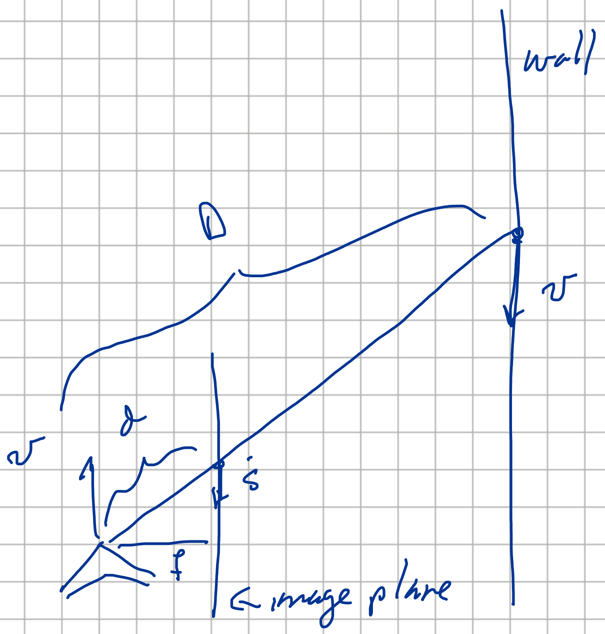
Optic Flow

Side looking camera



Let s = pixel and $\dot{s}$ = optic flow of side looking camera

Similar triangles:

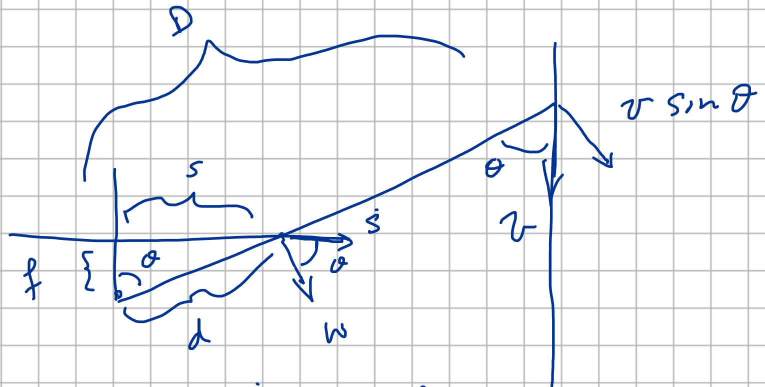
$$\frac{v}{D} = \frac{\dot{s}}{d} \qquad \text{where } d = \sqrt{s^2 + f^2}$$

$$\Rightarrow \dot{s} = \frac{d}{D} v = \frac{\sqrt{s^2+f^2}}{D} v = \sqrt{s^2+f^2}\,\frac{v}{D}$$

Relationship between velocity of vehicle and optic flow

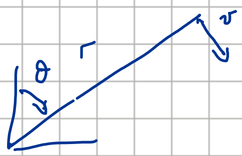
Note: If v-known, can compute D from optic flow.

Forward looking camera



$$\dot{s} = w \cos\theta$$

In general



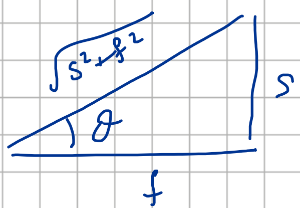
$$\underbrace{\dot{\theta}}_{\text{rad}/s} = \underbrace{\frac{v}{r}}_{\frac{m/s}{m} = \text{rad}/s} \Rightarrow \dot{\theta} = \frac{v \sin\theta}{D}$$

Also $\dot{\theta} = \dfrac{w}{d} = \dfrac{\dot{s}}{d\cos\theta}$

$$\therefore \frac{\dot{s}}{d\cos\theta} = \frac{v\sin\theta}{D} \Rightarrow \dot{s} = \frac{d}{D} v \sin\theta\cos\theta$$

where $d = \sqrt{s^2 + f^2}$

and $\theta = \tan^{-1}\left(\dfrac{s}{f}\right)$

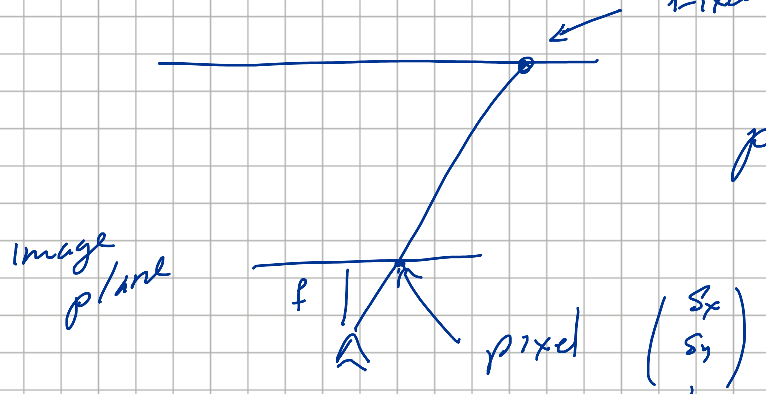


$$\Rightarrow \sin\theta = \frac{s}{\sqrt{s^2+f^2}} \qquad \cos\theta = \frac{f}{\sqrt{s^2+f^2}}$$

$$\Rightarrow \dot{s} = \frac{\sqrt{s^2+f^2}}{D} v \left(\frac{s}{\sqrt{s^2+f^2}}\right)\left(\frac{f}{\sqrt{s^2+f^2}}\right) = \frac{sf}{\sqrt{s^2+f^2}} \frac{v}{D}$$

$$\therefore \boxed{\dot{s} = \frac{sf}{\sqrt{s^2+f^2}} \frac{v}{D}}$$

Time to Collision



Focus on x-direction:

$$s_x = \frac{X}{Z}$$

$$\Rightarrow \dot{s}_x = \frac{Z\dot{X} - X\dot{Z}}{Z^2}$$

But object is fixed in the environment, so $\dot{X} = 0$

$$\Rightarrow \dot{s}_x = -\frac{X}{Z}\left(\frac{\dot{Z}}{Z}\right) = -s_x\left(\frac{\dot{Z}}{Z}\right)$$

$$\therefore \underbrace{\frac{Z}{\dot{Z}}}_{m/m/s = s} = -\frac{s_x}{\dot{s}_x}$$

$\frac{Z}{\dot{Z}}$ is the time until p crosses the image plane

called "time to contact" or "time to collision"

$$\therefore \boxed{\tau = -\frac{s_x}{\dot{s}_x}}$$

time to collision

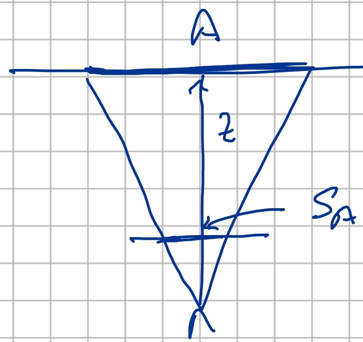
Therefore, for any point, we can find the time to collision with that point from optic flow.

- Calculation Assumes that Focus of expansion FOE is along optical axis (0,0)

- If not then 1st compute FOE as S_{FOE} and then compute time to collision as

$$\tau = -\left(\frac{S - S_{FOE}}{\dot{S}}\right)$$

Note that this also works if we compute a region containing an object



From similar triangle we have

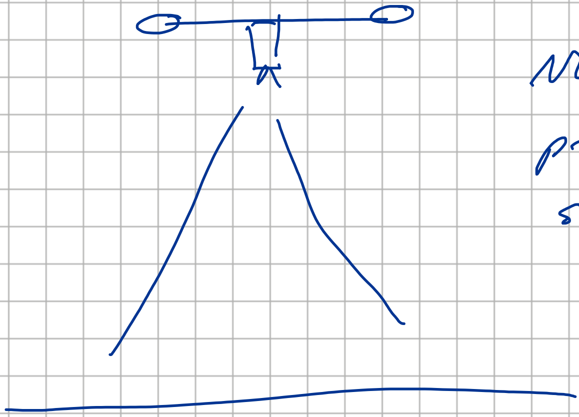
$$\frac{S_A}{f} = \frac{A}{z}$$ where A and f are constant. Then

$$\dot{S}_A = \frac{-fA\dot{z}}{z^2} = -f\left(\frac{A}{z}\right)\left(\frac{\dot{z}}{z}\right) = -S_A\left(\frac{\dot{z}}{z}\right)$$

$$\Rightarrow \boxed{\tau = \frac{z}{\dot{z}} = -\frac{S_A}{\dot{S}_A}}$$

UAV control using Optic Flow

Landing with Down looking camera



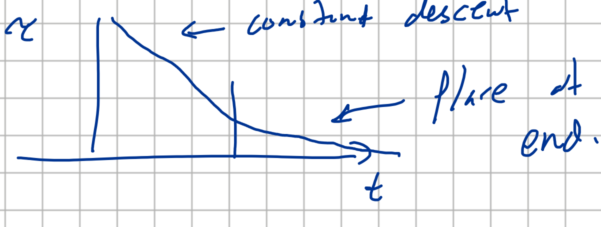
Non zero roll produces side to side optic flow.

Use roll angle to zero optic flow.

In wind $\Rightarrow$ roll angle may not be zero.

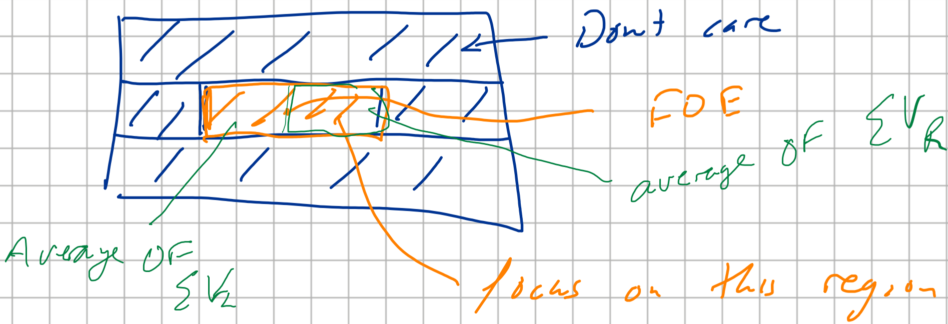
As descend:

- Find center of expansion
- compute time to collision
- regulate thrust & maintain desired time to collision profile



UAV control – Collision Avoidance

- Find TTC at sparse set of features.
- Find pixel with smallest TTC in

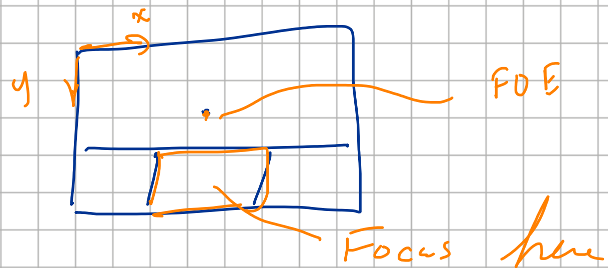


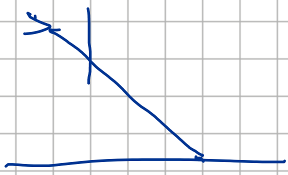
- When smallest TTC crosses threshold, command yaw rate unil obstacle moves outside of region
- Could probably do inversly proportional to TTC
- Assuming $+r^c$ yaws quadrotor to right

$$r^c = k\left(\frac{\Sigma v_L - \Sigma v_R}{\Sigma v_L + \Sigma v_R}\right)$$

control gain

UAV control - Ground Following





we have already seen that

$$\dot{S}_y = \frac{S_y f}{\sqrt{S_y^2+f^2}}\left(\frac{v}{D}\right)$$

$$\therefore \quad \frac{D}{v} = \frac{f}{\sqrt{S_y^2+f^2}}\left(\frac{S_y}{\dot{S}_y}\right)$$

The idea is to regulate this quantity. using thrust.

Higher speed implies D goes up,

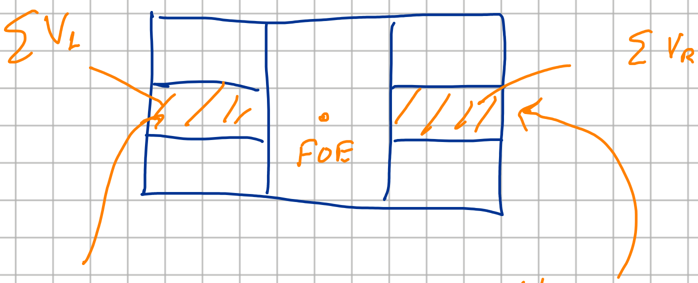
lower speed implies D goes down.

If v is known, then can regulate D directly.

In holodeck will need to regulate h.

$$h = K\left(\underbrace{\left(\frac{D}{V}\right)_0}_{\text{constant}} - \left\{\frac{f}{\sqrt{S_y^2+f^2}}\left(\frac{S_y}{\dot{S}_y}\right)\right\}\right)$$

UAV Control - Canyon Following



compute average optic flow

$$\boxed{\phi_c = k\left(\frac{\sum V_L - \sum V_R}{\sum V_L + \sum V_R}\right)}$$