Public Goods

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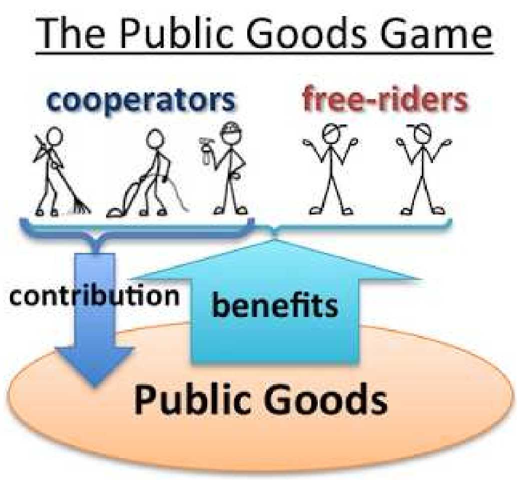
1 Introduction

In traditional economics, a public good is usually defined as something that has two properties - non-excludability and non rivalrousness. Non-rivalrousness means that if one person consumes a public good, he or she doesn't take it away from anyone else. We don't need to compete for public goods - they are there for all to consume. Non-excludability means that it is impossible to prevent people from consuming the public good once it is produced. Almost nothing fits the definition well - maybe clean air. Some goods are non-excludable - a fireworks display, loud music, but not obviously goods. Many public services are at least potentially enjoyed by all, for example a bridge across a river, but potentially subject to congestion.

Economists have long argued that government is needed to provide non-excludable, non-rivalrous goods. Here is a really cute slide from Benjamin Allen at Harvard University that sums up the argument.



In the figure, you can see the selfless cooperators working tirelessly for no personal gain to make the world a better place. Evil free riders take advantage of this selfless contribution of others and waste their time in the pursuit of leisure (I guess that from the picture because they are wearing baseball caps) which has no benefit to anyone but themselves. The policy implication is that a public institution financed by taxation is needed to force the free riders to pay. If nothing else, this just makes things fairer for the cooperators.

Economic theory goes a bit further to argue that in the situation described above too little of the public good will actually be provided. The reason is that the selfless cooperators will actually stop producing the public good at a point where the greedy free-riders would be willing to pay them to produce more.

The point of this reading is two fold. First, to suggest why the story above is misleading. Second, it explains why it is so important to try to get this story right.

Public goods are often treated as a fringe element of microeconomic theory - sometimes relegated to special topics courses. In fact, many of the goods that are most important to us are at least non-rivalrous. Modern technology makes most them non-excludable. The story above suggests that it is those two properties - non-rivalrous and non-excludable - are bad things that we need to try to get rid of just because they are bad. In fact, they are the very properties that give goods value.

A software program is an example, is something that is non-rivalrous and non-excludable by nature. As in the picture above, it takes effort to produce. If you run the same program as I do, I am not hurt by that. The program is on a digital file, so it can be freely distributed to everyone. Google, Facebook, Twitter are produce enormous amounts of code that produce services that most of us seem to love. We can't have evil free riders using that software to produce the same thing.

Maybe you can see the problem with that argument. The first is that all three companies seem to be free-riders. They use linux based tools that are open source and free. According to a survey of the top 1 million websites in 2015 done by a tech analysis company called W3cook (http://www.w3cook.com/), 98% of the servers behind those websites were using a Linux based (that is free) operating system. Your Wordpress website is driven by open source software. The html and javascript you use to make up your webpage are free open source software. At least in the US, the power of the tech sector and silicon valley seems to exist because software is non-rivalrous and non-exclusive. Those are the properties that make the whole tech industry valuable.

Secondly, you might see that the simple distinction between cooperators and free riders is completely wrong. The companies that support open source software do it for their own benefit and produce software that is useful to Google, etc. At the same time the software that Google produces is offered back to the producers of the linux operating system. The point being, that all of us are both cooperators and free-riders.

The nice diagram given above, which perfectly summarizes what most economists think about non-exclusiveness and non-rivalrousness, really misses the point when it tries to describe things that are important to us.

Perhaps a better way to describe a non-rivalrous good is to say that it is a good that is produced at a possibly high fixed cost, but thereafter has zero marginal cost. For software, one copy and a million copies cost exactly the same thing. Music comes on mp3 files which can be costlessly redistributed once they are produced. Researchers in universities produce data and concepts that are useful to a lot of people. An idea is non-rivalrous and non-excludable, as are jokes, fashion ideas, newspaper articles, etc etc.

To understand the differences between all these examples, we can use some of the basic ideas we have developed so far in consumer theory. We'll use the old standby techniques 2 goods and two traders. One good we'll just call money, denoted x, used for consuming other stuff. The other good y will be the one that is non-excludable and non-rivalrous (in other words, produced at a cost, but they freely available). We'll refer to y as a public good from time to time. Consumers have preferences $u(x, y)$ over these two goods as they always do. Consumers can produce more of the public good by giving up money.

The second idea is that since the public good y is non-rivalrous, if one consumer gives up money to produce more of the public good, the other consumer will enjoy this new public good as well. This gives us our selfless cooperator, and our selfish free rider. The difference is that, as in real life, both consumers play both roles. Production of the public good creates a very special type of positive externality in the story that follows.

Now we can start with a description of something called the *voluntary contribution game*, which is a common way to think about how public goods are provided. It explains why the amount of the public good in the voluntary contribution game is too small (because the outcome is not Pareto optimal: there is another outcome that will make both consumers better off).

1.1 The voluntary contribution game

Let $f(x)$ denote the amount of the public good that can be produced from x units of the private good. Suppose the two consumers have utility functions $u_1(x, y)$ and $u_2(x, y)$ respectively. Their endowments of the private good are ω_1 and ω_2. The set of points $\{(x, y) : y = f(\omega_1 + \omega_2 - x)\}$ is the *production possibilities frontier*. It looks exactly like the production possibilities frontier that we studied before.

If the first consumer decides to consume x_1 (and devote the rest of his endowment ω_1 to production of the public good) while consumer 2 decides to consume x_2 the utilities of each of the consumers are given by

$$u_1(x_1, f(\omega_1 + \omega_2 - x_1 - x_2))$$

for consumer 1 and

$$u_2(x_2, f(\omega_1 + \omega_2 - x_1 - x_2))$$

for consumer 2. The important point is that if consumer 1, say, decides to consume a bit less of the private good and produce a bit more of the public good, then consumer 2 will enjoy the additional public good too without any cost at all - that is what non-rivalrous means.

At this point, we need to make some changes to what we have done before. Each of the consumers simply picks the amount of the private good they want on their own. This is a bit hard to do because the amount that each consumer will choose to contribute depends on how much they expect the other consumer to contribute. To handle this, we have to replace *Walrasian Equilibrium* with *Nash equilibrium.* Instead of taking prices to be fixed, each consumer takes the contribution of the other consumer to be fixed and chooses the contribution that maximizes his utility given this expectation. In the Nash equilibrium each consumer must simultaneously choose his consumption of the private good (analogously, his contribution to production of the public good) while correctly forecasting what the other player will do.

Formally, a Nash equilibrium for the voluntary contribution game is a pair of private consumptions x_1^* and x_2^* such that

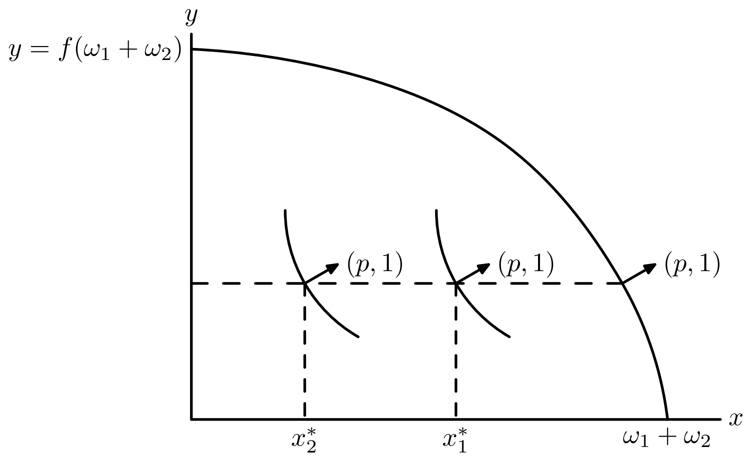
$$u_1\left(x_1^*, f\left(\omega_1+\omega_2-x_1^*-x_2^*\right)\right) \geq u_1\left(x', f\left(\omega_1+\omega_2-x'-x_2^*\right)\right) \tag{1}$$

for any alternative contribution $x' \in [0, \omega_1]$ and

$$u_2\left(x_2^*, f\left(\omega_1+\omega_2-x_1^*-x_2^*\right)\right) \geq u_2\left(x', f\left(\omega_1+\omega_2-x_1^*-x'\right)\right) \tag{2}$$

for any alternative contribution $x' \in [0, \omega_2]$.

One way to view the outcome of this game is given in Figure 1 where the two consumers choose x_1^* and x_2^*. The 'budget line' that consumer 1 faces, for example, when consumer 2 chooses consumption x_2^* is the set of all pairs $\{(x_1, y) : y = f(\omega_1+\omega_2-x_1-x_2^*)\}$. The slope of this is exactly the same as the slope of the production possibilities frontier at the point $(x_1^*+x_2^*, y^*)$. The same is true for consumer 2. So, in the equilibrium of the voluntary contribution game, each consumer has the same marginal rate of substitution and the same marginal rate of transformation in production.



With private goods, this is exactly what you want. Recall that, in the Edgeworth box, both consumers' indifference curves were tangent and the common slope of their indifference curves was equal to the slope of the production possibilities frontier. With public goods, this is not the outcome that you want.

You might be wondering what happened to the Edgeworth Box. Recall that in the Edgeworth box, one point could be used to represent the outcome for both consumers. The reason is that increasing the amount of good x that 1 consumes automatically lowers the amount that 2 consumes. Good x is rivalrous. In the usual story the same is true of good y. Here, however, good y is non-rivalrous - if 1 consumes more good y, then so does 2.

So lets modify the approach to account for this. For consumer 1, the indifference curve is just the collection of (x, y) pairs that satisfy

$$u_1(x, y) = K.$$

The graphs of these curves look no different than they do in any other problem. Higher indifference curves (further away from the origin) represent higher values of K in the equation above, and correspondingly higher payoffs to consumers.

The leap we are going to make here is that when we choose a bundle (x, y) for consumer 1, this bundle induces a corresponding bundle for consumer 2 given by

$$\left(\omega_1 + \omega_2 - f^{-1}(y) - x, y\right).$$

In words, we figure out how much money is required to produce y, that is the $f^{-1}(y)$, then subtract that from the total endowment $\omega_1 + \omega_2$ to get the total amount of money left over after producing y units of software. Subtract the x we want to give to consumer 1, then give the rest to consumer 2. The function $f^{-1}(y)$ is sometimes referred to as the cost function for the public good.

Given this logic, we could define an indifference curve for *consumer 2* by finding all the consumption bundles for *consumer 1* that induce the same payoff for player 2. Formally, an indifference curve for player 2 is the collection of all bundles for player 1 that satisfy

$$u_2\left(\omega_1 + \omega_2 - f^{-1}(y) - x, y\right) = K.$$

To figure out what these indifference curves look like is a bit daunting. Holding y constant, the less money consumer 1 has, the more is left over for consumer 2. In this sense, player 2's indifference curves represent higher payoffs for 2 the closer they are to the origin.

For the rest, we'll rely on the idea that the public good provides diminishing marginal utility to consumer 2. What that means is that the more of the public good that is being produced, the less valuable an additional unit of the public good will be. Fix the consumption of the private good of consumer 1 at some value x_0 and travel up the vertical line through x_0. Remember that changing the public good in this case means that all the cost of producing the public good is borne by consumer 2 since the amount of money consumer 1 has is being held constant at x_0. Initially as you increase the public good, this makes consumer 2 better off since the public good is very valuable to her when there isn't much of it. Eventually the thrill will wear off, and as the production of the public good gets higher and higher, consumer 2's payoff will begin to decline. What that means is that consumer 2's indifference curves will typically cross any vertical line twice.

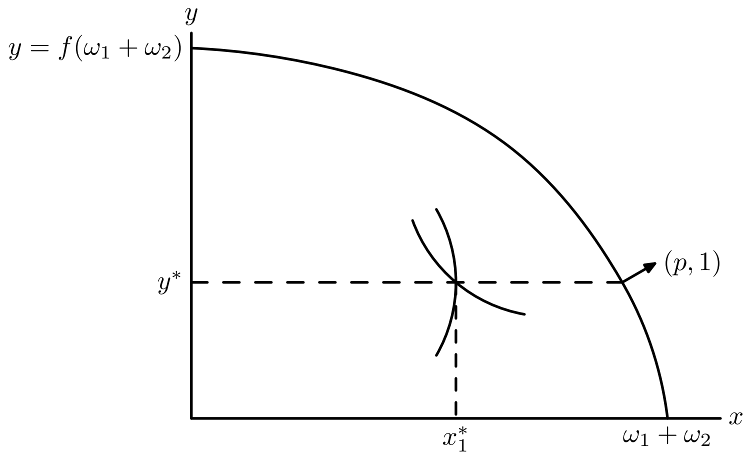
Of course, there will be one indifference curve that is just tangent to the vertical line. This one is illustrated in the picture below.

In a Nash equilibrium, consumer 2 will choose how much of the public good to produce assuming that the consumption of the public and private good by consumer 1 are fixed. What that means is that consumer 2 will choose a level of the public good at which her indifference curve as we have just described it is vertical.

The indifference curves should look like backward C's as in the following diagram, where I have superimposed consumer 2's indifference curve into the original diagram depicting the Nash equilibrium of the voluntary contribution game. Let me explain why. At the point where the indifference curve is just tangent to the vertical line, consumer 2 could increase her production of software y and travel further up the vertical line. If she travels directly upward, it means that she is holding consumer 1's holdings of money constant. In other words, if she increases production, she pays all the costs herself. At the tangency, she doesn't want to do this - the marginal benefit she gets from increasing production is exactly equal to the marginal cost. Formally

$$\frac{\partial u_2\left(\omega_1+\omega_2-f^{-1}\left(y^*\right)-x_1^*,y^*\right)}{\partial x}\frac{1}{f'\left(y^*\right)}=\frac{\partial u_2\left(\omega_1+\omega_2-f^{-1}\left(y^*\right)-x_1^*,y^*\right)}{\partial y}.$$

Then if you travel up the vertical line a bit above (x_1^*,y^*), then consumer 2 is strictly worse off. How to restore her payoff? You have to give her more money, which means reducing the amount of money you leave for consumer 1. That means that you have to travel left of the vertical line to restore 2's payoff. The indifference curve must lie to the left of the vertical line at every point except at the tangency.



Since the point (x_1^*, y^*) is supposed to coincide with the Nash equilibrium in the voluntary contribution game, consumer 1's indifference curve has the same slope as the production possibilities frontier at the point where y^* is produced. In other words, it is downward sloping, not vertical. The lens between the curves represents a situation in which both consumers could be made better off.

We draw the conclusion that the Nash equilibrium of the voluntary contribution game is not pareto optimal. Remember what I have said about 'pareto optimal' - it has nothing to do with optimal. An outcome where consumer 2 gets whatever she likes, while consumer 1 gets nothing is pareto optimal. Second, remember the lesson of Google, Facebook, Twitter, all your social media products - they come from an equilibrium that is not pareto optimal. It seems unlikely you would be too upset about the products that are produced in that market, so maybe something that isn't pareto optimal really isn't so bad.

Notice why it isn't so bad. All consumer are producers of the public good - they are cooperators and free riders at the same time - they help each other. Many important products have another property that is relevant here. In the story above, if consumer 1 produces more software, decreasing returns means that it is more expensive for consumer 2 to produce an *extra* unit of software than it was before. Software isn't like that, if consumer 1 produces more software, it becomes cheaper for producer 2 to produce new software. If you let me write more software, you will begin to want to write software that was too hard or expensive to produce before. Music is also like that, the more that is produced the easier it is to produce. Research is like that, jokes are like that, newspaper articles are like that, and many more. That probably explains why goods that are non-rivalrous and non-exclusive are so often produced in such abundance - despite the fact that the corresponding equilibrium outcome isn't pareto optimal.

We won't develop this formally because we only have two goods, but we can go much further. Benefiting from the efforts of another (what we started out calling free riding) allows consumers to devote their resources to other activities, these other activities may involve production of public goods. Probably every organization on earth is based on this principle. When one person volunteers to do some work on something, the others in the organization don't free ride on this, they use the time that has been freed to do other useful things. Most organizations don't work perfectly, nonetheless many of them work.

2 Intellectual Property

The reason it is so important to think about public goods and the narrative of cooperators and free riders is because of the awful policies it has spawned. The voluntary contribution game provides a sheen of logic to a narrative which can then be easily misused. The outcome in the voluntary contribution game is not pareto optimal, even though it looks like all the other problems you encounter in intermediate microeconomics. The reason appears to be that when a consumer produces the public good, other people can use it. Other people can't use your car because it is your property. Therefore we should turn public goods into property - in other words, try to make a non-exclusive good into an exclusive one. Then things would be pareto optimal again as they are when all goods are private goods.

There are a bunch of laws that people now associate with intellectual property - copyright law, trademarks and patents. They seem on the surface to be associated with things that people thought up, so they must be intellectual - therefore innovative. The story then goes that we need laws to protect intellectual property to promote innovation. There is a nice article by Richard Stallman at http://www.gnu.org/philosophy/not-ipr.en.html which explains in a pretty simple way how these laws relate to one another, and how none of them have anything to do with intellectual property or promoting innovation.

You can read about US copyright and patent laws in a lovely (free) book called "Against Intellectual Monopoly" by David Levine and Michele Boldrin (http://www.dklevine.com/general/intellectual/againstfinal.htm). The book is full of historical anecdotes and simple economic models to explain what is going on. The gist of their argument is that copyright and patent laws in the US are designed to give corporations levers that they can use to prevent entry and suppress competition. Evidence that these laws encourage creation of public goods is basically non-existent, what evidence there is suggests the opposite. This is important for Canada since the US almost always insists that countries adopt US copyright and patent law before they will engage in trade negotiation.

So lets continue with the example of software patents, and try to explain the effect that they have using our second year economic theory.

The way patents work is that one of the two consumers applies for a patent, and is then given a monopoly over production of the patented product, in this case our software. For example, Mircosoft has a couple of strange patents - one is a patent for double clicking on an icon to launch an application, awarded in 2004. Another is patent for using the page up and page down buttons on a keyboard to shift the content of a page up or down (2008). These operations are basic to just about all software. Consistent with the patent, anyone writing software would then have to pay microsoft a fee to use those procedures in their software.

You can see from these two examples, that patents (at least software patents) have nothing to do with innovation. In Microsoft's defense, they do need a portfolio of patents that they can use to defend themselves against other large companies who also hold patents - often for no better reason that to litigate a competitor out of existence (see Apple's litigation against other smartphone makers https://en.wikipedia.org/wiki/Apple_Inc._v._Samsung_Electronics_Co).

To make things simple here, we'll just assume that people have to pay microsoft to produce their software for them. That really isn't any different than having them pay a fee to microsoft whenever they write their own software.

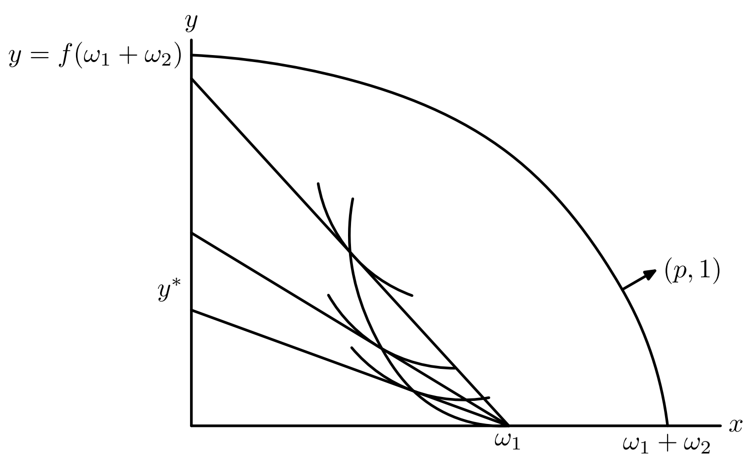
It might seem strange that microsoft could get a patent for a technique people have been using forever. It is important to realize that the US (and now Canada) has what is called a 'First to Patent' law, which means that whoever files a patent on something first gets the patent whether they developed the idea or not.

Lets just assume that consumer 2 wins this race. Consumer 2 now owns the public good. She can *exclude* consumer 1 and charge whatever price she likes for access to it. Suppose she sets the price p for the public good.

Normally we do all our stuff using the price of good x and letting the price of good 1 be normalized to 1. We'll switch that here, but this shouldn't create too much of a problem. The public good y has a price p , while the private good has price 1. Given the price set by consumer 2, consumer 1 now finds the best bundle of public and private goods he can afford. This is given by the point where his indifference curve is tangent to the budget line that passes through the point $(\omega_1, 0)$. Why care? The reason is that consumer 1 will no longer be able to produce the public good on his own (of if he does, he will have to pay consumer 2 a fee because she owns the public good). So the price that consumer 2 sets will change consumer 1's desire to have the public good. If consumer 2 sets a price too high, consumer 1 won't want any.

What this means is that the patent doesn't by itself give consumer 2 any incentive to produce more of the public good - production depends on price, which is chosen by consumer 2. What the patent does give her is a device that she can use to extract money from consumer 1.

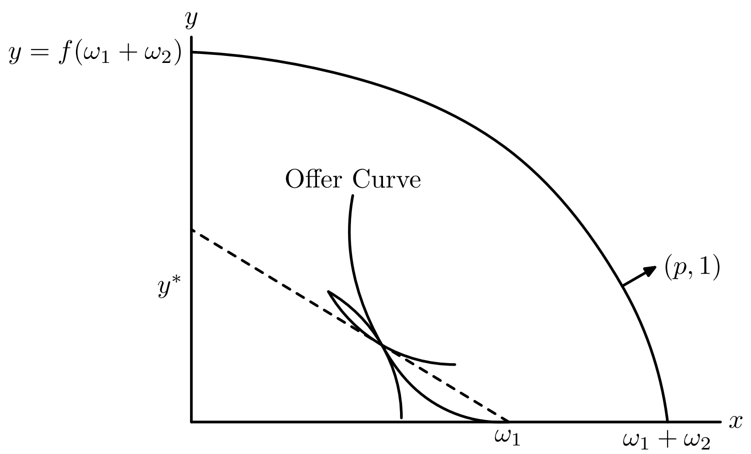
The figure below depicts what consumer 1 would want for three different prices.



The steepest budget line is the one that has the lowest price for the public good, the flattest budget line has the highest price. Since 2 now has the 'intellectual' monopoly, she can set any price that she likes. She will pick the price that maximizes her payoff.

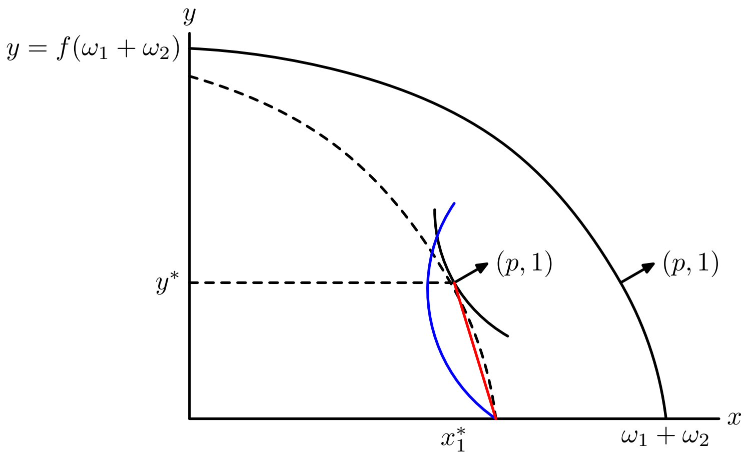
It would seem quite daunting to find this price, since the change in price will change consumer 1's consumption of the public good, which will in turn effect how much consumer 2 needs to produce.

Fortunately we have just the graphic device we need figure out what consumer 2 will do, since we just figured out what her indifference curves looked like in the space depicting consumer 1's consumption of the private and public good. She will pick the price that put her on the highest indifference curve consistent with consumer 1 maximizing subject to his budget constraint. You'll notice in the curve above that there is a smooth line that connects all the tangency points for different prices. This curve is sometimes referred to as the offer curve. The picture that follows shows what happens when consumer 2 chooses the point on this offer curve that is tangent to her own indifference curve.



The indifference curve for consumer 2 is now tangent to consumer 1's offer curve so consumer 2 is getting the best payoff she can get. Consumer 1 is getting the best payoff he can afford. Notice that because of the way the offer curve is drawn the two consumers indifference curves won't be tangent. Both consumers would be better off if more of the public good were produced. So the patent solution just produces another inefficient outcome. This is why proponents of patents will never talk about how well patents work. Instead they focus on how bad the outcome is likely to be without them - we went over that - not really so bad.

We can continue with graphic reasoning a bit more, but then we'll need to switch to algebra. The following figure reproduces the outcome for consumer 1 in the equilibrium of the voluntary contribution game.

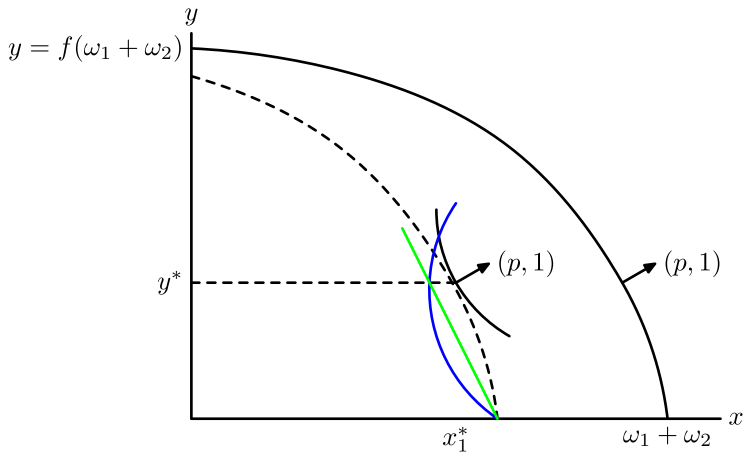


Recall that the budget line for consumer 1 with patents starts at his or her endowment point $(0, \omega_1)$. If consumer 2 sets a price for software which allows consumer 1 to buy the outcome he enjoyed in the voluntary contribution game, then the budget line consumer 1 faces would be given by the red line in the picture. Since this budget line always lies below the production possibilities curve that consumer 1 faces in the voluntary contribution game, it will be steeper than consumer 1's indifference curve at his allocation in the equilibrium of that game.

So far, this is completely working. If the price of the software were set so that consumer 1 could reproduce the outcome in the voluntary contribution game, then consumer 1 would be able to purchase the additional software that he wants from consumer 2. Consumer 2 would in turn be willing to produce it because she is making more than enough money to compensate her for her additional cost.

What patents get wrong is what they do next - they give consumer 2 control over price. Most economics students understand at some level that markets don't work when some market participant can control the price. In this example, it is easy to see why this is the case. The patent holder now has a new device for earning money that has nothing at all to do with having to produce software. It would first occur to her than since she controls price, she can actually get the same level of the public good y^* as in the voluntary contribution game, while ending up with a lot more money for herself.

In our formalism, we can use consumer 1's offer curve to understand this. Since the budget line that consumer 1 faces when the price is set in such a way that he can choose the equilibrium outcome in the voluntary contribution game is steeper than his indifference curve at that point, it means that consumer 1's offer curve cuts the horizontal line through y^* at a point to the left of consumer 1's original equilibrium allocation. The offer curve should look like the solid blue line in the figure above. So consumer 2 can raise the price of software until the budget line faced by consumer 1 looks like the green line in the next figure.



If you recall that the definition of the offer curve is all the points at which consumer 1's indifference curve is tangent to a line that runs from that point back to the endowment point, consumer 1 will voluntarily choose y^* units of software when the price is set to the green line is the budget line. By allowing consumer to control of the price, the patent basically creates a subsidy to the patent holder.

Consumer 2 might not be satisfied with this subsidy. She has really embarked at this point on a new venture - surplus extraction. Software is somewhat secondary at this point and she receives her reward by manipulating price. It is theoretically possible that she might want to raise output of software, but as in the figures above, this isn't her main objective, it is to find the place where her indifference curve is tangent to consumer 1's indifference curve.

Whether she does or not can be determined by traveling along the dashed horizontal line to the left of the equilibrium point in the voluntary contribution game until you reach the offer curve, given by the solid blue line in the figure above. Since consumer 2 can achieve any point on the offer curve that she wants, what she does will depend on what her indifference curve looks like through that point. Generally, the tangency with the offer curve may involve either more or less software than in the voluntary contribution game. So patents are as likely from a theoretical perspective to lower output of software as they are to raise it.

Whether software patents work well or not is not a theoretical issue - it all depends on preferences and costs. You might think that this means that theory has nothing to say about whether patents are good or bad. Yet the message of the theory is unambiguous - having a blanket policy where everything is patented (or otherwise protected as 'intellectual property') is going to do a lot of harm.

We can illustrate this with a couple of familiar examples. We'll start with one where patents act exclusively as a tax who proceeds are transferred to the patent holder, then follow up with an example in which patents can be beneficial to everyone (but more often than not just act as a way of transferring income to patent holders). In each case we'll follow the assumptions above, but assume that each consumer has money income ω to begin. We'll also assume that each unit of software can be produced for one unit of income. You should work out for yourself what implications those assumptions have for the diagrams above. All we'll change in the following two examples are consumer preferences.

Quasi-linear preferences

Assume that consumer preferences are given by $u(x, y) = x + \ln(y)$, where as above x is money income, y is software. In words preferences are quasi-linear in income, a very common assumption in economics. In the voluntary contribution game, each consumer's best reply is determined by maximizing

$$x + \ln(2\omega - x - x_2)$$

where x_2 is the amount of money the other player retains for herself. The solution is given by solving

$$1 = \frac{1}{2\omega - x - x_2},$$

so in the symmetric equilibrium of the voluntary contribution game

$$x = \omega - \frac{1}{2}.$$

This gives output of software in the voluntary contribution game as 1.

If consumer 2 is given a patent and sets the price p for software, consumer 1 will maximize

$$\omega - py + \ln(y)$$

which, as you know, has solution $y = \frac{1}{p}$. We can write consumer 2's payoff for any price p as

$$\omega - \frac{1}{p} + 1 + \ln\left(\frac{1}{p}\right).$$

Consumer 2 starts with ω, but has to produce $\frac{1}{p}$ units of software as requested by consumer 1. This costs $\frac{1}{p}$. She then receives p times $\frac{1}{p}$ dollars of revenue from consumer 1, which gives her $\omega - \frac{1}{p} + 1$ dollars for herself, and $\frac{1}{p}$ units of software. It is straightforward that consumer 2 will choose the price $p = 1$, so that the same amount of software is produced with patents as is produced in the voluntary contribution game.

What changes in the solution with patents is the distribution of income. Consumer 2 ends up with payoff $\omega + \ln(1) = \omega$ while consumer 1 ends up with $\omega - 1$. In other words, the patent simply allows player 2 to charge player 1 for all the software, whereas in the voluntary contribution game they would have split the cost.

Cobb-Douglas preferences

Things work a bit better for patents when consumers have Cobb-Douglas preferences. Under the right conditions, they can actually benefit both consumers, though, for the most part, their main role is still as a redistribution device. When preferences are Cobb-Douglas we have

$$u(x, y) = x^{\alpha} y^{1-\alpha}.$$

Consumer 1 maximizes

$$x^{\alpha}(2\omega - x - x_2)^{1-\alpha}$$

where x_2 is the amount of income consumer 1 expects consumer 2 to retain for herself (which makes her contribution to the production of software $\omega - x_2$). This is done subject to the constraint that $x \le \omega$.

The first order condition gives

$$x = \alpha(2\omega - x_2)$$

which has a solution between 0 and ω provided 2α is less than 1 and $x_2 < \omega$. The symmetric equilibrium for the voluntary contribution game is then

$$x = \frac{2\alpha}{1+\alpha}\omega$$

for each of the two players. The total output of software would then be

$$2\omega - \frac{4\alpha}{1+\alpha}\omega = 2\omega\frac{(1-\alpha)}{1+\alpha}. \tag{3}$$

On the other hand, if 2 has the patent, then as you know, consumer 1 will keep the fraction α of his money income and choose to buy $\frac{(1-\alpha)\omega}{p}$ units of the public good. Since $\frac{2}{1+\alpha}\alpha\omega$ is obviously strictly larger than $\alpha\omega$ you can see that consumer 1 will have a lot less income with patents.

So when 2 charges p for the public good, her payoff is

$$\left(\omega(2-\alpha) - \frac{(1-\alpha)\omega}{p}\right)^{\alpha}\left(\frac{(1-\alpha)\omega}{p}\right)^{(1-\alpha)}.$$

To figure out whether 1 will end up with more of the public good, we have to figure out what price 2 will choose. The price that maximizes the expression above will also maximize

$$\alpha \ln\left(\omega(2-\alpha) - \frac{(1-\alpha)\omega}{p}\right) + (1-\alpha)\ln\left(\frac{(1-\alpha)\omega}{p}\right)$$

which is a little simpler. Differentiate it to get

$$\frac{\alpha}{\omega(2-\alpha) - \frac{(1-\alpha)\omega}{p}}\frac{(1-\alpha)\omega}{p^2} - \frac{1-\alpha}{p}.$$

This means that the price 2 will set satisfies

$$\alpha\omega = p\left(\omega\left(2-\alpha\right) - \frac{\left(1-\alpha\right)\omega}{p}\right).$$

This has solution.

$$p = \frac{1}{2-\alpha}.$$

Now we can compare the two outcomes. Since consumer 1 determines the output of software, the total amount of software produced with be

$$\frac{\left(1-\alpha\right)\omega}{p} =$$

Now lets use the diagram above to figure out whether 2 will want more of less software than is created in the voluntary contribution game. We can decompose her price adjustment into two parts. The first thing we could imagine is that she takes the subsidy and raises the price to the point where 1 will want the same level of the public good as he did in the equilibrium of the voluntary contribution game. With Cobb-Douglas preferences, the consumer will always want to keep $\alpha\omega$ of his income. So we want to find a price that make consumer 1 purchase the level of output in the voluntary contribution game. From (3) this is $2\omega\frac{(1-\alpha)}{1+\alpha}$. So we want a price such that

$$2\omega\frac{(1-\alpha)}{1+\alpha} = \frac{\left(1-\alpha\right)\omega}{p}$$

or

$$p = \frac{1+\alpha}{2}.$$

It isn't too hard to show that $\frac{1+\alpha}{2} > \frac{1}{2-\alpha}$ for all $0 < \alpha < 1$, so consumer 2 will set a lower price for software than the price that would have supported the same output of software as in the voluntary contribution game.

We could have figured this out more directly. Once consumer 2 gets a patent, she automatically receives a transfer of income from consumer 1. However, the size of the transfer doesn't change as 2 varies the price because of the properties of Cobb Douglas preferences. So if consumer 1 has Cobb Douglas preferences, the only way consumer 2 can increase her payoff is by using the transfer to pay for more software for herself.

More generally, if the consumer has more reasonable preferences for which the proportion of income that she spends on software rises as software prices rise, then consumer 2 will be able to extract even more income from consumer 1 by raising software prices.

The fact that consumer 2 produces more software looks good, but it doesn't mean that consumer 1 is any better off than she would have been without the patent.

If consumer 2 has the patent, then consumer 1 has payoff

$$\left(\alpha\omega\right)^{\alpha}\left(\omega\left(1-\alpha\right)\left(2-\alpha\right)\right)^{1-\alpha} =$$

$$\omega\left(2-\alpha\right)\left(\frac{1}{2-\alpha}\right)^{\alpha}\left(\frac{\alpha}{\left(1-\alpha\right)}\right)^{\alpha}\left(1-\alpha\right).$$

In the voluntary contribution game, consumer 1 has payoff

$$\left(\frac{2\alpha\omega}{1+\alpha}\right)^{\alpha}\left(2\omega\frac{1-\alpha}{1+\alpha}\right)^{1-\alpha} =$$

$$\frac{2}{1+\alpha}\omega\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1-\alpha\right)$$

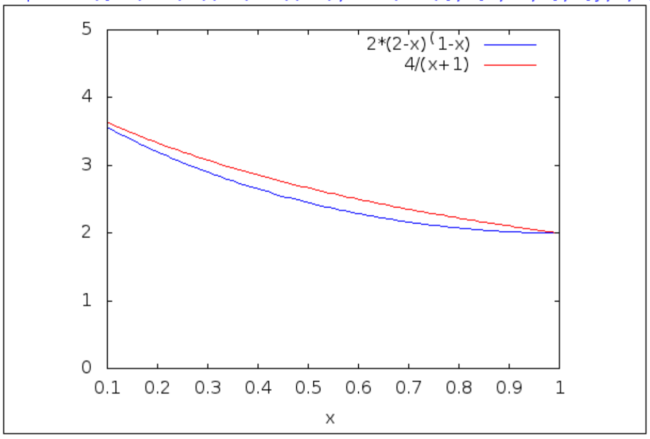
To compare the payoffs, we need to compare $\omega\left(2-\alpha\right)\left(\frac{1}{2-\alpha}\right)^{\alpha}$ and $\frac{2}{1+\alpha}\omega$. We can't really figure this out analytically, so we'll turn to the computer.

The next picture shows a plot of the payoffs of consumer 1 as they vary with his propensity for keeping is own income, α, when $\omega = 2$. This calculation, and the draft are drawn using wxMaxima, which is a free computer algebra program. If you want to try it for yourself, I left the code for the calculation at the top of the diagram.

Recall, that with CD preferences, α is the weight that the consumer gives to retained income x - if α is small, the consumer desperately wants software and is willing to pay a lot for it. If α is large, the consumer isn't so interested in software. This is consistent with the *demand* for software $\frac{1-\alpha}{p}$ - at any price, the consumer will buy less software the larger is α. The red curve in the figure below represents the function $\frac{2}{1+\alpha}\omega$ which is the factor associated with the equilibrium of the voluntary contribution game. The blue figure represents the value of the factor associated with patents. The implication of the fact that the red curve lies above the blue curve is that consumer 1 is never better off than he was in the equilibrium of the voluntary contribution game. He gets more software, but the price of the software is much more expensive to him than the software he would have produced on his own.

$$\left(\alpha\omega\right)^{\alpha}\left(\frac{\left(1-\alpha\right)\omega}{p}\right) = \left(\alpha\omega\right)^{\alpha}\left(\frac{2\left(1-\alpha\right)\omega}{1+\alpha}\right)^{(1-\alpha)}$$

```
wxplot2d([2*(2-x)*((1/(2-x))^x),2*2/(1+x)], [x,.1,1], [y,0,5])$
```



We conclude that though the patent does result in more software being produced, its primary purpose is still to transfer income to the patent holder - essentially a tax.

Problems to work on.

One way in which the model describe above differs from open source software is that, with software, public good production tends to involve increasing returns - the more software other people write, the more software you can write for the same amount of money. Try to carry out the analysis in the section with quasi-linear software above when the production function for public goods is given by

$$y = x^2$$

where y is output of the public good and x is money spent on developing the public good. Go as far as you can, but at the very least, verify for yourself that when production is subject to increasing returns, a lot more software will be produced in the voluntary contribution game.