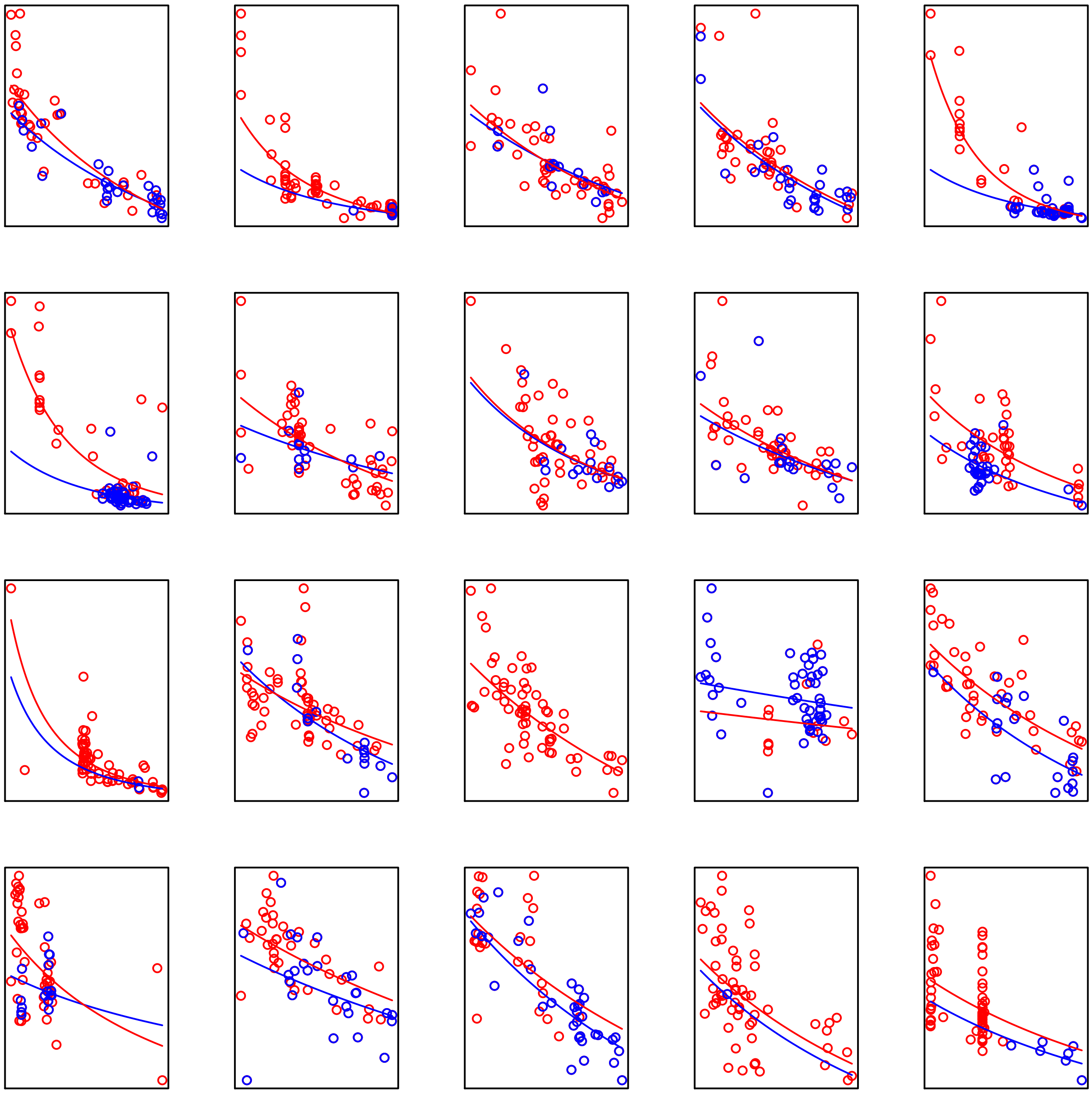


Figure 7: Demand Curves for 88 Stores

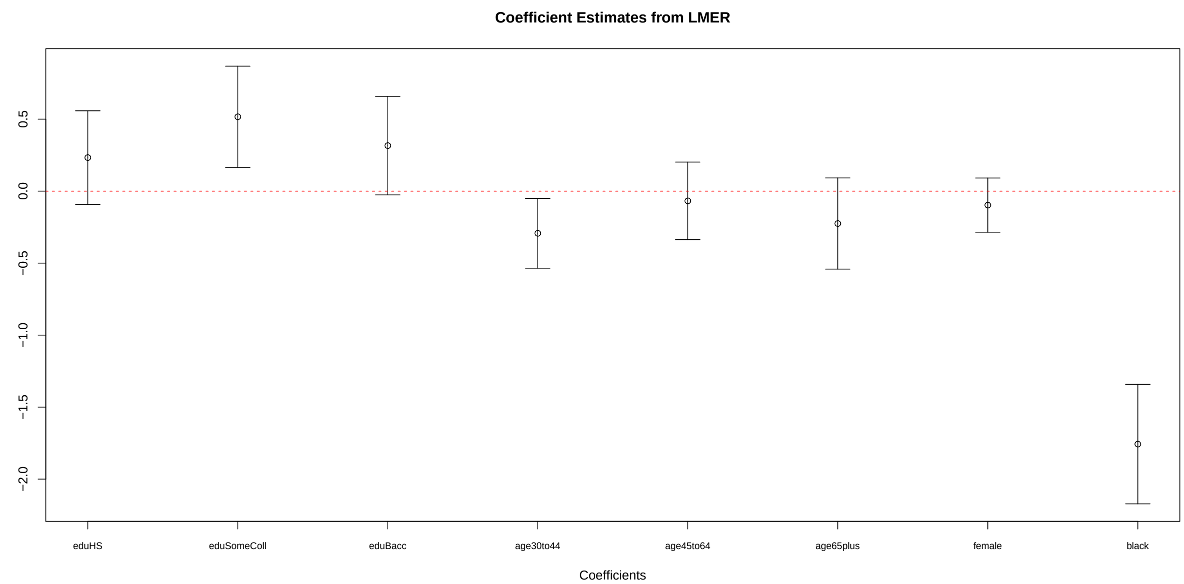
A Hierarchical Probit Model via Data Augmentation

Empirical Bayes Analysis using LMER

We can fit a hierarchical model with state-varying intercepts and a fixed β model using glmer, with $family =' binomial'$. This model is fit using the following call. (wt.sc indicates scaled and centered weight.)

```
hlm = glmer(bush ~ edu + age + female + black  + wt.sc + (1|state),
            data=data, family='binomial')
```

The variance estimate for the state-varying intercept term is 0.1732. Confidence intervals for fixed effects are below.



Fully Bayesian Hierarchical Augmented Model using Gibbs Sampler

Model:

Original probit model:

$$P(y_{ij} = 1) = \Phi(z_{ij})$$
$$z_{ij} = \mu_i + x_{ij}^T \beta$$

The trick proposed by Albert and Chib (1993) is to introduce a latent variables z_{ij}, where we observe the y_{ij} but the underlying z_{ij} are normally distributed. In this case, the $z_i j$ in the model formulation above can act as our latent variable.

We can define the model as follows.

$$z_i \sim N_{n_i}(W_i \mu_i + X_i \beta, I_{n_i})$$
$$\mu_i \sim N_1(0, \tau^2)$$
$$\beta \sim N(\mu_\beta, \Sigma)$$

This model allows for a state-varying intercept, with other covariates fixed. In this formulation, X_i is the $(n_i x p)$ matrix of demographic covariates for each state's observations, including a column of 1's for the intercept. Then W_i is a $(n_i x 1)$ matrix; a single column of 1's, to hold the intercept offset for each state.

Then we can easily that

$$P(Y_{ij} = 1) = \Phi(W_i \mu_i + X_i \beta)$$

And we can define y as

$$y_{ij} = \begin{cases} 1 \text{ if } z_{ij} > 0 \\ 0 \text{ if } z_{ij} < 0 \end{cases}$$

The Gibbs Sampler will then proceed as follows:

(1) Update $\mu_1, \ldots, \mu_s$, β, τ^2 by drawing from full conditionals.

(2) Update z_{ij} by drawing from the truncated normal, based on whether each observed y_{ij} is greater than zero.$^{(**)}$

(3) Calculate update $P(y_{ij} = 1)$ based on all updated parameter values.

Likelihood

$$(z_1, \ldots, z_n) \propto \exp\left[-\frac{1}{2} \sum_{i=1}^{s} (z_i - W_i \mu_i - X_i \beta)^T (z_i - W_i \mu_i - X_i \beta) \right]$$

$(**)$ Note: For updating the z_{ij}, as shown below, this 'likelihood' acts as the prior, and we update z_{ij} with the observed y likelihood to obtain its posterior. The latent z_{ij}s must be included in the sampler.

Full Conditionals, including z_i

$(\beta|\ldots)$

$$(\beta|\ldots) \propto \exp\left[-\frac{1}{2}\left(\beta-\mu_\beta\right)^T\Sigma^{-1}\left(\beta-\mu_\beta\right)\right]\exp\left[-\frac{1}{2}\sum_{i=1}^{s}\left\{\beta^T X_i^T X_i\beta - 2\beta^T X_i^T\left(z_i - W_i\mu_i\right)\right\}\right]$$

We recognize this as a normal-normal update.

$$(\beta|\ldots) \sim N_p\left(m^*, V^*\right), \text{with} \tag{13}$$

$$V^* = \left(\Sigma^{-1} + \sum_{i=1}^{s} X_i^T X_i\right)^{-1} \tag{14}$$

$$m^* = V^*\left[\Sigma^{-1}\mu_\beta + \sum_{i=1}^{s} X_i^T\left(z_i - W_i\mu_i\right)\right] \tag{15}$$

$(\mu_i|\ldots)$

$$\begin{aligned}(\mu_i|\ldots) &\propto \exp\left[-\frac{1}{2}\mu_i^T\mu_i\right]\exp\left[-\frac{1}{2}\left\{\mu_i^T W_i^T W_i\mu_i - 2\mu_i^T W_i^T\left(z_i - X_i\beta\right)\right\}\right] \\ &= \exp\left[-\frac{1}{2}\left\{\mu_i^2\left(1+n_i\right) - 2\mu_i^T W_i^T\left(z_i - X_i\beta\right)\right\}\right]\end{aligned}$$

We recognize this as a normal-normal update.

$$\begin{aligned}(\mu_i|\ldots) &\sim N_1(m^*, v^*), \text{with} \\ v^* &= \frac{1}{\tau^2 + n_i} \\ m^* &= v^*\left[W_i^T\left(z_i - X_i\beta\right)\right]\end{aligned}$$

$(\tau^2|\ldots)$

$$(\tau^2|\ldots) \propto (1)\left(\tau^2\right)^{-\frac{s}{2}}\exp\left[-\frac{1}{2}\sum_{i=1}^{s}\mu_i^2\right]$$

We recognize this as the inverse gamma kernel.

$$(\tau^2|\ldots) \sim IG\left(\frac{s}{2}, \frac{1}{2}\sum_{i=1}^{s}\mu_i^2\right) \tag{16}$$

$(z_i|\ldots)$

$$\begin{aligned}(z_{ij}|y_{ij}=1) &\sim \left[\mathbb{1}\left(y_{ij}=1\right)\cdot N(\mu_i + X_{ij}\beta, 1)_+\right] \\ (z_{ij}|y_{ij}=0) &\sim \left[\mathbb{1}\left(y_{ij}=0\right)\cdot N(\mu_i + X_{ij}\beta, 1)_-\right]\end{aligned}$$

Where

$$\begin{aligned}N(\mu_i + X_{ij}\beta, 1)_+ &= \text{ Truncated Normal } (0, \infty) \\ N(\mu_i + X_{ij}\beta, 1)_- &= \text{ Truncated Normal } (-\infty, 0)\end{aligned}$$

We can confirm mixing of the Gibbs sampler using trace plots for a handful of the predictors. Trace plots are also shown for a few of the state intercepts, where the fixed intercept and the offset are combined.

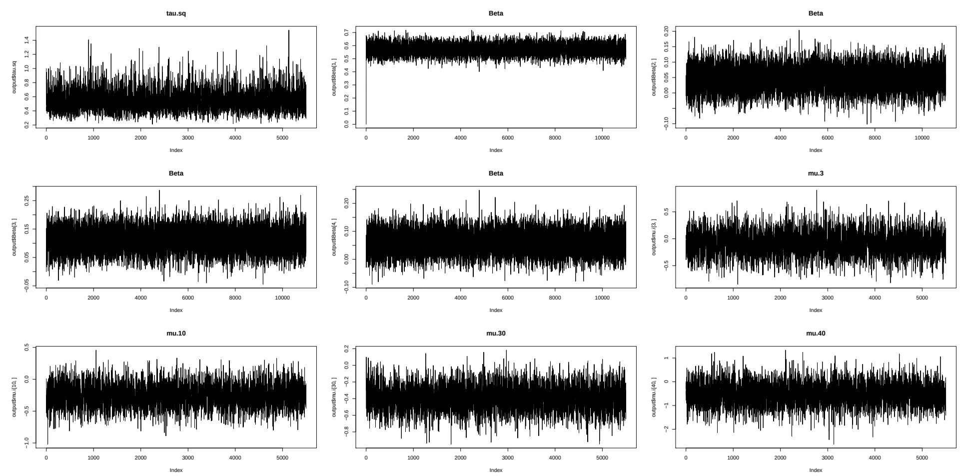


Figure 8: Trace plots for selected predictors

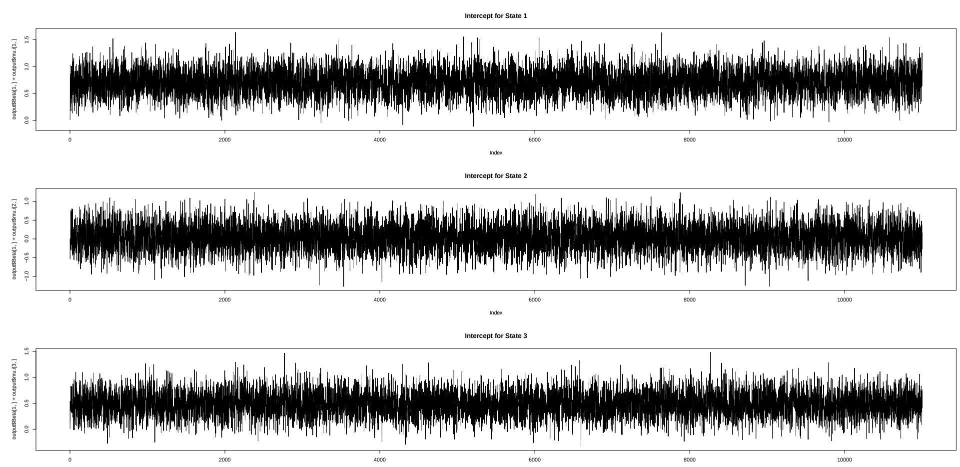
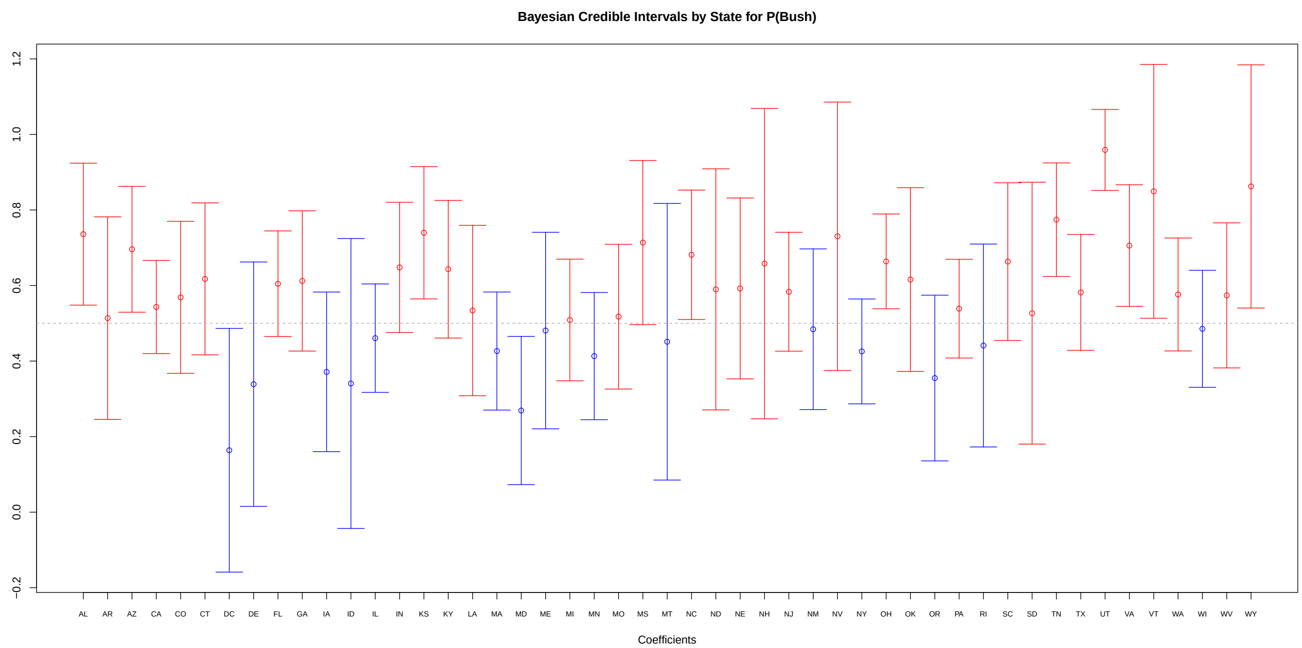


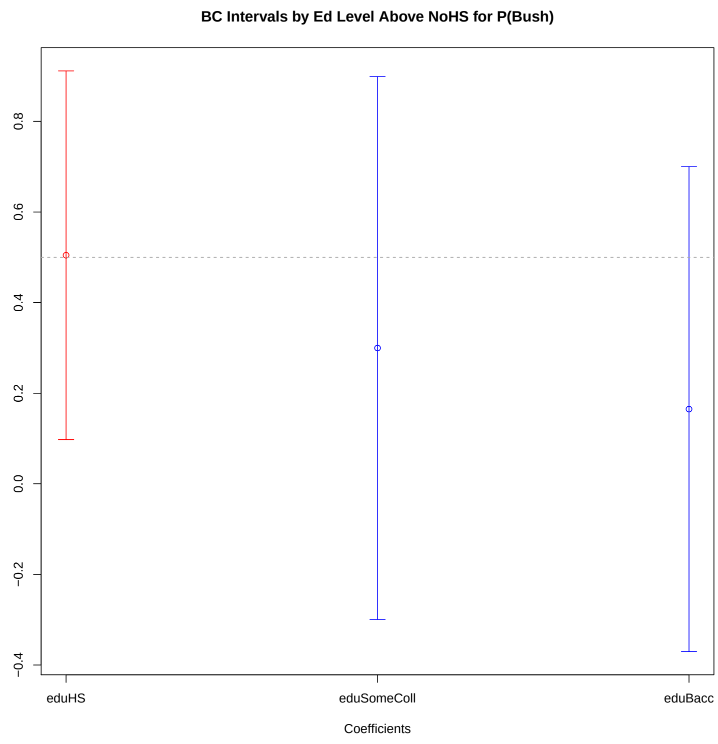
Figure 9: Trace plots for selected intercepts, including fixed and offset terms

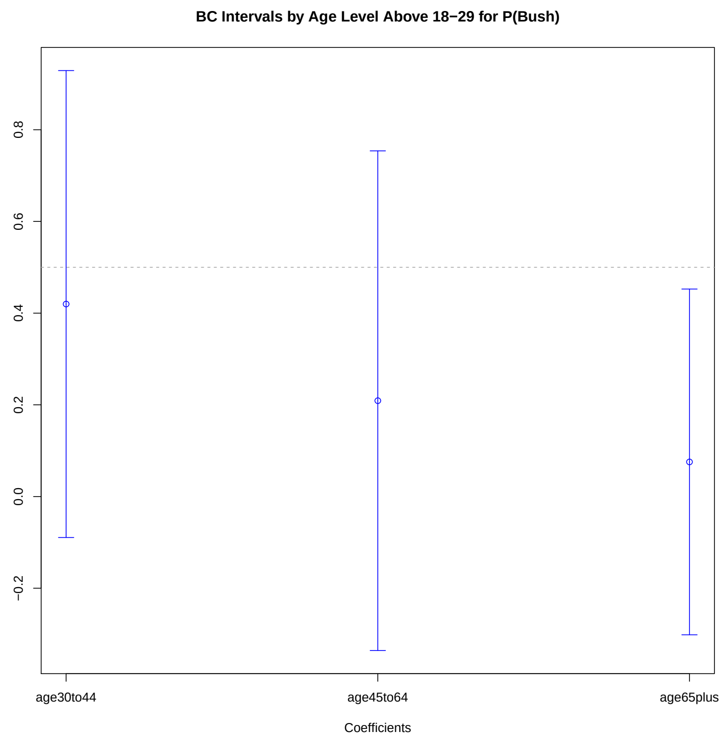
Results

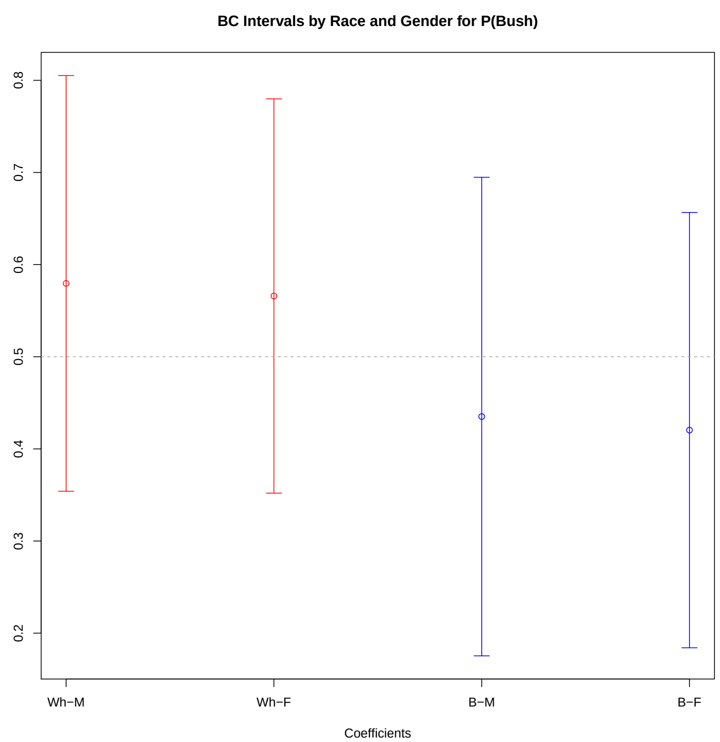
The goal of the analysis is to understand how demographic information, including state, relate to the probability that someone will support Bush in the 1988 presidential election.

As demonstrated in the plots below, state appears to vary the most in terms of probability of supporting Bush. There are also effects for education level and race. Age, weight and gender do not appear to relate to the probability. There does not appear to be a race-gender interaction. These results are interesting based on how different politics are today than in 1988, in relation to the age and gender covariates.


Bayesian Credible Intervals by State for P(Bush)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
AL AR AZ CA CO CT DC DE FL GA IA ID IL IN KS KY LA MA MD ME MI MN MO MS MT NC ND NE NH NJ NM NV NY OH OK OR PA RI SC SD TN TX UT VA VT WA WI WV WY
Coefficients


BC Intervals by Ed Level Above NoHS for P(Bush)
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
eduHS
eduSomeColl
eduBacc
Coefficients


BC Intervals by Age Level Above 18−29 for P(Bush)
−0.2
0.0
0.2
0.4
0.6
0.8
age30to44
age45to64
age65plus
Coefficients


BC Intervals by Race and Gender for P(Bush)
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Wh−M
Wh−F
B−M
B−F
Coefficients

Gene Expression Over Time