Macroeconomics

w. Josep Lluís Carrion-i-Silvestre
Universitat de Barcelona

Complementary notes for the slides
by Thor Donsby Noe *

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*Department of Economics, University of Copenhagen, Øster Farimagsgade 5, DK-1353 Copenhagen K, Denmark, (e-mail: jwz766@alumni.ku.dk)

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1 Theoretical issues

Convergence in distribution

$$\hat{\beta}_{OLS} \xrightarrow{d} \text{N}(\beta, V(\beta))$$
$$(\hat{\beta}_{OLS} - \beta) \xrightarrow{d} \text{N}(0, V(\beta))$$

$$\begin{aligned}\hat{\beta}_{OLS} &= (x'x)^{-1}x'y \\ &= \beta + (x'x)^{-1}x'u\end{aligned}$$

Asymptotic independence

If independent, the joint probability equals the product of probabilities, i.e.

$$P(A \cap B) = P(A) * P(B)$$

2 ARIMA modelling

Univariate TS strategy

Not based on Economic Theory
(reduced form modelling $\rightarrow$ the past is informative about the present)

$$y_t = f(y_{t-1}, y_{t-2}, y_{t-3} \cdot)$$

The performance in terms of **forecasting** is key!

If you want to learn about main drivers, you need to use other models.

1. Stationary time series:

Sometimes 1st difference is required to ensure stationarity.

2. Identification:

Specify the model that explains the behaviour the best

AR: Autoregressive model

MA: Moving Average model

ARMA: AutoRegressive Moving Average model

To initially decide between them, plot the

- **ACF:** The Simple Autocorrelation Function.
  Check if there is persistence/memory:

  $$\frac{Cov(x_t, x_{t+k})}{\sqrt{V(x_t)}\sqrt{V(x_{t+k})}}, \text{ for } k = 0, \pm 1, \pm 2, \cdots$$

  Related to the asymptotic confidence interval

  $$\left(\frac{-1.96}{\sqrt{(T)}}, \frac{1.96}{\sqrt{(T)}}\right)$$

  or degrees of freedom corrected

  $$\left(\frac{-1.96}{\sqrt{(T-k)}}, \frac{1.96}{\sqrt{(T-k)}}\right)$$

- **PACF:** The Partial Autocorrelation Function
  e.g. The The 3rd partial autocorrelation coeffcient of the PACF, ϕ_{33}:

  $$\tilde{x}_t = \phi 31\tilde{x}_{t-1} + \phi 32\tilde{x}_{t-2} + \phi 33\tilde{x}_{t-3} + \varepsilon_t$$

→ Using both functions helps us to make an initial specification.

- Helps us to decide on the number of lags.
- Too many lags can cause multi-colliniarity.

Better to choose an $f(t) \neq 0$ to test it.

3. Estimation
..

4. Validation
How well is the model specified?

-
- Residuals

Re-specify if not correctly validated (repeat step 2-4)

5. Forecasting
When model is valid, use it to forecast.

2.1 AR(p) process

Stationarity conditions for the AR model:

1. The deterministic component f does not depend on t (rules out MA)
2. All roots of the autoregressive polynomial must, in module, be greater than one, e.g.

$$x_t = \phi_1 x_{t-1} + \varepsilon_t$$

$$\underbrace{(1-\phi_1(L))}_{\text{AR polynomial}} x_{t-1} = \varepsilon_t, \quad L > 1 \text{ required!}$$

$$V(x_t) = V(\phi_1 x_{t-1}) + V(\varepsilon_t)$$

$$\sigma_x^2 = \phi_1^2 \sigma_x^2 +$$

$$\sigma_x^2 = \frac{1}{1-\sigma_1^2}\sigma_\varepsilon^2, \text{ requires } \sigma_1^2 < 1$$

2.2 MA(q) model

If MA is invertible

$$\Rightarrow MA(q) \equiv AR(\infty)$$

$$\simeq AR(p), \quad p \text{ is large}$$

Use the $\hat{\varepsilon}_t^0$ estimation algorithm.

3 Order of Integration Analysis

#1. Iterative Strategy: DIY.

#2. Model Selection Strategy:
Automated procedure using AIC or BIC.

- $p_m ax = 3$
- $q_m ax = 3$

"Autometrics".

Models that rely on x_t being stationary and invertible (for consistent $\hat{\beta}$):

	ACF	$PACF$
$AR(p)$		$x_{(p)}$
$MA(q)$	x_q	
$ARMA(p,q)$	$?_{\hat{p},\hat{q}}$	$?_{\hat{p},\hat{q}}$

Usually x_t is non-stationary.

→ $\Delta^d x_t, d = 0,1,2,3,\cdots$ is stationary?

→ $\hat{\beta}$ is consistent!

- Integrated of order "d".
  - $prices_t$
  - $\Delta prices_t \simeq$ Inflation.

- $\Delta^2 prices_t \simeq$ Acceleration prices.

A non-stationary proces $\simeq$ Random walk process features (uniroot).

$$\begin{aligned} x_t &= \phi x_{t-1} + \varepsilon_t, \qquad \phi = 1 \\ &= x_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j} \end{aligned}$$

With the variance

$$\begin{aligned} V(x_t) &= V(x_0) + V\left(\sum_{j=0}^{t-1} \varepsilon_{t-j}\right) \\ &\vdots \\ &= \sigma_\varepsilon^2 t \end{aligned}$$

3.1 Unit root tests

t-Student distrubution

- Zero mean.
- Broad tails with a low degree of freedoms k.

$\rightarrow$ Tends towards the normal distribution when $k \rightarrow \infty$.

Dickey-Fuller (DF) test

- Only valid for an AR(1) process!
- Statistical inference: Objective:
  - Test statistics:
    $H_0: \; x_t \sim I(1)$
    $H_1: \; x_t \sim I(0)$
  - "Confirmatory analysis":
    $H_0: \; x_t \sim I(0)$
    $H_1: \; x_t \sim I(1)$

DF:

$$\Delta x_t = f(t) + \alpha x_{t-1} + u_t$$

- DF assumes $u_t \sim iid$

Corrections to DF:

- Philips-Perron (PP) test
  - Non-parametric correction.
  - More general but difficult to estimate the s^2.
    * Allowing a high number of covariates: test statistics looses power
    * Allowing a low number of covariates?
- ADF test:
  - Parametric correction.
  - More efficient for finite samples.
- Ng-Perron M test statistics (2001)
  3 legs:
  1. Parametric longrun estimate of S_{AR}^2.
  2. $\bar{\alpha}$ estimated numerically (Elliot, Rothenburg & Stock, 1996) and Power Envelope Function (PEF) using GLS.
  3. Number of lags k to include: decided using the Midified Information Criteria (MIC) strategy:
     * MAIC
     * MSIC

ADF-test
Capturing any information that might be hidden in the u_t that creates and effect in the λ-term, $\lambda = \sigma^2 - \sigma_\varepsilon^2/2$

$$\Delta x_t = f(t) + \alpha x_{t-1} + \sum_{j-1}^{p-1} \gamma_j \Delta x_{t-j} + u_t$$

$$\Rightarrow \rightarrow \varepsilon_t \sim iid$$

Philips-Perron (PP) test

- Nice correction to the DF test.
- In practice, it's difficult to calculate the short-run variance s_u^2 and the long-run variance s^2.

Complex hypothesis (less feasible):

$$\begin{cases} H_0: & \alpha = 1 & = & x_t \sim I(1) \\ H_1: & |\alpha = 1| & = & x_t \sim I(0) \end{cases}$$

Simple alternatives:

$$\begin{cases} H_0: & \alpha = 1 \\ H_1: & \alpha = \bar{\alpha}, \quad \bar{\alpha} = 1 + \frac{\bar{c}}{T} \end{cases}$$

i.e. 'local alternatives' in the neighborhood of H_0.

Ng-Perron principle:

$$H_1: \ L(\bar{\alpha}) - L(1) \tag{3.1}$$

Using the likelihood (LM test).

Def. 'Power Envelope Function' (PEF): Chooses $\bar{c}$ such that the $PEF = 0.5$
$\Rightarrow$ use **GLS**.