Feature Tracking and Optical Flow

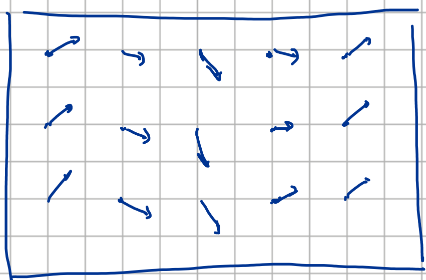
How do we detect and track motion in a scene.

Two basic methods:

1. Optic Flow
2. Feature tracking.

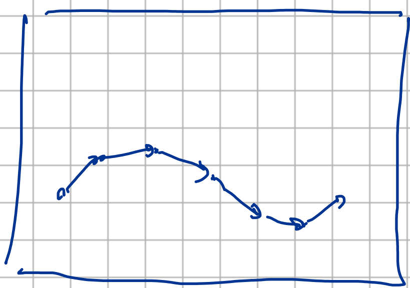
The two methods are intimately related.

Optic Flow:



motion of pixels is computed at fixed locations in the image

Feature trackin



A fixed feature is tracked over time.

Both methods require a model for for how pixels move from one image to the next.

Small baseline

$$\underbrace{I_1(x)}_{\text{image intensity in image \#1 at pixel } x} = \underbrace{I_2(x + \Delta x)}_{\text{image intensity in image \#2 at } (x+\Delta x)}$$

image intensity in image #1 at pixel x

image intensity in image #2 at $(x+\Delta x)$ where Δx is new location of x in image #2.

For short period of time we have

$\Delta x = u\, dt$

u ← pixel motion (pixels/sec)

Then

$$I(x(t), t) = I(x(t) + u\,dt,\ t + dt)$$

$$\simeq I(x(t), t) + \left(\left.\frac{\partial I}{\partial x}\right|_{x(t),t}\right)^T u\,dt + \left(\left.\frac{\partial I}{\partial t}\right|_{x(t),t}\right) dt + HOT$$

↑ Taylor series

Neglecting the H.O.T and letting

$$\nabla I \triangleq \begin{pmatrix} I_x \\ I_y \end{pmatrix} = \left(\frac{\partial I}{\partial x} \Big|_{x(t), t} \right)$$ and

$$I_t = \frac{\partial I}{\partial t} \Big|_{x(t), t}$$ gives

$$\boxed{\nabla I^T u + I_t = 0}$$

Note 1-equation in two unknowns.

∇I is called the image gradient and can be found using, for example, the Sobel operators:

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$ (Horizontal changes)

$$G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$ (vertical changes)

ie $I_x(x)$ is found by multiply the 3x3 window around x by G_x

I_t is found as

$$I_t \simeq I_2(x, t+\Delta t) - I_1(x, t)$$

Since this is one equation in two unknowns, and is susceptible to noise, the motion vector u is found as the solution of a least squares problem:

$$u = \arg\min \sum_{\tilde{x} \in W(x)} \left| \nabla I(\tilde{x})^T u + I_t(\tilde{x}) \right|^2$$

$\therefore$ u satisfies

$$\underbrace{\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}}_{G \in \mathbb{R}^{2\times 2}} u + \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{b \in \mathbb{R}^{2\times 1}} = 0$$

or $\boxed{u = -G^{-1} b}$

Of course this depends on G being full rank.

If $G(x)$ is invertible then we say that x is a "feature point."

< Algorithm 4.1 Feature Tracking / Optic Flow >