Macroeconomics

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Complementary notes for the slides
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Contents

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1 Stochastic processes

Convergence in distribution

$$\hat{\beta}_{OLS} \xrightarrow{d} \text{N}(\beta, V(\beta))$$
$$(\hat{\beta}_{OLS} - \beta) \xrightarrow{d} \text{N}(0, V(\beta))$$

$$\begin{aligned} \hat{\beta}_{OLS} &= (x'x)^{-1}x'y \\ &= \beta + (x'x)^{-1}x'u \end{aligned}$$

The Central Limit Theorem (Greene, 1996)

$$\sqrt{T}(\bar{x} - \mu) \xrightarrow{d} \text{N}(0, \sigma^2)$$

Lindberg-Feller Central Limit Theorem: Generalizations to a multivariate framework with constant or varying covariance matrix.

Asymptotic independence

If independent, the joint probability equals the product of probabilities, i.e.

$$P(A \cap B) = P(A) * P(B)$$

2 ARIMA modelling

Univariate TS strategy

Not based on Economic Theory
(reduced form modelling $\rightarrow$ the past is informative about the present)

$$y_t = f(y_{t-1}, y_{t-2}, y_{t-3} \cdot)$$

The performance in terms of **forecasting** is key!

If you want to learn about main drivers, you need to use other models.

1. Stationary time series:

Sometimes 1st difference is required to ensure stationarity.

2. Identification:

Specify the model that explains the behaviour the best

AR: Autoregressive model

MA: Moving Average model

ARMA: AutoRegressive Moving Average model

To initially decide between them, plot the

- **ACF:** The Simple Autocorrelation Function.
  Check if there is persistence/memory:
  $$\frac{Cov(x_t, x_{t+k})}{\sqrt{V(x_t)}\sqrt{V(x_{t+k})}}, \text{ for } k = 0, \pm 1, \pm 2, \cdots$$
  Related to the asymptotic confidence interval
  $$\left(\frac{-1.96}{\sqrt{(T)}}, \frac{1.96}{\sqrt{(T)}} \right)$$
  or degrees of freedom corrected
  $$\left(\frac{-1.96}{\sqrt{(T-k)}}, \frac{1.96}{\sqrt{(T-k)}} \right)$$
- **PACF:** The Partial Autocorrelation Function
  e.g. The The 3rd partial autocorrela-

tion coeffcient of the PACF, ϕ_{33}:

$$\tilde{x}_t = \phi 31 \tilde{x}_{t-1} + \phi 32 \tilde{x}_{t-2} + \phi 33 \tilde{x}_{t-3} + \varepsilon_t$$

→ Using both functions helps us to make an initial specification.

- Helps us to decide on the number of lags.
- Too many lags can cause multi-colliniarity.

Better to choose an $f(t) \neq 0$ to test it.

3. Estimation

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4. Validation

How well is the model specified?

-
- Residuals

Re-specify if not correctly validated (repeat step 2-4)

5. Forecasting

When model is valid, use it to forecast.

2.1 AR(p) process

Stationarity conditions for the AR model:

1. The deterministic component f does not depend on t (rules out MA)
2. All roots of the autoregressive polynomial must, in module, be greater than one, e.g.

$$x_t = \phi_1 x_{t-1} + \varepsilon_t$$

$$\underbrace{(1 - \phi_1(L))}_{\text{AR polynomial}} x_{t-1} = \varepsilon_t, \quad L > 1 \text{ required!}$$

$$\begin{aligned} V(x_t) &= V(\phi_1 x_{t-1}) + V(\varepsilon_t) \\ \sigma_x^2 &= \phi_1^2 \sigma_x^2 + \\ \sigma_x^2 &= \frac{1}{1-\sigma_1^2} \sigma_\varepsilon^2, \text{ requires } \sigma_1^2 < 1 \end{aligned}$$

2.2 MA(q) model

If MA is invertible

$$\begin{aligned} \Rightarrow MA(q) &\equiv AR(\infty) \\ &\simeq AR(p), \quad p \text{ is large} \end{aligned}$$

Use the $\hat{\varepsilon}_t^0$ estimation algorithm.

3 Order of Integration Analysis

#1. Iterative Strategy: DIY.

#2. Model Selection Strategy:
Automated procedure using AIC or BIC.

- $p_m ax = 3$
- $q_m ax = 3$

"Autometrics".

Models that rely on x_t being stationary and invertible (for consistent $\hat{\beta}$):

	ACF	$PACF$
$AR(p)$		$x_{(p)}$
$MA(q)$	x_q	
$ARMA(p,q)$	$?_{\hat{p},\hat{q}}$	$?_{\hat{p},\hat{q}}$

Usually x_t is non-stationary.

→ $\Delta^d x_t, d = 0, 1, 2, 3, \cdots$ is stationary?

→ $\hat{\beta}$ is consistent!

- Integrated of order "d".

- $prices_t$
- $\Delta prices_t \simeq$ Inflation.
- $\Delta^2 prices_t \simeq$ Acceleration prices.

A non-stationary proces $\simeq$ Random walk process features (uniroot).

$$\begin{aligned} x_t &= \phi x_{t-1} + \varepsilon_t, \qquad \phi = 1 \\ &= x_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j} \end{aligned}$$

With the variance

$$\begin{aligned} V(x_t) &= V(x_0) + V\left(\sum_{j=0}^{t-1} \varepsilon_{t-j}\right) \\ &\vdots \\ &= \sigma_\varepsilon^2 t \end{aligned}$$

3.1 Unit root tests

t-Student distrubution

- Zero mean.
- Broad tails with a low degree of freedoms k.

$\rightarrow$ Tends towards the normal distribution when $k \rightarrow \infty$.

Dickey-Fuller (DF) test

- Only valid for an AR(1) process!
- Statistical inference: Objective:
  - Test statistics:
    $H_0: \ x_t \sim I(1)$
    $H_1: \ x_t \sim I(0)$
  - "Confirmatory analysis":
    $H_0: \ x_t \sim I(0)$
    $H_1: \ x_t \sim I(1)$

DF:

$$\Delta x_t = f(t) + \alpha x_{t-1} + u_t$$

- DF assumes $u_t \sim iid$

Corrections to DF:

- Philips-Perron (PP) test
  - Non-parametric correction.
  - More general but difficult to estimate the s^2.
    * Allowing a high number of covariates: test statistics looses power
    * Allowing a low number of covariates?
- ADF test:
  - Parametric correction.
  - More efficient for finite samples.
- Ng-Perron M test statistics (2001)
  3 legs:
  1. Parametric longrun estimate of S^2_{AR}.
  2. $\bar{\alpha}$ estimated numerically (Elliot, Rothenburg & Stock, 1996) and Power Envelope Function (PEF) using GLS.
  3. Number of lags k to include: decided using the Midified Information Criteria (MIC) strategy:
     * MAIC
     * MSIC

ADF-test
Capturing any information that might be hidden in the u_t that creates and effect in the λ-term, $\lambda = \sigma^2 - \sigma_\varepsilon^2/2$

$$\begin{aligned} \Delta x_t = f(t) + \alpha x_{t-1} + \sum_{j-1}^{p-1} \gamma_j \Delta x_{t-j} + u_t \\ \Rightarrow\rightarrow \varepsilon_t \sim iid \end{aligned}$$

Philips-Perron (PP) test

- Nice correction to the DF test.

- In practice, it's difficult to calculate the short-run variance s_u^2 and the long-run variance s^2.

Complex hypothesis (less feasible):

$$\begin{cases} H_0 : & \alpha = 1 & = & x_t \sim I(1) \\ H_1 : & |\alpha = 1| & = & x_t \sim I(0) \end{cases}$$

Simple alternatives:

$$\begin{cases} H_0 : & \alpha = 1 \\ H_1 : & \alpha = \bar{\alpha}, \quad \bar{\alpha} = 1 + \frac{\bar{c}}{T} \end{cases}$$

i.e. 'local alternatives' in the neighborhood of H_0.

Ng-Perron principle:

$$H_1 : \; L(\bar{\alpha}) - L(1) \tag{3.1}$$

Using the likelihood (LM test).

Def. 'Power Envelope Function' (PEF): Chooses $\bar{c}$ such that the $PEF = 0.5$
$\Rightarrow$ use **GLS**.

4 Cointegration Analysis

Spurious regression

$\hat{\mu}, \hat{\gamma}$ inconsistent (random variables)

$t_{\hat{\mu}}, F^*$ inconsistent (diverge towards $\pm\infty$)

$\bar{R}^2$ inconsistent (random variable)

Best (incomplete) cure
Model the relationship using differences

$\hat{\beta}_{OLS}$ consistent.

$t_{\hat{\mu}}$ $\sim N(0,1)$

F^* $\sim$ F-snedecor

$\bar{R}^2$ ok.

4.1 Estimation of cointegration relationships

$r = 0 \Rightarrow$ Spurious regression
$0 < r < m \Rightarrow$ Cointegration relationships
$r = m \Rightarrow y_t \sim I(0)$

Engle & Granger estimates a single equation $\Rightarrow r = 0, r = 1, (0 < r < m)$?

1st Estimate the LR relationship and obtain the residuals.

2nd Cointegration test. ADF $\rightarrow$ critical values for the standard DF test (79; 81).

- $H_0 : z_t \sum I(1) \equiv$ No cointegration.
- $H_0 : z_t \sum I(0) \equiv$ Cointegration.

- We only estimate one equation
  - $\rightarrow$ We loose information (regressors might not be exogenous)
  - $\rightarrow$ Inefficient estimation, can't do inference.

Johansen's procedure allows you to consider all possible equations together.

$\rightarrow$ All variables in system are assumed to be ***endogenous***.

Consistent + efficient estimation of $\hat{\alpha}$ in single equation framework

- Dynamic Ordinary Least Squares (DOLS) of Saikkonen (1991) and Stock and Watson (1993).
  - Works best in a finite sample (in the limit they're the same).
  - Including k_2 lacks and k_1 leads.
  - → A very simply way to include endogeneity in the sample!
  - Using AIC and BIC to choose the model (might be available in R→URCA)
- Fully Modified (FM) OLS of Philips and Hansen (1990).
- Canonical Cointegration Regression (CCR) of Park (1992).

⇒ allows to do inference.

5 VAR modelling

Introduced by Sims (1980) as an alternative to structural models.

- Estimates all possible relations in the system (complete endogeneity).
- Allows to model multivariate time series with a minimum of economic theory.

Uses

→ Delivered very good forecasts without being bound by specification of structure.

→ Allows to do 'causality' analysis.

→ Impulse response analysis. How would the variables react to a chock to the system?

VAR(p) model is a VAR version of the AR(p) model

y_t is a vector of variables that depends upon the lag of the vector.

There are no ACF or PACF for the VAR(p) model.

→ Use an IC.

- Use hypothesis testing

$$\begin{cases} H_0: & p = p_0 \\ H_1: & p = p_1 \end{cases}$$