

Macroeconomics

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Complementary notes for the slides
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1 THEORETICAL ISSUES

Convergence in distribution

$$\begin{aligned}\hat{\beta}_{OLS} &\xrightarrow{d} N(\beta, V(\beta)) \\ (\hat{\beta}_{OLS} - \beta) &\xrightarrow{d} N(0, V(\beta))\end{aligned}$$

$$\begin{aligned}\hat{\beta}_{OLS} &= (x'x)^{-1}x'y \\ &= \beta + (x'x)^{-1}x'u\end{aligned}$$

Asymptotic independence

If independent, the joint probability equals the product of probabilities, i.e.

$$P(A \cap B) = P(A) * P(B)$$

2 ARIMA MODELLING

Univariate TS strategy

Not based on Economic Theory
(reduced form modelling → the past is informative about the present)

$$y_t = f(y_{t-1}, y_{t-2}, y_{t-3} \dots)$$

The performance in terms of **forecasting** is key!

If you want to learn about main drivers, you need to use other models.

1. Stationary time series:

Sometimes 1st difference is required to ensure stationarity.

2. Identification:

Specify the model that explains the behaviour the best

AR: Autoregressive model

MA: Moving Average model

ARMA: AutoRegressive Moving Average model

To initially decide between them, plot the

- **ACF:** The Simple Autocorrelation Function.
Check if there is persistence/memory:

$$\frac{Cov(x_t, x_{t+k})}{\sqrt{V(x_t)}\sqrt{V(x_{t+k})}}, \text{ for } k = 0, \pm 1, \pm 2, \dots$$

Related to the asymptotic confidence interval

$$\left(\frac{-1.96}{\sqrt{(T)}}, \frac{1.96}{\sqrt{(T)}} \right)$$

or degrees of freedom corrected

$$\left(\frac{-1.96}{\sqrt{(T-k)}}, \frac{1.96}{\sqrt{(T-k)}} \right)$$

- **PACF:** The Partial Autocorrelation Function
e.g. The 3rd partial autocorrelation coefficient of the PACF, ϕ_{33} :

$$\tilde{x}_t = \phi_{31}\tilde{x}_{t-1} + \phi_{32}\tilde{x}_{t-2} + \phi_{33}\tilde{x}_{t-3} + \varepsilon_t$$

→ Using both functions helps us to make an initial specification.

- Helps us to decide on the number of lags.
- Too many lags can cause multicollinearity.

Better to choose an $f(t) \neq 0$ to test it.

3. Estimation

..

4. Validation

How well is the model specified?

-
- Residuals

Re-specify if not correctly validated (repeat step 2-4)

5. Forecasting

When model is valid, use it to forecast.

2.1 AR(p) process

Stationarity conditions for the AR model: Use the $\hat{\varepsilon}_t^0$ estimation algorithm.

1. The deterministic component f does not depend on t (rules out MA)
2. All roots of the autoregressive polynomial must, in module, be greater than one, e.g.

$$x_t = \phi_1 x_{t-1} + \varepsilon_t$$

$$\underbrace{(1 - \phi_1(L))}_{\text{AR polynomial}} x_{t-1} = \varepsilon_t, \quad L > 1 \text{ required!}$$

$$V(x_t) = V(\phi_1 x_{t-1}) + V(\varepsilon_t)$$

$$\sigma_x^2 = \phi_1^2 \sigma_x^2 +$$

$$\sigma_x^2 = \frac{1}{1 - \phi_1^2} \sigma_\varepsilon^2, \text{ requires } \sigma_1^2 < 1$$

2.2 MA(q) model

If MA is invertible

$$\Rightarrow MA(q) \equiv AR(\infty)$$

$$\simeq AR(p), \quad p \text{ is large}$$

3 ORDER OF INTEGRATION ANALYSIS

#1. Iterative Strategy: DIY.

and invertible (for consistent $\hat{\beta}$):

#2. Model Selection Strategy:

Automated procedure using AIC or BIC.

$$\bullet p_{max} = 3$$

$$\bullet q_{max} = 3$$

"Autometrics".

Models that rely on x_t being stationary

	ACF	PACF
$AR(p)$		$x_{(p)}$
$MA(q)$	x_q	
$ARMA(p, q)$	$?_{\hat{p}, \hat{q}}$	$?_{\hat{p}, \hat{q}}$

Usually x_t is non-stationary.

→ $\Delta^d x_t, d = 0, 1, 2, 3, \dots$ is stationary?

→ $\hat{\beta}$ is consistent!

- Integrated of order "d".

– $prices_t$

– $\Delta prices_t \simeq$ Inflation.

– $\Delta^2 \text{prices}_t \simeq$ Acceleration prices. Corrections to DF:

A non-stationary process \simeq Random walk process features (unirroot).

$$\begin{aligned} x_t &= \phi x_{t-1} + \varepsilon_t, \quad \phi = 1 \\ &= x_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j} \end{aligned}$$

With the variance

$$\begin{aligned} V(x_t) &= V(x_0) + V\left(\sum_{j=0}^{t-1} \varepsilon_{t-j}\right) \\ &\vdots \\ &= \sigma_\varepsilon^2 t \end{aligned}$$

3.1 Unit root tests

t-Student distribution

- Zero mean.
 - Broad tails with a low degree of freedoms k .
- Tends towards the normal distribution when $k \rightarrow \infty$.

Dickey-Fuller (DF) test

- Only valid for an AR(1) process!
- Statistical inference: Objective:
 - Test statistics:

$$\begin{aligned} H_0 : x_t &\sim I(1) \\ H_1 : x_t &\sim I(0) \end{aligned}$$
 - "Confirmatory analysis":

$$\begin{aligned} H_0 : x_t &\sim I(0) \\ H_1 : x_t &\sim I(1) \end{aligned}$$

DF:

$$\Delta x_t = f(t) + \alpha x_{t-1} + u_t$$

- DF assumes $u_t \sim iid$

• Philips-Perron (PP) test

- Non-parametric correction.
- More general but difficult to estimate the s^2 .
 - * Allowing a high number of covariates: test statistics loses power
 - * Allowing a low number of covariates?

• ADF test:

- Parametric correction.
- More efficient for finite samples.

• Ng-Perron M test statistics (2001)

3 legs:

1. Parametric longrun estimate of S_{AR}^2 .
2. $\bar{\alpha}$ estimated numerically (Elliot, Rothenburg & Stock, 1996) and Power Envelope Function (PEF) using GLS.
3. Number of lags k to include: decided using the Modified Information Criteria (MIC) strategy:
 - * MAIC
 - * MSIC

ADF-test

Capturing any information that might be hidden in the u_t that creates an effect in the λ -term, $\lambda = \sigma^2 - \sigma_\varepsilon^2/2$

$$\begin{aligned} \Delta x_t &= f(t) + \alpha x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta x_{t-j} + u_t \\ &\Rightarrow \varepsilon_t \sim iid \end{aligned}$$

Philips-Perron (PP) test

- Nice correction to the DF test.
- In practice, it's difficult to calculate the short-run variance s_u^2 and the long-run variance s^2 .

i.e. 'local alternatives' in the neighborhood of H_0 .

Ng-Perron principle:

$$H_1 : L(\bar{\alpha}) - L(1) \quad (3.1)$$

Complex hypothesis (less feasible):

$$\begin{cases} H_0 : \alpha = 1 & = x_t \sim I(1) \\ H_1 : |\alpha| = 1 & = x_t \sim I(0) \end{cases}$$

Using the likelihood (LM test).

Def. 'Power Envelope Function' (PEF):

Chooses \bar{c} such that the $PEF = 0.5$

\Rightarrow use **GLS**.

Simple alternatives:

$$\begin{cases} H_0 : \alpha = 1 \\ H_1 : \alpha = \bar{\alpha}, \quad \bar{\alpha} = 1 + \frac{\bar{c}}{T} \end{cases}$$

3.2 Cointegration Analysis

Spurious regression

Model the relationship using differences

$\hat{\mu}, \hat{\gamma}$ inconsistent (random variables)

$\hat{\beta}_{OLS}$ consistent.

$t_{\hat{\mu}}, F^*$ inconsistent (diverge towards $\pm\infty$)

$t_{\hat{\mu}} \sim N(0, 1)$

\bar{R}^2 inconsistent (random variable)

$F^* \sim F\text{-snedecor}$

Best (incomplete) cure

\bar{R}^2 ok.