Points and lines

Equation for a line:

$$y = mx + d$$

slope (m), y-intercept (d)

A more general expression is

$$ax + by + c = 0$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}^T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

Let $\bar{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ and $l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ then

Fact The point $\bar{x}$ is on the line l

iff $\bar{x}^T l = 0$

Now suppose that there are two lines

l_1 and l_2 and let

$$\bar{x} = l_1 \times l_2$$

Then $l_1^T \bar{x} = l_1^T (l_1 \times l_2) = 0$

and $l_2^T \bar{x} = l_2^T (l_1 \times l_2) = 0$

which implies that $\bar{x}$ is on l_1 and l_2

ie

Fact The intersection of two lines l_1 and l_2 is the point $\bar{x} = l_1 \times l_2$

Note that the third element of $\bar{x}$ may not be 1.

We say that $\bar{x} \equiv \bar{y}$ (written $\bar{x} = \bar{y}$) for homogeneous coordinates if they are equal up to a scale factor, ie

$$\bar{x} = \lambda \bar{y}$$ ← only the ratio of points matters.

Similarly let $\bar{x}$ and $\bar{y}$ be two distinct points in the image, and let

$l = \bar{x} \times \bar{y}$, then $\bar{x}^T l = \bar{y}^T l = 0$

$\Rightarrow$ both $\bar{x}$ and $\bar{y}$ are on the line.

Fact The line through two points $\bar{x}$ and $\bar{y}$ is $l = \bar{x} \times \bar{y}$

Intersection of parallel lines

$$l_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \underbrace{l_2 = \begin{pmatrix} a \\ b \\ c' \end{pmatrix}}$$

same slope, different y-intercept.

The intersecting point is

$$\bar{x} = l_1 \times l_2 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c' \end{pmatrix} = \begin{pmatrix} bc' - bc \\ ca - c'a \\ 0 \end{pmatrix}$$

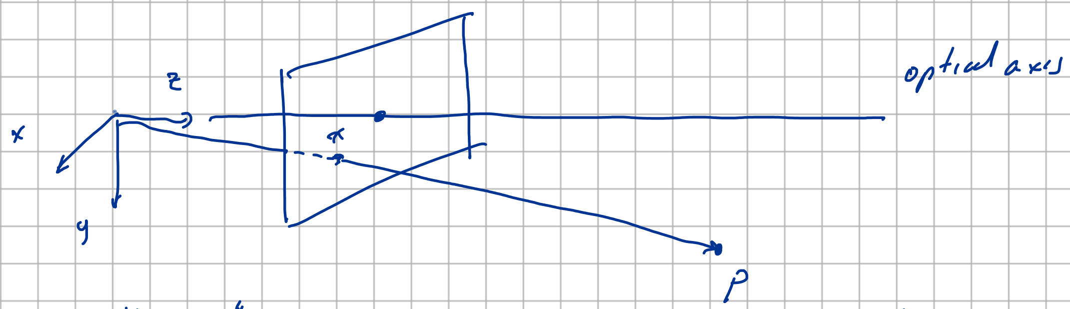
$$= \underbrace{(c'-c)} \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}$$

Note that the inhomogeneous coordinates are $\begin{pmatrix} b/0 \\ -a/0 \end{pmatrix}$ which is a point at infinity.

<u>Fact</u>: In homogeneous coordinates, a zero in the third element is a point at infinity.

Image, pre image, co-image.

Figure 3.10



The "image" of point p is x, ie the pixels that show up in the image.

The "pre-image" of x, is the set of all points that project to x.

$$\text{i.e.} \quad \text{pre-image}(\bar{x}) = \text{span}(\bar{x})$$

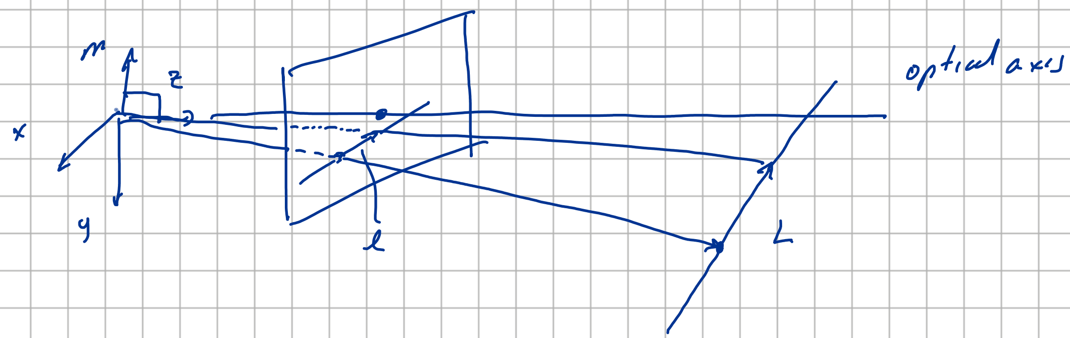
The co-image of x is the linear space that is orthogonal to the preimage:

$$\text{coimage}(x) = \text{preimage}(x)^{\perp}$$

For $\bar{x}$: $\quad \text{coimage}(\bar{x}) = \text{span}(\text{columns of } \hat{x})$

Lines

Figure 3.10



l is the image of L.

However the preimage of l is the plane containing l and passing through the origin.

preimage(l) is a 2D space in 3D

Therefore the coimage(l) is a 1D space or line in 3D, that is perpendicular to the plane preimage(l)

The line m is computed as follows:

- find three points on the line in the image, denoted x_1, x_2, x_3. m satisfies

$$\begin{pmatrix} x_1^T \\ x_2^T \\ x_3^T \end{pmatrix} m = 0$$

i.e. $m \in \mathcal{N}\begin{pmatrix} x_1^T \\ x_2^T \\ x_3^T \end{pmatrix}$ ← easy to find using SVD.