Optimal taxation
Homework 1 [*]

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Contents

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1 Optimal linear income taxation

For an economy with only two individuals ($h = 1, 2$) one's utility from consumption C and leisure $L = 16 - l$ is given by the utility function

$$U^h = \alpha^h \ln C + (1 - \alpha^h) \ln L, \qquad \alpha^1 = \frac{1}{2}, \alpha^2 = \frac{1}{3} \tag{1.1}$$

The linear tax function includes a universal transfer R and the wage w^h per working hour l^h is the same for each individual

$$T(y) = -R + ty = -R + tw^h l^h, \qquad w^1 = w^2 = 6 \tag{1.2}$$

a) Set and solve the maximization problem of each individual, where C and L are the control variables.

From the tax function (1.2) we can set the budget constraint of individual h to be:

$$C = R + (1 - t)w^h l^h = R + (1 - t)w^h(16 - L) \tag{1.3}$$

The maximization problem for individual h can be written as

$$\max_{C,L} U^h = \alpha^h \ln C + (1 - \alpha^h)L \text{ s.t. } C = R + (1 - t)w^h(16 - L) \tag{1.4}$$

To solve the maximization problem (1.4), we set up the Lagrangian

$$\mathcal{L} = \alpha^h \ln C + (1 - \alpha^h)L - \lambda[C - R - (1 - t)w^h(16 - L)] \tag{1.5}$$

Taking the FOCs

$$\frac{\delta \mathcal{L}}{\delta C} = 0 \Rightarrow \lambda = \frac{\alpha^h}{C} \tag{1.6}$$

$$\frac{\delta \mathcal{L}}{\delta L} = 0 \Rightarrow \frac{1 - \alpha^h}{L} = \lambda(1 - t)w^h \tag{1.7}$$

First inserting λ from (1.6) in (1.7) gets us

$$\frac{1 - \alpha^h}{L} = \frac{\alpha^h}{C}(1 - t)w^h \tag{1.8}$$

Next we insert the budget constraint C from (1.3) and isolate L

$$
\begin{aligned}
\frac{1-\alpha^h}{L} &= \frac{\alpha^h}{R+(1-t)w^(16-L)}(1-t)w^h \\
\frac{1-\alpha^h}{L} &= \frac{\alpha^h(1-t)w^h}{R+(1-t)w^h(16-L)} \\
\frac{L}{1-\alpha^h} &= \frac{R+(1-t)w^h(16-L)}{\alpha^h(1-t)w^h} \\
\frac{L}{1-\alpha^h} &= \frac{R}{\alpha^h(1-t)w^h} + \frac{(1-t)w^h(16-L)}{\alpha^h(1-t)w^h} \\
\frac{L}{1-\alpha^h} &= \frac{R}{\alpha^h(1-t)w^h} + \frac{16-L}{\alpha^h} \\
L &= \frac{(1-\alpha^h)R}{\alpha^h(1-t)w^h} + \frac{(1-\alpha^h)(16-L)}{\alpha^h} \\
L &= \frac{(1-\alpha^h)R}{\alpha^h(1-t)w^h} + \frac{(1-\alpha^h)16}{\alpha^h} - \frac{(1-\alpha^h)L}{\alpha^h} \\
L\left(1+\frac{1-\alpha^h}{\alpha^h}\right) &= \frac{(1-\alpha^h)R}{\alpha^h(1-t)w^h} + \frac{(1-\alpha^h)16}{\alpha^h} \\
L\left(\frac{1}{\alpha^h}\right) &= \frac{(1-\alpha^h)R}{\alpha^h(1-t)w^h} + \frac{(1-\alpha^h)16}{\alpha^h} \\
L &= \alpha^h\left(\frac{(1-\alpha^h)R}{\alpha^h(1-t)w^h} + \frac{(1-\alpha^h)16}{\alpha^h}\right) \\
L^* &= \frac{(1-\alpha^h)R}{(1-t)w^h} + (1-\alpha^h)16 \qquad (1.9)
\end{aligned}
$$

Isolationg L in equation (1.8) and inserting the expression for the optimal value of leisure (1.9) we are now ready to isolate C^*

$$
\begin{aligned}
\frac{1-\alpha^h}{L} &= \frac{\alpha^h(1-t)w^h}{C} \\
\frac{L}{1-\alpha^h} &= \frac{C}{\alpha^h(1-t)w^h} \\
L &= \frac{1-\alpha^h}{\alpha^h(1-t)w^h}C \\
\frac{(1-\alpha^h)R}{(1-t)w^h} + (1-\alpha^h)16 &= \frac{1-\alpha^h}{\alpha^h(1-t)w^h}C \\
C^* &= \alpha^h R + \alpha^h(1-t)w^h 16 \qquad (1.10)
\end{aligned}
$$

Inserting values of $\alpha^h, wage^h$ for each individual h in (1.9) and (1.10) we have the solution to the individual optimization problem that

$$\begin{aligned} L^{1*} &= \frac{R}{12(1-t)} + 8, \qquad C^{1*} = \frac{R}{2} + 48(1-t) \\ L^{2*} &= \frac{R}{9(1-t)} + \frac{32}{3}, \qquad C^{2*} = \frac{R}{3} + 32(1-t) \end{aligned} \tag{1.11}$$

b) Provide the indirect utility function of each consumer.

Inserting the optimal values of leisure and consumption (1.11) in the utility function (1.1) we can write out the indirect utility functions of each consumer

$$\begin{aligned} U^1 &= \frac{1}{2}\ln\left(\frac{R}{2} + 48(1-t)\right) + \frac{1}{2}\ln\left(\frac{R}{12(1-t)} + 8\right) \\ U^2 &= \frac{1}{3}\ln\left(\frac{R}{3} + 32(1-t)\right) + \frac{2}{3}\ln\left(\frac{R}{9(1-t)} + \frac{32}{3}\right) \end{aligned} \tag{1.12}$$

c) Formulate the maximization problem of the social planner, assuming a strict utilitarian objective function

A strictly utilitarian social planner only seek to maximize overall utility in the society, thus the maximization problem is

$$\begin{aligned} \max_{C,L} U^s =& \frac{1}{2}\ln C^1 + \frac{1}{3}\ln C^2 + \frac{1}{2}L^1 + \frac{2}{3}L^2 \\ &\text{s.t. } C^1 + C^2 = 2R + (1-t)w^1(16-L^1) + (1-t)w^2(16-L^2) \end{aligned} \tag{1.13}$$

d) Provide the FOC's of the problem set in c)

To solve the maximization problem (1.13) we would first set up the Lagrangian

$$\begin{aligned} \mathcal{L} =& \frac{1}{2}\ln C^1 + \frac{1}{3}\ln C^2 + \frac{1}{2}L^1 + \frac{2}{3}L^2 \\ &- \lambda[C^1 - C^2 - 2R - (1-t)w^2(16-L^2) - (1-t)w^2(16-L^2)], \qquad w^1 = w^1 = w = 6 \end{aligned} \tag{1.14}$$

And then take the FOCs of (1.14)

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta C^1} &= 0 \Rightarrow \lambda = \frac{1}{2C^1} \\ \frac{\delta\mathcal{L}}{\delta C^2} &= 0 \Rightarrow \lambda = \frac{1}{3C^2} \\ \frac{\delta\mathcal{L}}{\delta L^1} &= 0 \Rightarrow \frac{1}{2L^1} = \lambda(1-t)w^1 \\ \frac{\delta\mathcal{L}}{\delta L^2} &= 0 \Rightarrow \frac{2}{3L^2} = \lambda(1-t)w^2 \end{aligned} \tag{1.15}$$

2 Optimal non-linear income taxation: top marginal tax rate

Piketty and Saez (2013) find the optimal tax rate to be

$$\tau^g = \frac{1-g}{1-g+a\cdot e} \tag{2.1}$$

Assuming that the government is utilitarian and the marginal utility of consumption declines in income z, then in the limit the social preferences of the government would be $g \to 0$ for $z \to 0$. That is, for the very top income earners the government does not care about their marginal consumption but only wants to optimize the tax rate in order to maximize tax revenue collected from the top bracket subject to their real responses e, giving us the simpler tax function

$$\tau = \frac{1}{1+a\cdot e} \tag{2.2}$$

Where a is a parameter for the shape of the income distribution. Relaxing the assumption that all responses are real (due to either a change in productivity or working hours) the optimal tax rate with rent seeking is

$$\tau^* = \frac{1+a\cdot e_b}{1+a\cdot e}, \qquad e_b = s\cdot e \tag{2.3}$$

Here a fraction s of the behavioral responses to taxes e is due to bargaining where employees are able to get overpaid or underpaid relative to their marginal productivity conditional on their bargaining. e_b is then the bargaining component of the elasticity of taxable income e. The reason bargaining should respond to the marginal tax rate is the assumption that one gets disutility from the amount of effort put into bargaining, thus there is a trade-off depending on the net after-tax value of increasing one's wage through bargaining.

2.1 Benchmark

In the benchmark case where the wage equals marginal productivity, i.e. top income owners are not overpaid nor underpaid relative to their productivity

$$s = 0 \Rightarrow e_b = 0 \Rightarrow \tau^* = \tau$$

Which should be expected as the difference between 2.2 and 2.3 is only rooted in the correction for bargaining effects.

2.2 Trickle up

Assuming that top earners are overpaid relative to their marginal productivity then the optimal taxation of the higher income bracket (2.3) would be higher than without rent-seeking (2.2) as $s > 0 \Rightarrow e_b > 0 \Rightarrow \tau^* > \tau$. In the extreme presence of labour market frictions or where the utility cost of increasing productivity or hours tend to infinity while bargaining costs are modest, then the bargaining share s of the total behavioral response e would tend to 1, thus, $e_b \rightarrow e \Rightarrow \tau^* \rightarrow 1$.

That is, the utility-maximizing tax rate for top income owners τ^* should be increased in order to minimize the source of bottom-up redistribution that is due to the bargaining component. The reasoning being nested in the fundamental assumption that an individual has diminishing returns to consumption $u(z) \xrightarrow[z \rightarrow \infty]{} 0$.

2.3 Trickle down

'Trickle down' corresponds to the phenomenon where top earners are underpaid relative to their marginal productivity, thus, the more their income is increased through real responses to taxes, the more income does 'trickle down' to the other employees through increased production. This relies on the assumption of zero profit of the companies, so that underpayment of the top earner would directly translate into overpayment of the remaining employees. This situation would establish a second channel for redistribution where a tax cut on top income owners relative to the optimal tax without rent seeking (2.2) actually could redistribute economic resources to other employees. Analytically this is evident as equation (2.3) for the optimal tax rate under rent seeking implies

$$s < 0 \Rightarrow e_b < 0 \Rightarrow \tau^* < \tau$$

3 Optimal consumption taxation

Proof analytically if the following statement is true: "goods strongly complement with leisure should be more heavily taxed".

The Ramsey rule for optimal taxation of good i with income and substitution (cross-price) effects for consumption of other taxable goods $j \neq i$ as well as redistribution concerns is given by

$$\frac{\sum_{j=1}^{n} t_j \cdot \frac{\delta x_j^C}{\delta q_i}}{x_i} = -(1-b), \qquad i \in 1, 2, \cdots, n \tag{3.1}$$

Where for a good i the commodity tax rate for the good is t_i, the quantity is given by x_i, and the price is q_i. On the right-hand-side b is the net social marginal utility of income, i.e. taking into account that transferring money to the private sector also generates a return in increased tax revenues.

Goods $i = 1, 2$ are two taxable goods and $i = 0$ is leisure which is an untaxable 'good'. Multiplying with x_i on both sides of equation (3.1) the FOCs with respect to the price of each of the taxable goods are given by

$$\begin{aligned} i = 1: \quad & t_1 \frac{\delta x_1^C}{\delta q_1} + t_2 \frac{\delta x_2^C}{\delta q_1} = -(1-b)x_1 < 0 \\ i = 2: \quad & t_1 \frac{\delta x_1^C}{\delta q_2} + t_2 \frac{\delta x_2^C}{\delta q_2} = -(1-b)x_2 < 0 \end{aligned} \tag{3.2}$$

Isolating the implicit optimal tax rates of goods $i = 1, 2$ we have from (3.2)

$$\begin{aligned} t_1^* &= \frac{-(1-b)x_2x_1}{-Dq_2}(\varepsilon_{22} + \varepsilon_{11} + \varepsilon_{10}) \\ t_2^* &= \frac{-(1-b)x_2x_1}{-Dq_1}(\varepsilon_{22} + \varepsilon_{11} + \varepsilon_{20}) \end{aligned} \tag{3.3}$$

Where $D < 0$ is the negative semi-definite substitution matrix. Subtracting the optimal tax rate for each of the two goods $i = 1, 2$ given by (3.4) gives

$$\begin{aligned} t_1^* - t_2^* &= \frac{-(1-b)x_2x_1}{-Dq_2}(\varepsilon_{22} + \varepsilon_{11} + \varepsilon_{10}) - \frac{-(1-b)x_2x_1}{-Dq_1}(\varepsilon_{22} + \varepsilon_{11} + \varepsilon_{20}) \\ &= \frac{-(1-b)x_2x_1}{-Dq_2q_1}(\varepsilon_{20} - \varepsilon_{10}) \end{aligned} \tag{3.4}$$

For the case where good $i = 1$ is more complementary with leisure $(i = 0)$ than good $i = 2$ then the cross-price elasticity between good 1 and 0 is relatively small compared to the cross-price elasticity between good 2 and 0, implying that

$$\varepsilon_{20} - \varepsilon_{10} > 0 \Leftrightarrow t_1 - t_2 > 0 \Rightarrow t_1^* > t_2^* \tag{3.5}$$

That is, the Ramsey rule for optimal commodity taxation implies that it is efficient that the good more complementary with leisure is taxed more heavily in order to deter people from enjoying leisure.

As an example that could be increases commodity taxes on alcohol, non-business flight-tickets and other leisure-time activities such as cinemas and fitness-subscriptions. While the former two might help reaching other goals for health and greenhouse gas emissions, the latter two might also cause negative externalities regarding culture and health.

References

Piketty, Thomas and Emmanuel Saez (2013). "Optimal labor income taxation." In: *Handbook of public economics*. Vol. 5. Elsevier, pp. 391–474.