Macroeconomics

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Complementary notes for the slides
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1 Theoretical issues

Convergence in distribution

$$\hat{\beta}_{OLS} \xrightarrow{d} \mathrm{N}(\beta, V(\beta))$$
$$(\hat{\beta}_{OLS} - \beta) \xrightarrow{d} \mathrm{N}(0, V(\beta))$$

$$\hat{\beta}_{OLS} = (x'x)^{-1}x'y$$
$$= \beta + (x'x)^{-1}x'u$$

Asymptotic independence

If independent, the joint probability equals the product of probabilities, i.e.

$$P(A \cap B) = P(A) * P(B)$$

2 ARIMA modelling

Univariate TS strategy

Not based on Economic Theory (reduced form modelling → the past is informative about the present)

$$y_t = f(y_{t-1}, y_{t-2}, y_{t-3}\cdot)$$

The performance in terms of **forecasting** is key!

If you want to learn about main drivers, you need to use other models.

1. Stationary time series:

Sometimes 1st difference is required to ensure stationarity.

2. Identification:

Specify the model that explains the behaviour the best

AR: Autoregressive model

MA: Moving Average model

ARMA: AutoRegressive Moving Average model

To initially decide between them, plot the

- **ACF:** The Simple Autocorrelation Function.
  Check if there is persistence/memory:
  $$\frac{Cov(x_t, x_{t+k})}{\sqrt{V(x_t)}\sqrt{V(x_{t+k})}}, \text{ for } k = 0, \pm 1, \pm 2, \cdots$$
  Related to the asymptotic confidence interval
  $$\left(\frac{-1.96}{\sqrt{(T)}}, \frac{1.96}{\sqrt{(T)}}\right)$$
  or degrees of freedom corrected
  $$\left(\frac{-1.96}{\sqrt{(T-k)}}, \frac{1.96}{\sqrt{(T-k)}}\right)$$
- **PACF:** The Partial Autocorrelation Function
  e.g. The The 3rd partial autocorrelation coeffcient of the PACF, ϕ_{33}:
  $$\tilde{x}_t = \phi 31\tilde{x}_{t-1} + \phi 32\tilde{x}_{t-2} + \phi 33\tilde{x}_{t-3} + \varepsilon_t$$

- $\rightarrow$ Using both functions helps us to make an initial specification.
  - Helps us to decide on the number of lags.
  - Too many lags can cause multicolliniarity.

Better to choose an $f(t) \neq 0$ to test it.

3. Estimation

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4. Validation

How well is the model specified?

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- Residuals

Re-specify if not correctly validated (repeat step 2-4)

5. Forecasting

When model is valid, use it to forecast.

2.1 AR(p) process

Stationarity conditions for the AR model:

1. The deterministic component f does not depend on t (rules out MA)
2. All roots of the autoregressive polynomial must, in module, be greater than one, e.g.

$$x_t = \phi_1 x_{t-1} + \varepsilon_t$$

$$\underbrace{(1-\phi_1(L))}_{\text{AR polynomial}} x_{t-1} = \varepsilon_t, \quad L > 1 \text{ required!}$$

$$\begin{aligned} V(x_t) &= V(\phi_1 x_{t-1}) + V(\varepsilon_t) \\ \sigma_x^2 &= \phi_1^2 \sigma_x^2 + \\ \sigma_x^2 &= \frac{1}{1-\sigma_1^2}\sigma_\varepsilon^2, \text{ requires } \sigma_1^2 < 1 \end{aligned}$$

2.2 MA(q) model

If MA is invertible

$$\begin{aligned} \Rightarrow MA(q) &\equiv AR(\infty) \\ &\simeq AR(p), \quad p \text{ is large} \end{aligned}$$

Use the $\hat{\varepsilon}_t^0$ estimation algorithm.

3 ...