

# **Panel Data**

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Complementary course notes

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## 1 INTRODUCTION

Example macro panel data:  
Maddison, 2007, and IFS

- Formatting data
- Descriptive statistics
- Graphs
- Maps

## 2 FIXED EFFECTS ESTIMATORS

### 2.1 Simultaneous equations models with exogenous explanatory variables

Four different models:

M1:  $\alpha \neq, \beta \neq$ . Estimation individual by individual (GLS).

M2:  $\alpha =, \beta =$ . Equal constant terms and slopes (presence of homogeneity).

M3:  $\alpha \neq, \beta =$ . Equal slopes, different constant terms.

M4:  $\alpha =, \beta \neq$ . Equal constant terms, different slopes.

Choosing between them using

#### Test for Homogeneity:

- Estimate the extended/unrestricted model.
- Estimate the restricted model.

- $H_0$  : Homogeneity (the unrestricted is not better than the restricted).  
Reject  $H_0$  if the F-value is higher than the critical value of the F-distribution.

$$F = \frac{(SSR_R - SSR_{UR})/r}{SSR_{UR}/df}$$

In ML we maximize the probability.

In OLS we don't care about the variance.

### 2.2 The fixed effects model

$\alpha_i$  is a parameter capturing the individual effect (time-invariant!)

$$y_{it} = i\alpha_i + X_{it}\beta + \varepsilon_{it}$$

$$\Rightarrow y = \begin{bmatrix} i & X \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \varepsilon$$

Leasy Squares Dummy Variables (LSDV):

- FE model: All variables are within-transformed e.g the deviations from the mean.

Test the homogeneity analysis:

$$H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_M$$

### 2.3 Within and between estimators

The overall variance: Weighted variation between the within-variance and between-variance.

### 2.4 Effects of group and time

## 3 RANDOM EFFECTS ESTIMATOR

- 3.1 The random effects model
- 3.2 The generalized least squares estimation
- 3.3 Feasible Generalized Lest squares (unkown )

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4 FIXED EFFECTS VS. RANDOM ESTIMATOR

- FE allows for correlation with  $y_{it}$  - and has explanatory power.
- RE doesn't allow for correlation with  $y_{it}$  - the effect is random. Nor does is allow for correlation with  $\varepsilon_{it}$ .

Check if there are individual effects;

- $H_0 : \alpha = 0$ ? poolability test / homogeneity test.
- $H_0 : \sigma^2_{\xi} = 0$ ? Breush-Pagan test.

	No endogeneity	Endogeneity
R.E.	Consist. & Efficient	Inconsistent
F.E.	Consist. but inefficient	Consist. but ineff.

Table 1: Endogeneity problems

- 4.1 The Breush Pagan test

4.2 The Hausman test

  - FE -  $\beta^{FE}$
  - FE -  $\beta^{RE}$
  - $(\beta^{FE} - \beta^{RE})?$
- Diff(FE-BE) → estimate 'pseudo' Hausman test.

→ Allows for including 'fixed' variables (with no time-variation)

RE:

Problems:

$$y_{it} + x_{it}\beta + \psi_i + \varepsilon_{it}$$

- Hausman assumptions
- We cannot use "fixed" variables (no time variation).

Mundlak regression:  
Including both RE and BE.

The Mundlak estimation:

$$y_{it} + (x_{it} - \bar{x}_i)\beta^w + \bar{x}_i\beta^b + \psi_i + \varepsilon_{it}$$

- Allows joint estimation F.E. / BE
- Is just an instrumental regression!

Standard errors are unreliable and  $R^2$  is lower.

- FE: We capture anything permanent for any individual in  $\alpha_i$ 
  - Get rid of anything permanent.
  - Good for controlling.
    - We don't know what the individual effects mean though!
- Mundlak: We 'only' capture anything permanent considered in  $\bar{x}_i$

Increasing the data set (information)

- Hausman test: Is likely to find endogeneity though the difference in coefficients is very small.

## 4.3 Long run vs. short run effects

Baltagi & Griffin (1984) "Short and Long Run Effects in Pooled Models".

- Between est. → LR effect.
- Within est. → SR effect.
- OLS and RE → average of SR & LR effect.

Inequality and growth

- Positive in the SR
- Negative in the LR

## 5 HETEROSKEDASTICITY AND AUTOCORRELATION IN PANEL DATA

- Hausman test
- Non-spherical disturbances.
  - Heteroscedasticity
  - Correlation
  - Autocorrelation

Heteroscedasticity over time:  $\sigma_\varepsilon^2$

### 5.3 Autocorrelation in the FE model

In eq. 5.1 it can be that

$$\varepsilon_{it} = V_{it} + \theta V_{it-1}$$

$$\rightarrow \text{AR}(1) \rightarrow \varepsilon_{it} = \rho_1 \varepsilon_{it-1} + v_{it}$$

### 5.1 Heteroskedasticity in FE model

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it} \quad (5.1)$$

Heteroscedasticity over time:  $\sigma_\varepsilon^2$

Assumption: Everyone is homogenous in their autocorrelation parameter.

Consistency can come from either a high number of N or T.

### 5.2 Heteroskedasticity in RE model

$$y_{it} = x_{it}\beta + \zeta_i + \varepsilon_{it} \quad (5.2)$$

Heteroscedasticity over individuals:  $\sigma_\zeta^2$

### 5.4 Autocorrelation in the RE model

We need to start with the FE estimation

→ To get a consistent estimate of  $\rho$ .

- Proceed to estimate either a FE or RE model.

## 6 INCOMPLETE/UNBALANCED PANELS

Incomplete panels are similar to heteroscedasticity issues.

- Some issues can be fixed with weighting observations.

- For some procedures we will need complete panels though.

## 7 DYNAMIC PANELS

Became the standard in the 90s for economic growth. Has some issues, so they're not the state of the art anymore. (Can be a column in the regression table).

Autocorrelation in  $y_{it}$ .

$$y_{it} = \alpha y_{it-1} + \alpha_{it}\beta + \dots$$

## 7.1 GMM estimation

(some have started using ML, but requires the use of strongly nonlinear algorithms)

Allows us to impose restrictions

e.g. for  $y = x\beta + \varepsilon$ :

$$\text{cov}(x, \varepsilon) = 0$$

$$\text{cov}(y, \varepsilon) = 0$$

This is possible as  $u$  and thus  $\beta$  is restricted to follow a normal distribution.

We can use past realizations of  $y$  as instruments for  $y_{it-1}$

## Limitations to GMM:

- Need at least 150-200 observations (per time period)  $\rightarrow$  or

## 7.2 Validation of instruments

Three ways of **validation** of the instruments

- Hausman Test

- Incremental Sargan: Can be problematic (and boring)
- Looking at the autocorrelation of the modified residuals - in STATA valid if

$$m_1 : \text{corr}(\varepsilon'_{it}, \varepsilon'_{it-1}) \Rightarrow \text{p-val} < 0.05$$

$$m_2 : \text{corr}(\varepsilon'_{it-1}, \varepsilon'_{it-2}) \Rightarrow \text{p-val} > 0.10$$

$$\text{as } \varepsilon'_{it} = \varepsilon'_{it} - \varepsilon'_{it-1}$$

## 7.3 GMM vs. within estimator

**GMM:** The closer our data is to a normal distribution  $\rightarrow$  the faster it converges into normality.

The F.E. within estimator:

- Small  $T \rightarrow \Rightarrow$  F.E. biased
- $T \rightarrow \infty \Rightarrow$  F.E. is ok.

inference of  $Z$ ,  $t$  is fine, if close to normal and/or observations are high,  $N \rightarrow \infty$ .

## 7.4 Testing overidentifying restrictions

Sargan (1958): The error in his test is "proportional to the number of instrumental variables" (counter-intuitive) as the partial  $R^2 \rightarrow 0.99$

"Forward orthogonal transformation"

(Arellano and Bover, 1995) introduce "orthogonal deviations" The basis for the

- Subtracts the average of all future available observations of a variable.
- Instead of subtracting the previous observation from the contemporaneous one

### **The Blundell Bond (1998) estimator:**

"The system GMM estimator". Uses  $y_{t-k}$  as instrument for growth  $\Delta y$  AND  $\Delta y_{t-k}$  as an instrument for levels  $y_t$ .