Linear regression

A Short Course on Data Analysis Using R Software

Wan Nor Arifin
Unit of Biostatistics and Research Methodology,
Universiti Sains Malaysia.

Email: wnarifin@usm.my
Website: wnarifin.github.io

Contents

1 Introduction

1. A statistical method to model relationship between:
    - outcome: numerical variable.
    - predictors/independent variables: numerical, categorical variables.

2. A type of Generalized Linear Models (GLMs), which also includes other outcome types, e.g. categorical and count.
3. Basically, the linear relationship is structured as follows,

$$numerical\ outcome = numerical\ predictors + categorical\ predictors$$

2 Simple linear regression (SLR)

2.1 About SLR

1. Model *linear* (straight line) relationship between:
   - outcome: numerical variable.
   - a predictor: numerical variable (only).

   Note: What if the predictor is a categorical variable? Remember, we already handled that with one-way ANOVA.
2. Formula,

$$numerical\ outcome = intercept + coefficient \times numerical\ predictor$$

   in short,

$$\hat{y} = \beta_0 + \beta_1 x_1$$

   where $\hat{y}$ is the predicted value of the outcome y.

2.2 Analysis

2.2.1 Libraries

```
# library
library(foreign)
library(epiDisplay)
```

```
## Loading required package: survival
```

```
## Loading required package: MASS
```

```
## Loading required package: nnet
```

```
library(psych)
```

```
##
## Attaching package: 'psych'
```

```
## The following objects are masked from 'package:epiDisplay':
##
##     alpha, cs, lookup
```

```
library(lattice)
```

```
##
## Attaching package: 'lattice'
```

```
## The following object is masked from 'package:epiDisplay':
##
##     dotplot
```

```
library(rsq)
library(MASS)
library(car)
```

```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:psych':
##
##     logit
```

```
library(broom)
```

2.2.2 Data set

```
# data
coronary = read.dta("coronary.dta")
str(coronary)
```

```
## 'data.frame':    200 obs. of  9 variables:
##  $ id    : num  1 14 56 61 62 64 69 108 112 134 ...
##  $ cad   : Factor w/ 2 levels "no cad","cad": 1 1 1 1 1 1 2 1 1 1 ...
##  $ sbp   : num  106 130 136 138 115 124 110 112 138 104 ...
##  $ dbp   : num  68 78 84 100 85 72 80 70 85 70 ...
##  $ chol  : num  6.57 6.33 5.97 7.04 6.66 ...
##  $ age   : num  60 34 36 45 53 43 44 50 43 48 ...
##  $ bmi   : num  38.9 37.8 40.5 37.6 40.3 ...
##  $ race  : Factor w/ 3 levels "malay","chinese",..: 3 1 1 1 3 1 1 2 2 2 ...
##  $ gender: Factor w/ 2 levels "woman","man": 1 1 1 1 2 2 2 1 1 2 ...
##  - attr(*, "datalabel")= chr "Written by R.              "
##  - attr(*, "time.stamp")= chr ""
##  - attr(*, "formats")= chr  "%9.0g" "%9.0g" "%9.0g" "%9.0g" ...
##  - attr(*, "types")= int  100 108 100 100 100 100 100 108 108
##  - attr(*, "val.labels")= chr  "" "cad" "" "" ...
##  - attr(*, "var.labels")= chr  "id" "cad" "sbp" "dbp" ...
##  - attr(*, "version")= int 7
##  - attr(*, "label.table")=List of 3
##   ..$ cad   : Named int  1 2
##   .. ..- attr(*, "names")= chr  "no cad" "cad"
##   ..$ race  : Named int  1 2 3
##   .. ..- attr(*, "names")= chr  "malay" "chinese" "indian"
##   ..$ gender: Named int  1 2
##   .. ..- attr(*, "names")= chr  "woman" "man"
```

2.2.3 Data exploration

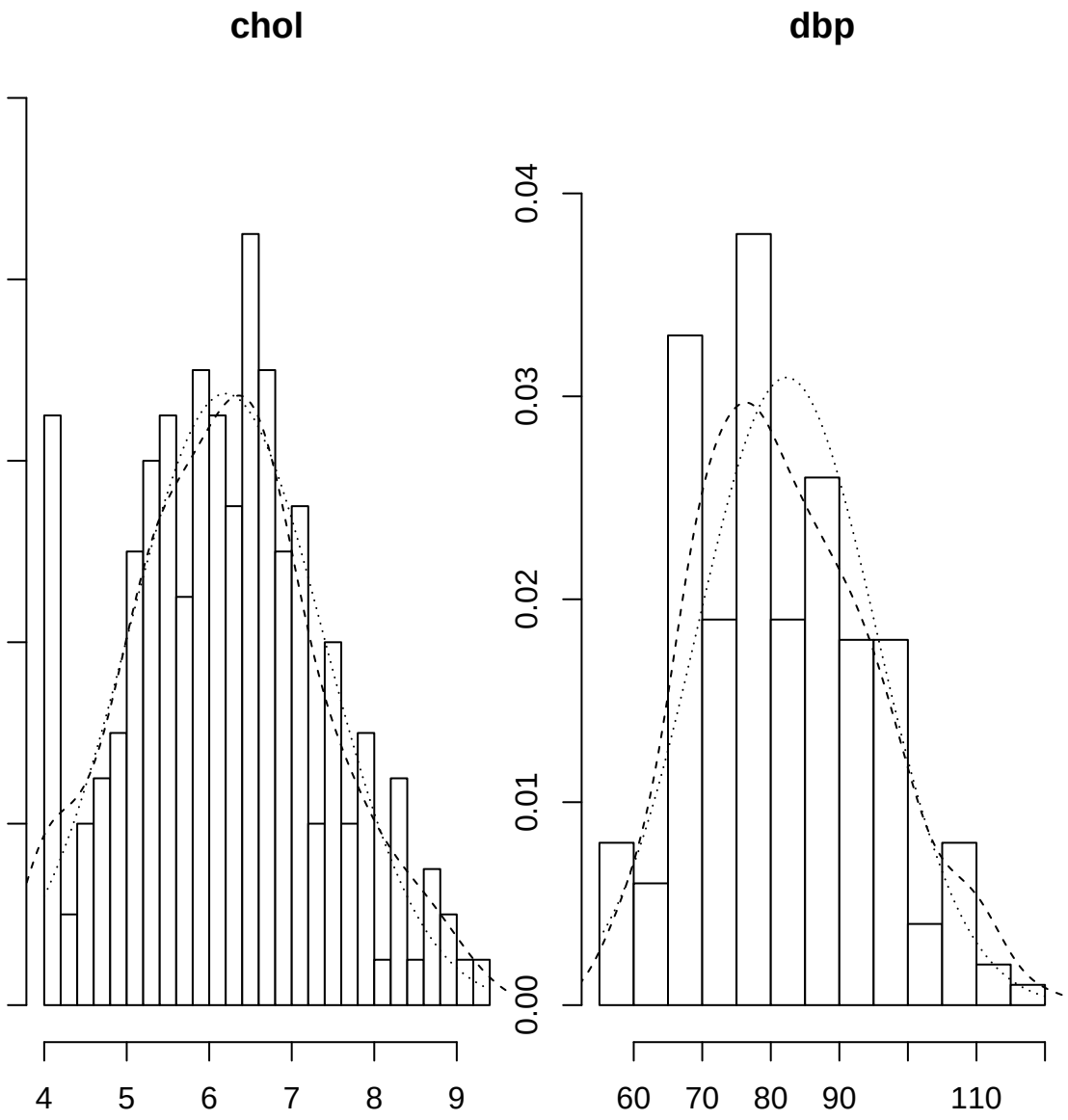
2.2.3.1 Descriptive statistics

```
summ(coronary[c("chol", "dbp")])
```

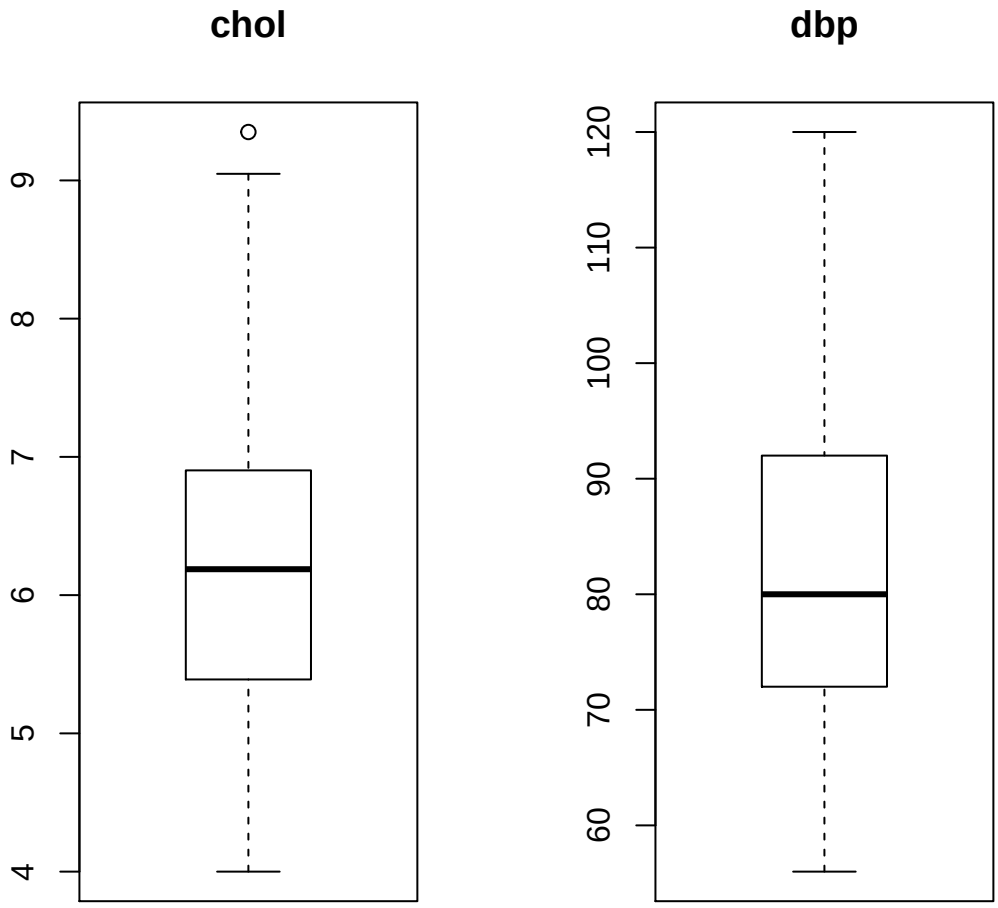
```
##
## No. of observations = 200
##
##   Var. name obs. mean   median  s.d.   min.   max.
## 1 chol      200  6.2    6.19    1.18   4      9.35
## 2 dbp       200  82.31  80      12.9   56     120
```

2.2.3.2 Plots

```
multi.hist(coronary[c("chol", "dbp")], ncol = 2)
```




```
par(mfrow = c(1, 2))
mapply(boxplot, coronary[c("chol", "dbp")],
       main = colnames(coronary[c("chol", "dbp")]))
```




```
##       chol      dbp
## stats Numeric,5 Numeric,5
## n     200       200
## conf  Numeric,2 Numeric,2
## out   9.35      Numeric,0
## group 1         Numeric,0
## names ""        ""
```

```
par(mfrow = c(1, 1))
```

2.2.4 Univariable

Fit model,

```
# model: chol ~ dbp
slr_chol = glm(chol ~ dbp, data = coronary)
summary(slr_chol)
```

```
##
## Call:
## glm(formula = chol ~ dbp, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.9967  -0.8304  -0.1292   0.7734   2.8470
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.995134   0.492092   6.087 5.88e-09 ***
## dbp         0.038919   0.005907   6.589 3.92e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.154763)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 228.64  on 198  degrees of freedom
## AIC: 600.34
##
## Number of Fisher Scoring iterations: 2
```

```
Confint(slr_chol)  # 95% CI
```

```
##               Estimate      2.5 %     97.5 %
## (Intercept) 2.99513427 2.03065127 3.95961727
## dbp         0.03891876 0.02734161 0.05049591
```

Important results,

- Coefficient, β.
- 95% CI.
- P-value.

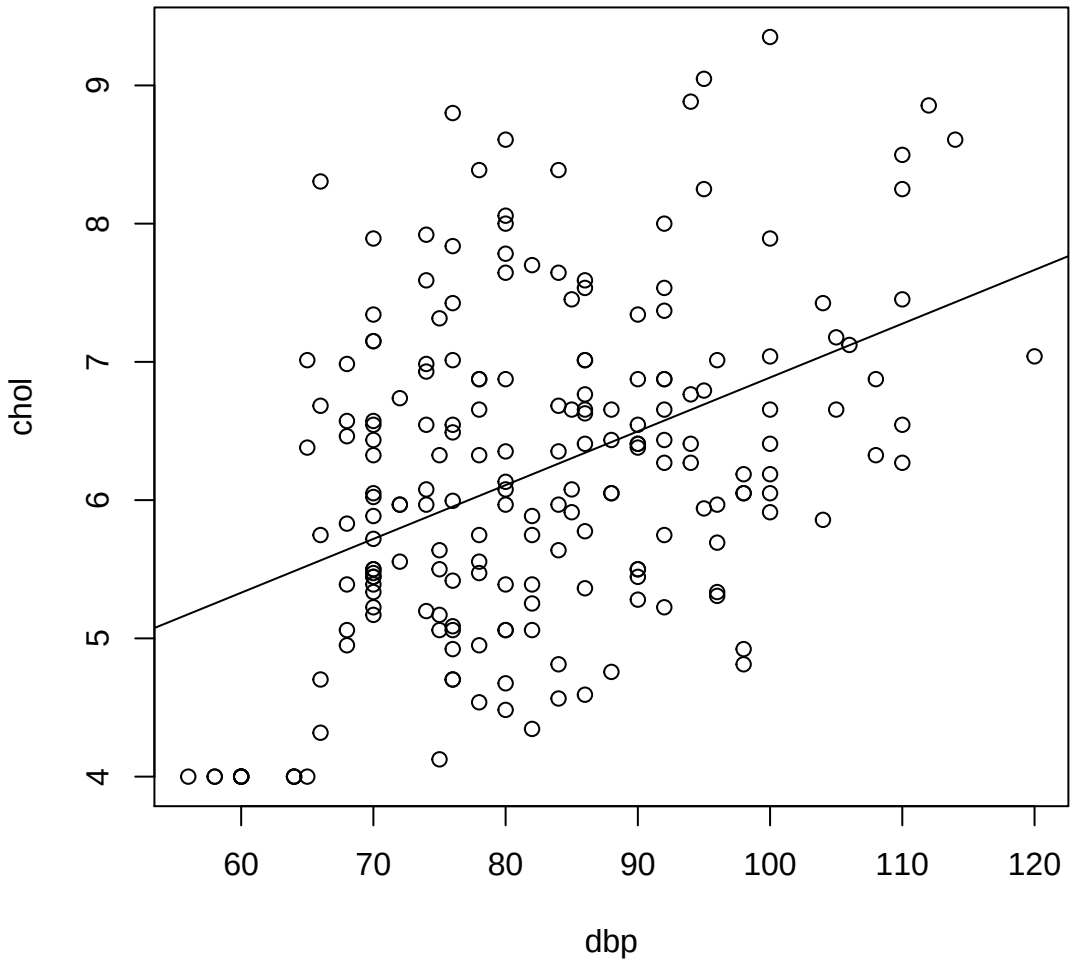
Obtain R^2, % of variance explained,

```
rsq(slr_chol, adj = T)
```

```
## [1] 0.1756834
```

Scatter plot,

```
plot(chol ~ dbp, data = coronary)
abline(slr_chol)
```



this allows assessment of normality, linearity and equal variance assumptions. We expect eliptical/oval shape (normality), equal scatter of dots on both sides of the prediction line (equal variance). Both these indicate linear relationship between `chol` and `dbp`.

2.2.5 Interpretation

- 1mmHg increase in DBP causes 0.04mmol/L increase in cholestrol.
- DBP explains 17.6% variance in cholestrol.

2.2.6 Model equation

$$chol = 3.0 + 0.04 \times dbp$$

3 Multiple linear regression (MLR)

3.1 About MLR

1. Model *linear* relationship between:
   - outcome: numerical variable.
   - predictors: numerical, categorical variables.

   Note: MLR is a term that refers to linear regression with two or more *numerical* variables. Whenever we have both numerical and categorical variables, the proper term for the regression model is *General Linear Model*. However, we will use the term MLR in this workshop.

2. Formula,

$$numerical\ outcome = intercept + coefficients \times numerical\ predictors + coefficients \times categorical\ predictors$$

   in a shorter form,

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k$$

   where we have k predictors.

Whenever the predictor is a categorical variable with more than two levels, we use dummy variable(s). This can be easily specified in R using `factor()` if the variable is not yet properly specified as such. There is no problem with binary categorical variable.

For a categorical variable with more than two levels, the number of dummy variables (i.e. once turned into several binary variables) equals number of levels minus one. For example, whenever we have four levels, we will obtain three dummy (binary) variables.

3.2 Analysis

3.2.1 Review data set

```
# data
str(coronary)
```

```
## 'data.frame':    200 obs. of  9 variables:
##  $ id    : num  1 14 56 61 62 64 69 108 112 134 ...
##  $ cad   : Factor w/ 2 levels "no cad","cad": 1 1 1 1 1 1 2 1 1 1 ...
##  $ sbp   : num  106 130 136 138 115 124 110 112 138 104 ...
##  $ dbp   : num  68 78 84 100 85 72 80 70 85 70 ...
##  $ chol  : num  6.57 6.33 5.97 7.04 6.66 ...
##  $ age   : num  60 34 36 45 53 43 44 50 43 48 ...
##  $ bmi   : num  38.9 37.8 40.5 37.6 40.3 ...
##  $ race  : Factor w/ 3 levels "malay","chinese",..: 3 1 1 1 3 1 1 2 2 2 ...
##  $ gender: Factor w/ 2 levels "woman","man": 1 1 1 1 2 2 2 1 1 2 ...
##  - attr(*, "datalabel")= chr "Written by R.              "
##  - attr(*, "time.stamp")= chr ""
##  - attr(*, "formats")= chr  "%9.0g" "%9.0g" "%9.0g" "%9.0g" ...
##  - attr(*, "types")= int  100 108 100 100 100 100 100 108 108
##  - attr(*, "val.labels")= chr  "" "cad" "" "" ...
##  - attr(*, "var.labels")= chr  "id" "cad" "sbp" "dbp" ...
##  - attr(*, "version")= int 7
##  - attr(*, "label.table")=List of 3
##   ..$ cad   : Named int  1 2
```

```
##   .. ..- attr(*, "names")= chr  "no cad" "cad"
##   ..$ race  : Named int  1 2 3
##   .. ..- attr(*, "names")= chr  "malay" "chinese" "indian"
##   ..$ gender: Named int  1 2
##   .. ..- attr(*, "names")= chr  "woman" "man"
```

We exclude `id`, `cad` and `age` from our data for the purpose of this analysis, keeping only `sbp` , `dbp`, `bmi`, `race` and `gender`. We will add `age` later in the exercise.

```
coronary = subset(coronary, select = -c(id, cad, age))
# remove id, cad, age from our data since we're not going to use them,
# easier to specifiy multivariable model.
```

3.2.2 Data exploration

3.2.2.1 Descriptive statistics

```
summ(coronary[c("chol", "sbp", "dbp", "bmi")])
```

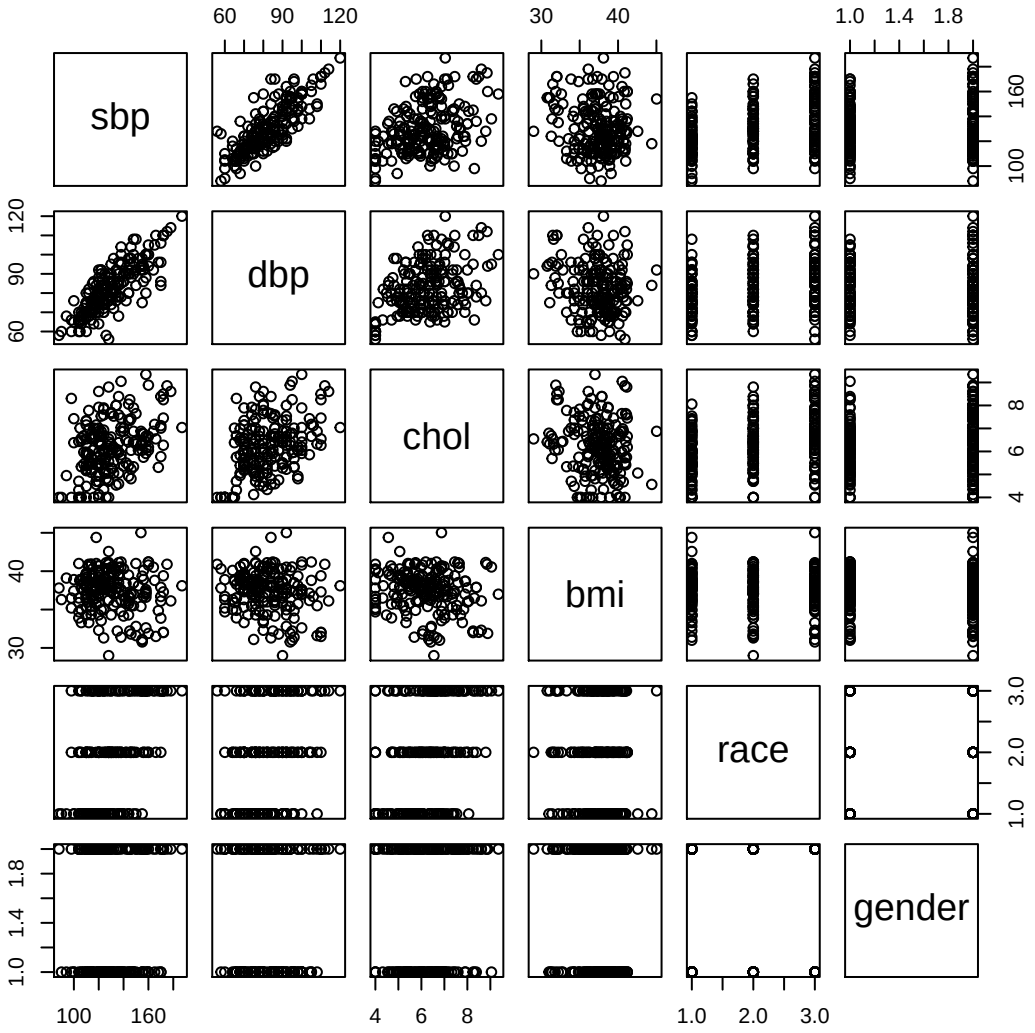
```
##
## No. of observations = 200
##
##   Var. name obs. mean   median  s.d.   min.   max.
## 1 chol      200  6.2    6.19    1.18   4      9.35
## 2 sbp       200  130.18 126     19.81  88     187
## 3 dbp       200  82.31  80      12.9   56     120
## 4 bmi       200  37.45  37.8    2.68   28.99  45.03
```

```
codebook(coronary[c("race", "gender")])
```

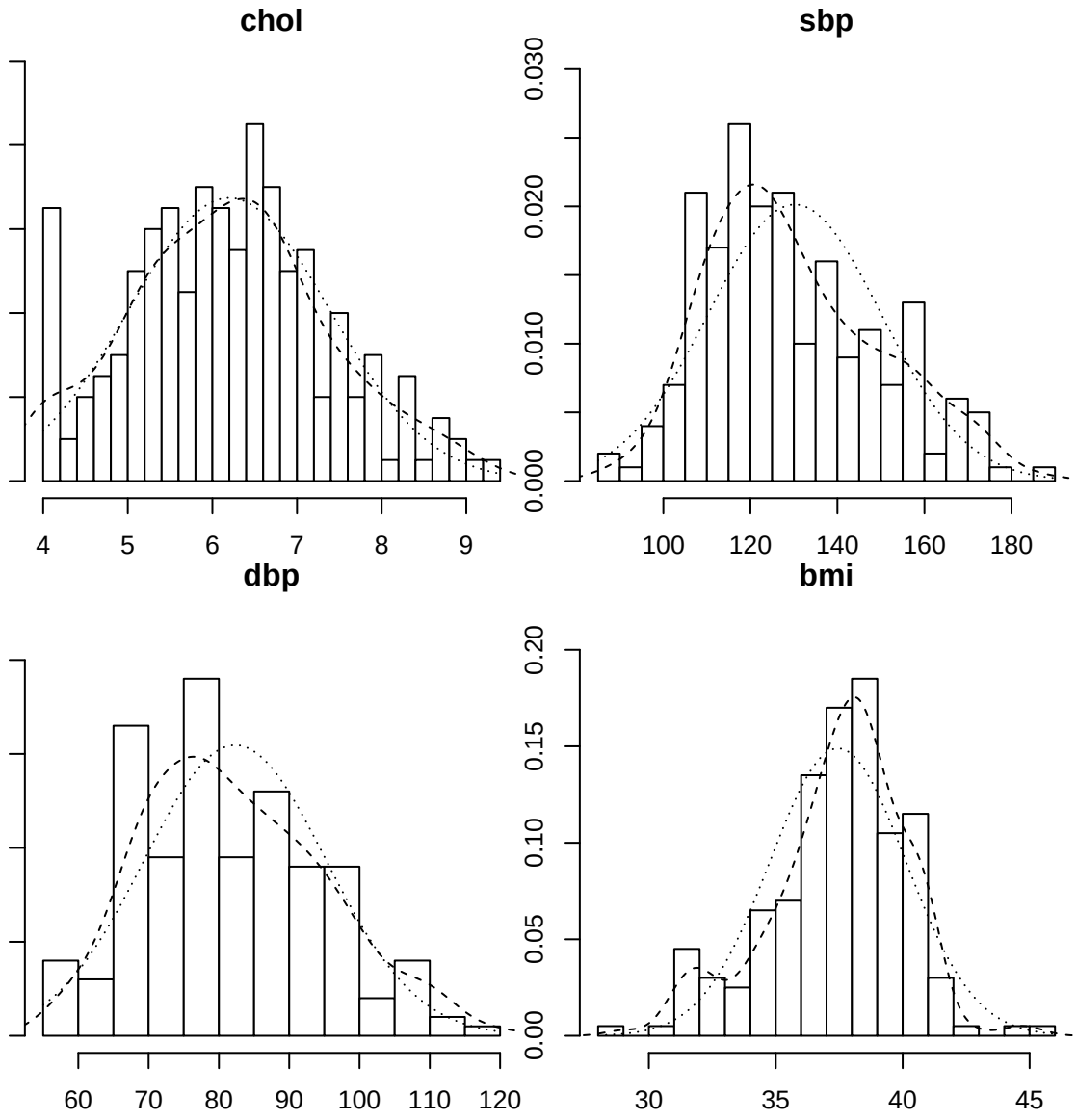
```
##
##
##
## race      :
##         Frequency Percent
## malay          73    36.5
## chinese        64    32.0
## indian         63    31.5
##
##  ==================
## gender    :
##       Frequency Percent
## woman       100      50
## man         100      50
##
##  ==================
```

3.2.2.2 Plots

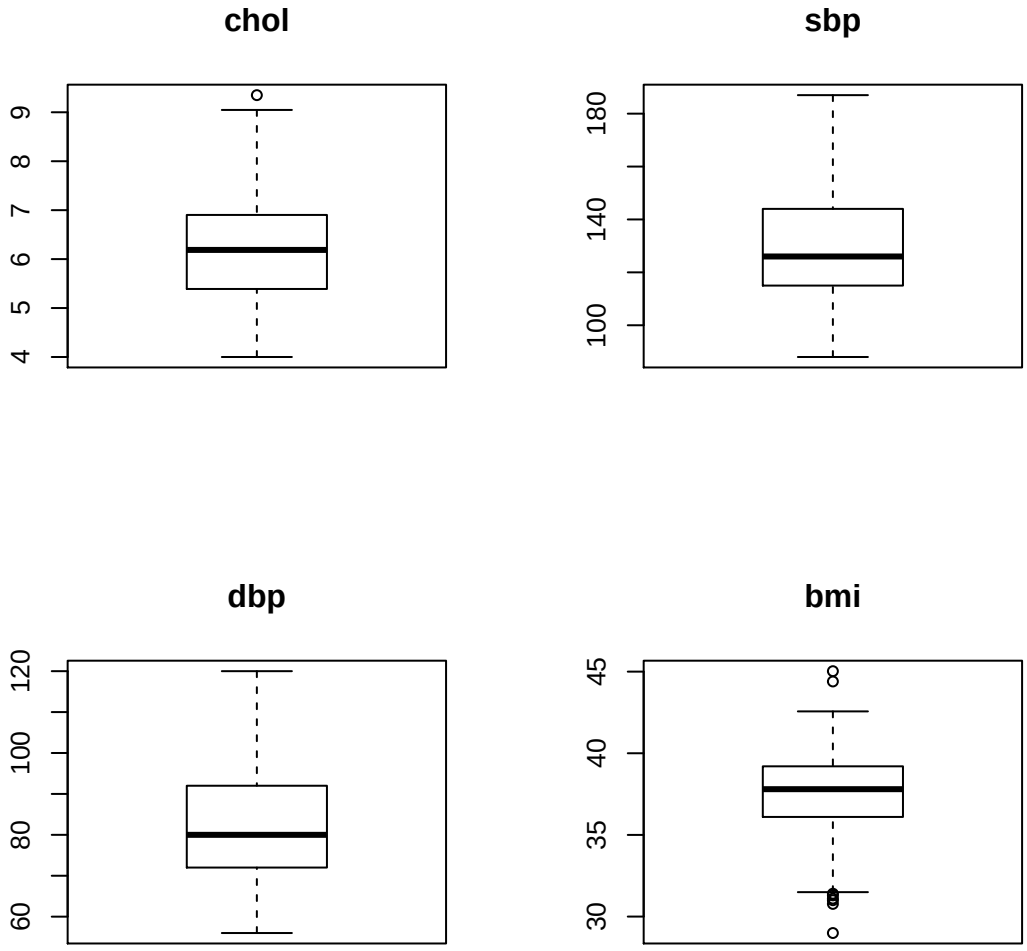
```
plot(coronary)
```




```
multi.hist(coronary[c("chol", "sbp", "dbp", "bmi")])
```

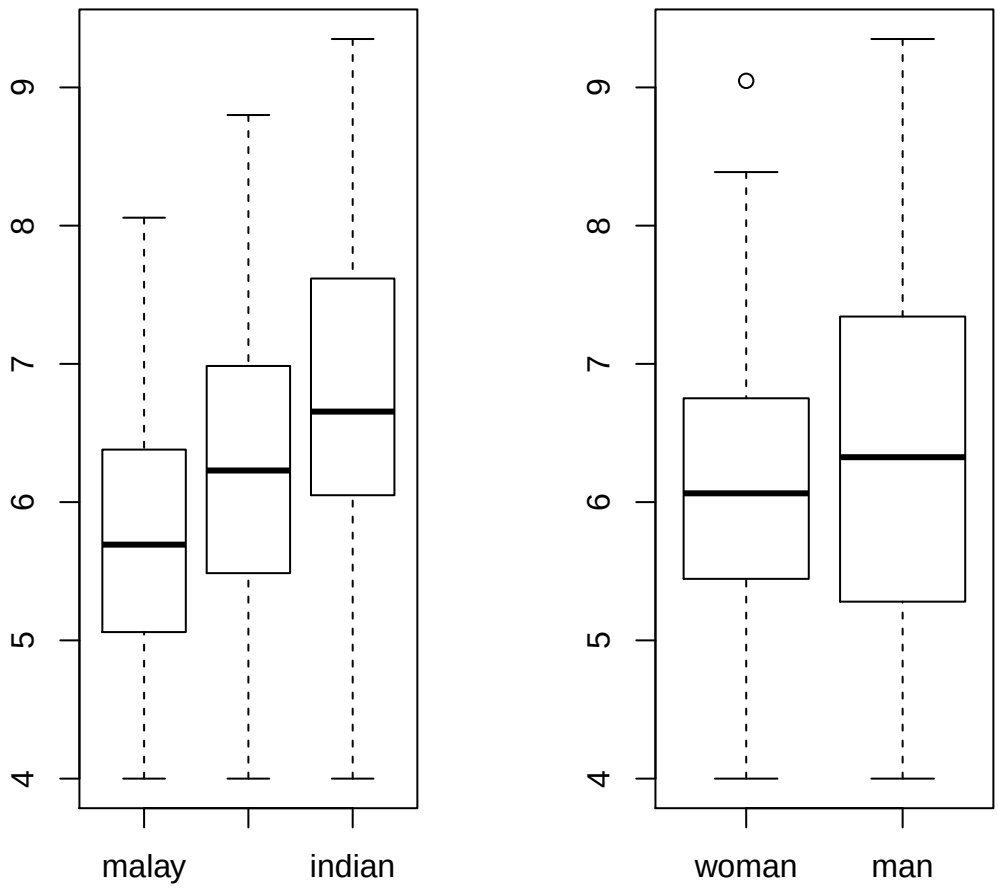



```
par(mfrow = c(2, 2))
mapply(boxplot, coronary[c("chol", "sbp", "dbp", "bmi")],
       main = colnames(coronary[c("chol", "sbp", "dbp", "bmi")]))
```




```
##       chol      sbp       dbp       bmi
## stats Numeric,5 Numeric,5 Numeric,5 Numeric,5
## n     200       200       200       200
## conf  Numeric,2 Numeric,2 Numeric,2 Numeric,2
## out   9.35      Numeric,0 Numeric,0 Numeric,8
## group 1         Numeric,0 Numeric,0 Numeric,8
## names ""        ""        ""        ""
```

```
par(mfrow = c(1, 1))
par(mfrow = c(1, 2))
boxplot(chol ~ race, data = coronary)
boxplot(chol ~ gender, data = coronary)
```




```
par(mfrow = c(1, 1))
```

3.2.3 Variable selection

3.2.3.1 Univariable

Perform SLR for `chol, sbp, dbp` and `bmi` on your own as shown above. Now, we are concerned with which variables are worthwhile to include in the multivariable models.

We want to choose only variables with P-values < 0.25 to be included in MLR. Obtaining the P-values for each variable is easy by LR test,

```
slr_chol0 = glm(chol ~ 1, data = coronary)
summary(slr_chol0)
```

```
##
## Call:
## glm(formula = chol ~ 1, data = coronary)
```

```
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.19854  -0.80854  -0.01104   0.69021   3.15146
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.19854    0.08369   74.06   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.400874)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 278.77  on 199  degrees of freedom
## AIC: 637.99
##
## Number of Fisher Scoring iterations: 2
```

```
names(coronary)
```

```
## [1] "sbp"    "dbp"    "chol"   "bmi"    "race"   "gender"
```

```
add1(slr_chol0, scope = ~ sbp + dbp + bmi + race + gender, test = "LRT")
```

```
## Single term additions
##
## Model:
## chol ~ 1
##        Df Deviance    AIC scaled dev.  Pr(>Chi)
## <none>      278.77 637.99
## sbp     1   235.36 606.14      33.855 5.938e-09 ***
## dbp     1   228.64 600.34      39.648 3.042e-10 ***
## bmi     1   272.17 635.20       4.792   0.02859 *
## race    2   241.68 613.43      28.561 6.280e-07 ***
## gender  1   277.45 639.04       0.952   0.32933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All variables are significant and $< .25$ except `gender`. So proceed with the rest of the variables, excluding `gender`.

3.2.3.2 Multivariable

Perform MLR with *all* selected variables,

```
# all
mlr_chol = glm(chol ~ sbp + dbp + bmi + race, data = coronary)
#mlr_chol = glm(chol ~ ., data = coronary)  # shortcut
summary(mlr_chol)
```

```
##
## Call:
## glm(formula = chol ~ sbp + dbp + bmi + race, data = coronary)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
```

```
## -2.17751  -0.73860  -0.02674   0.63163   2.90926
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.842338   1.265149   3.827 0.000175 ***
## sbp          0.000975   0.006990   0.139 0.889210
## dbp          0.028350   0.010327   2.745 0.006615 **
## bmi         -0.038537   0.028170  -1.368 0.172879
## racechinese  0.354039   0.183169   1.933 0.054710 .
## raceindian   0.716327   0.200346   3.575 0.000441 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.089387)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 211.34  on 194  degrees of freedom
## AIC: 592.61
##
## Number of Fisher Scoring iterations: 2
```

```
rsq(mlr_chol, adj = T)
```

```
## [1] 0.2223518
```

Focus on,

- Coefficients, βs.
- 95% CI.
- P-values.

For model fit,

- R^2 – % of variance explained by the model.
- Akaike Information Criterion, AIC – for comparison with other models. This is not useful alone, but for comparison with other models. The model with the lowest AIC is the best model.

3.2.3.3 Stepwise

As you can see, not all variables are significant. How to select? We proceed with stepwise automatic selection,

```
# stepwise
# both
mlr_chol_stepboth = step(mlr_chol, direction = "both")
```

```
## Start:  AIC=592.61
## chol ~ sbp + dbp + bmi + race
##
##        Df Deviance    AIC
## - sbp   1   211.36 590.63
## - bmi   1   213.38 592.53
## <none>      211.34 592.61
## - dbp   1   219.55 598.23
## - race  2   225.30 601.40
##
## Step:  AIC=590.63
## chol ~ dbp + bmi + race
```

```
##
##        Df Deviance    AIC
## - bmi   1   213.40 590.55
## <none>      211.36 590.63
## + sbp   1   211.34 592.61
## - race  2   227.04 600.94
## - dbp   1   235.88 610.58
##
## Step:  AIC=590.55
## chol ~ dbp + race
##
##        Df Deviance    AIC
## <none>      213.40 590.55
## + bmi   1   211.36 590.63
## + sbp   1   213.38 592.53
## - race  2   228.64 600.34
## - dbp   1   241.68 613.43
```

```
summary(mlr_chol_stepboth)  # racechinese marginally sig.
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

```
# forward
mlr_chol_stepforward = step(slr_chol0, scope = ~ sbp + dbp + bmi + race + gender,
                            direction = "forward")
```

```
## Start:  AIC=637.99
## chol ~ 1
##
##          Df Deviance    AIC
## + dbp     1   228.64 600.34
## + sbp     1   235.36 606.14
## + race    2   241.68 613.43
```

```
## + bmi     1   272.17 635.20
## <none>        278.77 637.99
## + gender  1   277.45 639.04
##
## Step:  AIC=600.34
## chol ~ dbp
##
##          Df Deviance    AIC
## + race    2   213.40 590.55
## <none>        228.64 600.34
## + gender  1   226.64 600.58
## + sbp     1   226.96 600.87
## + bmi     1   227.04 600.94
##
## Step:  AIC=590.55
## chol ~ dbp + race
##
##          Df Deviance    AIC
## <none>        213.40 590.55
## + bmi     1   211.36 590.63
## + gender  1   212.47 591.67
## + sbp     1   213.38 592.53
```

```
summary(mlr_chol_stepforward)  # same with both
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

```
# backward
mlr_chol_stepback = step(mlr_chol, direction = "backward")
```

```
## Start:  AIC=592.61
## chol ~ sbp + dbp + bmi + race
```

```
##
##        Df Deviance    AIC
## - sbp   1   211.36 590.63
## - bmi   1   213.38 592.53
## <none>      211.34 592.61
## - dbp   1   219.55 598.23
## - race  2   225.30 601.40
##
## Step:  AIC=590.63
## chol ~ dbp + bmi + race
##
##        Df Deviance    AIC
## - bmi   1   213.40 590.55
## <none>      211.36 590.63
## - race  2   227.04 600.94
## - dbp   1   235.88 610.58
##
## Step:  AIC=590.55
## chol ~ dbp + race
##
##        Df Deviance    AIC
## <none>      213.40 590.55
## - race  2   228.64 600.34
## - dbp   1   241.68 613.43
```

```
summary(mlr_chol_stepback)  # same with both
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

Looking at all these results, we choose:

```
chol ~ dbp + race
```

which has the lowest AIC.

```
mlr_chol1 = glm(chol ~ dbp + race, data = coronary)
summary(mlr_chol1)
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

3.2.3.4 Confounder

If we include a variable and it causes notable change ($> 20\%$) in the coefficients of other variables, it is a confounder. When the confounder is significant and the main effect variable is also significant, we keep the confounder in the model.

Formula for % change,

```
100 * (model_small - model_large) / model_large
```

Hosmer, Lemeshow, & Sturdivant (2013)

Start by including common demographic adjustment, gender,

```
# + gender
mlr_chol2 = glm(chol ~ dbp + race + gender, data = coronary)
summary(mlr_chol2)  # higher AIC, gender insig.
```

```
##
## Call:
## glm(formula = chol ~ dbp + race + gender, data = coronary)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.06350  -0.71634  -0.04471   0.64533   2.70974
##
## Coefficients:
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.203032   0.497111   6.443 8.94e-10 ***
## dbp         0.031533   0.006124   5.149 6.37e-07 ***
## racechinese 0.353052   0.182369   1.936   0.0543 .
## raceindian  0.692724   0.192293   3.602   0.0004 ***
## genderman   0.137663   0.148790   0.925   0.3560
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.089578)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 212.47  on 195  degrees of freedom
## AIC: 591.67
##
## Number of Fisher Scoring iterations: 2
```

```
coef(mlr_chol2); coef(mlr_chol1)
```

```
## (Intercept)         dbp racechinese  raceindian   genderman
##   3.2030318   0.0315331   0.3530516   0.6927239   0.1376627
```

```
## (Intercept)         dbp racechinese  raceindian
##  3.29802826  0.03110811  0.35996365  0.71369024
```

```
100 * (coef(mlr_chol1) - coef(mlr_chol2)[1:4])/coef(mlr_chol2)[1:4]  # change < 20%
```

```
## (Intercept)         dbp racechinese  raceindian
##    2.965828   -1.347773    1.957792    3.026647
```

```
# no notable change in coeffs, gender is not a confounder
```

Now, we can try adding `sbp` & `bmi` to `mlr_chol1` and see what happens to the coefficients. We will use `update()` function here.

```
mlr_chol3 = update(mlr_chol1, . ~ . + sbp)
summary(mlr_chol3)  # higher AIC, sbp insig.
```

```
##
## Call:
## glm(formula = chol ~ dbp + race + sbp, data = coronary)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.12850  -0.71572  -0.03242   0.59676   2.77189
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.269724   0.529556   6.174 3.78e-09 ***
## dbp         0.029978   0.010281   2.916 0.003963 **
## racechinese 0.357407   0.183561   1.947 0.052963 .
## raceindian  0.705445   0.200635   3.516 0.000545 ***
## sbp         0.000958   0.007005   0.137 0.891365
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.094256)
```

```
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.38  on 195  degrees of freedom
## AIC: 592.53
##
## Number of Fisher Scoring iterations: 2
```

```
coef(mlr_chol3); coef(mlr_chol1)
```

```
##  (Intercept)          dbp  racechinese   raceindian          sbp
## 3.2697237312 0.0299783153 0.3574065705 0.7054452332 0.0009580065
```

```
## (Intercept)         dbp racechinese  raceindian
##  3.29802826  0.03110811  0.35996365  0.71369024
```

```
100 * (coef(mlr_chol1) - coef(mlr_chol3)[1:4])/coef(mlr_chol3)[1:4]  # change < 20%
```

```
## (Intercept)         dbp racechinese  raceindian
##   0.8656550   3.7687027   0.7154536   1.1687670
```

```
# no notable change in coeffs, sbp is not a confounder
```

```
mlr_chol4 = update(mlr_chol1, . ~ . + bmi)
summary(mlr_chol4)  # slighly higher AIC, bmi insig.
```

```
##
## Call:
## glm(formula = chol ~ dbp + race + bmi, data = coronary)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.18698  -0.73076  -0.01935   0.63476   2.91524
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.870859   1.245373   3.911 0.000127 ***
## dbp          0.029500   0.006203   4.756 3.83e-06 ***
## racechinese  0.356642   0.181757   1.962 0.051164 .
## raceindian   0.724716   0.190625   3.802 0.000192 ***
## bmi         -0.038530   0.028099  -1.371 0.171871
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.083909)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 211.36  on 195  degrees of freedom
## AIC: 590.63
##
## Number of Fisher Scoring iterations: 2
```

```
coef(mlr_chol4); coef(mlr_chol1)
```

```
## (Intercept)         dbp racechinese  raceindian         bmi
##  4.87085865  0.02950027  0.35664168  0.72471631 -0.03853042
```

```
## (Intercept)         dbp racechinese  raceindian
##  3.29802826  0.03110811  0.35996365  0.71369024
```

```r
100 * (coef(mlr_chol1) - coef(mlr_chol4)[1:4])/coef(mlr_chol4)[1:4]  # change < 20%
```

```
## (Intercept)         dbp racechinese  raceindian
##  -32.290619    5.450250    0.931459   -1.521432
```

```r
# no notable change in coeffs of other vars (ignore intercept!)
# bmi is not a confounder
```

Our chosen model:

mlr_chol1: chol ~ dbp + race

```r
summary(mlr_chol1)
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

```r
Confint(mlr_chol1)  # 95% CI of the coefficients
```

```
##                Estimate       2.5 %     97.5 %
## (Intercept) 3.29802826 2.345067995 4.25098852
## dbp         0.03110811 0.019143668 0.04307255
## racechinese 0.35996365 0.002958566 0.71696873
## raceindian  0.71369024 0.339566932 1.08781356
```

Compare this model with the no-variable model and all-variable model by LR test and AIC comparison,

```r
# LR test
anova(slr_chol0, mlr_chol1, test = "LRT")  # sig. better than no var at all!
```

```
## Analysis of Deviance Table
##
## Model 1: chol ~ 1
## Model 2: chol ~ dbp + race
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1       199     278.77
```

```
## 2       196     213.40  3   65.373 5.755e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# model with no var at all is called Null Model
anova(mlr_chol, mlr_chol1, test = "LRT")  # no sig. dif with all vars model,
```

```
## Analysis of Deviance Table
##
## Model 1: chol ~ sbp + dbp + bmi + race
## Model 2: chol ~ dbp + race
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1       194     211.34
## 2       196     213.40 -2  -2.0593   0.3886
```

```
# model with 2 vars (dbp & race) is just as good as full model (with all the vars)
# model with all vars is called Saturated Model
```

```
# AIC
AIC(slr_chol0, mlr_chol1, mlr_chol)
```

```
##           df      AIC
## slr_chol0  2 637.9921
## mlr_chol1  5 590.5459
## mlr_chol   7 592.6065
```

```
# our final model has the lowest AIC
```

3.2.3.5 Multicollinearity, MC

Multicollinearity is the problem of repetitive/redundant variables – high correlations between predictors. MC is checked by Variance Inflation Factor (VIF). VIF > 10 indicates MC problem.

```
vif(mlr_chol1)  # all < 10
```

```
##          GVIF Df GVIF^(1/(2*Df))
## dbp  1.132753  1        1.064309
## race 1.132753  2        1.031653
```

3.2.3.6 Interaction, *

Interaction is the predictor variable combination that requires interpretation of regression coefficients separately based on the levels of the predictor (e.g. separate analysis for each race group, Malay vs Chinese vs Indian). This makes interpreting our analysis complicated. So, most of the time, we pray not to have interaction in our regression model.

```
summary(glm(chol ~ dbp*race, data = coronary))  # dbp*race not sig.
```

```
##
## Call:
## glm(formula = chol ~ dbp * race, data = coronary)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.10485  -0.77524  -0.02423   0.58059   2.74380
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)      2.11114    0.92803   2.275 0.024008 *
## dbp              0.04650    0.01193   3.897 0.000134 ***
## racechinese      1.95576    1.28477   1.522 0.129572
## raceindian       2.41530    1.25766   1.920 0.056266 .
## dbp:racechinese -0.02033    0.01596  -1.273 0.204376
## dbp:raceindian  -0.02126    0.01529  -1.391 0.165905
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.087348)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 210.95  on 194  degrees of freedom
## AIC: 592.23
##
## Number of Fisher Scoring iterations: 2
```

```
# in R, it is easy to fit interaction by *
# dbp*race will automatically include all vars involved i.e. equal to
# glm(chol ~ dbp + race + dbp:race, data = coronary)
# use : to just include just the interaction
```

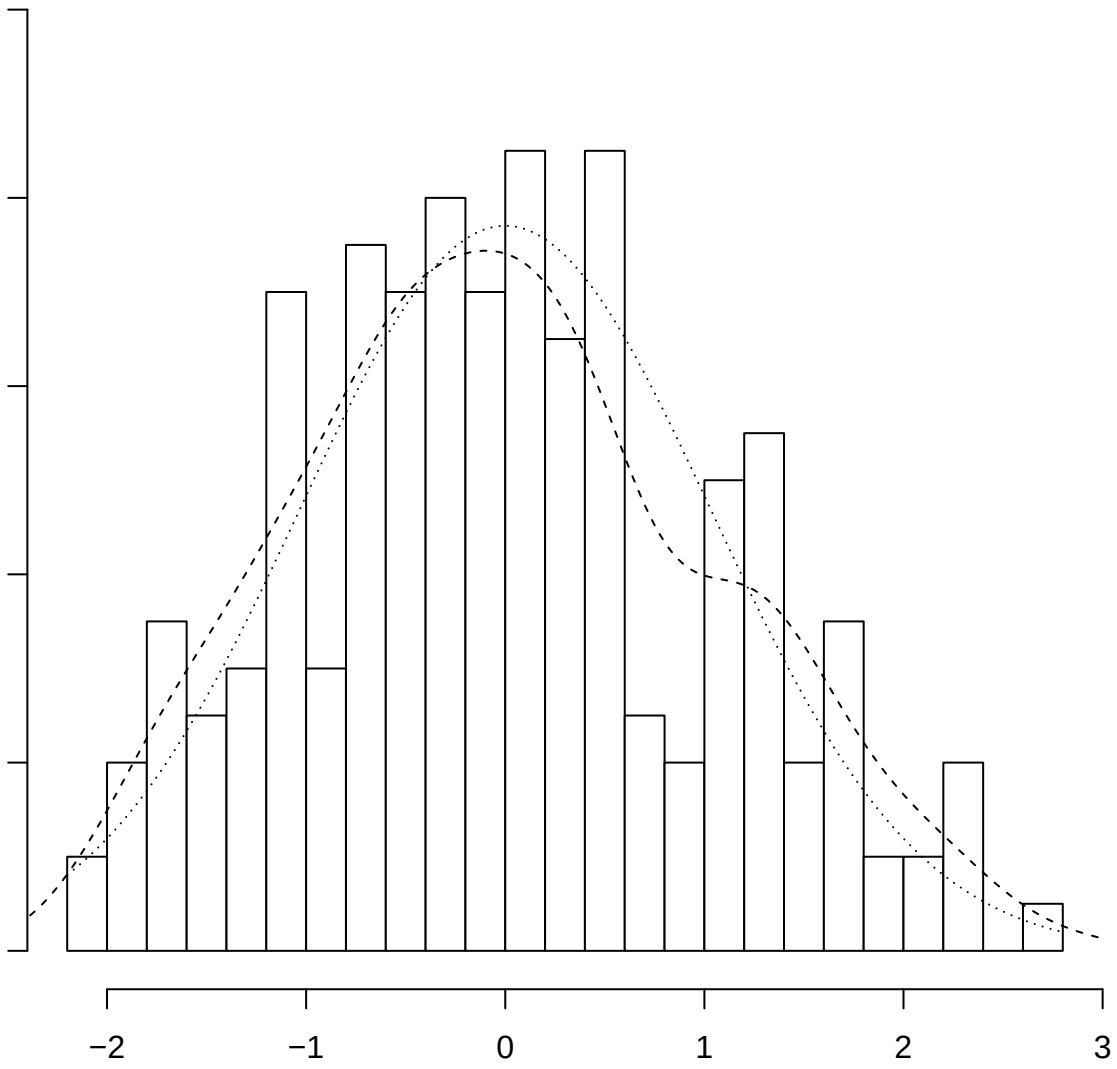
There is no interaction here because the included interaction term was insignificant.

3.2.4 Model fit assessment: Residuals

3.2.4.1 Histogram

Raw residuals: Normality assumption.

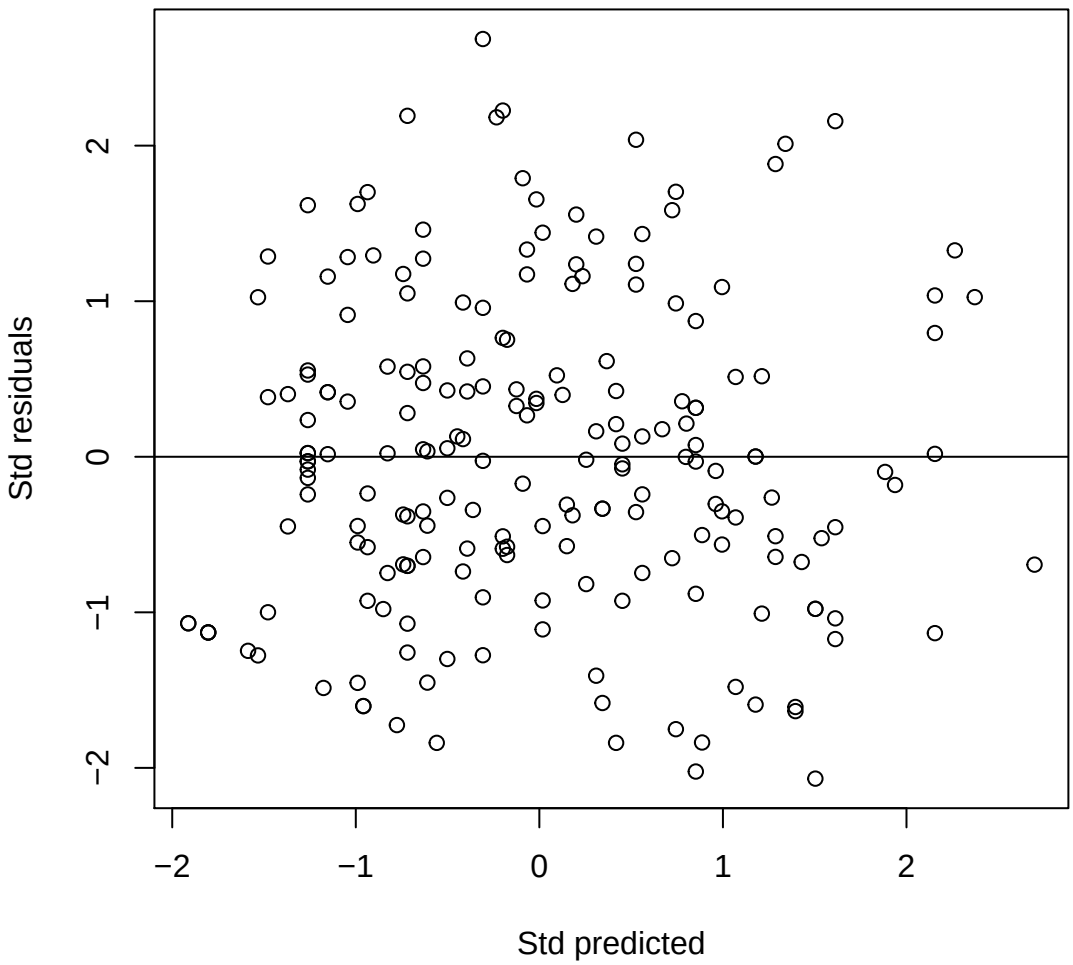
```
rraw_chol = resid(mlr_chol1)  # unstandardized
multi.hist(rraw_chol)
```



3.2.4.2 Scatter plots

Standardized residuals vs Standardized predicted values: Overall – normality, linearity and equal variance assumptions.

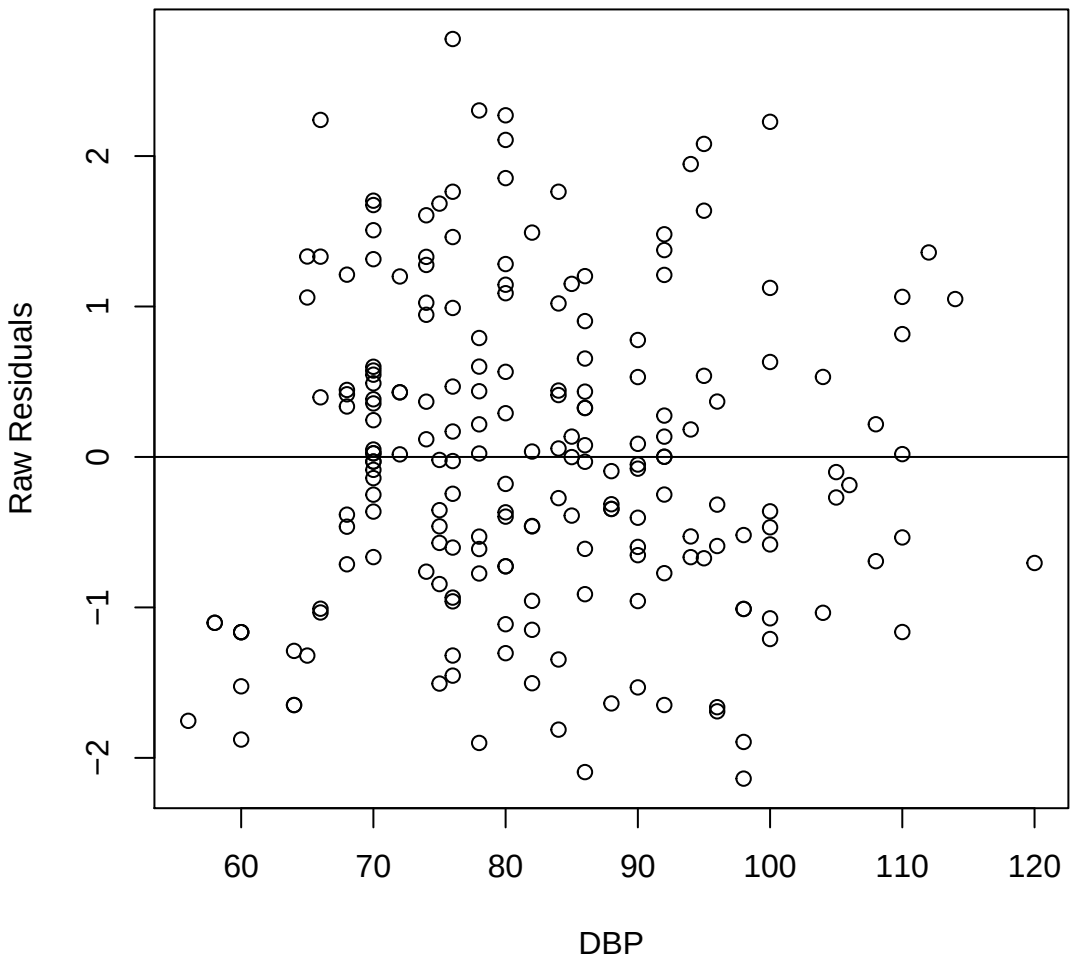
```
rstd_chol = rstandard(mlr_chol1)  # standardized residuals
pstd_chol = scale(predict(mlr_chol1))  # standardized predicted values
plot(rstd_chol ~ pstd_chol, xlab = "Std predicted", ylab = "Std residuals")
abline(0, 0)  # normal, linear, equal variance
```



The dots should form elliptical/oval shape (normality) and scattered roughly equal above and below the zero line (equal variance). Both these indicate linearity.

Raw residuals vs Numerical predictor by each predictors: Linearity assumption.

```
plot(rraw_chol ~ coronary$dbp, xlab = "DBP", ylab = "Raw Residuals")
abline(0, 0)
```



3.2.5 Interpretation

Now we have decided on our final model, rename the model,

```
# rename the selected model
mlr_chol_final = mlr_chol1
```

and interpret the model,

```
summary(mlr_chol_final)
```

```
##
## Call:
## glm(formula = chol ~ dbp + race, data = coronary)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1378  -0.7068  -0.0289   0.5997   2.7778
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.298028   0.486213   6.783 1.36e-10 ***
## dbp         0.031108   0.006104   5.096 8.14e-07 ***
## racechinese 0.359964   0.182149   1.976 0.049534 *
## raceindian  0.713690   0.190883   3.739 0.000243 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.088777)
##
##     Null deviance: 278.77  on 199  degrees of freedom
## Residual deviance: 213.40  on 196  degrees of freedom
## AIC: 590.55
##
## Number of Fisher Scoring iterations: 2
```

```
Confint(mlr_chol_final)  # 95% CI of the coefficients
```

```
##                Estimate       2.5 %     97.5 %
## (Intercept) 3.29802826 2.345067995 4.25098852
## dbp         0.03110811 0.019143668 0.04307255
## racechinese 0.35996365 0.002958566 0.71696873
## raceindian  0.71369024 0.339566932 1.08781356
```

```
rsq(mlr_chol_final, adj = T)
```

```
## [1] 0.2227869
```

- 1mmHg increase in DBP causes 0.03mmol/L increase in cholestrol, controlling for the effect of race.
- Being Chinese causes 0.36mmol/L increase in cholestrol in comparison to Malay, controlling for the effect of DBP.
- Being Indian causes 0.71mmol/L increase in cholestrol in comparison to Malay, controlling for the effect of DBP.
- DBP and race explains 22.3% variance in cholestrol.

Turn the results into data frames results using `broom`,

```
tib_mlr = tidy(mlr_chol_final); tib_mlr
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   3.30     0.486        6.78 1.36e-10
## 2 dbp           0.0311   0.00610      5.10 8.14e- 7
## 3 racechinese   0.360    0.182        1.98 4.95e- 2
## 4 raceindian    0.714    0.191        3.74 2.43e- 4
```

```
tib_mlr_ci = tidy(Confint(mlr_chol_final)); tib_mlr_ci
```

```
## Warning: 'tidy.matrix' is deprecated.
## See help("Deprecated")
```

```
## # A tibble: 4 x 4
##   .rownames   Estimate  X2.5.. X97.5..
##   <chr>          <dbl>   <dbl>   <dbl>
## 1 (Intercept)   3.30   2.35     4.25
```

```
## 2 dbp           0.0311 0.0191   0.0431
## 3 racechinese   0.360  0.00296  0.717
## 4 raceindian    0.714  0.340    1.09
```

Combine the estimates and CIs together,

```
df_mlr_ci = data.frame(tib_mlr[1:3], tib_mlr_ci[3:4], tib_mlr[4:5]); df_mlr_ci
```

```
##          term   estimate   std.error      X2.5..    X97.5.. statistic      p.value
## 1 (Intercept) 3.29802826 0.486213151 2.345067995 4.25098852  6.783091 1.355900e-10
## 2         dbp 0.03110811 0.006104419 0.019143668 0.04307255  5.095998 8.135024e-07
## 3 racechinese 0.35996365 0.182148798 0.002958566 0.71696873  1.976207 4.953419e-02
## 4  raceindian 0.71369024 0.190882748 0.339566932 1.08781356  3.738893 2.425248e-04
```

Display the results using `kable` in a nice table,

```
knitr::kable(df_mlr_ci)
```

term	estimate	std.error	X2.5..	X97.5..	statistic	p.value
(Intercept)	3.2980283	0.4862132	2.3450680	4.2509885	6.783091	0.0000000
dbp	0.0311081	0.0061044	0.0191437	0.0430726	5.095998	0.0000008
racechinese	0.3599636	0.1821488	0.0029586	0.7169687	1.976207	0.0495342
raceindian	0.7136902	0.1908827	0.3395669	1.0878136	3.738893	0.0002425

We can export the results into a .csv file for use later (e.g. to prepare a table for journal articles etc.),

```
write.csv(df_mlr_ci, "mlr_final.csv")
```

3.2.6 Model equation

Cholestrol level in mmol/L can be predicted by its predictors as given by,

$$chol = 3.30 + 0.03 \times dbp + 0.36 \times race\ (chinese) + 0.71 \times race\ (indian)$$

3.2.7 Prediction

It is easy to predict in R using our fitted model above. First we view the predicted values for our sample,

```
coronary$pred_chol = predict(mlr_chol_final)
head(coronary)
```

```
##   sbp dbp   chol  bmi   race gender pred_chol
## 1 106  68 6.5725 38.9 indian  woman  6.127070
## 2 130  78 6.3250 37.8  malay  woman  5.724461
## 3 136  84 5.9675 40.5  malay  woman  5.911109
## 4 138 100 7.0400 37.6  malay  woman  6.408839
## 5 115  85 6.6550 40.3 indian    man  6.655908
## 6 124  72 5.9675 37.6  malay    man  5.537812
```

Now let us try predicting for any values for `dbp` and `race`,

```
str(coronary[c("dbp", "race")])
```

```
## 'data.frame':    200 obs. of  2 variables:
##  $ dbp : num  68 78 84 100 85 72 80 70 85 70 ...
##  $ race: Factor w/ 3 levels "malay","chinese",..: 3 1 1 1 3 1 1 2 2 2 ...
```

```r
# simple, dbp = 90, race = indian
predict(mlr_chol_final, list(dbp = 90, race = "indian"))
```

```
##        1
## 6.811448
```

More data points

```r
new_data = data.frame(dbp = c(90, 90, 90), race = c("malay", "chinese", "indian"))
new_data
```

```
##   dbp    race
## 1  90   malay
## 2  90 chinese
## 3  90  indian
```

```r
predict(mlr_chol_final, new_data)
```

```
##        1        2        3
## 6.097758 6.457722 6.811448
```

```r
new_data$pred_chol = predict(mlr_chol_final, new_data)
new_data
```

```
##   dbp    race pred_chol
## 1  90   malay  6.097758
## 2  90 chinese  6.457722
## 3  90  indian  6.811448
```

4 Exercises

1. Present the results in a table (follow Arifin et al. (2016))
2. Obtain the coefficient for 5mmHg increase in DBP.
3. Add `age` to the multivariable model. What happens?

References

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